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NPTEL ONLINE CERTIFICTAION COURSE

Course
On
Analog Communication

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Lecture 26: PLL (Contd.)

Okay so in the last class we have started discussing about PLL and some analysis of PLL right so how good it is tracking we already discussed that very important that we track the incoming frequency and phase well and we were seen that detrimental effect if we cannot track either the phase of frequency specially the frequency is very important because that case modulation term if you just think about SBSC.

So and of course phase is also important because it gives attenuation if we do not track it properly so we have already started doing the analysis of PLL we have already given that basics circuit diagram of PLL means it is we have already told that its probably of operates an feedback loop with a special component that is called VCO right voltage control oscillator.

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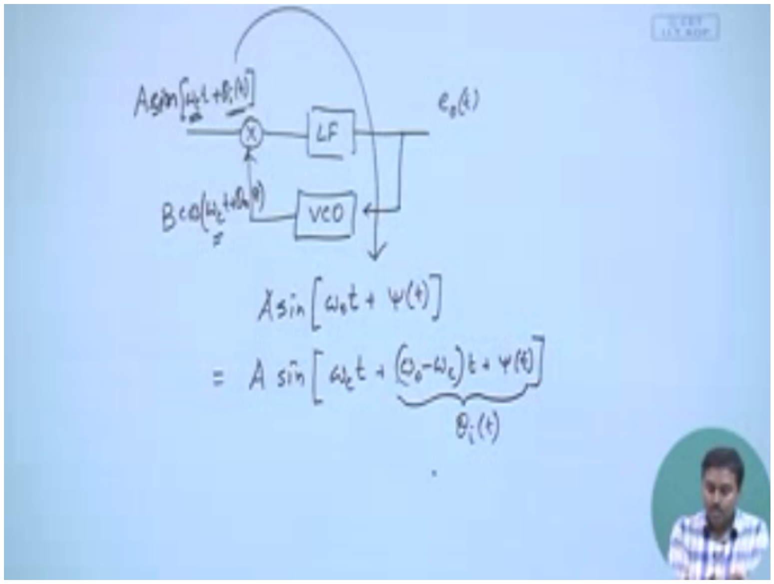

So whatever input voltage you give accordingly it will oscillate okay and the frequency of the oscillation will be a linear response with respect to the input voltage right so that is what we have done and what we have done the basic PLL circuit that we have given was having a multiply followed by loop filter.

And this is a rod signal that was getting generated and then there is a feedback in the feedback loop you have that VCO okay so whatever VCO produces that is being cos of ω_Ct+ ϕ output T oaky and the input signal which is A cos or sin ω_Ct+ θit so this is how we represent it so sinusoidal is coming we generate sinusoidal signal over here from VCO and all we wish to do is this two phase we want to get proper synchronism in this two phase input phase as well as output phase okay.

So one thing we have not probably covered that here if you see we are already assuming where if the incoming frequency is ω_c then the phase of the frequency of the VCO also should be ω_c let say we have already talked frequency of VCO let say that is ω_c okay or Fc, so that is already given and my incoming frequency is not matching to that it might be closed but it is not matching then what happen okay.

Then will this VCO technique has still operate at that point let us try to see that okay so let say I have an incoming signal which is A sin instead of ω_ct that has a different frequency which is that ω_ot+ some input phase let say ψt is coming over okay so this is just a modified one of this one. We have just assuming over here at this loop even no matching in frequency idea is whenever we started analyze it you are saying that in analysis you will assume that the frequencies are matched.

And then there is a time varying phase and we want to match that phase okay so that was our target now we are saying that can be really extend that same analysis for a frequency mismatch case okay so if this is case I can actually rewrite this I can write it this way $\omega_c t + \omega_c 0 - \omega_c t + \psi t$ no problem in doing that right I can always do that and what I can do I can actually take this part and define that has the input phase $\theta_i t$.

So I can always do that because any way this was not a constant thing this was the variable of time so I can add another time varying part to it and I can consider that the whole thing composite thing has $\theta_i t$ and then what happens immediately whatever frequency comes I can actually represent that in terms of the frequency of VCO and then I can accordingly manipulate the input phase okay.

And then tracking will be on that phase only right so I can always frequency mismatch I can actually put that in phase mismatched out and in then do the analysis similarly the way we have been doing it okay so alright after this we has we have now a good understanding that phase and frequency both we can treat similarly what we have done.

We have actually convert it this particular circuit that we have shown with the multiplier because that was actually a non linear circuit equivalent linear circuit where you are treating phase as the input and phase as the VCO generated things okay so immediately we can see that it will be a linear circuit.

Because whenever we multiply there will be a phase+ term and phase – term and then there is a loop filter which is assume to be low pass filter so the + term will be cancelled it just – term which will be there so if phase wise we think about that circuitry it just a input phase- the output phase so that is the linear circuitry right.

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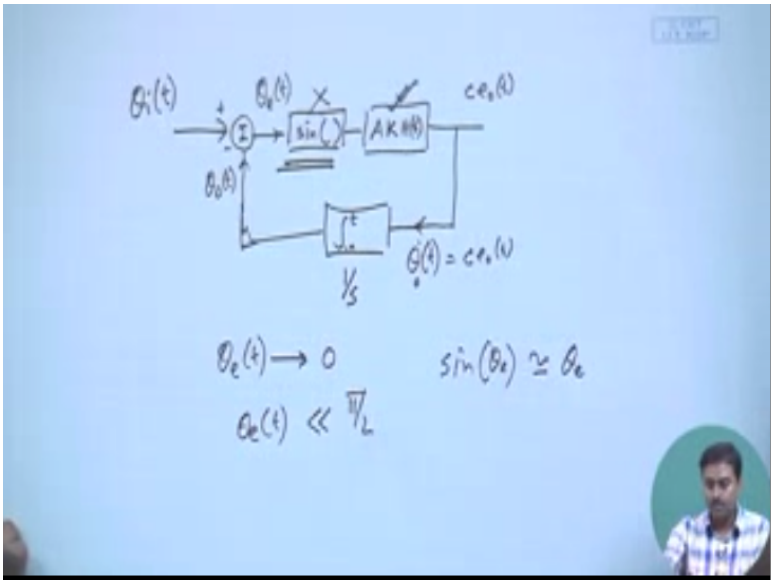

So that is what we have already employed so what we have said we will see it from the phase perspective not an actual signal perspective okay so immediately what happens so this is θ_it so there is θ_it which is coming in and that creates my θ_et after subtraction okay after the difference of course because the original signal was some cos and sin so you multiply there will be cos sin a+b and sin a-b and it implies terms get cancelled so the rest will be sin a-b so if this the phase difference.

There should be of circuitry which converts it into sin right that should be pass to the low pass filters according to our low pass into equation that we have derived in the previous class right so that should pass through the low pass filter and equivalently we have written that should AK and Hs right.

That Hs is a transfer function of the low pass filter and AK are system defined parameter hither it comes from VCO or it comes from the generally comes from the hither signal or VCO right so that is the system generated parameter okay so this should generate the error signal if we are calling as Ce_0t okay and then that gets feeder to the VCO we have also seen if we see from the phase perspective that becomes integrator.

These are the things we have already proven right so it hither – infinity or it is the signals tarts from 0 you can take it from 0 to t so as – infinity so this is just integrated circuit where as the input is actually the output phase the derivative of that to which is equal to Ce_0t right so that gets experiment to this because it is integrated immediately convert it to $\theta_o t$ and then gets feeder over this circuit right.

So this was the equivalence circuit that we have already means transform so this circuit transformation we have already done in the previous class right and that was easier now we wanted to do the analysis of this circuit whether there is stabilisable whether we get this error to the means in a time it will try to see what kind of filter designing will give me this error to 0 okay.

Because that was we want if you want the phase matching then θi should be equivalent to $\theta 0$ that has to happen then θe must be 0 right so if we wish to get that this must be 0 okay so phase must be matched and then this error signal must be 0 this is what we want actually so that is something we wanted to achieve right.

So if that happens then immediately there will be a locking to the input and the output phase okay so for that as you can see this K parameter that was typical to this VCO okay it was having the parameter at B which is the actually if you just see over here that B is the strength of the VCO output signal so that is the typical to VCO.

And then it was also having a parameter called C which is also part of that linear variation with respect to VCO input voltage and frequency that it will generate so this is also parameter typical to the VCO and K is actually half b*c we have proven that okay so that is the typical VCO related parameter okay.

So I cannot really touch that once the VCO is given I cannot touch that A is also something which is coming along with the input signal right I cannot really touch that so what I can touch is this Hs so that loop filter designing has to be done properly so that I can prove that this particular things actually track my input frequency and phase.

That should be an effect now on wards okay so let us try to see how do you do that this entire circuitry if you see if I just convert it into S domain this is fine that should be integration so it should be 1/S right we know that integration equivalent in domain and it should be 1/S that is all fine.

This is alright typical function of S and no problem in that addition no problem in that only problem or the non linear part that is where in the sin of this signal so the sinusoidal of something if you wish to calculate the transform function that will be non linear thing right so the transfer function calculation of transfer function is going to be very difficult.

So that is the only disturbing element now what will do is will try to think that whatever happens that incoming frequency and the corresponding phase is already very close to the one that is generated by the VCO okay so this small assumption I will take and immediately we can say θet is actually very close to 0 okay or it is very small I can just write that.

You can write that θet is much, much less than basically I can say π/2 okay if I write that so basically the error in phase is much smaller in that case the sinusoidal can be approximated it as angle itself so I can write sin θe as almost equivalent to θ once I write that immediately this sinusoidal goes away for small error whenever the error is within a very small amount I know that I can have some analysis will characterize how much small it should be so all those things will be characterized.

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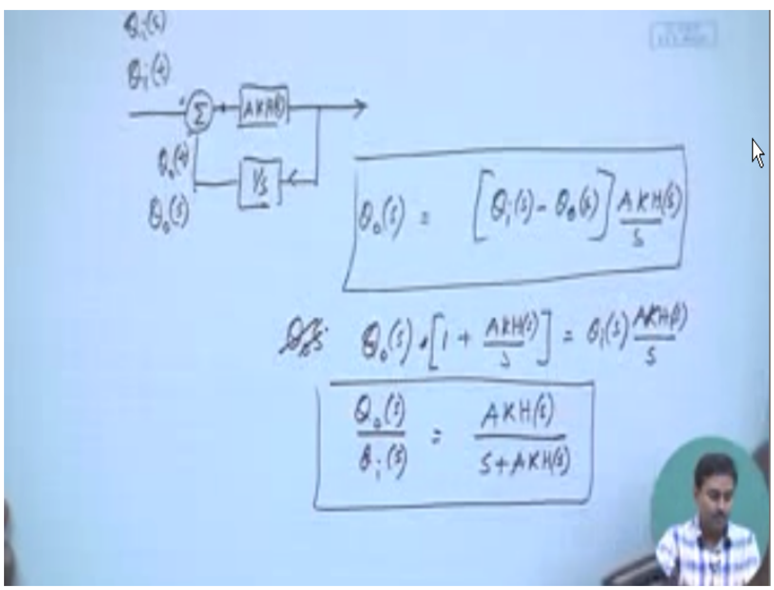

But right now you can see that if it is stage error is much smaller than π/2 we can actually approximate this sin as this signal itself immediately my transfer function of this particular things becomes very simplifier so I have a 1/S over here due to the integration and I have AK Hs over here right fine.

So this is actually my $\theta_i t$ or in S domain I can write it has $\theta_i s$ this is actually $\theta_o t$ or I can write it has $\theta_o s$ right so at this point I get $\theta_i s$ - $\theta_o s$ right this is + this is – so basically what happens $\theta_i s$-$\theta_o s$ right this if I multiply so this is that amount if I multiply by this and this then I get my $\theta_o s$ so I can write $\theta_o s$ is nothing but this into AK Hs/S this is overall equation I can write immediately I get a relationship between my output phase and input phase right.

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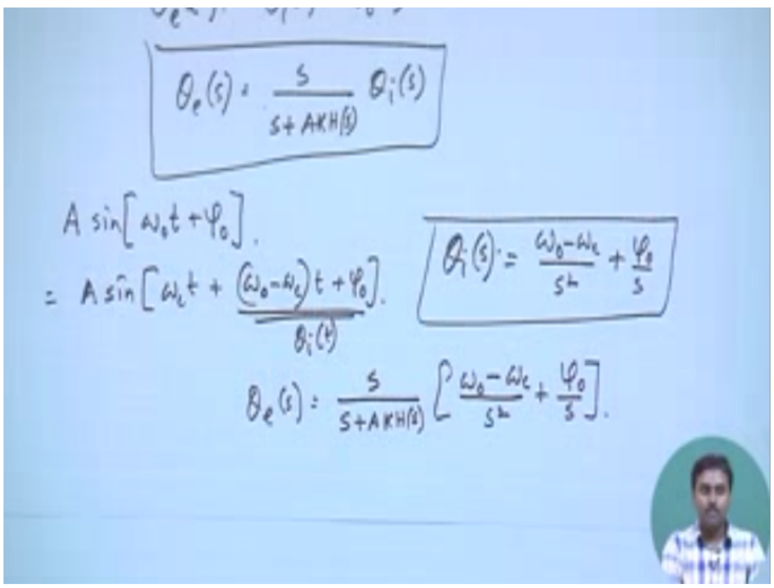

So I can write $\theta_o s$ should be this side AK Hs/S is equal to $\theta_i s$ and AK Hs/S so I can write $\theta_o s$/ $\theta_i s$=AK Hs/S+AK Hs right. That something like this what is $\theta_e s$ that is equal to $\theta_i s$-$\theta_o s$ okay and immediately from there I can get a also has I got a relationship between θ and θi I can get a relationship between θe and θi so immediately I can write this $\theta_e s$ is equal to S/ S+AK Hs that just get manipulation of $\theta_i s$ if just put that over here I will be able to gte this relationship also right so far it is all good but I have to now see is I give my input phase and iw nat to see that my θes in equivalent time domain if I put θt into infinity it should go to 0 oaky.

So that should eb my target or it should go to something whenever it goes that my actually phase error finally I will be getting that the steady state phase error because there might be oscillation and all other things but as θt infinity whatever I get that should be the phase error I will be getting right so because θ_es is actually phase error between the input and the output okay.

So I will try to characterize that one what do I do let us say my incoming signal has a frequency error okay let say it is A sin $\omega_o t+\psi o$ okay so basically what I am trying to do is I have a input signal now which is sinusoidal okay this I might have created by squaring it and then filtering it properly okay.

So which we have already discussed that double side band surface area if we get we have to square it filtering whatever frequency we get we actually make it half ten from ω_c we get ω_c or whatever it is that might be little bit deviated so whatever we do get we are that is actually at means we do not say ω_c so ω_0 let say so we get this right.

So whatever it is we will be getting that now at the VOC, VOC generates that ω_c okay so immediately I have to calculate the effective $\omega_i t$ with respect to that ω_c so I can write this as usual log as previous case I can write this is $\omega_c t+ \omega_0- \omega_c t+\psi 0$ right so that becomes my $\omega_i t$ right so $\omega_c t$, ω_c is already that frequency of the VCO.

So therefore this must to be my input phase okay so this is the input phase what is the corresponding S domain representation okay so $\omega_o- \omega_c t$ is $1/S^2$ right so it should be $\omega_o- \omega_c/S^2$ so it just go through the list of transform will be seeing that when the linear in time that gets $1/S^2$ and a constant will be 1/S right so I can right that has why do we get this right so θ_is is evaluated.

Now let see θ_es from this relationship so immediately I can write θ_es as S/S+AK Hs into θ_is right so that was what we have already written so θ_is I can write over here so that should be $\omega_o-\omega_c/S^2$ $\psi 0/S$ right so we get this now here all other things are unknown only thing that is not known is Hs so let try to put some Hs over here.

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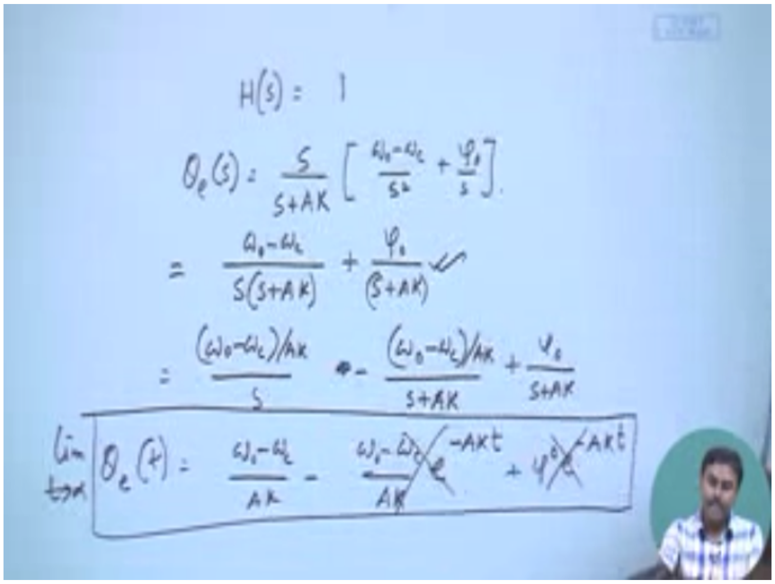

So let say first thing I do is a low pass filters okay so Hs must be one within the band of interest then it should be 0 so I can put Hs within the band of interest should be one okay so may just replies that so my θ_es must be S/S+Hs becomes 1 so it should be AK and I do have ωo-ωc/S^2 +ψ0/S right this I can write so this should be ω-ωc/S*S+AK right +ψo S gets cancelled so it is S+AK or I wish to do is θ_es I have got I want to go to time domain representation.

So inverse transform has to be done so for this I know how to evaluate transform okay which this is inform of 1/S+ some constant this is still not I can do partial fraction evaluation and I can again represent it this way and it turns out to be ω0-ωc/AK/S-ω0-Ωc/AK/S+AK right +ψ0/S+AK this is fine if you just add these two you can see that things will be cancelled and finally get ω0-ωc okay.

So this is the valid representation and immediately if I just wished to do from the list f PLL transform I can evaluate this as whenever it is juts 1/S that becomes that thing already whatever I have so it should be ω0-ωc/AK whenever it is S+AK it should have a exponential term okay it is power minus that into T so it should be ω0-ωc/AK that should be there and there should be term called e^{-AKT} so that should be ψe^{-AKT}.

So I have got my ψe^{-AKT} representation or $\theta_e t$ representation now all are to do is I to see the final value okay whenever I put times to be infinity that should be the steady state response of this particular means circuit right for steady state response all I have to do is I have to now put limit terms to infinity immediately I can see this term will survive.

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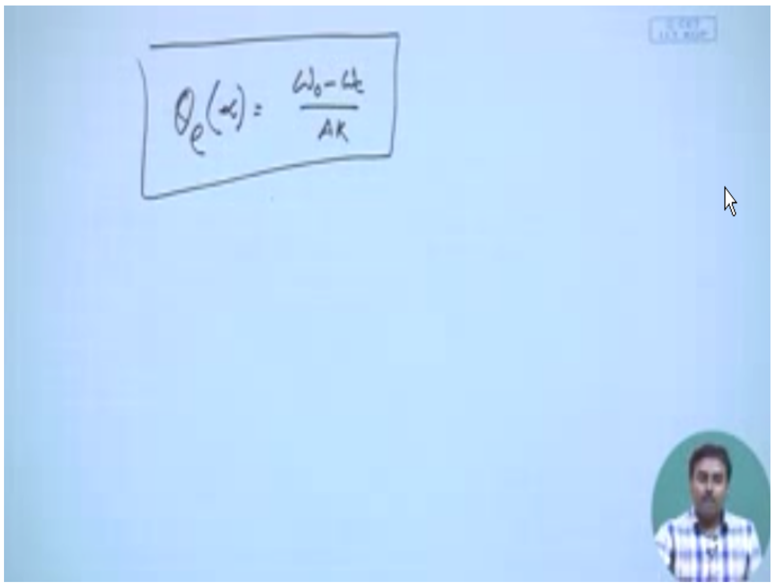

Because that is the constant term this has to go infinity this goes to 0 so this two terms will be cancelled out so what I get is the steady state phase error is called that θ_e at infinity that happens to be $\omega 0-\omega c/AK$ so what is happen I have put a first order low pass filters because Hs was 1 within the band of interest after that it must be a low pass filters so whenever I put that first order characteristics or within the band all pass characteristics.

What I can see that this should be the final phase error so there was a input frequency error right we have $\omega 0$ and the ωc so it can actually track that frequency there was no problem in that it has tracked that frequency where as only there will eb a phase error which si constant pahse error which is related to these parameters okay.

The input frequency the free domain frequency A which is the input amplitude and K which is typical to our phase lock okay or this VCO so that should be the overall pahse error fortunately this is constant right constant in time so will be getting the constant phase error if I just do this what does this says or our DSBSC modulation.

It says very simple thing that there will be phase error and that phase error is constant over time constant phase error so that means I will be getting attenuation in my signal we have seen that so if there si a phase error in the input VCO generated out or VCO whatever VCO generates if there si a phase error then there should be a amplitude degradation or I should say attenuation.

So attenuation should be cos of this term so whatever that phase error cos of that term only there should be a amplitude attenuation so as long as we can said ωo ωc into our known how to design all this okay and we know how much phase will be there if I have that kind of loop filter okay so next what will try to do will try to see that my changing filter characteristics.

We are first part of filter which was Hs equal to 1 if I just change the characteristics of the filter can be really do little bit later can we really take the phase error also out of the picture it is something we can achieve then we know that even that attenuation will not come into pictures so in the next class will try to evaluate that part. If we just give some different kind of filter characteristics can we get something better than this okay so that is our point of discussion thank you.