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NPTEL ONLINE CERTIFICTAION COURSE

Course
On
Analog Communication

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Lecture 27: PLL (Contd.)

Okay so in the last class we have discussed about the means characteristics of a PLL and how it can track these and frequency errors, so we have seen that just if I giving a first order filter which is h = 1 within the band of interest and it is a low pass filter of course because it has to reject that higher frequency term so if that we give we could see t hat it tracks frequency very nicely, so basically exactly tracks the frequency in phase it gives a constant phase error okay so this is what we have already observed right. So now what we wish to do is can be better this so our target should be will take a has.

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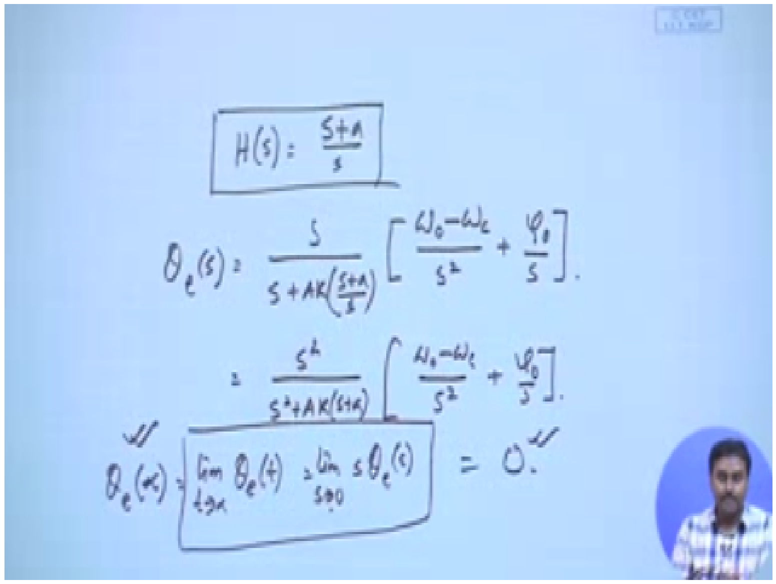

Which is of second order so let say the transfer function is something like this S + A / S so this is just a realistic filter we can design it with any RC circuitry so that is possible so let us say we will take this so immediately what will happen by θes which is nothing but this s / s + Ak hs so implies of hs we can write s + A / S that is my hs into θis which is ω0 – ωc / s2 + ψ0 to s this is something we have already proven, so we have said that it is the similar kind of input which is coming at a different frequency of ω0.

Not exactly maximum with the frequency and it has a constant phase is that is the type of input of course if it has something other than this characteristic will have to do something else, okay so as long as this is something which that happening that where exact frequency I do not know thre is a deviation and there is a phase that phase is whatever that phase is that remains same that is not the random phase okay.

So as long as this is happening we have the error signal as this okay so let us simplify this little bit so we get it should S2 and then S2 A + Ak S + A and we have ω0 – ωC S2 + ψ0 / S okay and most you has learned Laplace transform in network theory you might have learn the final value theorem so instead of doing this suppose I want to evaluate that θe at + ∝ right, so that I can do by evaluating θet and putting limit t turns to ∝ same thing is equivalent to if I just have s domain representation.

Θs multiplied by s limit s turns to 0 so that is the result of final value theorem so will be utilizing base we our target is to evaluate at time turns to ∝ what remains what is the phase error that remains that should be my final phase error, so let us try to evaluate this path so if I just do that you can see that if S turns to 0 I multiplying by S so for this term S2 gets cancelled S turns to 0 this will have some value but I will still have a if I just put that S turns to 0.

This particular part will go towards 0 same thing will be happening over here because here only one S will be cancelled so there is already 1s another S will be coming from here if I put s turns to 0 that part also will go to 0 so this must be = 0 if I put s turns to 0 okay, so μ final value is 0 so now you can see very nicely if I put a second order loop filter immediately frequency has been tracked I know that because the way I have represented already let us in the input for this one the means or the ratio generated.

One will be exactly tracking that ω0 okay no problem in that even in phase also there is no error say tracks the phase exactly accurately you might be are doing there is a some in difference the way we have if I just give you that one of the rd here example, where we were putting the VCO circuit so if you see, whenever my phase error is 0 that is this is exactly tracked okay no problem in that but the problem that will be occurring is this is sin and this is Cos so there will be no matter how well you track the phase and frequency.

So if you exactly track the phase and frequency there will be a ψ2 phase shift so if you wish to use that VCO output as a means demodulator, we have to give another ψ/2 phase shift to that particular thing so this is absolutely mandatory okay that you will have to again from Cos Sin you have to transmitted to sinusoidal or sinusoidal to Cos Sin so we will have to do that because then input whatever phase it has there will be after complete tracking of the loop there will be a ψ/2 phase shift that something we can see.

If this is Sin this is Cos okay so that ψ/2 phase shift has to be somehow irradiative and that can only be done by employing a ψ/2 phase shit again after the easier okay so whatever we see output we get that is completely insynchronized barring a ψ/2 phase shift so now I can employ that ψ/2 so earlier whenever we said there was a phase difference of that ω0 – ωC / AK when we were putting a first order loop with that means that was already there pus there will be another ψ/2 okay.

So that was the overall phase shift whereas here the phase difference has been track completely so you have exactly ψ/ 2 phase shift that has to be somehow taken out from the circuitry whenever you design your VCO right so this is what we have now design now you can just play around with it you can actually show that if suppose whatever that we have written it is this A sin.

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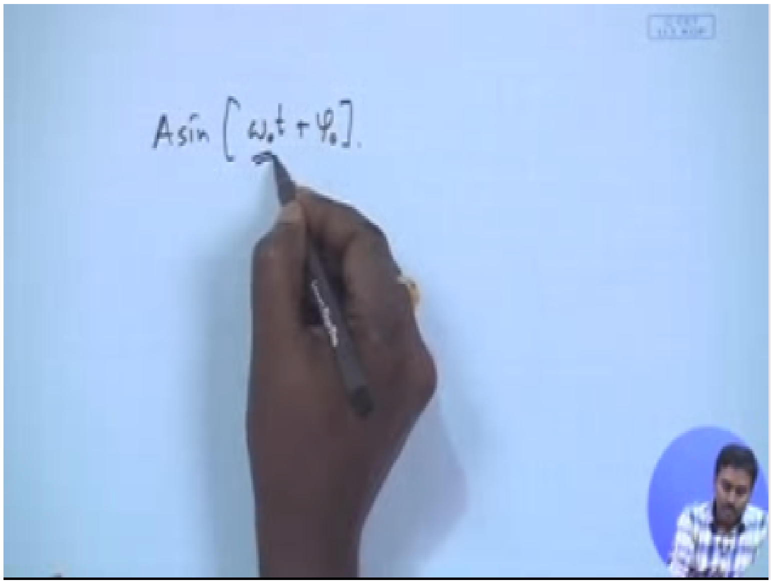

Ω0t + ψ0 we have written this right now if here this ω0 also where is with time let say it is linearly varies with Sin, so of course in modulation that will not be happen here but for some reason for a particular amount of time if that is the case your first order will not be able to when track it, so when frequency tracking will not be happening so that will fail you can immediately prove that you would not get to a final values.

That will still loss in that so you will not be able to track that where as your second order will track that and there will be a constant phase shift, will be able to prove that similar method you just apply employ juts put ω0 as a linear function of time okay so if you just do that will be able to show that second order will actually give you a constant phase shift, whereas if you now go to third order filter so higher the order.

Probably the tracking becomes better and better so if you put a third order filter you will you can prove that it will track the phase and frequency completely so that something which will be happening but remember the center analysis has been done for a small error signal or small phase error signals so if that is not happening you probably and not argue about this outcomes okay so this will be always happening is long as the phase and frequency are almost synchronized that means they are very close to each other.

Juts there is little bit of deviation and we know that even a little bit of deviation can be detrimental for our demodulation so we walk wish to make it completely synchronized that is why this PLL is being employed okay, so now we will try to see that part if there is a deviation okay so let say.

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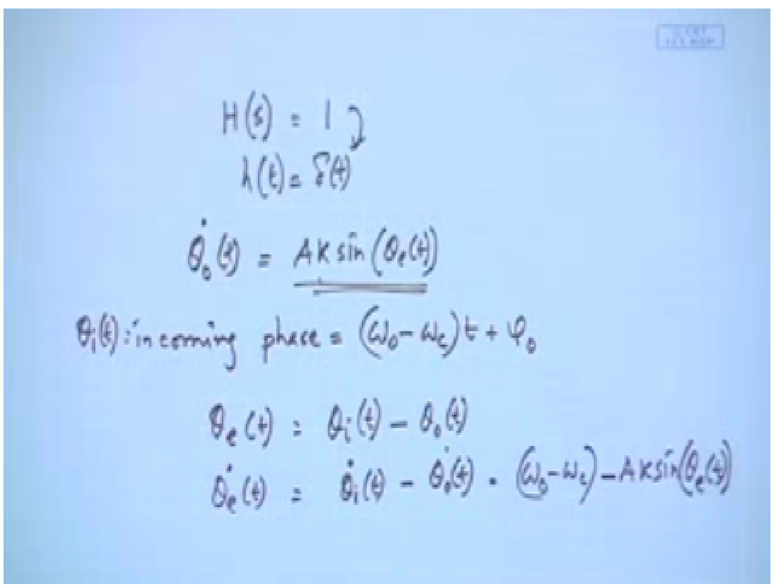

I have a loop filter okay which is equals to 1 okay so loop filter is just that first order filter we will trying to employ okay so this is something you are assuming of course it will have a low pas characteristics but what we can also assume that 2ωC is very high so I can almost assume it to be a band pass within the band of my interest and then accordingly I can write this ht to be a δ

function right because that is just a inverse Fourier transform okay hs is 1 so therefore ht will be just inverse the Laplace transform of this one.

So that should be δt okay so this is what we get now we do not actually put that Sinθ = θ because right now whatever analysis will be doing that is not good for means hold good for this small signal analysis, let us assume this part okay so if this is the case what we can write is we can write that our equation okay so the equation is if we just go back to our filter or is in this previous one okay.

So we just go back to this so there is sinusoidal after that I am just putting Ak hs is becoming one right because that is what I have employed okay if there is a sinusoidal then I cannot really call that as θe I can just write that as if that is a sinusoidal of θ3 multiplied by Ak so that should be filtered over here, so my θ output derivative should be that one okay so I can write immediately that θ output t the derivative of that must be Ak Sin of θet can I write this okay so tht directly comes from that.

Phase module where no approximation is being taken now earlier we were taking an approximation were saying Sin θet is equivalent to θet that was the approximation we were taking now we are not taking any approximation, okay so we just saying according to this particular phase lock loop whatever means from the phase perspective whatever linear circuitry we have or non linear circuitry we have put this should be the case okay so that is why.

So this is the case now I can also write my let say incoming phase like I was writing previously so that should be ω0 – ωC into t plus ψ0 so this is equivalent earlier whatever I was writing that it is coming at ωCt + ψ0 so if I just compare it with the Vc or free running then the phase input phase which is actually θit that must be equal to this we have already done that, okay so this is that phase now let us try to write what is the θe so θe is basically θit – θ over t right.

Now I can derive mean take a derivative over here so immediately θet derivative that becomes θit derivative – θ over t θit I already know the derivative of that should be ω0 – ωC because it is just linear of time and θot derivative I already know this so that must be AK Sinθet fine, so after this there is no problem I can write that write this one in this fashion okay so once I have got this now what I can do because we do not want to know the solving part will be difficult.

Because there is sinusoidal and the derivative of that solving will be little bit difficult but we do not need to solve it we can actually say something about it, so let us try to actually lot θet and means θe derivative with respect to θet let us plot that okay so if I just plot that it is just a shifted sinusoidal.

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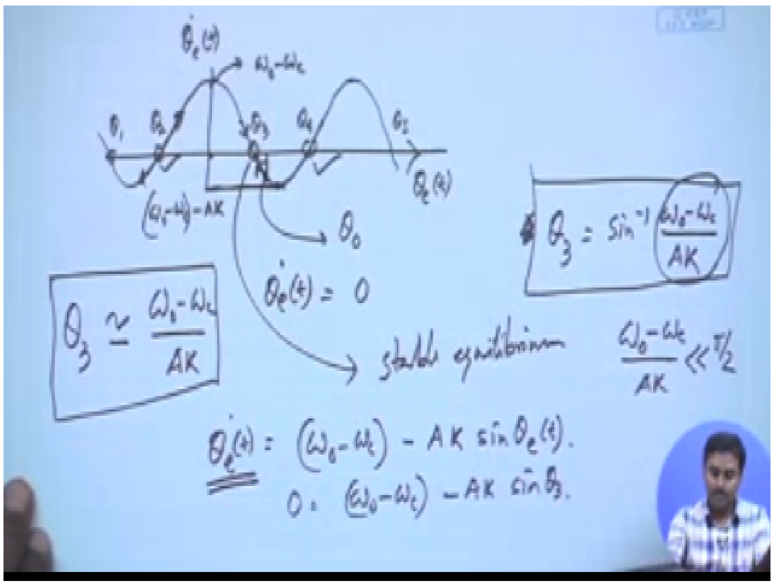

Okay so it will look like this whereas it will be touching at 0 at ω0 – ωC because see this is where θet derivative I am plotting and this where I am plotting θet whenever you put θet = 0 we get θe derivative t as ω0 – ωC so this must be cutting because at that point θet is 0 so this must be cutting at this point okay, and this particular value how much is this that should be ω0 – ωC – AK the maximum of the amplitude right okay now let us try to see where exactly I will gate that phase lock.

Okay or equilibrium so equilibrium means at the input to the VCO I have constant thing it is not changing okay so that means this θt derivative that is equal to 0 because that was the input to the if you see that is the input to the VCO right this was the input to the VOC if that is the input to the VCO if that input is 0, that means there is no point in changing the VCO now VCO is locked so whenever that inputs becomes 0 that is where it will be in equilibrium okay so whatever that is so whenever θe.

Derivative is 0 I know that those are the equilibrium upon now let us try to analyze where those equilibrium points are θe derivative is on the y axis so wherever it crosses this 0 in y those are the equilibrium so this must be let us mark them let say this is actually θ_1 this is θ_2 this is θ_3 this is θ_4 right like this there will be if I just keep on going there should be a θ_5 and so on and θ – 1 – 2 like that there will be many okay.

So there are infinite equilibrium point in this case that something we can see already now we have to see whether this equilibriums are stable, or unstable okay stable means was stable like that is our part because you derivate little bit it will try to come back to that semi equilibrium okay so that is a equilibrium and even if you put a bit with juts derivate it and it will again try to track it back and unstable is.

It is in a equilibrium in part of it, it will go away from that okay so it will go further away again and it will again if you tracking and go away from that is so those are the stable and unstable equilibrium so let us try see that okay so let say around, this there was a initial phase error okay, see whatever happens because I could see that the overall phase trajectory should be on this so it should be moving or along this whatever happens the loop equation actually gets bounded over there so it must be actually moving around that.

Sinusoidal this is what should be have happening so let say initially I have a phase error right, so that phase error let us called that as θ_0 at that point what is the derivative that is negative right so the derivative of that is negative so immediately what will try to do because the derivative is negative it must be trying to lift that phase up okay so that particular part so because the derivative is negative and it will try to negative and it will try to go towards the 0 okay so if that happens.

Then immediately what will happen to the phase it can go up only in this direction okay that derivative can go up only in this direction it is going towards 0 okay so if that happens the phase is actually getting reduced and it is whenever you put a phase over here, it will always try to go towards this same thing will be happening over here if you just put a phase error over there it will actually try to track it back to this equilibrium so that is why I can see that even I am at that equilibrium I give a part.

It will always be coming back to that particular equilibrium so I can immediately call that equilibrium to their stable equilibrium okay so this is happen it to be a right whereas you can also test this or this equilibrium we will see that they just go away once you put over there in those equilibrium if you put a part of bastion okay immediately you will see it will just take it on the other side so it will just take it, away from the equilibrium so you can immediately say that probably.

In this particular equilibrium gets plot so that will be happening so this for me $\theta 3$ is a stable to here okay at $3\theta 3$ what will be happening so I had a guiding equation right so the guiding equation was θe - = $\omega 0 - \omega c$ – AK Sin θet right, at θ this must be 0 that is what we are seeing that $\theta 3$ it is a equilibrium that means the derivative must be 0 so this I can put a 0 so 0 is = $\omega 0 - \omega C$ – AK Sin$\theta 3$ or I can write Sin or $\theta 3$ = Sin inverse $\omega 0 - \omega C$ / AK a very nice thing has happen if you now see what has happened if this is pretty small that means $\omega 0 - \omega C$ / AK is much smaller than $\psi/2$ I can write this $\theta 3$.

Almost approximately equal to $\omega 0 - \omega C$ / AK remember what was a filter I have put it was inches first order filter in first order filter when we did for a small signal analysis the final value means we put the final theorem or we put the at t turns to $\propto$ what will be the phase error that was this exactly we are again when we start putting this particular approximation again that this is much lesser than or phase error is much lesser than $\psi/2$.

Again see that we are getting back to same the so it is all consistent but what we also see that here probably you can tell about more or equilibrium, so what kind of equilibrium can get earlier we were just solving for one equilibrium because we have already saying that it is very closer okay that is much lesser then $\psi/2$ so we are already in this equilibrium and we are trying to solve that okay whereas here if you see we have multiple almost infinite number of equilibrium and we can immediately see that half of them are stable equilibrium and half of them.

Are unstable equilibrium or non stable equilibrium so this is something which is happening over here, so this is the extracting you can say and there also because for any this approximation analysis if you do the general analysis from there if you again could back the approximation probably you should be getting back the same result so that is what we are getting, okay so this characterizes the PLL mean almost in little bit of greater details.

Of course there are other mean non linear analysis of this one but that probably will not be required from this course but here at least we have understood what can happen to this particular PLL when it can track the frequency or phase what are the condition of doing that all those things here also we can also see that see when if you just try to see this curve where it crosses it crosses means what is the minimum point that is actually $\omega_0 - \omega_C - AK$ okay.

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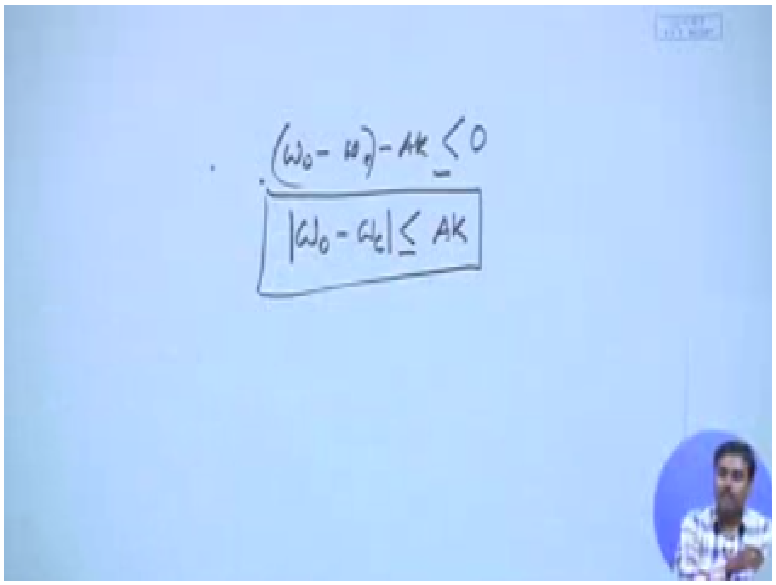

So it is actually $\omega_0 - \omega_C - AK$ right so this is what we are getting so for some reason if this particular sinusoidal goes above the 0 at x okay above the x axis will you have any equilibrium point will not have any point so that means there is a condition on this ω_0 ω_C and AK which are the parameter of incoming whatever phase you are getting or frequency or getting and the PLL phase have frequency as well as some of the parameters of the PLL okay like K so as long as these things.

It is this must be less than 0 okay so if this happening then only will have a tracking of PLL right, otherwise not because otherwise whatever car we have put it is just go above x axis and there will be no actually cutting point where your PLL that θ_e. become 0 is that is not 0 then the PLL VCO input will not be 0 so it will keep on changing the output, right as long as there is something at the input of VCO, VCO will keep on changing the frequencies so there will be no lock that will be achieved through the PLL so to achieve lock in PLL I need to have this sinusoidal coming below 0.

And for that I need this condition or I should say $\omega 0 - \omega C$ must be less than = AK okay or we should actually write a models of that that because it might happen that $\omega C > \omega 0$ so that might also happen whatever happens it should be this okay so that condition should be always satisfied so and that relationship, must be there to get a lock in the PLL so which also says that your frequency should not be too much deviating from the targeted phase lock look frequency that should be the case.

You cannot really hope to track back a particular input sinusoidal phase and frequency when your PLL is deviating sorry the PLL hs VCO and VCO frequency is deviating too much so that is a here design that you know your carrier is that 500 mega probably and you come up with the PLL which has a free learning frequency of 5 mega points it does not make sense, it must be around that 500mh you know that exactly I would not be able to track but it must be around that so I say that I will be creating a 500mh.

But of course there will be a deviation because of manufacturing and all those things so if this is 500.1 this might be 499.9 something like that and then the rest of the things it can track and that is very clear because we need this, otherwise you do not have a tracking okay so this probably ends our discussion on PLL so we have seen the effectiveness if PLL we have seen how to what are the design criteria of PLL.

What are the things we need to do and what are the consequence if we contract it properly that is also something we have already seen it is a very important integrated circuit so which should be studied in depth okay, so what will do in the next class is a we will try to see in a DS BSC demodulation how this PLL and the squaring of the signal which is the overall thing for carrier recovery can be a very nice way implemented so that there is a famous circuit called Cost test loop will try to explain that part.

That actually gives you a practical DS BSC demodulator now just multiplier because formality player we need to assume that there is a complete tracking of phase and frequency so what will be giving is our realistic practical circuitry. So we will try to see how the concept of PLL is almost being employed over there okay thank you.