

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

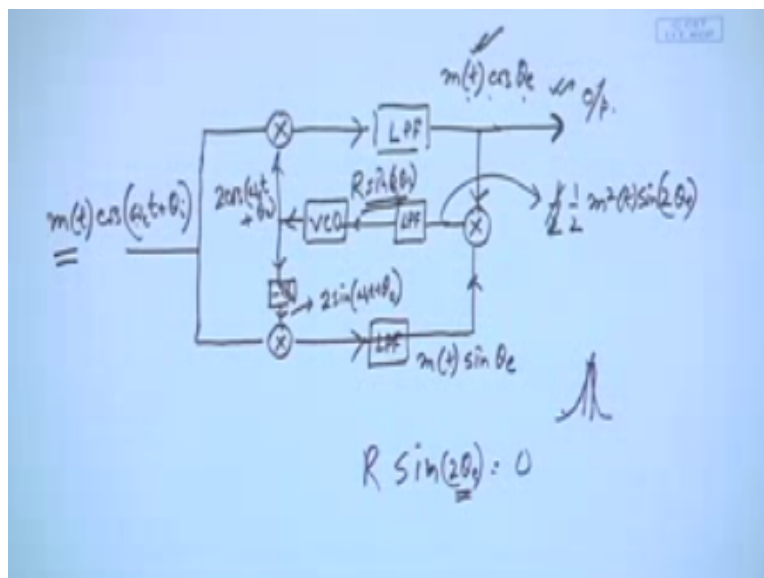
Course
on
Analog Communication

by
Prof. Goutam Das
G S Sanyal School of Telecommunications
Indian Institute of Technology Kharagpur

Lecture 28: PLL (Contd.) and LTI

Okay, so probably last two or three classes we have been discussing heavily on PLL so we have almost finished the analysis of PLL, now what we wish to do is try to see a practical application of it so that is actually DSB-SC demodulation where the PLL has been heavily used so we will see that we will later on when we will study FM we will see also this particular thing PLL can be also implied for FM demodulation we will explore that part and that is why probably PLL is one of the most important circuitry in communication system. So let us try to draw that circuit.

(Refer Slide Time: 01:07)



So it is nothing but your let us say $m(t)\cos(\omega_c t + \theta)$ that is what is coming okay, so this is actually just $m(t)$ is your message signal and we have done a DSB-SC modulation, so we have multiplied

with a $\cos\omega_c t$, but what try to happen if I have a phase okay, θ_i which might be included in it okay. So this is coming we take it into two arms and then we use to multiple a circuit over here that multiple circuit actually gets input from a VCO so that is that almost appearance circuitry is being created.

So VCO will definitely create another cosinusodial let us say okay, or it might create a it means sinusoidal whatever it is let us say we create a cosinusodial okay, so VCO will create a cosinusodial which is we termed at as $2\cos(\omega_c t) + \theta_o$ right, same thing is coming over here I give a $-\pi/2$ phase shift okay and then multiply so this will be sum $2\sin(\omega_c t + \theta_o)$ right, this is what we get when both the arms are passed through a low pass filter okay.

So what will happen that multiplication term will be creating a this plus this and this minus this right, let us say the minus because ω_c we have told already there is a matching even if there is no matching we know that how to deal with that ω_0 I can represent as $\omega_c - \omega_0$ and $+\omega_c$, so we can always do that okay. So frequency we do not have to really think about it is all about phase okay, so there will be a $2\omega_c + \theta_i + \theta_o$ will be there whatever it is and there should be a $\theta_i - \theta_o$ low pass filter will take $2\omega_c t$ term will be only having this $m(t) \cos\theta_e$ which is just the difference of these two.

Of course I am not writing that it is a function of t it must be a function of t okay, so this θ_e will be a function of t. so this must be my output and what I do I need to have a feedback so I take it over here fine, I put another multiplier circuit this is almost working like a squaring circuit so here also there should be a low pass filter what do I get because it is sin so cosinusodial multiplied by a sin it will be $\sin a + b \sin a - b$ then $\sin a - b$ will be surviving so $m(t) \sin\theta_e$ will be surviving.

I multiply these two right, and then I pass it through a low pass filter again which is a very narrow band low pass filter okay, it just around DC it is try to pick that DC term nothing else, okay. So that is this thing so this is a narrow band low pass filter so what do I get over here is $1/2$ or I should not say $1/2$ it is just m^2 yes, there should be a $1/2 m^2(t)$ it is $\cos\theta_e \times \sin\theta_e$ right, so that should be $\sin(2\theta_e)$ right, that must be get into this.

And now I am passing through a narrow band low pass filter so what will happen whatever that $m(t)$ variation around DC it will almost take a constant value, okay so $m(t)$ might have some variation right, $m^2(t)$, because I am just taking a very narrow band low pass filter so it will just

take a constant term so at the input of the VCO it should be a constant $\sin(2\theta_e)$ which be coming over here, okay then from the VCO the output goes, fine.

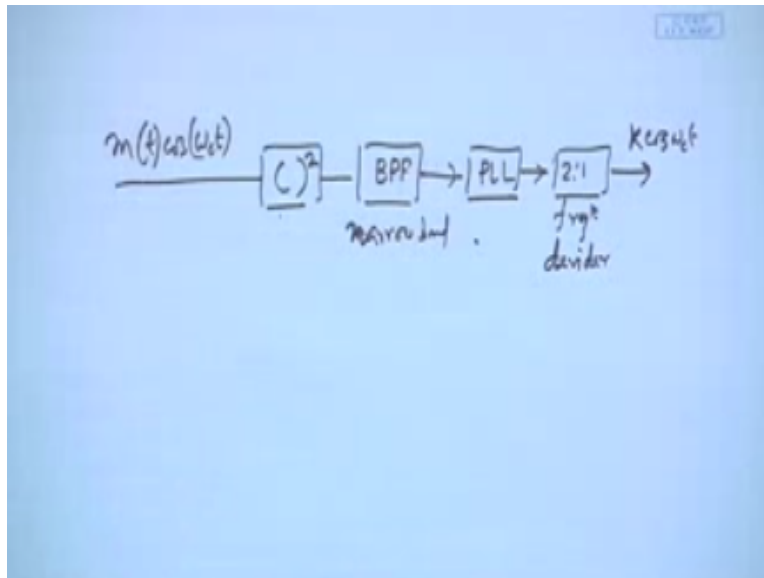
See what will happen, VCO will be keep on running till work time till it gets some input which is 0 right, if it does not get that he will keep on running right, so when it is 0 then what will happen to this $\sin R$ is a constant let us say $\sin(2\theta_e)$ that must be 0, so immediately what happens to the value of θ_e that must be 0 that means it gets a I means the VCO will stop deviating its phase that means this θ_o will stop deviating when this θ_e is becoming 0 that means we have a phase lock in θ_i and θ_e .

And once that is happening what will be the output, let us see that it is the modulated output as you can sorry, demodulated output. Because $m(t) \cos\theta_e$ if θ_e is 0 this should be 1 so I get $m(t)$ back, so this is where I get my output back, so that is the beauty of the costas loop. So costas loop actually is doing everything it is doing this $m^2(t)$ term as you can see this 2 gets multiplied and I get $m^2(t)$ so that part is happening which was the original proposal that you have to square it.

After squaring it is also putting a PLL circuitry because the main part of PLL which is a VCO followed by a loop in a, with a low pass filter with a loop filter that is all happening over here okay. So it has in a way both the things but they are combined in nice way so that it is even producing the modulated output sorry, the demodulated output that $m(t)$ is getting produced whenever there is a locking okay, of course here also you can do the same PLL around DSB-SC you can show that if the phase errors are very large you would not be able to track it so all those things you can do.

But of course we have already done that for PLL this will be just a duplication so we do not want to go into that direction, but what we can now see that a very useful practical circuit can be designed employing both the things the squaring, filtering then PLL okay. And our original proposal just to remind you was something like this.

(Refer Slide Time: 08:32)



We have our $m(t) \cos(\omega_c t)$ you do pass it through a squaring circuit and then put a band pass filter right, this band pass filter must be narrow band like that low pass filter and whatever you get okay, after this you will be getting at $2\omega_c t$ right, you put a PLL which has a free run in frequency around that $2\omega_c t$ then you will get a pure, purely tracked sinusoidal once you get that because this PLL will help you to track even if there are some phase error, so it will also track that once you get that then you do a 2:1 frequency divider okay, you get $k\cos\omega_c t$.

After this you need to put a demodulator so this was the original proposal which has in very nicely taken inside the PLL it means insider sorry, insider the costas loop so this was proposed by Costas and generally all the DSB-SC demodulator follows the costas loop because that is probably integrate the PLL squaring circuit and all this frequency divider and everything those are not required they integrated very nicely and give you the demodulated output finally, okay. So that will almost end our I means discussion of circuit as well as in short we should say system as well as signal for the amplitude modulated things, we have already I think few class back we have already summarized all those modulation technique and we have also talked about I means different possibility of imperfection that come within this we have characterized them in terms of their advantages, detective advantages these advantages in terms of power efficiency in terms of what kind of system we can use it for can it be useful wires or video we will also talked about can it be used for broadcasting or point to point transmission if there are if it is bandwidth efficient that is something we have characterized all those things we have already done.

Now the thing which is left and which is probably the most important part of communication is characterization of channel and the effect say it good or bad mostly bad, bad effect of channel and how you come back that channel that is something which will be I mean sin next few class we will dealing with, okay today we will probably start little bit but it is a channel characterization which is come next and in the channel characterization one of the most important part is noise characterization that in presence of noise.

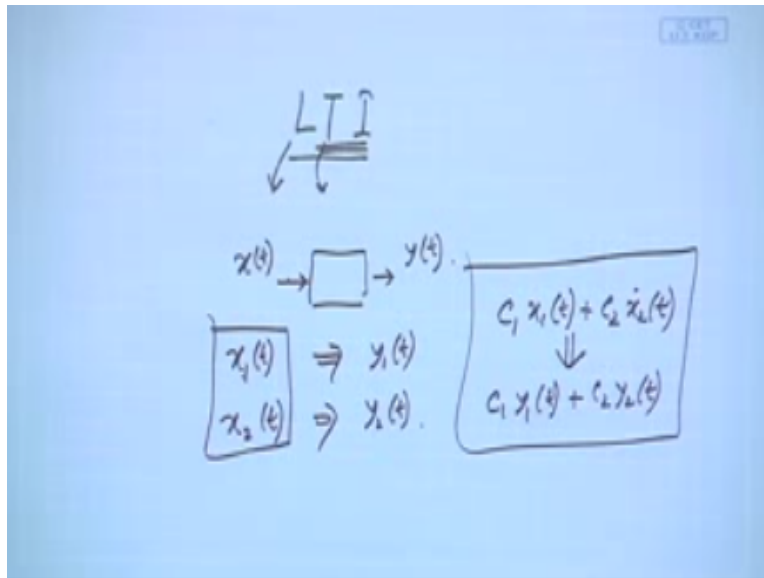
What happens to the modulation system I mean whenever I putted in the channel there will be we have talked about in our some of the previous classes that noise will be present even in transmitter as well as in channel and in the receiver, so that will be there plus channel will have some other imperfections will, which also will be characterizing.

In presence of all this what happens to our means all the modulation technique that we have discussed so far, so we will try to means we have done a comparison but now probably the most important comparison which is coming up is in presence of all this imperfection or impurities of channel who survives best okay, so for that two things will have to do one is we need to understand a system called linear time in variant system we need to understand the characteristics of that, that is one part.

And the second part will we need to understand ransom process because without ransom process or without the basic understanding of ransom process will not be able to characterize noise or even interference okay, so these two things if you wish to characterize we need to have a good understanding of random process. So we will first we will try to talk about the other than noise other imperfections which are little bit easier to deal with and not many will be in detailed deal with in this particular course but we will touch them at least.

And for the noise part we will do our very detailed discussion on random process before going into the analysis of all this systems okay, amplitude modulation to DSB-SC to SSB, VSB and all other systems, so before going into that we will do a very detail regress understanding or regress teaching of random process, so this is something which we do. So today what we will try to do is we will try to discuss about this LTI.

(Refer Slide Time: 13:47)



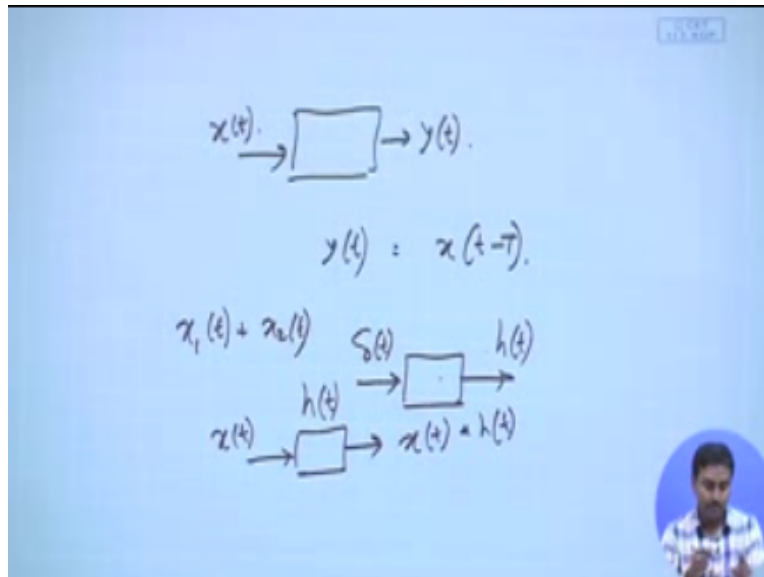
Which is called linear time in variant system, so there are two terms actually one is this many of you are already familiar with this but let us just give a brief over view there are two terms very important terms one is called the linear and the other one is this time in variant TI okay, so we will have to characterize these two things separately what do you mean by linear, linearity in a circuit so a circuit is linear if we say, suppose I give a input to a circuit and I get a output $y(t)$ okay.

Let us say if I give input as $x_1(t)$ I get a output corresponding output as $y_1(t)$ and I give another input $x_2(t)$ and I get a corresponding output of $y_2(t)$. Now what we can say if we give a linear combination of these two input so let us say $c_1x_1(t)+c_2x_2(t)$ a circuit will be linear probably you might have done that in your network theory course. If the output is also similarly means with similar scaling factor linear combination of the corresponding output, so if $x_1(t)$ produce $y_1(t)$ and $x_2(t)$ produce $y_2(t)$ then output of this must be $c_1y_1(t)+c_2y_2(t)$.

Whichever circuit actually provides this functionality we call them linear circuit okay, all those circuitry which are multiplier means or those quadratics sinusoidal will not provide this characteristics okay, it must have a linear relationship that is something which has to be there. So and you can do it for any number of signal okay, so as many linear combination you will take always you will be getting at the output the linear combination of the corresponding output.

So this is something which has to be characteristics of a linear circuit, so whenever the circuitry is like that we call that as a linear circuitry and what is time in variant.

(Refer Slide Time: 16:03)



That means if I give a signal $x(t)$ the output $y(t)$ should be just a replica of the input only with the delay okay, so if this particular signal whatever component it has okay, all those signals if I am talking about a time invariant circuit that means it might at most provide a delay to the input the output must be a delayed version of that nothing else, there should not be any distortion of the overall signal, okay.

So the output characteristics remain the same it just gets delayed as long as the circuit is time invariant. So basically what we can say if I give an input as suppose this $x(t)$ so $y(t)$ must be something which is the delayed version of that, so I can say $x(t-T)$ okay, and there must be some other coefficient that can be there. But time invariant means the signal quality should not be distorted okay, that is the most important part or I can say that every component should be equivalently delayed whichever corresponding component it has, so let us say I have $x_1(t)+x_2(t)$ what will happen if I pass it through it.

So it be if there is a delay it be equivalent delay to both those signals, okay so there will be equivalent delay nothing else. If there is a composite signal and at the output they are different delayed then the composition will actually be distorted it will not keep the similar structure of the input signal. So the circuitry which provides this particular functionality we call them as time in

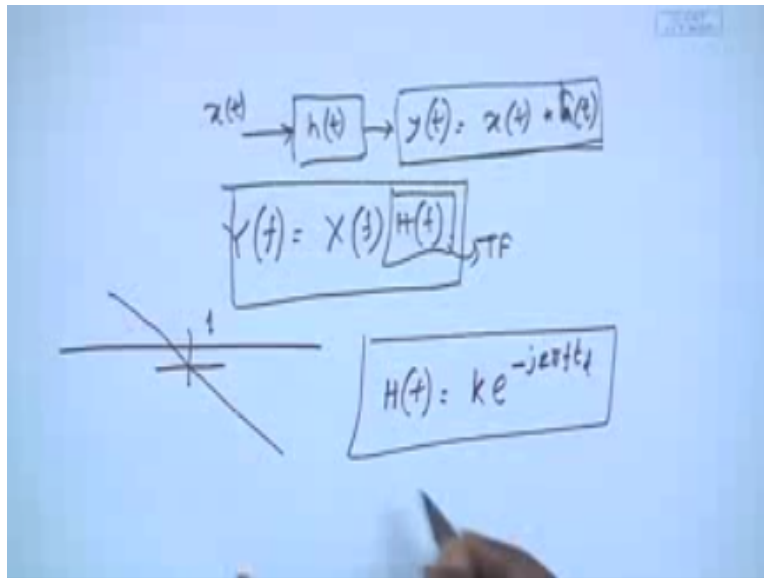
variant as well, okay. So all the circuitry that we will be looking for now onwards are those linear time invariant systems, okay.

So whichever is linear the way we have discussed and time invariant and then from these two properties it can be proven that suppose I have a system okay, now system is characterized by the impulse response of it so we just give a δ over here and of course there must be an output I will be observing, and this is called the impulse response of a system. So if I provide a δ function at the input that data δ function actually and then whatever output I get that is actually the means the impulse response of that system.

And we also know that if it is linear time invariant then it or we should say it can be proven that if I now for this system let us say I put a signal $x(t)$ then what should be the output, output should be the convolution of this impulse response and the input signal this can be just proven from those two properties that if the circuit is linear and if the circuit is time invariant then I will be always able to prove that means I am not doing that prove over here because that is this is not the forum for that.

But we will be always able to prove that it must be this the output should be this, whatever signal you put always the output will be this and the entire circuitry is completely characterized by the impulse response which is this $h(t)$ I do not need any other description. The circuitry is uniquely identified by its impulse response, and corresponding transfer function which is just the Fourier transform of it.

(Refer Slide Time: 20:08)



So basically if I say this is $h(t)$ if I give $x(t)$ I get a output $y(t)$ which is nothing but $x(t)$ convolution of this sorry, $h(t)$ okay, so if I just take a Fourier transform of this part $Y(f)$ is the corresponding Fourier transform of the output that must be convolution in time domain must be multiplication in Fourier domain that is something we have already proven so that must be $X(f)H(f)$ right, so this $H(f)$ is called as the as you all know called the transfer function of this particular thing whatever circuitry you are talking about or whatever system you are talking about.

So this either the $h(t)$ which is the impulse systems or the Fourier transform of this which is called the transfer function of that particular system this particular relationship should always hold for a linear time in variant system or in time domain I should say this particular relationship should always hold and the corresponding impulse response characterizes the whole system this is something for a linear time in variant system I will be always able to tell that, okay.

So now what I will talk about is about a system which is distortion less, so now my target will be to talk about a system which is distortion less, okay. So if I have to have a system which is distortion less so you can now see that this $X(f)$ and this $Y(f)$ must have similar characteristics okay, $X(f)$ sorry $Y(f)$ should exactly equivalent to the $X(f)$ okay, it should almost look like similar thing.

So if this has to happen then what do I first need, that the amplitude of this or amplitude spectrum of this must be 1 because otherwise there is no possibility that $X(f)$ and $Y(f)$ will have identical characteristics, so any other things will create distortion see any other things which are

which might not, might be still linear time invariant like a filter let us say okay, so if I just employ a filter, filter will have some characteristics $H(f)$ will be suppose let us say first order low pass filter.

So it will have some characteristics where some of the frequency element will be means having similar amplitude they will pass some of the frequency element will be surprised and if our signal targeted signal $X(f)$ is not band limited the high frequency term will be at inverted more than the low frequency term, so it is getting distorted at the output, okay so filter always create a distortion right.

So if I need to have distortion less signal at the output so therefore the distortion less things whatever I am characterizing that must be a all pass filter, it should have $H(f)$ which is constant 1 in all frequencies so it just passes all the frequency with equal or I should say no attenuation, right. So that is the amplitude part what should be happening to the phase let us try to see that part, okay. Earlier we have done an exercise, suppose we have at the input we have two sinusoidal okay, for those two sinusoidal if I need to have that signal not getting distorted that means both the sinusoidal should be equivalently delayed okay.

So what should be equivalent phase difference that should be created while passing through or what should be the equivalence phase difference in each of those sinusoidal that will be created by passing through this particular system. We have already seen that it actually linearly scales with the frequency, this is something we have already proven in one of the, so that means we can immediately see if we now start taking more number of sinusoidal so it must be linear whatever the frequency it must be proportional to that frequency, okay, the phase that will be created.

So therefore the phase spectrum should look like a linear thing okay, so it should be always a linear characteristics stretching from $-\infty$ to $+\infty$, okay. What does this mean this thing my $H(f)$ must have an amplitude spectra which is 1 okay, and there should be a phase which is $e^{-j2\pi f t_d}$ and there should be a constant thing right, because it should be linear with respect to this is the overall phase so phase has already f so it this must be constant so let us say some t_d I put. So this must be that $H(f)$ okay.

So whatever I do that must be the $H(f)$ if this is the $H(f)$ let us try to see the corresponding impulse response okay, or instead of 1 I can also write some k okay, so what should be the corresponding impulse response okay, or let us also forget about that let us say I have a $X(f)$.

(Refer Slide Time: 25:56)

$$Y(f) = X(f) k e^{-j 2\pi f t_d}$$

$\xrightarrow{\quad}$ $x(t)$

$y(t) = k x(t - t_d)$

\rightarrow 1

Forget about the impulse response that we will come back later on, so let us say I have a $X(f)$ which is the input that should be passing through a system which is $H(f)$ so that must be $k e^{-j2\pi f t_d}$ this must be my $Y(f)$ so whenever we multiply in a Fourier transform for a k is a constant term, so we do not have to worry about that whenever suppose this $X(f)$ has a corresponding $x(t)$, whenever we multiply with $e^{-j2\pi f t_d}$ what happens in the time domain.

I will be getting a delay okay, of value t_d so this must give me $y(t)$ as $k x(t - t_d)$, if k is 1 this is actually the distortion less signal transmission so if I have a system like that I can immediately corresponding time domain signal I can see that produces a distortion less signal, because it just

the whole signal it delays it that means the signal characteristics is not changing and whole signal either it will not attenuating at anything or if I have a constant attenuation it gives constant attenuation to every time or every frequency component, so it will again not change the overall signal quality.

So what do I, why I am talking about this so that means in the channel whatever I am transferring let us say $m(t) \cos(\omega_c t)$ that is what I am transmission at the receiver what I expect, I expect the same thing but in between there is channel if I characterize this channel as a system I need to ensure that channel is a distortion less channel, it should be linear time in variant of course more than that I should be saying that channel must give me this characteristics, so it must be a distortion less that means channel must be all pass filter which is ideal, it has linear phase spectrum.

If the channel deviates from there now we will be talking about the impairment of channel, if channel deviates either in amplitude spectra or the phase spectra I will have corresponding distortion that I have to deal with at the receiver end. So this is why we started with this thing, once we understand this we know that what kind of imperfection I should be getting and means what should I say for the misfortune of all the communication engineers the channel is not a all pass filter, it is a low pass filter whichever way you see it.

It is a low pass filter it has different amplitude attenuation at different frequency component and the phase is also not always linear. So there will be additional distortion which will be characterizing in the next class that will be happening whenever you pass signal through any channel, so we have to ready to compared those things at the other end, okay or we have be choose our channel carefully, okay where we can almost see this characteristics. So we will try to characterize that in the next class, thank you.