

NPTEL
NPTEL ONLINE CERTIFICATION COURSE

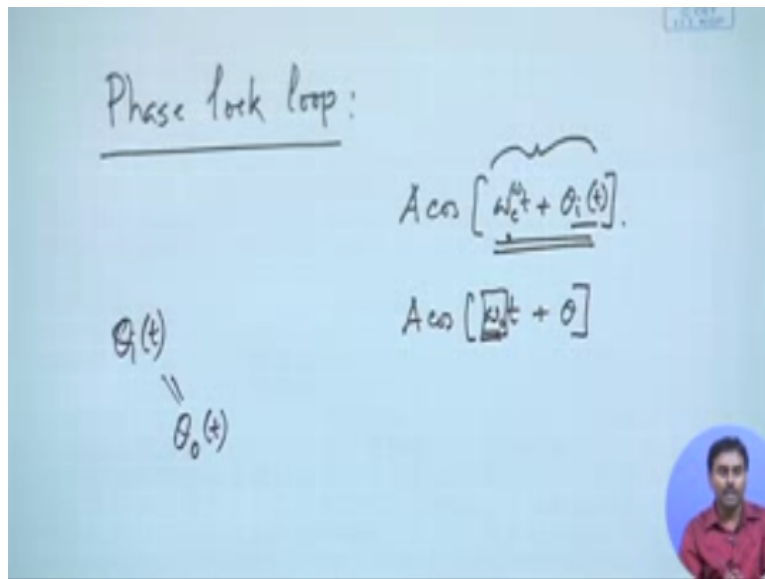
Course
On
Analog Communication

by
Prof. Goutam Das
G S Sanyal School of Telecommunications
Indian Institute of Technology Kharagpur

Lecture 25: PLL

Okay so has promised in the previous class let us start discussing about PLL which is called phase lock loop.

(Refer Slide Time: 00:35)



Okay this is a typically circuit which will be a mostly in many places will be used in communication so it has utilizations in a digital communication has well because there are also you need to do clock recovery for clock recovery PLL's one of the integral part or the topic we are discuss in this PLL that is where it is very important that carrier recovery we have already seen that if I wished to do coherent deduction or coherent demodulation where I need to generate a local carrier we have already seen the effect of that local carrier drift.

So we need to have something where the carrier is properly synchronized and locked to the incoming okay in with respect to frequency and phase okay and we will later on we will also see that this particular circuitry also being employed heavily or FM demodulation we will once we understand the mathematics of or the basic principle of this circuit will then we will discuss about frequency modulation and we will see that how this can very easily used for FM demodulation.

So this is one of the circuit for communication probably one of the most important circuit let us try to discuss about it is essentially a means we are saying phase lock that means we are trying to lock the phase okay so weather it is frequency or phase is suppose I have cos sinusoidal let us say $A \cos \omega_c t + \text{sum input phase}$ it is having right it might be time varying might not be even ω_c might be time varying might not be time varying whatever it is okay.

So we are saying that this is the phase and this is the frequency or over all we can say this is actually the time varying over all phase of that cos sin so this is the phase and we want to lock this overall phase that is our target okay within this there are two constituent part one is the frequency and another one is the phase okay so pure phase so and generally what happens if I have the representation like this where ω_c and θ are no time varying.

That means they are constant with respective time then that separation of frequency and phase comes into picture, okay otherwise it is the overall phase so once both of them are varying with time I do not really I cannot really discriminate what is frequency and what is phase where as if this is the way it is happening then I know that there is one part which is multiplied by t so that is my frequency + there is some constant part that is the phase okay so that is how we give a definition of frequency and phase is separate them extract them out.

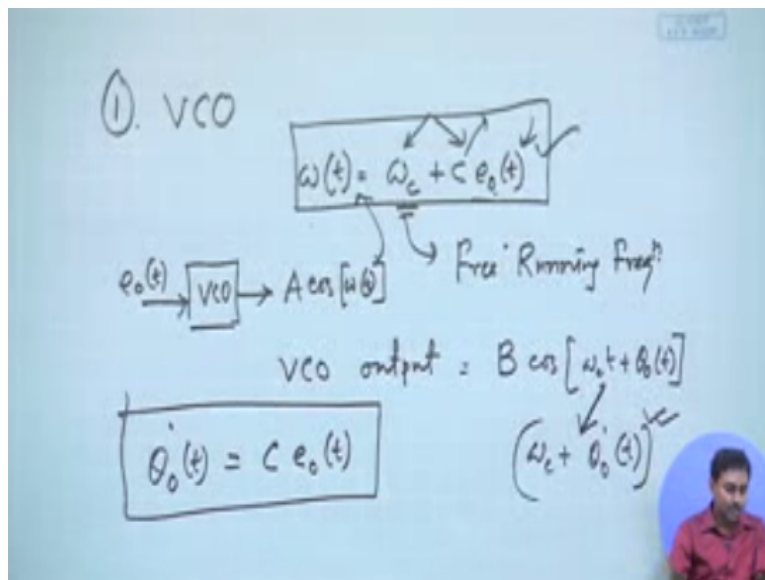
Otherwise in a completely time varying scenario where ω_c and θ are also variables of time I cannot really discriminate between them it is just overall phase of that cos sinusoidal okay which is time varying, so and that is why this particular circuitry is called phase lock loop that means whatever this entire phase is I want to lock it to that okay it might vary with time but my things also should have equivalent variation whatever I will be generating at my local oscillator.

That must have equivalent variation so this is what I want to do, so definitely if I wish to lock something immediately one thing comes into your anybody's mind that is feedback loop, so that

is it is called as a loop circuit okay so we will have a feedback and with that feedback we will try to actually minimize the phase error between the incoming and the outgoing.

So that is the overall principle the principle is very simple I will have a phase which is incoming so let us say I will have cos sinusoidal which is coming with a incoming phase of $\theta_i(t)$ and locally I will generating some phase $\theta_o(t)$ and my target should be that this difference θ_o to θ_i or θ_i to θ_o must go to 0 so I need to create a feedback loop, so that error which is the phase error must go to 0 that can only happen with the feedback. If I try to do that so my circuitry also will look like that it is a feedback circuit, so the circuit is something like this.

(Refer Slide Time: 05:21)



It hence before drawing the circuit we should talk about what or things it consists of 1 it consists of a VCO this is probably the first time you are hearing that is called voltage controlled oscillator this is the typical circuitry where you actually give a out input voltage and proportional to that voltage it will this is the oscillator that is why it is called voltage control oscillator.

So it will create a conciliation, but the frequency of oscillation so it will actually create a sinusoidal wave but the frequency of oscillation which is that ω_c must be proportional to the input voltage that we are giving okay so that is called VCO voltage control oscillator that means it is a topical circuitry where you give a input voltage and you can expect that outside or output will be getting a sinusoidal which has a frequency which is proportional to the input voltage okay.

So typically we write it like this so the $\omega_c t$ okay which is the frequency of oscillation of this voltage control oscillator, so this is the output okay output is actually I should say it is a $\cos \omega_c t$ or ωt okay so it is a cos sinusoidal which as a instances frequency of this one, and this must be sum ω_c constant I will talk about that + some constant into $e_0 t$ where $e_0 t$ is the input of the VCO it gives me some I can take $A \cos \omega t$ right.

Where ωt is this okay so you can immediately see that ωt that is the frequency of that cos sinusoidal which is coming out of VCO that is proportional to $e_0 t$ but there are proportional means it must be linear so this is a linear curve, with two constant which are specific parameter to the VCO one is the ω_c what is ω_c that means if I actually do not give any voltage as input and if I keep the VCO to run freely.

That means I give power VCO is operating so I keep him to run freely then this is the frequency at which he will there okay so this is the output frequency we will generating that is why it I called free running frequency okay so that is the typical parameter to VCO different can have a different standard and different free running frequency so accordingly we will be buying a VCO with your target carrier range right it should be around that your free running frequency should be around that.

So that is the ω_c and c is just a constant which is a by which the $e t$ is multiplied so that is probably that slope of that linear variation so that is also typical to a particular VCO okay so these two parameter will be given for a chosen VCO so what will happen if I have some C sorry some $e_0 t$ input to it will keep on running at different frequency now if I put the VCO in a loop circuit remember my target is this $e_0 t$.

If that is the error voltage let us or that is equivalent to the error phase difference the I will be targeting that error must be zero okay in the loop that is how I will be designing the loop we will see that later on but remember if I but in a feedback loop then this error will be 0 because the error will be generated by a phase difference between the input phase and output phase so that can only be 0 if these two frequency are locked that means the it will not be we the error is 0 VCO will not be a in a loop configuration VCO will not be operating at ω_c because the input error is 0 in a loop it will actually at the frequency.

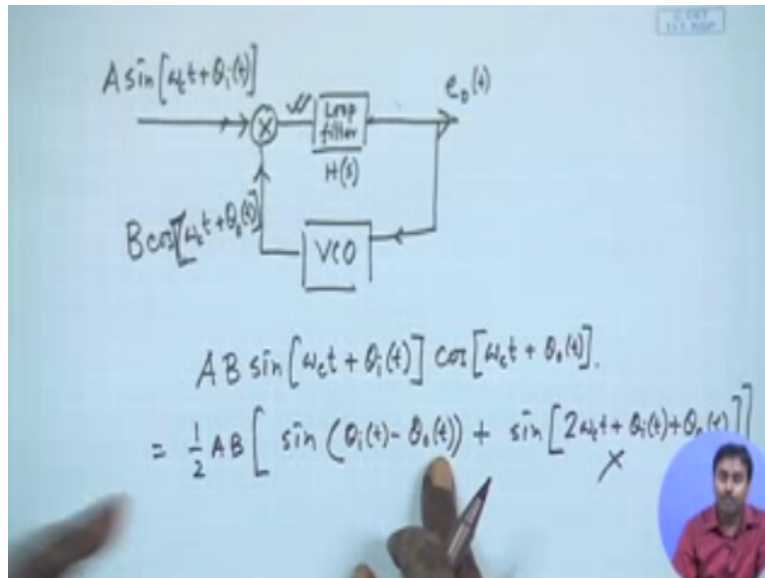
Which is equivalent to the incoming frequency and phase so thing you have to always consider once I write this equation you will be always thinking that as if I put $e^{j\omega t} = 0$ it will be always running at ω_c so that is not the case it is the case if I am not putting it inside the loop it will be it will not be the case if I put inside the loop because inside the loop it will actually at the frequency which is completely in synchronized with the incoming phase and frequency because then only the difference will be 0 and the VCO will not change further.

So it will be located to that so we will see that so I can I can see this now let us say the VCO output so the VCO if it is running at this frequency so what should I get let us say VCO output is according to my target is some $B \cos(\omega_c t + \theta)$ output t okay so this is my target I want a VCO which must run with this frequency and phase okay.

But what is happening if the VCO input is $e^{j\omega t}$ then that must be the ωt okay fine so from here this is a phase so what should be the instantaneous frequency I should differentiate it because when this phase term was constant it was $\omega_c t + \text{some } \theta$ if I differentiate I get the frequency so for instantaneous frequency from the instantaneous phase it is always the differentiation of it so that must be $\omega_c + \text{first derivative of } \theta \text{ w.r.t } t$ so this must be the VCO instantaneous frequency where as I have already said at this must be the VCO instantaneous frequency that ωt okay.

If I give $e^{j\omega t}$ as input single sp two must be same if I put that VCO inside so therefore I can always write that θ output t differentiation must be equal to this inside the loop okay when I put the VCO inside VCO output will be this that is my output phase and I assume that it is running at ω_c okay so if that is case this output correspondingly I will be having a instantaneous frequency sp I calculate that and I have also told that VCO instantaneous frequency must be this so therefore there must be a relationship between the output phase as well as the input voltage. So that is the relationship we were targeting and we have got this. Okay now let us see how do, we use the VCO for PLL.

(Refer Slide Time: 13:05)



So it is like this I will have a signal which is $A \sin \omega_c t + \theta_1$ which is coming okay remember this, this is the carrier I am getting you might be asking okay what where do I get this carrier so that is the first question because the PLL if this is the PLL we are designing so PLL will be at the receiver circuit do not have the carrier, so we have already talked about that squaring and narrow band pass filter passing it through that we get the carrier with some phase.

Whatever carrier phase as carrier frequency or carrier phases this is that so basically before PLL always you will employ that squaring circuit and for SSB and VSB we have told that PLL cannot even work because that squaring does not give me carrier so if I do not get a carrier work here so if I do not get a carrier work here pure carrier work here is your cannot help okay.

So it is very important that after squaring and putting a narrow band pass filter must get a carrier if I am getting that then I can employ receiver technique so VCO will take this carrier whatever it is θ phase and frequency.

And he will have a multiplier circuit okay so in that multiplier what he is doing is giving the output from VCO okay so VCO will generate as we have already discussed that it should be generating these $B \cos(\omega_c t + \theta)$ output t right so that is the signal VCO is generating we have already told that okay.

If the output phase is θ we will have multiplier circuit followed by a low pass nature of loop filter with the transfer function of $H(s)$ whatever it is it is a low pass filter we will see why low pass

filter is requiring and should produce the error signal which should be fade back because it is a feedback circuit to with VCO okay.

So this is all the, let us try to see if this is correct okay this is what we are targeting so what is happening whenever we multiply what do we get we will be getting a multiplication of these two so that should be so I have this and I have this so if I multiply I will getting after multiplication, so it should be $A \times B \sin \omega_c t + \theta_i t$ right that is the incoming signal with incoming phase and then the VCO generated wall.

Every carefully check I have already assumed that probably frequency are same okay that is not a requirement will just after sometimes we will prove that this is not a requirement okay we can always have any frequency that VCO is running at or any input frequency which is different from the VCO free running frequency we can still track it we will prove that but right now we are probably assuming that probably the frequency is already simonized it is the phase I want to track okay.

So if that is a case so I will multiply these two okay so after multiplication it is just half AB so we will have to $\sin A \cos B$ so I can just means put $\sin A + B$ and $A - B$ that formula okay so we will be getting \sin this – this so that should be $\omega_c t$ will be cancelled so I will get $\theta_i t - \theta_o t$ right and then $+$ \sin I will have this $+$ this so that should be $2 \omega_c t + \theta_i t + \theta_o t$ right I will have these two term now you can see after the multiplier this where I will be getting these two.

I want at this point just the phase difference term, so I do not want this what is the producer to reject that just a low pass filter so that is why this must be a low pass filter this will reject this part if it is designed probably so that $2 \omega_c t$ term it cancels out okay so immediately at this point I will be getting just the \sin this difference base difference pass through the filter transfer function, okay.

(Refer Slide Time: 18:16)

$$\begin{aligned}
 H(s) &\Leftrightarrow h(t) \\
 c e_0(t) &= h(t) \times \frac{1}{2} AB \sin[\theta_1(t) - \theta_0(t)] \\
 \dot{\theta}_0(t) &= \frac{cAB}{2} \int_{-\infty}^t h(t-x) \sin[\theta_1(x) - \theta_0(x)] dx \\
 &= AK \int \dots \quad \boxed{\frac{1}{2} cB} = K
 \end{aligned}$$

So let us say the filter transfer function is $H(s)$ and corresponding if I do inverse transform the corresponding impulse response is $h(t)$ so what should I expect at the output of the filter that must be convolution with this one of the sin signal so it must be by $e_0(t)$ which is generated as error that must be equals to that $h(t)$ that is the filter trans impulse response convolution with whatever signal I have got $\frac{1}{2} AB \sin \theta_1 t - \theta_0 t$ right because I know the filter is already a high pass sorry low pass filter.

I have negotiated the other term because that is not required so this must be my overall output $e_0 t$ I can write that as hence I can multiply this by c so I multiply by c what was $c e_0 t$ that was actually $\theta_0 t$ differentiation so I get this equation c of course $AB \frac{1}{2}$ comes out $\int_{-\infty}^t$ so I am just writing that $h(t)$ convolution this so that should be $h(t-x) \sin \theta_1 x - \theta_0 x$ so I have just put the convolution.

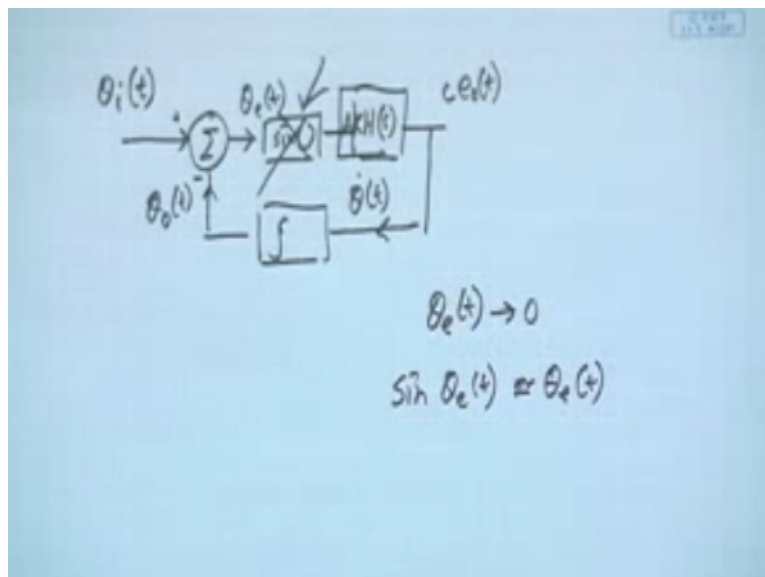
Okay where I will be now putting this $\frac{1}{2} CB$ that is a term completely or defined by the VCO because C is also a term of VCO and it generates amplitude of B so these are terminal means these are all parameter of VCO so therefore $\frac{1}{2} CB$ I can just represent as a parameter of VCO so I can write that as A , so I can have then A and integration this term right where this I can write now as θ_e or tx right.

So far I have done this now let us try to see if I can get a equivalence circuit of this one okay so the equivalence circuit now I wish to define is see earlier what I was having I was having a cos sinusoidal if I just put that circuit this is was the actual circuit of VCO I was putting two cos

sinusoidal I was putting a multiplier the circuit immediately becomes a nonlinear circuit right I have no way to actually analysis it through linear circuitry okay or linear circuitry measure.

So from there I wish to now transform this to a linear circuit right so what I do instead of taking this sinusoidal and co sinusoid I take it from the phase perspective as if my input is phase output is phase only okay so what is do is I transform this circuit.

(Refer Slide Time: 21:38)



As If I am giving a input of $\theta_e(t)$ okay instead of giving that sinusoidal I can just think about that I am the sinusoidal all other parameters are fixed on so that as nothing to do with the circuit A and ω_c those are not going to change it is just this phase which will be actually track and control okay, so this phase as to be probably designed so what I can say is this is the phase which is coming in and then after that I can put a adder circuit.

So this is where the multiplier becomes adder because if I see it from phase terminology it is just phase addition or subtraction okay so what does happen inside if you see I have a sinusoidal and I have a phase subtraction so this subtraction part only I will pick. So what I will say that I have a

output phase which is being generated from the VCO okay so VCO I will put equivalence circuitry okay.

Later on but this is the phase that is being generated I will put + here – here, so that should be the $\theta_e t$ but what I have got inside before the loop filter is sin of that right, so I pass through a sin converter so this is just take that phase and convert it into sin then I get a exactly what should be the input of that loop filter then I put this $H(s)$ but in that $H(s)$ now I should also have something which is if I have just done sin this then sorry I have just convoluted sin with this one this particular term should be part of it.

So what I can do now in $H(s)$ I can include that that is constant term so I can just put instead of $H(s)$ I can put $A k H(s)$ by then this sin will just convert it to $\sin \theta_i - \theta_0$ then $A k$ will be multiplied because either it is the signal or filter it does not matter because this is a constant term $H(s)$ is automatically there whatever I get over here that I can term as my $c e_{\theta} t$ right so that I can term as $c_{\theta} t$ so if this is $c e_{\theta} t$ because that is what we have told that is the error signal so this after convolution I get the error signal $c e_{\theta} t$ so if this is $c e_{\theta} t$ then what should be the output that is just a differentiation of the so this must be θ'

So what should be this circuit then this is a integrator that is the beauty of it so it can immediately see very nicely that non linear circuit if I just visualize it from the phase perspective I can actually transform it into a linear circuit I have added I have integrator in S domain this will be also a treated as linear part $H(s)$ is a realistic filter so no problem in all these things only problematic thing is this sinusoidal so we will now we will see in the next class probably we will try to see how to get rid of this what we will say a very simple assumption will get rid of this.

We will say we are doing a small error analysis that means the VCO as already all most tracked it or generally if I have a carrier signal I will actually give a input which is very close to the carrier because it cannot be that I have a carrier coming at high megahertz and I am just starting from 1 megahertz I will never do that because I know roughly it is 5 megahertz so VCO that I will be putting free running frequency will be already around 5 megahertz just that small error that needs to tracked.

So if that is the case the phase error should be very small so I can say this $\theta_e t$ is very close to 0 if that is the case what can I write about $\sin \theta_e t$ that must be almost equal to $\theta_e t$ that is it I do not need this and immediately I can say at entire circuit becomes just a linear circuit and I can just do

the equivalent transfer function calculation and the whole analysis whether I will be getting a means whether this particular thing should give me tracking and what kind of tracking it will be any track phase any track frequency and all those things so in the next class.

What we will try to do we will do this small signal analysis or small error analysis at try to see what kind of tracking is required only thing that is in my hand VCO it is integrator I cannot touch that all those constant I can change but that will not change the overall transfer function of the overall circuitry now only thing that is in my hand is the filter transfer function $H(s)$ so we will see what kind of filter to be employed and correspondingly what kind of tracking I get in terms of frequency FAS so in the next class we discuss that thank you.