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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

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Lecture 03: Fourier series (Contd.)

So we have so far discussed about basic communication okay, and the course time is another communication tool of course will be slowly going into analog communication but we have so far discussed the modules of communication that the transmitter side receiver side and how a transmitter side receiver side communicate to each other so today what we will see that what a particular communication system requires. So it generally requires two things or if we try to analyze it we really require two things one is called signal.

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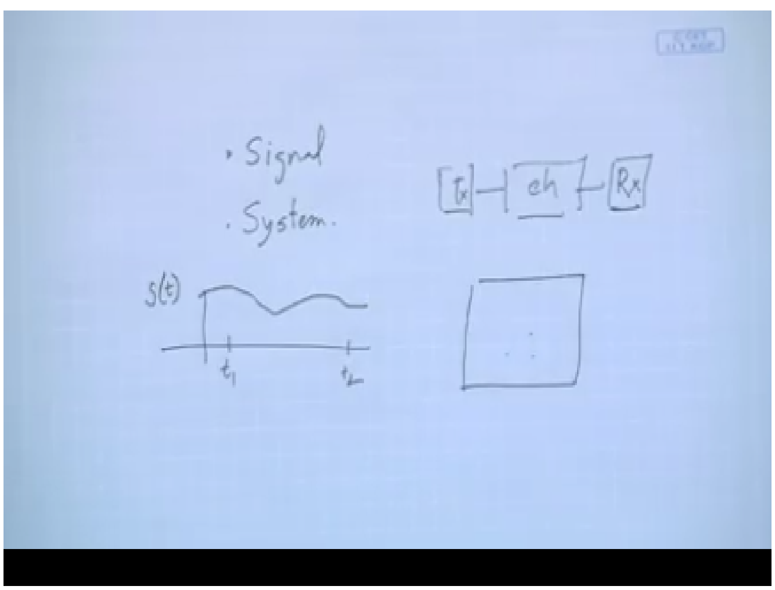

So a little bit of understanding of signal and the other part is system so basically we were talking about a transmitter followed by a channel and then a receiver right so this transmitter and receiver that is actually part of system and whatever we put as input to the transmitter side that is actually signal and what that transmitter does that system actually operates on signal and produce another signal this is how it and then the signal propagates through the channel goes to the fever again receiver is a system it operates on.

That signal and generates something which is desirable okay, so this is how it the communication goes through but let us try to understand system probably we will discuss later on it is probably the hardware that has to be designed which will do some desired operation but initially we will be more concerned about signals so what is signal is something where a particular parameter let us say it might be if it is electrical signal probably to the voltage or current okay which varies with time and this is the stress with respect to time so how it varies with respect to time so let us say if I just put it as a function of time so let us say GT and it varies with respect to time the Σ actually the description entire description over the time of our concern if the time is truncated let us says tarts from p1 and ends at p2.

So it is within the time limit you want to t2 whatever the signal at rest or let say we are talking about voltage or we are talking about current so it is the amplitude of that thing that is actually a signal description now signal can be of multiple pattern okay and it can be means we are here probably in communication system or in communication analysis we are talking about time varying or the independent variable is time it is not it is not necessarily a time always it might be a special variable.

So even a signal might be just like let us say in television the pixel values okay, it has a special variation so in a two dimensional space like in a television screen at every location what is the signal strength so there the signal is actually a means independent variable in space whatever it is to coordinate if the space is taken as 3 coordinate it is a volume then it should be 3 coordinate so with that space how the whatever parameter that we are discussing about how that varies okay.

So that is the signal can be off means it can vary with respect to anything but for our case or for our purpose and communication will be mostly discussing about signal varying with time okay.
So now let us first try to see how do we means anything we define suppose signal we have defined the way we have defined right now so that must be parameterized that means there should be some measurable quantity in that signal so how do I suppose.

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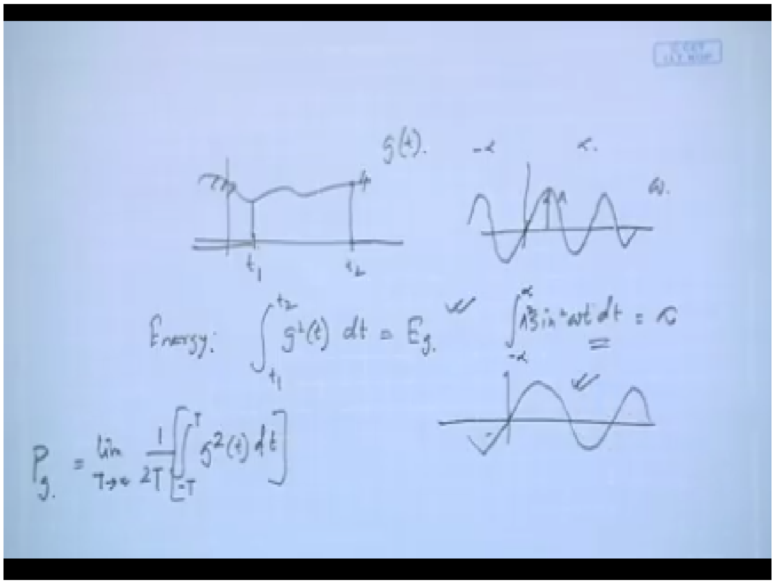

I give just a signal let us say this is GT okay, now if I just say that let us say it is defined from t1 to t2 rest of the part we don't define it we can take it as 0 okay let us say this is the signal definition now how do we actually measure some parameter of the signal or how do I characterize this signal one way is this graphical or pictorial depiction that is alright can we give one parameter to measure some property of this signal. So that one important property is probably the energy of the signals we will see that has a big consequence later on.

We will see that okay ,so how do we define energy this is something very easy if you take GT as voltage and if you pass it through a unit resistance so we will be able to calculate that it should be just G square t and you integrate it right fromt1 to t2 okay so that is actually the energy of this signal G okay so as long as it is time bounded probably we can we can integrate it from t1 to t2 that is fine we will get some value and that is actually characterizing that is a specific property of a signal different.

Signal will have different things does not mean that every signal will have a unique energy multiple signal might have same energy it's just that integration value has to be same for different signals okay, if it is not time bounded then the integration goes from - infinity to plus infinity it might happen that this might go to infinity so signal might also have in finite energy that is possible okay so this is one way of characterizing a signal the other way of characterizing a signal is like this suppose the signal.

We are talking about it is a sinusoidal okay, so it goes like this something like this it stretches from minus infinity to plus infinity okay so of course if we try to measure that energy of the signal you can immediately see that we will go to infinity okay so if I just put this sin square suppose the frequency is ω TDT and suppose it has a amplitude of a it will be a square sin square and then if you just integrate it from minus infinity to + infinity this is going to be infinity so this is one example of infinite energy okay, then actually I have failed to characterize this because it is going to infinite energy any sinusoidal I will be drawing suppose I draw it with some other ω so it will have different frequency and again.

I try to characterize it will just have be infinite energy again so basically what has happened I have failed to characterize the signal with respect to that particular parameter that I have defined that is energy can we do something else can we put another measurement parameters which will characterize the signal so that is called power so whenever we are talking about power what we generally say that will take the overall signal energy and will also measure the time and will divide it by time okay, so it is like this suppose I have signal G so G Square t I integrate I integrate it from- (t) +(t) okay.

And then I divide by 1 by 2 T and then what I do I put a limit T tends to infinity so that means by this from - (t) +(t) I calculate the energy of the signal I divided by the time period which is 2 so I get the power of this signal and then I stretch time to infinity that means I capture the whole signal right so this is the power of the signal g so this is another way of characterizing a signal wherever you can see that some of the signal which has infinite energy that might have finite power like this one sinusoidal if you just try to do it will be just means whether you do it for the entirety or you do it for one period.

And whatever calculation you get it will be the same because that just keeps on repeating so more number of so g Square t you get that many number of t also you will be getting so it will get balanced and you will get similar powers okay, so accordingly we will classify signals later on so that that part will see but these are just two parameter with which we can actually characterize. The signals this is something I wanted to tell you okay so the next part is how do we classify signal so the classification okay.

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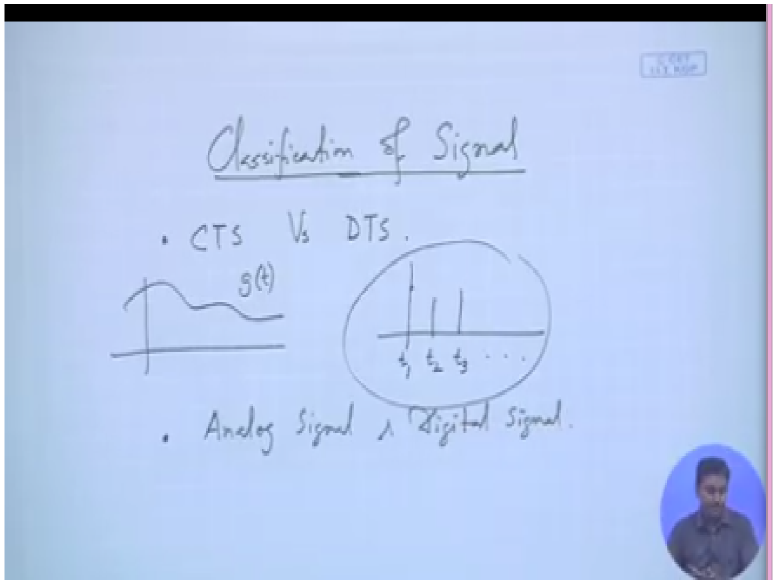

So one way to classify is see any signal we were describing we are having us independent variables which is time okay for our case it might be other things also so for our case at least it is time so whenever we classify the signal with respect to time we can say there are two things one is if I take it in discrete time that means the signal is defined in discrete time and the other one is the signal is defined always that is continuous time okay so I can have a continuous time signal versus discrete time signal.

So the typical example is the g(t) we have drawn or even sinusoidal input that is a continuous time signal okay instead of that if I just sample this signal at different location so let us say it is then this and this in between suppose this is time t1 t2 t3 and so on so these are this particular signal is only defined at time t1 at time t2 and t3 in between there is no definition of that signal so this is called discrete time signal okay, will been countering this particular kind of signal because we will be doing sampling of signal that you will see later on this is a typical way of representing a signal so that will be dealing with this later on and the other part is continuous time signal.

Where it signal is defined at every time instants between the limit we are considering ok so the second way of classifying a signal now we had this independent variable time we also have this dependent variable which might be voltage for our case ok so the signal can be also classified with respect to that the voltage can be discrete level so it can take any value okay so accordingly we will be talking about analog systems and digital C so analog signal that is very important for our analog communication we are mostly will be dealing with analog signal so do not get confused with respect to analog signal and discrete time or continuous times.

It does not mean that analog signal has to be always continuous-time or discrete-time it can be anything only thing is that it will be analog or it will be classified as analog signal if the voltage level or whatever that dependent variable we are defining with a voltage level current level whatever it is what whatever we are measuring that can take any value in a particular range okay so if it can take any value there is no restriction within that or it is not taking some discrete values then it is called analog signal if that is not the case it only takes few discrete levels voltage levels then it is a digital signal typical example of digital signal is binary signals.

Where we are probably transmitting 0 volt and 5 volt okay, so it only has two levels so that is why it is called binary you can have other ways of representing signal multiple voltage level you can put so accordingly those are also characterized as digital signal okay ,only thing is that the levels number of levels are more okay, so that is another way of classifying signals and do not get confused with the analog signal digital signal or continuous-time and discrete-time the other part is which will encounter quite.

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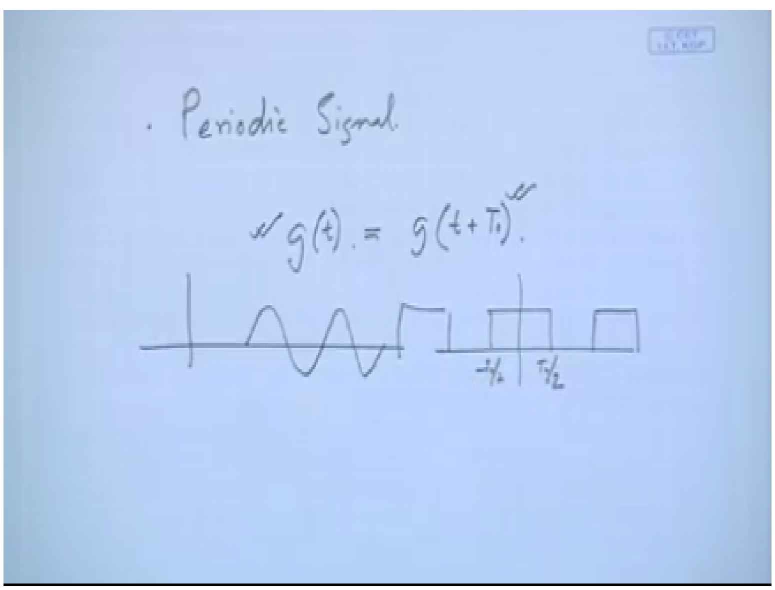

A lot this is periodic signal and the counterpart a periodic so what do we mean by periodic signal it's like this suppose we have a signal g(t)which characterizes this or which has this quality that if I translate the signal by a particular value t0 it looks like that means in time if I shift the signal overall signal if I shift by t0 okay so whichever direction it might be+ t0 it might be - t0 okay so which ever direction I shift the signal the signal just looks similar the typical example is sinusoidal if it is defined from - infinity to infinity okay, so if you just shift by one period of a sinusoidal it is just look same so if that this particular criteria is satisfied then we call that particular signal periodic signal you will see this periodic signal also will pla.

An important role in terms of signal characterization will today only we will discuss about Fourier series you will see periodic signal has a big role to play in Fourier series the way we represent signals so that is one thing of course if a signal is time bounded it cannot be periodic because if a signal is time bounded it will definitely start from somewhere after that suppose it has a periodic nature but whatever it is if I just translate.

It will either start from somewhere it will end somewhere if I just shift it then at least either the last or beginning will not match so for a signal to be periodic always the criteria is signals must be stretched to - infinity and on the other side + infinity this has to happen now whichever is not fulfilling this criteria that is called a periodic signal all other signals are repeated if I just put a pulse like this is nothing else is defined from- t0 by2 to + t0 by 2 it is like this the signal is defined as this so this is a periodic signal because whatever shift you give you will never get back the same thing.

That is not possible if the same + is repeated after every maybe let us say t0 then probably if it is repeated and if it is stretched to – infinity + infinity then it is again a periodic so that is the difference between periodic signals and a periodic signal next I am just giving all the definition you will see that when we will be discussing the properties of signal these things will come back so next is energy signal and power signal.

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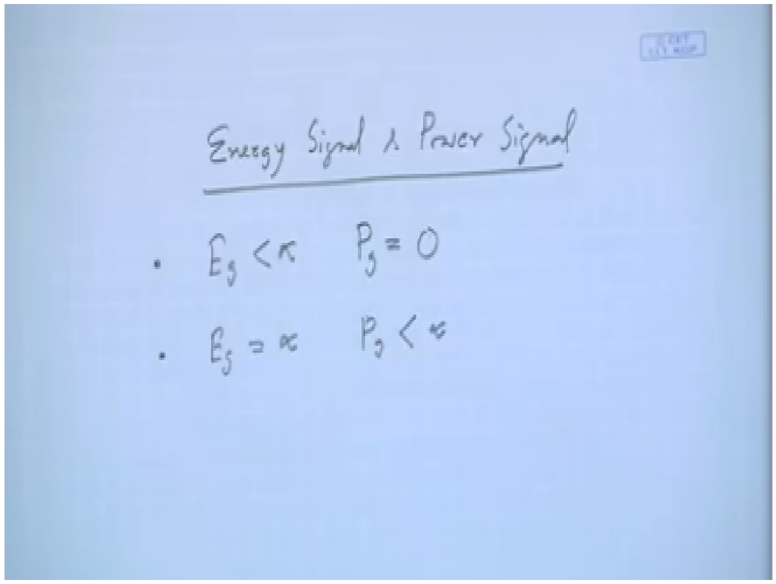

So what we mean by energy signal we have already seen that signals can be characterized by two parameters we have defined one is energy of the signal and the other one is all of the signals defined by if signal is g(t) defined by and PG we have already seen that okay, we have characterized that so if for particular signal energy is finite okay so that means it is less than infinity and power is 0 if this criteria happens most probably you can see already the signal must be truncated it should be limited in time it should not stretch to- infinity + inside and somehow the energy is also finite if it is truncated probably we can always calculate the energy if it is not going to infinity okay, in between.

The signal level is not going to infinity if that is not happening all the signal levels are finite then if time is also truncated then probably we will be able to evaluate the energy it should have a infinite energy and power automatically because time can be stretched to infinity and if I divide by that time of course power will become 0. So those signals are called energy signal mostly energy example of energy signals are time truncated signals so whichever has a finite duration those are generally the energy signal and the other side area is the signals levels lead voltage or current it should not go to infinity okay.

Then it is energy signal the power signal is all those periodic signal we are talking about where the energy goes to infinity because for all those periodic signal if you calculate energy that is always going to infinity but the power is finite okay so that is actually the example of power signal this is energy signal will see the difference between these two it is actually if you see almost periodic a periodicity has something to do with energy signal power signal how do you characterize them okay, and the fourth part or sorry fifth part.

Which is also our important part that where we will be actually our communication will be mostly concerned with that that is called deterministic signal and random signal so deterministic signal means that the entire time duration I know exactly how the signal behaves okay so I know the levels or the level it attains at every instance of time is that something I do not know whoever is characterizing the signal whoever is analyzing the shape signal if that person does not know about what will be the value of the signal at a particular time instants then we call that as random signal or we characterize that as random signal.

So if do not know the value of that signal then what we are doing here and I will also give you a typical example noise is a typical example of this random signal so what happens even though we do not know exactly what is the value of that signal at different time instants but what we know is the statistical property of that city so that means we actually know that if the signal level is defined by a random variable what's the PDF of that random very so these are the things.

And some more things of course right now probably we have not discussed about that so when we'll be dealing with this in detail form we will be knowing that what exactly is required to define a random signal and deterministic is what we know like sinusoidal or pulse train or any kind of just single pulse so the other deterministic signals where we know the definition of the signal for the entire duration okay.

So this is typically means what we talk about signal and a classification of signal different kinds of signals how do we classify them so these other typical example of that so right now what we will try to do which has been done many years before by four years that can we have a different representation of signal so we know the time representation of the signal can we now try to characterize the signal in different way a different representational together of the signal.

So this is what we will to appreciate whatever our courier has done what came into four years mind and how he means logically develop that methodology we will try to explain that okay so that that will be our next target so what couriers have seen at that time signal is almost similar to vector we all are aware of vectors so what we will try to do first we will try to characterize how do we define vectors and then from that vector analogy we will try to build up the theory of signals which for years has developed so this is something we will try to do now so investor theory what we know we know that suppose we have a vector.

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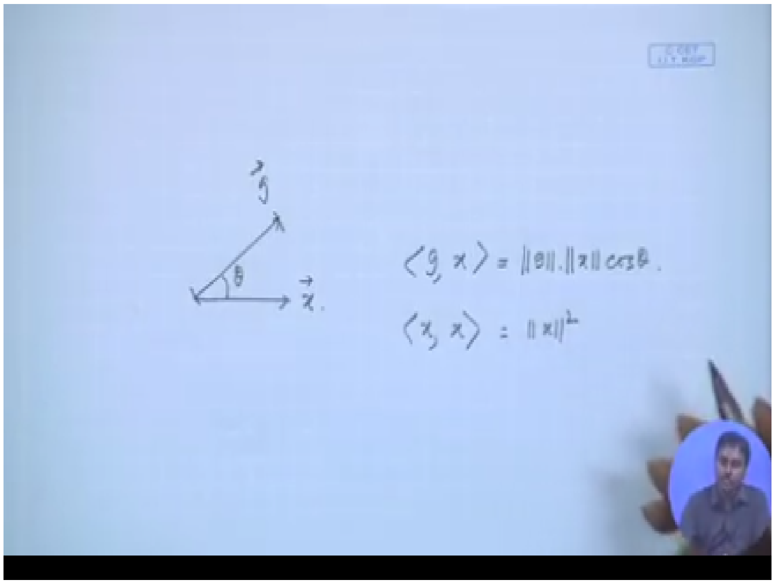

Let us say this is g okay and then we have another vector that is in another direction let us say this is X okay so these are two vectors in different direction okay in vector q few parameters are defined so we just for recapitulation most of you all aware of this so I will just define so whenever we say a vector space it has a dot product defined along with that vector space so that what we mean by dot product the definition of dot product you already know.

So if I say g,x this dot product I want to get so generally it is this G that's actually the amplitude of g okay so it's actually this value into amplitude of X into in-between whatever angle they have vector means it has a direction so they must be having some angle between the COS of that angle so this is typical concept of vectors right the dot product concept .

Okay if we try to do dot product of the vector along with itself so that must be distance multiplied by his own distance so distance Square and θ is 0 because it is the same vector so θ 0 because 0 is 1 so generally distance square is characterized and distance Square comes from dot

product on D so a vector dot product with itself gives the distance square right so this is something I know already know what I will try to do now is something new so basically what I am trying to do.
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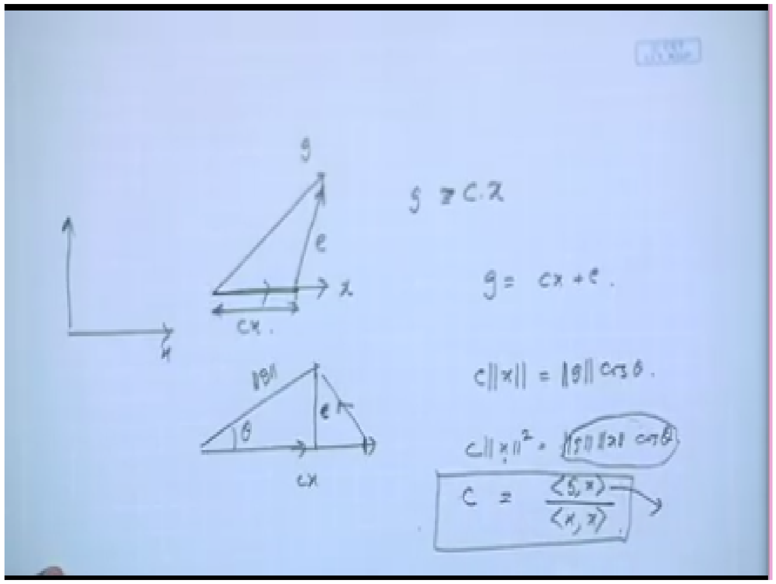

Suppose I have a vector x which is a known vector let us say this vector I know already now I have another vector g I want to represent with respect to vector x it is just a representation so I have a arbitrary vector I am trying to represent best representation that I can do I want to represent this vector g with respect to this vector x what will be my representation so basically if I have to represent with respect to this vector as always in this direction I cannot go any other direction because I have to represent with that vector only so I can only put a scalar multiplication to that vector right so let us say that scalar multiplication is c so I approximate g with c into x that c is a scalar okay.

So now it depends on the value of c what value of c I put so if I put c this much then it will be let us say c is defined in such a way that c into x is up to this okay or I can I can also define it in such a way that c is up to this okay so this is c x and here this after this is c x okay so I wish to represent g with respect to x so I have come up with this value c x right so how good my representation was I need to know the era's of this representation so error must be how much I should add with c x to get G that is should be my error.

So if I just put this vector which I call e that must be the error vector so I can immediately write in vectorial term g = c x +e where e is the error right so if I just put it this way then my error will be this one if I just put it this way my error will be this one now it is very trivial to see how do I minimize the error when it will be minimized when the vector gets its projection on X it will be exactly perpendicular so c is chosen in such a way that I get the projection of that vector immediately on that point okay ,so that should be my best choice I cannot do anything better than this right.

So this is what I wish to do and this is how I minimize the error the and just trying my best to represent that vector given this other vector I am not doing anything else okay but you will see with this a beautiful representation will come and all okay so this is what I can do and of course this becomes my error so that is the best error I can do now whenever this error is the best one then what I can say about this is how do I evaluate this particular c where the error is minimized right.

So this is something I wish to now means derive right so whenever I am trying to derive that what I can say that let us say this c x this particular value when the thing is minimize the error is minimized what is that that must be that value of g into cos θ because it is exactly the projection so g into cos θ must be the c x okay the modulus I am talking about of course now I am not talking about the direction.

Because I have already taken the θ so c x must be g into cos θ so this is true this should be the case now what I will do both sides I will multiply by the modulus of x or the length of x so c mod x^2 that must be mod g or norm g norm x cos θ right now go back to our earlier definition so we get see this is nothing but dot product of v and x so that is t and x dot products and this is nothing but dot product of x with itself.

So this is how the optimal c can be represented and when we say a vector g is orthogonal such the definition of orthogonality when we say a vector g is orthogonal to x if g cannot be represented by x that means the projection will be the value of c will be 0 so I cannot represent that vector anyway by the other vector if this is happening then only I can say they are basically also them.

So immediately what should happen c should be 0 so that means the g vector should be directly perpendicular to this x vector and I will not be able to represent anyways ok I cannot characterize even the error because I do not have any representation c value is 0 so if c is 0 if c has to be 0immediately I can see the condition of orthogonality that means g dot x must be0 so that is the famous of the unity condition in vector.

All these things you already know I just have demonstrated this to mean so that you can appreciate that it's nothing but a representation technique that we are trying to employ where we are trying to all we are trying is trying to minimize the error of representation nothing else now what we'll do on this paper which is a two dimensional thing I will just take two vector and you will see very nicely the entire representation will be done this is how the coordinate system has been designed so in the next section of this class we will try to do that and then we'll go to signal with this analogy.