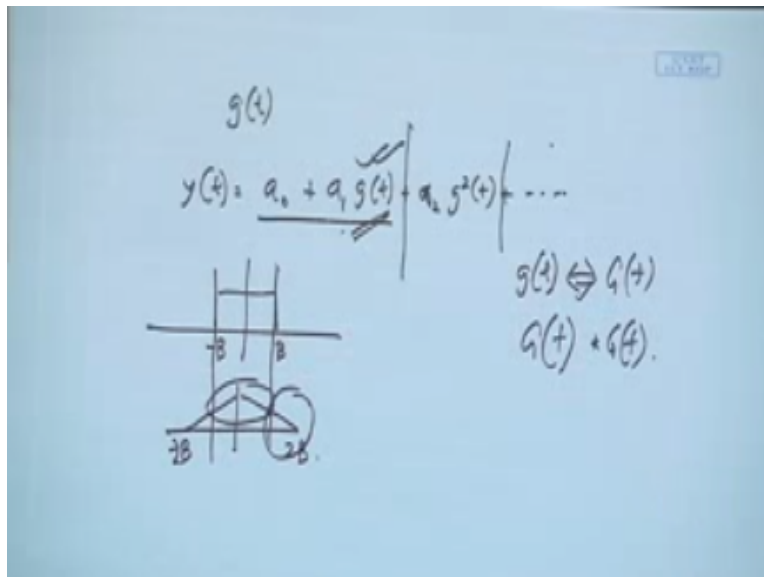


NPTEL
NPTEL ONLINE CERTIFICATION COURSE
Course
On
ANALOG communication
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Lecture: 30 Channel nonlinearities and
Multipath effects

Okay so let us now try to see the nonlinear effect of channel so when we say a channel is nonlinear we can say if suppose I have a signal called $G(t)$.

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Which is being transferred over the channel and after passing through the channel if expect the output $Y(t)$ which is just not the linear means part of it or linear means this transformation of it. it also gives some square Q and all other thumbs so if this looks like this some a zero which is the constant term then a 1 into $G(t) +$ some a 2 into $G^2(t) +$ order dot has many things are there.

So up to this the channel is linear because whatever you give you get the linear means all the linearity property of that LTI system we have talked about that will be satisfied but if you just go to this whenever it starts taking the square term or any other higher term you will see that it is basically creating means that LTI the property of that is not no longer holding and it will start creating some spurious frequency term that is the something we are interested now so that's what will be happening we means it's little bit clearer because you can see if as long as it is just up to this part the linear term or the first-order term then what happens.

If I do a Fourier transform so the frequency component that will be involved in it will be still the frequency component that is already involved in GT ok so it will just be equivalent frequency component in the output whereas if I start taking this term what does this means G^2 Square infrequency domain what that will be $G^2 F$ means if the GT has a Fourier transform of $G f$ then it should be convolution of $g f$ with itself whenever we convolute very simple example is if I take that box function okay.

If I convolute this box function with itself this will create a triangular function so it will look like this and if this is band limited to B that box function the way it is this will go up to $2B$ + to be the Bandwidth gets doubled and so on if you start creating taking G^3 cube it will actually occupy 3 times the bandwidth and to keep on increasing one after another okay so I can see already if I take the nonlinear part nonlinear portion first of all just think about this signal this particular signal okay.

This is the Box signal coming from sinc function so if my signal was sinc okay sign see corresponding $G F$ should be this if I just take that second order term what will happen I will get this so within the band what is happening there is a distortion already the box function has been transformed to a triangular function so there is a distortion within the channel but the good means bad part is it is now starting to create some distortion out-of-band okay.

So suppose I had expectation that my particular thing like voice it is band limited it is up to B + B I modulate it and put it in the channel right and the neighboring channel I also put another voice so those two voices are being carried over the channel if the channel is linear then for this signal as well as the neighboring signal which is also suppose a box signal okay

infrequency domain so for both of them start creating this nonlinear effect so they will spread in frequency domain.

The earlier one we have seen when it was having means dispersion it was getting spreaded in time domain here what we are seeing due to nonlinear effect probably we are getting spread in frequency domain and that is now creating a inter channel interference so earlier one was interaction an interference this is called inter that is something which will be happening whenever we pass it through a nonlinear channel but of course what will happen the channel is it might be non-ideal but these coefficients will be very small because generally.

Channel will not do too much off means will not create too much of non-linearity it will create some amount of non-linearity butte coefficients will be pretty small and accordingly you will see the effect of it okay but whatever it is you have to always keep this in mind that channel might show some non-linearity and accordingly you have to come back okay so how we will come back will later on will see but this something you have to be sure that the channel might be the ideal channel distortion less channel we have talked about it might not be this.

So it might have that dispersion which we have discussed already if the phase is not non-ideal or the amplitude is non-ideal in the transfer function or the channel might be just a nonlinear thing and which will create this kind of interference so let us just give one example so if we just have a signal $Y.T$.

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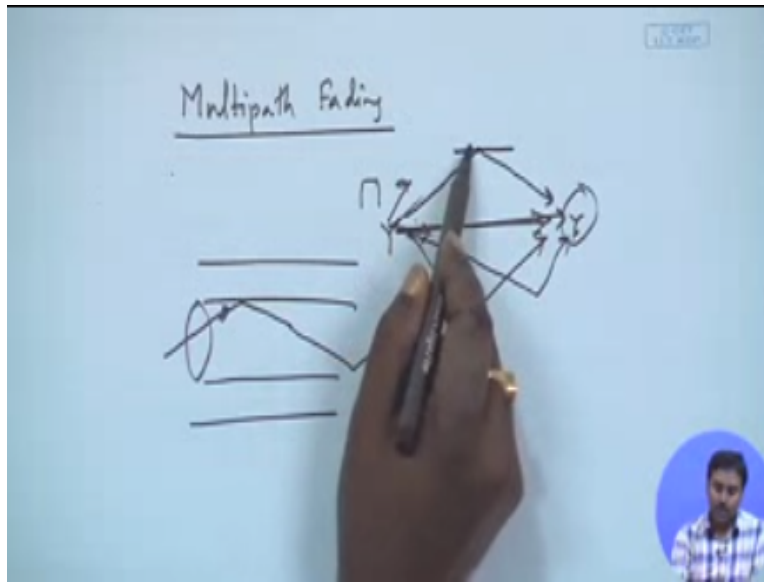
$$\begin{aligned}
 x(t) &= 2000 \operatorname{sinc}(2000\pi t) \\
 X(f) &= \Pi\left(\frac{f}{2000}\right) \\
 y(t) &= 2000 \operatorname{sinc}(2000\pi t) + 0.316 \times 2000 \operatorname{sinc}^2(4000\pi t) \\
 Y(f) &= \Pi\left(\frac{f}{2000}\right) + 0.316 \Delta\left(\frac{f}{4000}\right)
 \end{aligned}$$

Which is something like this which is let us say X_T + some this is just an example and I have taken from the books let's say this is this is my Y_T as you can see it's already the nonlinear coefficient is pretty small okay so that $X^2 T$ the corresponding coefficient this is small that is what will be happen generally okay now let us see that is examine what kind of effect will be getting with this.

So let us say I put a X_T which is something like them sink 2005 T okay so it's a sin function in frequency domain why we have taken a sink function because we want to represent it as a box function okay infrequency domain so this is our X_T so definitely X_F will be a box function of duration 2000 so that that is the case in I pass this thing function over the channel then my Y_T should be 2000 sink 2000 T + this will be there and then 2,000 square so 2,000 square into this will give you this number 0.316 okay or let us say 2,000 square not 2000square you multiply this by 2,000 you get this and there will be another 2,000 which should be left and then you get a sink square okay.

So I can write this as this is just a means if I take Y_F this is just a box function okay of strength 2000 or sorry the width 2000 and this is just three one six if I take and this is actually a triangular function so I can take a triangular function of this one 4000 notice because 2,000 convoluted with 2000 box function so that creates a 4000 band triangular function so just slot them they look like this the first part will be that this part that remains the same function.

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Which is a box function running from this 1000 to +1000 okay and the next one is if I see this one it has a strength of 0.316 so at this tip it should be 0.316 and it should be going as a triangular function going from - mm 2 + mm okay what will be happening hi will have addition of these two so if I just see it should look like this and there should be some part right so this should be my overall output function YS.

Now you can see there is a distortion because ideally I should have expected this thing this is the distortion part which is coming in bed so non the ID actually has to affect it corrupts your signal and it corrupts other signal as well so it has a distortion and not only that out-of-bandit has created some spurious thing which will create distortion to others okay so you have to be very careful about anon linear channel.

Because nonlinear channel creates not only problem for you it will also additionally create problem for all your neighboring FDM channels or it starts creating inter channel interference that is something we should be very careful since the channel is nonlinear we should be very careful about it okay and accordingly we should come back so there are methods of combating nonlinear channel but right now we are just stating what should be the ideal channel and if there is deviation what should be the effect on the signal.

Nothing because whenever we have said we wish to do communication we have almost assumed as if the channel was ideal so you have characterized what do we mean by ideal that means

whatever we transmit we are almost expecting although time that at the receiver I will be receiving almost similar thing this is probably not true as you can now see okay so it can have dispersion we have already talked about that it can have non-linearity this is something we are now talking about next we will talk about something else.

Okay which is called especially being observed in a wireless channel it is called multipath effect or fading okay so what is this whenever you transmit a signal with your antenna most probably you would not be able to restrict means if the antenna is Omni directional then you would not be able to restrict the signal to a particular direction and generally in many cases that is not the intention because you do not know exactly where your receivers you target your antenna to a particular direction with a very pencil besides pencil beam probably you miss the target you do not get you do not transmit a signal where the user is okay.

So that is not very good so generally I would not bother myself to see where my recipient is I will just broadcast the signal on the air so that means I need only directional antenna and which will broadcast in every direction the signal and what will happen when you are broadcasting you are not sitting in a free space right means and then transmitting it you are sitting in some locality where there are buildings there are trees there are many other things which can be act which can act as a reflector to your signal.

So what might happen suppose I have reflector over here this is my receiver antenna this is my transmitter antenna so whenever I transmit it goes in every direction so the one that goes over this direction will directly be received one that goes in this direction okay this should be reflected that or some portion of that will be reflected back it might absorb something it might scatter something another direction but some portion of that will be reflected back and that is still be received over here.

This particular model is called multipath effect okay even in other scenario where we have a guided communication like let us say fiber-optic communication okay so if we have a fiber-optic communication there also it will be it will have some diameter of your core where you are launching the light and then due to total internal reflection.

So basically the core looks like this and there is a cladding of that fiber and you launch the optic means light and then it gets totally internally reflected and gets guided but what will happen

there also there might be different rays launched at different angles and they will go through different path length okay.

So there also the same phenomena will be happening so whatever happens this might be a very low dimension that is why there will be not much difference between those rays whereas here there will be huge amount of difference today that goes directly to a receiver and the Ray that takes $2d$ and there might be multiple reflectors multiple such things can be happening okay so all those things will get inside this effectively what is happening effectively suppose let us say I again.

Let us take a means example of pulse I transmit a pulse some portion of its power is getting directly linked so the delay on this will be much lesser whereas some portion of that power is getting on this link so there the delay will be higher so basically a multipath means it's different delayed signal replica of that same signal is coming and sitting at the receiver okay this is what happens whenever we talk about multipath fading okay.

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Diagram of a multipath channel model showing a direct path with delay d and a reflected path with delay $2d$ and gain α .

$$H(f) = e^{-j2\pi f d} + \alpha e^{-j2\pi f (2d)}$$

$$= e^{-j2\pi f d} (1 + \alpha \cos(2\pi f d) + j\alpha \sin(2\pi f d))$$

$$= \sqrt{[1 + \alpha \cos(2\pi f d)]^2 + \alpha^2 \sin^2(2\pi f d)}$$

$$= \sqrt{1 + \alpha^2 + 2\alpha \cos(2\pi f d)} \exp(-j\{2\pi f d + \tan^{-1}(\frac{\alpha \sin(2\pi f d)}{1 + \alpha \cos(2\pi f d)})\})$$

So let us just try to see a channel model where only two days we can think about okay so that to Ray model one is straightly going to the receiver and the other one is getting reflected from another reflector and coming back okay so the first one the Ray is which is going directly then

have some delay because transmitter receiver are not co-located so there should become delay of TD after which you will be receiving okay and there can be further signal which gets a little bit more attenuation let's say α why that is happening because it is taking a longer path then getting reflected that might absorb some amount of signal.

So that will have higher attenuation probably and after that it will also go through a delay which is definitely greater than TD because it is taking a longer path so that should be $PD + \text{some } \beta$ let us say and these two signal will be added simply at the receiver because receiver cannot separate them they are just coming from different direction receiver also has omnidirectional antenna because he also does not know from which direction his signal will be coming.

So he has to keep our migration antenna so all the things which should be coming from that particular transmitter will all be received to him they are all in same frequency so you have no way to separate them so they will all come into your receiver and will be received so if I now see the channel even though it was usual channel that has a different characteristics now so this is now my channel characteristics due to this propagation model okay.

If just to raise our there if there are multiple raise I have to put that many arms with all different values and different Δ values I have to put all those multiple arms and all of them should be summed over here and that is why we probably do not call that as two paths it is multi path so multiple paths can be there all of them should be accounted for my signal versus for simplicity and to get some insight were just taking two remote or two path model okay.

So if this is the case what the transfer function of this filter let us try to first identify this is just a delay element so that should give me if I try to characterize HF that should be just $e^{-j2\pi FTD}$ it is just a delay element of fixed delays PD so it should give means I have not taken any attenuation over here so it should give me $|H| = 1$ as the amplitude and the delay should be this+ because there is a adder so +there should be a attenuation of β so this should be β into $e^{-j2\pi FTD}$ this delay should be $TD + T$ right.

So this is what we get this is my HF now let us try to see the amplitude part of HF and the phase part of HF okay so I can take out $e^{-j2\pi FDT}$ what do I get I get $1 + \dots$ this I can write with the Euler's theorem again as $\cos + j \dots$ okay so I can write $\cos 2\pi FDT$ so

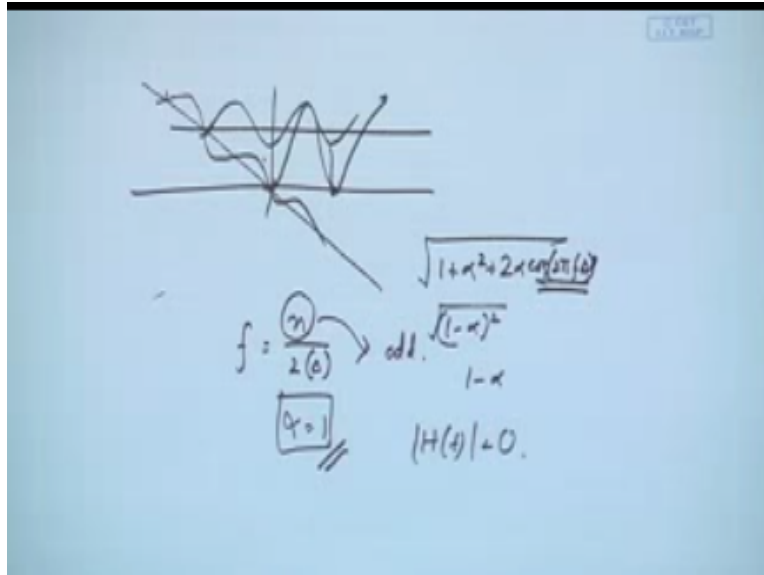
what if I could $2 F T D$ is already gone so $2 \pi F T + J \sin 2 F T$ right I can write it this is T or ΔT or sorry ΔI I have just it that should be π this .

So I can write it this way right now I have to get the phase part and the amplitude part so this is already a complex one I need to first try to evaluate the amplitude part of that so that should be square root of this square so $1 + \cos 2 F$ square + this square right so sine squares $2 F$ right and there should be a β right somewhere we have missed that β by no β okay so that should be β square is we missed that.

So if I just simplify this what we get root over $1 +$ this should be β square \cos square + β square sine square that should be giving me β square + 2β this should be means that is $2 \beta \cos$ this one right that is my amplitude and also similarly I can evaluate the phase that should be just an inverse this $_$ this right so I get this phase is already there +that tan inverse part so I should be getting exponential $_ J$ I get $2 F T D$ which is this phase + tan inverse that this thing divided by this whole thing right.

So I can just write that part it is just this part comes in the numerator and this whole part goes in the denominator ok so I have now evaluated for that HF the amplitude and phase vector or phase part right now let us see what kind of amplitude and phase it has so this amplitude if I carefully observe it what is happening it is actually with respect to Δ it has a modulation or it is a sinusoidal okay basically if I wish to plot that amplitude it's a periodic signal I should say well this will be happening in the amplitude.

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So this is HF there should be some with some period it's actually oscillating so that depends on this and then phase if I wish to plot so this is suppose the phase so phase has a linear part this part + tan inverse something which is also oscillating because we have a cost term and sign term so that is also oscillating so phase will also be with some other oscillation will have oscillatory part because it is also a periodic signal whatever oscillation that will be that might not be exactly cos sinusoidal or sinusoidal.

But that will have some oscillatory part so whenever we do a multipacks or two path fading so this is the impairment or this is the impurities that we see in the channel characteristics and if you try to characterize it little bit more what can we see let us say I had that amplitude spectra right that is one + β square + $2\beta \cos 2\pi \delta \Delta$ right so this was the case this is something we were happy ok so in this if we just put this F equals to N by 2 okay if you just put this okay so what do we expect over here if I am putting this.

So immediately what do we get so if we have this n now I can put at a start putting n as odd and even right so let us say n I put as odd immediately what will be the value of this must be $\frac{1}{2}$ right that whole thing right and then if you just calculate this how much this will be $\frac{1}{2} \beta$ whole square root so that is $\frac{1}{2} \beta$ now let us say β equals to 1 I am just approximating that both the paths are having similar strength if this is happening now what do I get I get nothing because my HF modulus of that gets 0 so all the frequency term which has this value where n is odd will give me nothing at the output and if I just take it as even.

And if I take again equal to 1 this will be just instead of becoming $1 - 1$ this will be $1 + 1$ okay so that will be a huge strength that will be coming to my receiver so what I can now see depending on this if I have a different characteristics basically the HF is this undulation is there that means some of the frequencies are really getting heavily attenuated okay if this one this will almost go like this okay so this will be zero in some of the frequency in solve the frequency is very high so that is exactly what is happening which is it which is termed as you might have heard about this term that's called frequency selective fading.

So what my channel the way I have characterized it I have taken two path model and with that two paths I have characterized the overall characteristics of the channels okay so that HF I have evaluated then I could see that that HF phase and frequency response both have some undulations or some modulation or I should say it's periodic in nature and then going deep into the details of it what I could do is I could set this β equal to one and then try to see if I put this F as n by two β .

So two gets cancelled and this is PI Interviewer: on right and now n if I put odd or even I get different result for this cost it might be $+ 1$ or $- 1$ whenever I put odd it is $- 1$ whenever I put even it is $+ 1$ and accordingly I can see a destructive interference and constructive interference is being created so different frequency term because different n I put different frequency I will be getting for different frequency I will be getting at different characteristics this exactly is termed as frequency selective fading that means your channel acts differently over different frequency band okay and there are methods to actually combat these things.

So what you do if you really wish to transmit a very broadband signal over a channel which has this kind of characteristics What will happen some of the selectively it will actually attenuate some of the some portion of the signal and selectively it will enhance some portion of the signal or it will transmit as it is some portion of the signal that's not very good instead of that if you can segregate your entire frequency band into smaller bands and in each band you transmit something then it will have less effect of frequency selective fading and that is what is being done in orthogonal frequency-division by the brain which probably is not the topic of our discussion over here.

But this is what people do instead of utilizing the whole band they subdivide the band into smaller bands and they try to transmit something on that those smaller bands independently so

that they can be detected independently in a of course in smarter way which is part of that OFDM but this is the basis for doing that and that is why in a typical wireless channel people do this because you can see that multipath fading comes from a wireless channel and we could also understand the frequency selective fading part of archangel this will always come in multipath channel how do we come back immediately this comes to our mind because in this particular course.

We would be actually dealing with multipath fading in most of the time so maybe we can just give some hint how we can comeback this so whatever has happened to the channel can we reverse that so that means a particular portion if we just go back to this same channel suppose let us say some part of the signal has been delayed but less delayed some part of the signal has been higher delayed and some part of the signal has not been attenuated some part of the signal hastener attenuated okay.

If I just do the reverse thing okay so I also almost the reverse because whatever has happened in the channel I want to really negate that or nullify that what I can do if I can just do the reverse thing that means whoever has got lower delay can I give higher delay to him and whoever has got higher delay can I give the ordinary to him whoever has got higher attenuation I give lower attenuation so this is actually called channel equalization this is something which you will be seeing in your digital communication course have been channel liquid is equalization is Avery important factor for your receiver designing.

But that as what is channel equalization that means you assume that channel will have this multipath effect and you try to equalize or that means you try to reverse that and that is being generally realized with a tapped delay line so that means you actually take the signal you tap at different power levels and then you actually adjust with a particular attenuation factor as well as a delay factor and try to again add them together to negate the channel now what will be the coefficient optimal coefficient of this delay.

And as well as this β that you have to actually understand all you have to know that is why what they do they train the channel that means you first transmit a known signal over the channel try to adjust this parameters try to see whether that known signal can be better realized after doing all these things adjust all those parameters of β that means the attenuation factor that you are

putting in the delay factor you are putting you equalize them this is actually called the equalization you equalize them.

So that you get a better response then once you have characterized the inverse of the channel and assuming that the channel is not time varying that means the channel is not varying over time this reflection and all those things are almost fixed it is almost coming from building not from a car or something which is moving up object if this is the assumption underlying assumption then the channel will remain the same and then the equalization that you have done that you can use for unknown signal detection okay.

So this is what people door doing channel equalization and of course in this course will not be dealing with that but this is a very important phenomena that happens in the channel and you have to means I have just given this example to let you know that how you can actually combat it physically okay so this is something which almost means tell us what are the things that can be there in the channel and these are the things which you have to combat well probably we still have not touched something which is another impairment which will detrimental affect our transmission that is called the noise.

So next class onward what we'll try to do is we will try to characterize noise first because that means even if you do not have wireless channels because why it can be been generated at receiver so even if you do not have a channel still noise can be there so it is that detrimental so if you are transmitting means the transmitter and receiver area the same location you do not have channel no impairments coming from the channel still you can have noise because your transmitter receiver can generate noise and we will try to characterize noise.

So and that is why probably noises the most important thing which has to be combated and we will devote quite number of classes towards understanding the noise better and then towards understanding how we come back noise or in presence of noise what kind of things you should expect so those things will try to analytically devise those things and we have to really to get as good understanding of those things we need to have a good understanding of random process so our next few classes will be devoted towards understanding random process okay thank you.