

NPTEL

NPTEL Online Certification Course

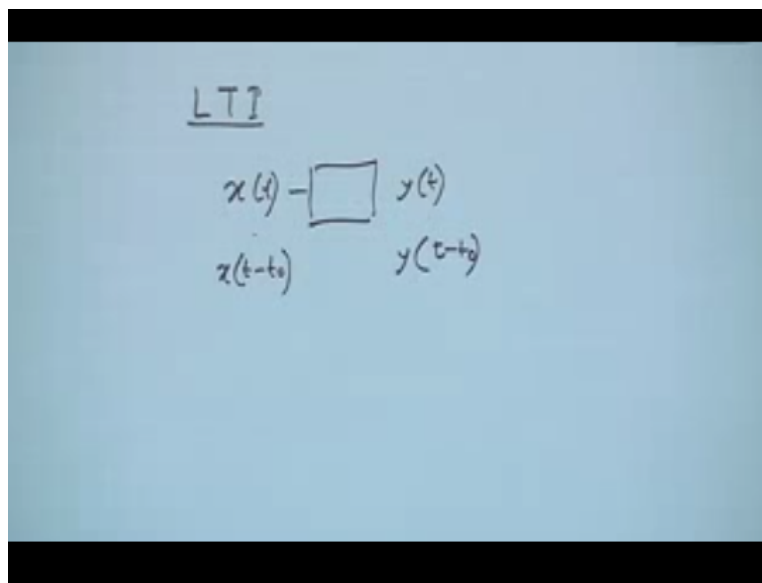
**Course
On
Analog Communication**

**by
Prof. Goutam Das
GS Sanyal School of telecommunications
Indian Institute of Technology Kharagpur**

Lecturer 29: Dispersion

So I think we have started character the channel the last class we discussed in the time variance system and what kind of system we need for realizing and starting this system this is something we have done so for linear time in variance system we have already told that it has to be linear that means input whatever you give corresponding output if you know if you give the linear combination of the input so you should be expecting the linear combination of the output and time in variance means suppose a particular delay X_T within a system gives you Y_T if I delay the input so if I pass the t_0 so the output will be just similar just delayed by same.

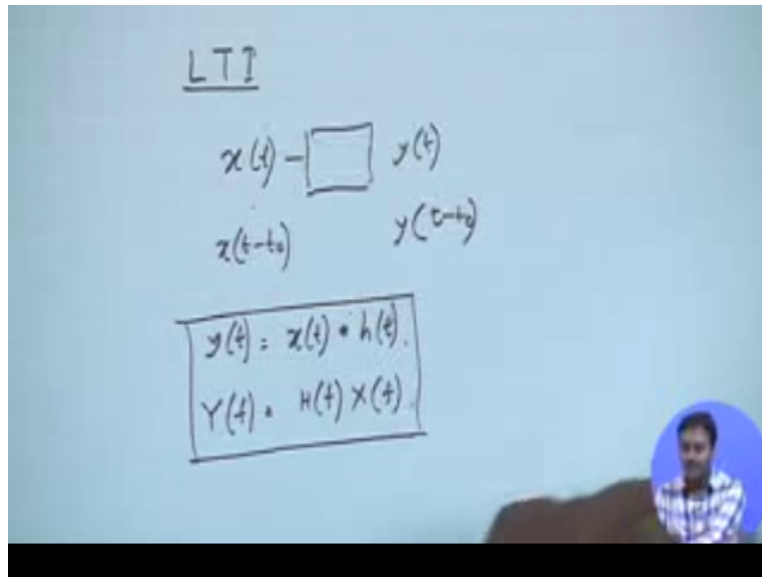
(Refer Slide Time: 01:14)



So it will be just a same output so basically if I give a delay it remains time in variant now output also will be just delayed by that signal that signal strength and the output signal the way it looks

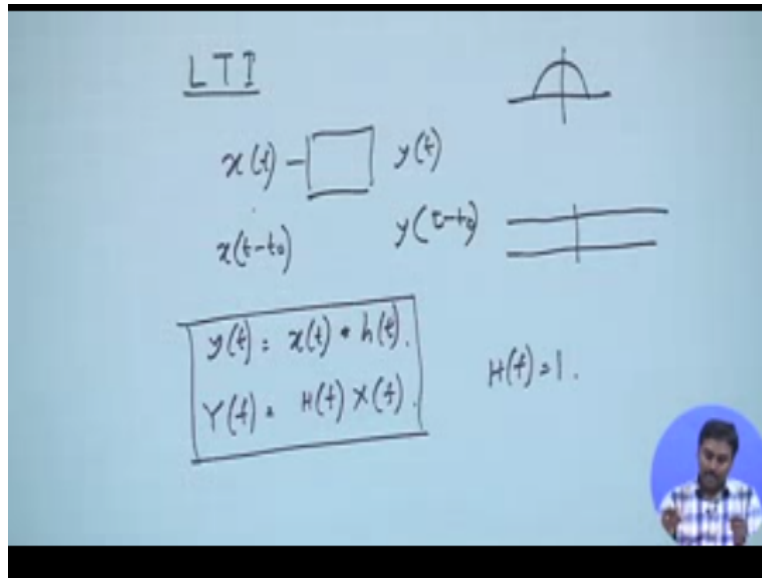
there will be no change in that okay so that was the linear time in variant system we have told that it can be characterized by only one thing which is the transfer function or the impulses of it and always we can prove that the if I put a input as XT the output will be always if I know the impulse response of that particular system so it should be convulsion of those impulse response and input function.

(Refer Slide Time: 02:00)



So always yt must be xt*ht okay so this is always true and the corresponding Fourier transform should be trans function so this two things are always true for a linear time in variant or system right so this is something we have told and then we went to a special class of linear time in variant system so where we were talking about dispersion sorry distortion less system so for that we have told that distortion less means the amplitude should remain the same okay so what does that means amplitude remains the same means that HF should gives me the same character tics hence after passing it through HF

(Refer Slide Time: 02:52)



The YF should have the same characteristics HF sorry XF so therefore HF must be 1 so I can write HF must be 1 if it is any signal I have to say that for all frequency value HF must be 1 so it should look like amplitude spectra of the trans function the amplitude representation of that must look like this all frequency component if XF is band limited so that means that XF you are talking about is something like this then if I wish to get YF.

Similar to XF I only need to ensure that within the band HF is 1 beyond that whatever happens I do not have because any way I do not have any frequency component of that signal so therefore I need to at least ensure that if it is a band limited signal that within that band so $-b$ to $+b$ HF must be at least 1 beyond that it can be and if it not band limited signal then I have to say that HF must be 1 at all frequency or I should say mode HF must be 1 for all frequency okay.

(Refer Slide Time: 04:01)

$x(t) \rightarrow \boxed{\cdot} \rightarrow x(t-t_0)$
 $Y(f) = \underline{X(f)} \underline{e^{-j2\pi ft_0}}$
 $H(f) = 1 \cdot \underline{e^{-j2\pi ft_0}}$
 $\underline{\underline{-2\pi ft_0}}$

The graph shows a coordinate system with a horizontal axis and a vertical axis. A straight line with a negative slope passes through the origin, representing the phase response of the system.

So notice this size because phase we have not talked about and then distortion less means there is also we have to talk about the phase and the effects of phase so what will happen if we have already told that distortion less means I put a input to a particular system okay which does not create any distortion so therefore the input should look same and the only thing that can happen it must get transmitted in time nothing else okay so there might be a certain delay to the signal okay but the exact signal should be replicated at the output the whole signal might get transmitted not some portion of it get transmitted to other portion does not get transmitted then probably the signal.

So that as to happen therefore my output should be whatever XT I give I must get something like $X(t-t_0)$ so immediately I know for the corresponding HF we can now do the Fourier transform of this file so output should be YF which is we already know that should be $X(f)e^{-j2\pi ft_0}$ so this is something we know so this is the XF so therefore this system HF must be this in so I can write my HF as $1 \cdot e^{-j2\pi ft_0}$ so this 1 is the amplitude part of that and this is the phase part of this so I should say $2\pi ft_0$ is case so if I plot that case it should look like this okay with the slope of okay.

So that is all that is happening for a hence now we have characterized distortion less system that means if you give a input to the distortion less system I should be expecting just the replica of that maximum I can once take it into a account of it so this is what I have done XT I told output must be $X(t-t_0)$ if you can say I can still take a attention because that might be variable if I transmit it to the signal about a channel this is the channel character tics we are talking about so

if we are transmitted there must be a alteration but the altercation should be with respective off frequency that means all the frequency must have similar alteration so I should be able to.

(Refer Slide Time: 06:38)

$$x(t) \rightarrow \boxed{k} \rightarrow kx(t-t_0)$$

$$Y(t) = \underline{kX(t)} e^{-j2\pi f t_0}$$

$$H(f) = \underline{k} e^{-j2\pi f t_0}$$

$$= -2\pi f t_0$$

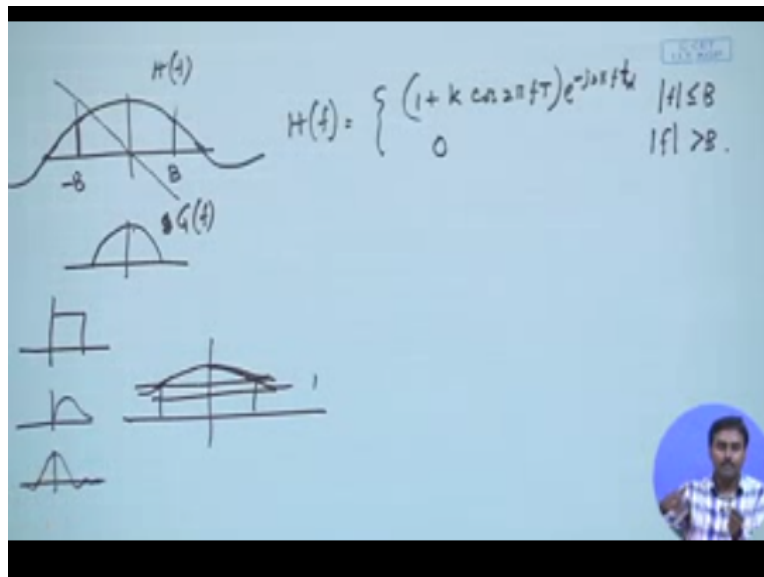
If XT is there I will be able to see K is less than 1 okay if k will come out then this will become k so basically what happens my HF is mode HF is still the same instead of 1 it is k okay and the phase is period so this is the character tics of the distortion less system or I should now start talking about channel so because we started characterizing the channel what I want from the channel whatever I transmit I need to receive in signal that is why the channel is characterizes it should be ideally not filtered that means it is neither creating any deformities or distortion in the overall signal character tics it might just give you some delay off course that will be there because if I transmit over a long distance I know that the inverse the fastest of career which is light in free space probably that will still have a velocity okay a finite velociuty c.

And of course there will always a delay so anything I will be transmitting there will be a delay okay so that delay I can take no problem with that but signal there will be no distortion okay so if I wish that my channel should have this character tics which almost like all past character tics and the phase should be linear so I really I want back and if I say it is already we have talked about that if I say that if it is band limited then I just need to ensure that it is flat.

So the channel character tics or channel transfer function is flat over that band of interest okay now what will try to see is we will try to see if this is possible in the channel and if somehow that

is not the characteristics of the channel what happens ideally we will be expecting this but probably we would not get that so let us try to see this is not happen so I will just go through two examples

(Refer Slide Time: 08:49)

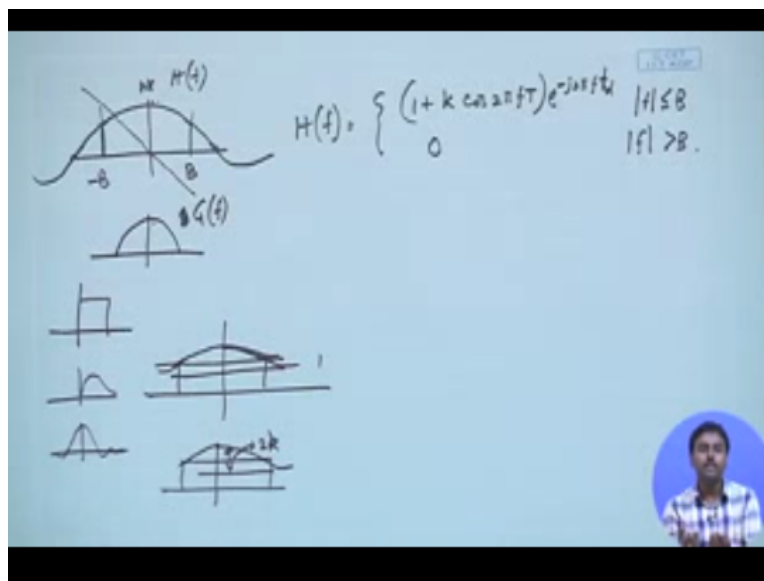


One is that my amplitude is something like this so I'm expecting a signal of band width b so okay so this is not the signal this is actually HF suppose I have a signal which is anything let us say okay this is actually G or let us say GF corresponding there is a signal which is band limited within $-b$ to $+b$ now I pass this signal through this which is my channel characteristics so the channel characteristics is the phase is the near but the amplitude has a curvature so that is not fixed which is idea if this happens what will be my overall reception that is what I want to say okay so let us say I will be transmitting particular pulse equable pulse so let us say this the pulse so this is that pulse or may be some other pulse which looks like some other pulse okay.

So which ever pulse it is I will be transmitting this pulse and assume that it means almost band limited that means a whatever is outside fixed band okay that is surprised or I might even represent then pulse with the sink okay so I represent the pulse with the sink then it is band width but of course the pulse is not the time limited that is something I know but the sink will probably will die down so this is suppose if this is my pulse okay. So that pulse will be band limited that is something I know already because I have been represented it as same pulse now suppose that sink pulse is been transmitted over this channel characteristics which is something like this.

So it is actually one $1+k\cos 2\pi fT \cdot e^{-2j\pi ft}$ okay, then $f < b$ where $f > b$ okay so this is the character tics I have taken as sinusoidal its just cutter truncated side so it was like this and I have just truncated it up to $-b$ and $+b$ and this t is the period of that sign okay so that us my overall trans character tics or I should actually write it as this so it is let us say 1 the sinusoidal actually is like this okay so or I can say it is about one I have something called 1 above 1 there is a swing of strength k right so which is what is happened so that is the sinusoidal I have a sinusoidal and about 1.

(Refer Slide Time: 12:28)



There is a swing of strength k lets say this swing is $2k$ so at this point it is $1+k$ so this is what it is at $f=0$ it actually $1+k$ so this much be $1+k$ and b whatever value it will get and this kind of I am just creating a shape okay and so it is not flat and I wish to actually see what happens to my input pulse so this is my HF

(Refer Slide Time: 13:11)

The whiteboard shows the following content:

- Graphs of $H(f)$ (a cosine wave) and $G(f)$ (a sinc function) centered at $f=0$ with limits $-B$ and B .
- Equation for $H(f)$:

$$H(f) = \begin{cases} (1 + k \cos 2\pi f T) e^{-j2\pi f t_0} & |f| \leq B \\ 0 & |f| > B \end{cases}$$
- Equation for $Y(f)$:

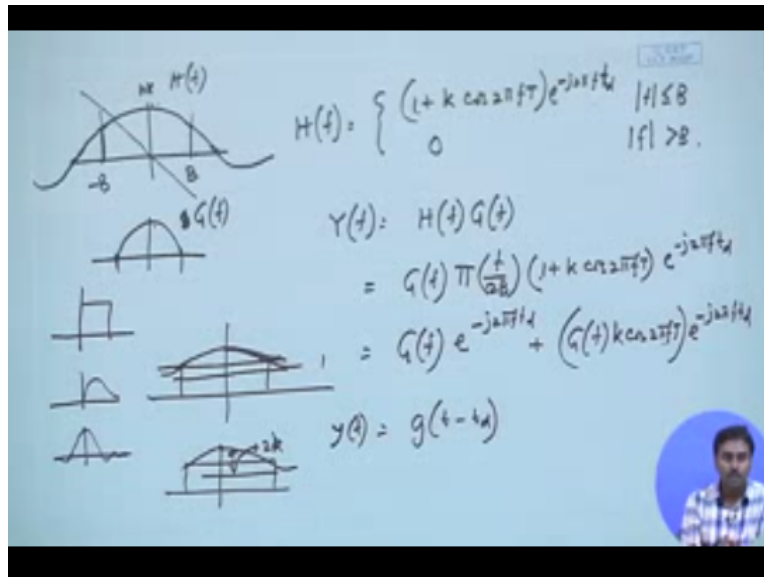
$$Y(f) = H(f) G(f)$$

$$= G(f) \Pi\left(\frac{f}{2B}\right) (1 + k \cos 2\pi f T) e^{-j2\pi f t_0}$$

$$= G(f)$$
- Diagrammatic representation of the multiplication process, showing the sinc function $G(f)$ being multiplied by the cosine wave $H(f)$ to produce the modulated sinc function $Y(f)$.

So therefore what should be my YF I will be passing GF through that so it should be HF .GH right so within the band of my interest okay what I will be getting I will be getting this so this particular thing is HF is actually if I multiply this is should be multiplied by this one over a box of $-b$ to $+b$ so I can write this as GF which is $2b$ okay so $-b$ to $+b$ is multiplied by that because rest of that parts its 0 so if I just multiply this particular function with a box function then I will be getting that particular part rest of the part will be 0 so this I will be getting into that whole function so $1+k\cos 2\pi f T \cdot e^{-2j2\pi f t_0}$ I can write in this way so now I have two terms I can have GF now what I can write because this multiplied by one will actually give me GF only because outside nothing else is there so that will be all zero

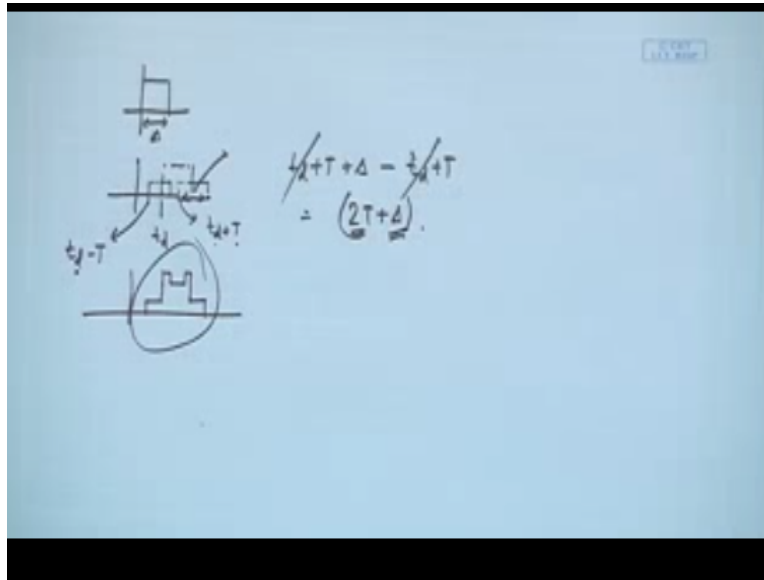
(Refer Slide Time: 14:34)



So I can whenever I multiply GF with the box function I will be getting just GF so $Gf e^{-j2\pi ft} + GF$ is band limited and it should be written as this box function beyond $-b+b$ or this bandwidth so I will be getting $k \cos 2\pi ft \cdot e^{-j2\pi ft}$ fine so this is what I am getting now I wish to see the output signal that means after transferring through this transfer function which is my channel. Now what do I expect at the other end so YT must be Fourier inverse transform of this Fourier inverse transform of GF is GT if I multiply this we know the Fourier transform of that should be GT right now this is the other thing where this cos function I can write as summation of 2 exponential.

So it will be $e^{j2\pi ft}$ and $e^{-j2\pi ft}/2$ so I will get a $k/2$ outside and then Gf and therefore I should get something which is again shifted that shifted $T \cdot t_d - t$ and there should be another term $K/2$ g shifted as $t/t_d + t$ that is all I will be getting if I just put the theorem and put $e^{j2\pi fT} + e^{-j2\pi ft}$ so that's why I am getting $K/2$ so what do I get now I had a pulse ideally I should have expected this right my target was that if I want distortion less channel then I should be getting the delayed version of that pulse that was all good but now I have two Fourier star which are further advanced delayed or came little bit earlier okay so if I just have a pulse.

(Refer Slide Time: 17:09)



Let us say that is this thing which is this pulse so what will happen so first part will be just the delayed version of that pulse so this will be delayed so it will be delayed by that amount the other part will be it will be further delayed by $t_d + T$ right and attenuation there will be some attenuation which is $K/2$ so there should be some attenuated pulse of same duration where this point is $t_d + T$ and there should be another point where this is probably $t_d - T$ right so the summation of these two things whichever way you will present it will look like this right the pulse will look like this take summation of all this things.

So what has happened now it actually starts from this $t_d - T$ and it goes up to $t_d + T$ pulse duration let us say that duration is Δ okay so overall pulse duration is now happening to be $t_d + T + \Delta$ that is this end point $t_d - T$ right so it happens to be t_d gets cancel so your pulse rate is $2T + \Delta$ which should be ideally just Δ but now I am getting a distorted pulse this is definitely a distorted pulse and the pulse rate has been extra stretched by this sum of 2.

And you can see that just coming from this okay so make this flatter this cos side will go away and you will get your pulse back exactly this particular part that we have described is called dispersion whenever you will be discussing means later on in other courses when in we will be discussing about digital communication will see that has a profound effect in pulse communication on digital communication so whatever pulse will be putting what happens whenever you have not ideal channel.

The way we have described that you have some amplitude variation of the transduction of a particular filter or particular channel then you will get dispersion that means it actually broadens your pulse so if you have a continuous stream of pulses which are carrying information okay the pulses are carrying ideal bits 0 or 1 accordingly the pulses are included if you have that and if the pulses are getting broadened what will happen it will actually spill over the next pulse okay because you have a dispersion or the pulse broadening.

So it will spill over the next pulse the problem is more it spills then the next pulse detection will become difficult so this is a particular thing which is called inter symbol interference ISI here in this term in digital communication, more often of course the same effect will be there in another communication so in another communication you do not have pulse you will see whatever is transmitted that is getting actually stretched so every bit in time whatever is there that is getting stretched.

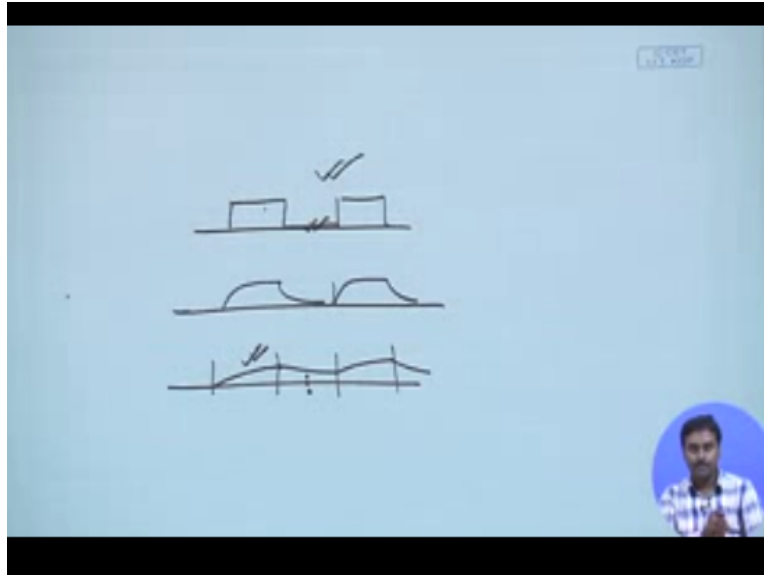
And they are getting distorting the other time signal in a means which are in advance in time okay they will keep distorting it okay so that distortion probably in another communication will be there you would not be able to take them out in digital what happens because it is just encoded signal either 1 or 0 so you still have possibility of detecting it even though there is a distortion okay so suppose I'm transmitting 1 or 0 it's just 1 or 0 that's all I have to detect if there is a spill over if I can still say that okay by the pattern of the signal I can detect that it is 1 or it is 0.

So if I can do then it is good enough okay but there will be a point when I will not be able to detect that and that is where the inter symbol interference actually becomes very serious so we will see the kind of channel impairments we are talking about here where the channel transfer function is a little bit non ideal there will be creating this is kind of interference the kind of interference that we are creating that is within the same channel that is means the frequency band we are there we are continuously transmitting signal the suppose it is a pulse train so single pulse is giving interference to the next pulse that is what called within inter channel interference so the interference is created by the same signal.

So the signal itself is creating interference to himself so that's the problem whenever you have a non ideal characteristics of a particular channel okay and this particular phenomenon is called as dispersion so what will generally happen if I get this given example so if you see I have

suppose a pulse string of this transmitted so what will happen once if it is a non ideal channel then what you will see.

(Refer Slide Time: 23:15)



Once the pulse is transmitted we will have this kind of distortion because most of the time we will have low pass effect in the channel so as low pass effect will start creating this thing it will start spooning the sharper edges okay so it will start creating this kind of line okay now this is 0 if this sharpening sorry this de sharpening of the edges is too much then what will happen it will go like this so 1 & 0 will be its almost like capacitor charging and capacitor discharge because we will be anyway low pass filter you can always reline with circle right.

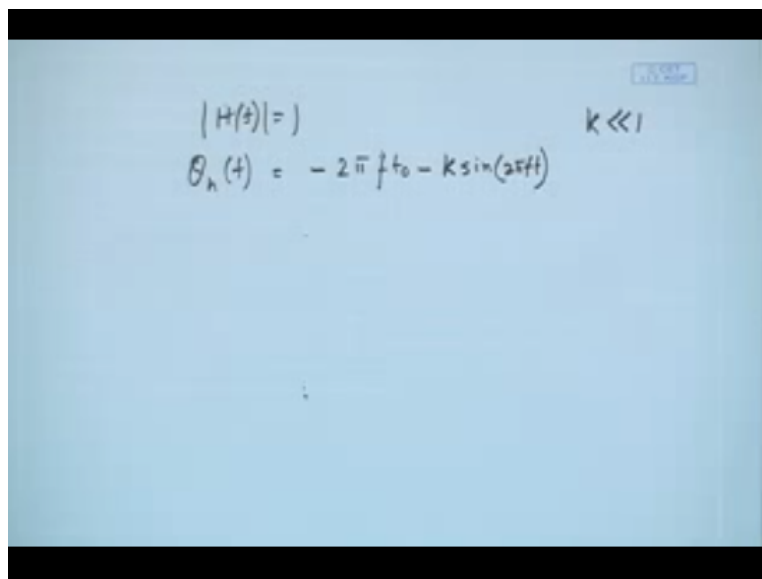
So if the channel is low pass filter it will be almost like passing a pulse through a RC circle and this is what will be happened so what is eventually happening you can see now the effect of inter symbol interferes it is just happening because of the non ideal character tics of the channel because you are passing this pulse through the channel and this is what we are getting and then in this portion can you now detect whether its if the pulse was transmitted like this you can easily detect it is 0 can you now detect whether this is 0 or 1 you cannot prove because for detecting whether this is 1 or 0 you will probably put a threshold.

And you will try to see whether the signal level is beyond that threshold okay its below threshold you say it is 0 if its above threshold you say it is 1 but that is getting mark here over there you will would not be able to detect so this is what hence you previous signal its actually because the

Fourier the speeded part of the previous signal has not really vanished and that is creating interferences over here okay so this is what happens.

So within the same channel it is just the previous part of your signal is creating interferences to you which will be happening if you have no idea okay so let us try to see if the same thing is happening in phase so I have a modified phase if that is happening okay so let us say I have a channel which looks like this.

(Refer Slide Time: 25:30)



The image shows a handwritten note on a light blue background. The text is as follows:

$$|H(f)| = 1$$
$$\theta_h(f) = -2\pi f t_0 - k \sin(2\pi f t)$$

$k \ll 1$

I have channel which is ideal in amplitude this is 1 but the phase part how should be it should be linear with respect to f but suppose it is not so it is $-2\pi f t_0 - k \sin 2\pi f t$ and we are assuming that this $k \ll 1$ the most of the time we will see this non ideality of the channel will not be too heavy in the previous example also there was in the amplitude there was $k \cos$ so k value will be pretty

low okay once the k value is pretty low we will not be still some spreading but it will not be that severe okay so the other two pulse delayed pulse we have generated if I can just give those bring that example back.

(Refer Slide Time: 26:31)

Handwritten notes on a whiteboard:

$$|H(f)| =$$

$$\theta_h(f) = -2\pi f t_0 - k \sin(2\pi f T)$$

$$Y(f) = G(f) e^{-j2\pi f t_0} \cdot e^{-k \sin(2\pi f T)}$$

$$\approx G(f) e^{-j2\pi f t_0}$$

A circled note in the top right corner says $k \ll 1$.

So if this particular k value is very low they will have a lower strength and the corresponding this one will be of low strength the pulse spreading that is being happening will be pretty low instead right so that is what will be happening but so this is something where the non ideality will not be very severe okay so that is why I wanted to tell this but here also probably the non ideality is coming due to this portion that will probably not be very severe so whatever that is if you just consider this lets try to see what will be the effect of it okay.

So again let us try to compute the YF that should be let us say I have a GF multiplied by the filter transfer function okay which is one so it should be just $e^{-j2\pi f t_0} \cdot e^{-k \sin(2\pi f T)}$ now we will be putting some approximation over here so GF is $e^{-j2\pi f t_0}$ now this you expand in Taylor 6 so will be

we are trying to put this approximation we expand in Taylor series and anything because k is the argument right so there should be terms of k, k^2, k^3 and all those things in all of those terms because k is very small add on the terms we can neglect so I can just take the first term and the second minutes just varying with this one k that terms so first and second term.

So I can write this as $1 - jk \sin 2\pi ft$ now just expand it so this is as it is $e^{-j2\pi ft}$ so the first term should be as it is the second term should be $-jk$ I can write that as the previous one again there should be a $2j$ coming out and I should be getting this $gfe^{-j2\pi ft} e^{-j2\pi fT}$ and then that should be $jk/2j$ and then j gets cancelled everywhere and $gf e^{-j2\pi ft} e^{-j2\pi fT}$ right this is what we get again I want to see the pulse that exactly the same you see just there will be minus sign so this becomes $gt-t_0$ this will be your Fourier transform or inverse Fourier transform this one will be $-k/2gt-t_0+t$ and then this should be $+k/2gt-t_0-T$ so I'm getting three pulses again similarly there will have they will be creating some distortion.

And there will be a pulse ready so whether we get a pulse transmitted to a trans function which is non ideal either in the amplitude or in the phase I almost get the similar effects that there will be a distortion which getting created in the pulse and the distortion is nothing but it broadens the pulse and it start reading in the symbol interface okay so this is something we have seen and we are calling this as dispersion next will see if the channel is little bit defiantly non ideal that means if it is showing non linear character tics then we will probably see that it will not only create interference to his own channel it will start creating interferences to other channels okay.

So will see that will be more detrimental in our system were frequency division multiplying is be used so if you have multiple signals the transmitted over multiple frequency band 1 particular thing if the channel is non ideal we will start creating Fourier frequency term into the other channels okay so that's the other effect of channel we will try to examine in the next file.