NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

by
Prof. Goutam Das
GS Sanyal School of Telecommunication
Indian Institute of Technology Kharagpur

Lecture 14: PSD of Random Signal

Okay so in the last class we have probably defined what is energy spectral density what is power spectral density and we have devised way to evaluate energy spectral density and power spectral density, so there was simple direct way you directly do the furrier transform of the signal and then take mod square of the you definitely get spectral density power spectral density just you have to do a truncation and then do it okay.

Almost similar method but the problem with this let us try to see where we will have problems so the first part is which is the most important part most of our signals are random in nature that means the signals will not be deterministic so whenever we are doing this furrier transform so far whatever we have talked about it is all deterministic symbol we have exactly defined it over time that it should be at this particular time it should transit from 0 to 1 And remain 1 the come to zero and so on.

So everything was completely deterministic the entire time means definition of the signal was completely explained what will generally happen see the signals are important mostly for a receiver, because the transmitter when they he is transmitting suppose I am trying to transit a voice, my own voice to myself as no information okay it gives me nothing because I already know I have created that voice so that will not give me any extra information.

Generally whenever we transmit signal the signals are important for the receiver because receiver does not what I am going to transmit so for him that carries information and means of course in this course we will not be doing but if you read little bit of information theory it will

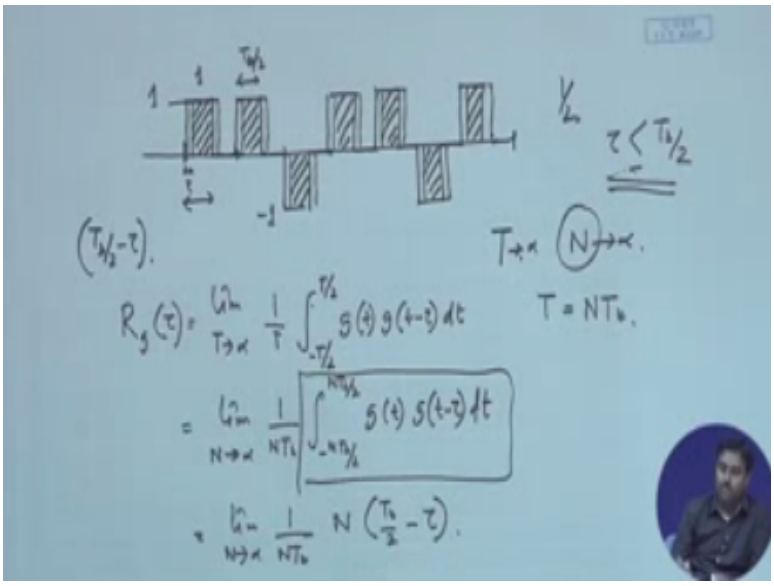
know means more uncertainty you have in the signal more amount of information it is carried which is very simply we can state suppose in the newspaper everyday due see that it is return sun rise in the east okay.

So that is almost know information we do not want that to be newspaper because that is something you know so will that be a news that will be nonsense for you so any information which is coeternity or which will eventually occur and you are aware of that will never give you information, but if you say suppose that means IS as bombed in let us say London okay, that might be very important information for you okay weather important not important it is good bad that is a different thing.

But that is a information because you do not see that happening everyday it is very uncertain to you and that is suddenly happen and that particular thing we will give some information so for a receiver part whichever it is we form communication prospective from general communication like I have given that example of newspaper so all, this if you have some amount of uncertainty or I should say some amount of randomness associated to the signal the only it carries information.

So therefore all real signal if you see receiver perspective it should be random to you it should not before and now there is no point in transmitting a sinusoidal because what does he get extra from that it is just a sinusoidal if we gets a first period he known's that it will be just keep on repeating, so exact pattern of sinusoidal we knowing to him, so there will be no information in first carrying out a sinusoidal if I just say a pulse strain okay so let us say it is like this.

(Refer Slide Time: 04:31)



It becomes one then 0 and it is a pulse strain only so it is always 1 then 0, then 0 to 1 is that a information for him it will never be information for him because he knows form the first pattern if he observes this pattern he knows this pattern will be keep on repeating in time afterwards so there is no information he will not be even interested in looking into that.

So where ever the receiver will be interested that means the signal that receiver will be interested will be the most realistic signal that we wish to communicate and those signals from the receiver perspective will be random so if receiver wish to do some processing on that it must be a random signal which is not expected to him does not it should have any pattern on it should not be a deterministic signal for him.

So mostly whenever we say we are doing all this signal analysis so far we happen doing for deterministic signal only for some purpose of course because we have to first understand that deterministic signal analysis then only we will be able to appreciate the random signal analysis so as long as the signal is deterministic so far wherever we are doing that is not very interesting and we know that whatever tools so far we have devised like let us say furies transform so for a deterministic signal.

There is a existence of fourier transmit we can do fourier because if you wish to do fourier transform that first thing you have to do is we have to take gt it is for – j2 ϕfd and we have to integrate it from - ∝ to + ∝ that means if you wish to get the gf you need to know the exact behavior gt over entire duration of time from - ∝ to + ∝ this is absolutely required so if you

wish to do Fourier transform you need to know then signal completely complete definition of signal is required to be there with you.

But now we are seeing that most of the realistic signals suppose receiver which you know what kind of signal i have what are the frequency component it has but unfortunately the receiver does not know about the signal atom it does not if you know the already the signal pattern we will not be interested in receiving as we have already stated so this is the case then we have a diagonal, can we now in show that or a realistic random signal okay which carry some information.

Where I do know exactly what the pattern will be so the signal is not no long the deterministic will can I take that other path of evaluating energy spectral density or prospect density and I still be able to get the spectrum component all spectrums can I told okay so that will be on next start, so we will just take one example, so very simple example is generally we transmit seen in from here we are just now going towards the just to demonstrate it.

In a easier way we just going towards from analog communication and digital communication okay, so in digital we will be knowing that what we do generally if we have even the digital signal also comes from analog signal how it come. I will tell you so if we have a signal suppose let say this is gt that just let say this is voice okay so what we generally do we first sample this signal.

And then we note the amplitude of those signals and then we note the amplitude of those signals then we also note that what should be the maximum seeing of this signal okay, so we know the maximum of g_{max} for amplitude max that it can go up to if I around a signal for the entire duration and the g mean of the signal if we know this range then we know that the signal whatever sample will be take those samples will be within this.

Okay so once I know this range what I will do I will divide this entire range into smaller, smaller ranges okay as when we have wish so that is how many I will be defining that depends on what kind of things are we want design and then what I do, for each of this samples I want to see which that smaller range it corresponds to and then for every range I give of equivalent digital representation I give a equivalent digital representation okay so what happens suppose I have this length I sub divide this.

Into let say 256 small, small ranges now this number 256 these actually 2^8 so basically I know that if I take 8 bits and if my signals are only binary can take only one or 0 then with that 8 bits I can actually represent 256 different patterns okay so this is already known to you, so basically 0 0 all 80 will be1 pattern 70 last 111 will be this next pattern and so on, we can represent upto 256 such patterns okay so then what will happen where ever this sample stands I can immediately say okay it is in this particular one.

That has a group in the binary representation I will use that code book and I represent that sample with code to like this every sample I will be able to represent in some binary representation and if I just put them serially one after the other I will be getting a binary signal strain, which in a way represent this one and later on when will be talking about sampling or mbccm we will see that it actually faithfully represent the signal so you can actually come there form that binary representation you can actually take out extract that entire signal the quality signal of course there will be some noise associated with it will also discuss about those things which is called noise.

Right now we should not be worried about that part but what we know is that we can for any signal we can have equivalent binary representation or these days computer communication is very famous so in computer communication what happens every key you let say a it has a s key representation and finally a binary representation again, so whatever we are generating over t here that also is directly generating some binary strains so let say those binary streams that is the digital communication actually.

I wish to just transmitted those binary that means I will be transmitting either a high signal or whichever way I represent that a high signal for let say if it is 1 and a low signal is be 0 and I do not know have to represent any other it has some other advantage will discussed that later on why digital communication is better than analog communication so right now we are not bothered about that I just wish to say that okay, this is the valid communication which can also transmit some analog data that is generated like this voice signal that we have generated so it can represent those data also.

So let us say we represent this 1 and 0 in this fashion, so this is my 0 voltage level 1 is represented by a pulse like this of duration T_b where for the first $+T_b/2$ or 1/2 duration it will be having some positive voltage it can be 5 volt or something like that, okay and rest of the time it

returns back to 0, okay so this is called a return to 0 pulse we will later on if we have to do line coding or in digital communication probably we will between line coding then you will be knowing all these things.

But right now it is just the representation so 1 is represented by this and our corresponding 0 is represented let us say so this is 1 and binary 0 is represented by this so it will for first $T_b/2$ or 1/2 cycle it will be negative of that same voltage so if this was +v this will be –v and the rest of the part it will be 0 so this is by representation of 1 and 0.

Now let us think of our random sequence which actually carries information at the receiver so that has this binary I mean random binary schemes that means 1s and 0s are all mixed it can be 1 it be 0 I do not have bad information, so it can be of any way so it can be a suppose first one is 1 then the next one is 1, next one is 0, next one is 1 again another 1 then one 0 so it can be of any pattern you can see that the patterns are broken randomly okay.

So it is not a usual pattern it can be anything there might be a stream of five 1s and then followed by seven 0s whatever it is all there, only thing is that any time you transfer this there is a statistical property associated with it okay, so whenever we talk about random signal there should be a statistical property. So the only statistical property we are just means exacting over here is that it does have all this bits does not have any correlation that means if this is 1 does not have any correlation that the next one should be 1 or 0 okay.

So it can be with equal probability 1 or 0, and also we have equal distribution of 1s and 0 so that means if I see observe this pulse it is almost like tossing a coin if we toss it for enough number of time you will see probably head and tail number of that frequency relation number of heads and number of tails will be almost equivalent here also same thing if I just stretch this pulse up to $-\infty$ to $+\infty$ we will see that as many numbers of 1 will be coming almost similar so there is no biasness towards 1 or 0, okay.

So they are equal probability with probability half right, so these two statistical property is given for this pulse so that means the pulse has a statistical definition or statistical property associated with it when the pulse is no longer deterministic it can have any pattern okay, so this is something I know. Now if I just say can you evaluate the Fourier transform of this not possible

because I do not know it is a ransom pulse I only know about the statistical characteristics that if this one is 1 I know that the next with equal probability might be 1 might be 0.

But for sure I do not know, and also when this how this 1s and 0s are arranged I have no information so if I just say okay, I need to have in the receiver side I need to know what are the frequency component it has for that I need to evaluate the power spectral to density of this one but if I say okay give me the power spectral density will not be able to evaluate because the first stabling block will be you would not be able to do Fourier transform of this particular to see.

So if you cannot Fourier transform you cannot even evaluate mod df^2 if this is g(t) moddf^2 is not evaluated, can I now take the other way I have defined to actually get a Fourier it means power spectral density of this one. So this will be our next target, so for auto correlation function what I need to do of course I can see this is a power signal because it is stretches for -∞ to +∞ okay. I will do truncation and all those things I will do that but it is a power signal.

Because I am not saying that this 1s and 0s are up starting from here and ends over there I am just saying okay, it is a continuous stream of bits which are going on and see the time infinity also you must always argue that why we are doing this, why we are calculating I mean straitening this as a power signal and why we are evaluating power spectral density you might be always asking because any signal actually starts at a finite time and it will end at a finite time no signals are actually if it is not that the signals ready sorry, radio waves that are transmitted from big band and it keeps on coming.

If that is not the signal we know all are the signals man generate a signals if you speak you are not speaking or talking from minus infinity and you will not be you cannot even survive up to plus infinity so it will never happen it will always be time bounded then it should be all energy signal and then we should do energy spectral density only but the counter argument is that the receiver will be seen the signal for a very small duration and within that you wish to know okay what is the power and all those things.

The time duration he is evaluating it compare to that the signal the time duration for which the signal lost is much bigger so in that perspective or in that relative sense you can always say that this is almost like a plus infinity and minus infinity to me because when I am trying the evaluate

the signal probably compare to that the signal existed much bigger time so I can always treat that as if it stretches to minus infinity and plus infinity and I can treat that as a power signal.

So that is why most of the time you will be actually going towards power spectral density rather than in a spectral density okay. So anyway those are the motivation why we are doing this so we wish to so this is a power signal for to us so all we wish to do is we wish to calculate the time auto co relation function right so that is our target. So that means what we have to do we have to shift this signal little bit slightly and then we have to multiply so let us say we have shifted by a part τ okay.

So what will happen? If as long as this τ is less than this tb /2 tb / 2 is the duration or tb is the overall bit duration so as long as it is less than all is than equal to this tb/2 I know that there will be some over lapping and each pulse will have same overlap. Because it is not going beyond that pulse duration so it should be if I just shift by only that τ which is less that tb/ 2 it still have similar overlap at every pulse so this pulse will also have similar overlap so that is the overlap region this is the overlap region and same thing will be happening over here.

That is the overlap region and so on so all shifted by τ right. Now let us say because it is power signal I need to Tran Kate it get it so let us say I have Tran Kate it up to t, and I have chose by for these of calculation I have chose by t in such a way that some n number of pulses exactly will sit within this okay so I will get my t is defined in such a way that I will get exactly suppose N number of pulse okay.

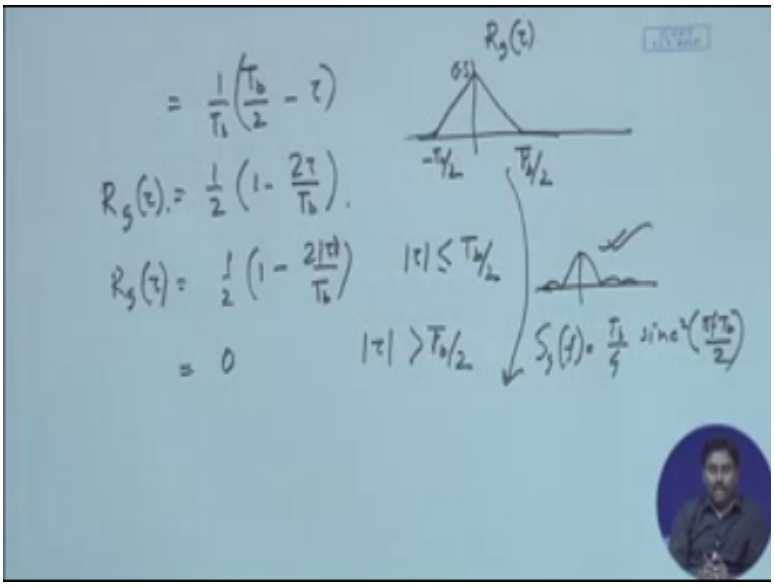
So I am free to chose any t there is no problem in that and we also know as t tens to infinity this n will also go to infinity infinite number of bits will be coming so infinite number of such pulses will be coming okay. So now my auto co relation function the way we have defined it, it should be because it is a power signal so it should be limit takes in to infinity 1/t okay7 and then we have to integrate right from –t/2 to + t/2 and then we will be doing this gt – τ dt right this is what we will be doing.

Now what is my t now t is n x tb because there are exactly n number of pulse so I can just re write this n tens to infinity 1/ because t tens to infinity means N tens to infinity instead t I can write N tb integration this goes –NTb /2 to + NTb/ 2 and then gt gt- τ dt right I can write it this

way, now let us say what is the overlap that we are getting so I have already assume that τ is less than tb/2 right.

So by overlapping area is this was tb / 2 – τ if you take so it must be if the amplitude level is +1 and -1 taken as that so then my overlapping area should be tb/2 – τ that is the overlapping area right because the amplitude is 1 so it will be just multiplied by 1 will get this value all the right. How many such pulses I will be getting N number of so this whole integration I can actually write as n x this area right so I can write limit intends to infinity 1/NTb nx tb /2 – τ I can writer it this way.

(Refer Slide Time: 23:48)



So now what happens n gets cancelled so my limit is no long that required because nothing else is there which is putting limit on N. So therefore I get this = T_b/ 2 – τ and there is 1/ T_b or I can just rearrange it, ½ 1- 2τ/ T_b right, I can write this. So this becomes my R_G τ which is nothing but a linear function, starts from 0.5 and goes to 0 oaky, so goes to 0 at T_b/ 2 and because it is a R_G τ we have already proven that it is a even symmetric function, so it should be exactly like this upto - T_b/ 2 that something we know, so we can write R_G τ should be ½ 1- 2τ/ T_b as long as 1- 2τ ≤ T_b /2 good upto this we have the value.

But what about τ can take any value, if now I wish to take τ > T_b /2, so if I just see this will happen, if τ > T_b /2 this will shifted beyond this pulse. So his corresponding pulse will be shifted

beyond the corresponding pulse. So it will now have overlapping with the next pulse or next to next pulse whatever it is. Whatever happens it is not his own pulse, it will be next pulse. Now there si exactly half probability that will be same.

That means 1 will overlapping with 1 or 0 will overlapping with 0 because the 1 and 0 first of all not correlated and equally probably, so there is exactly half probability or I should say ¼ probability that both will be one, so if I just add them half probability then both will be either 1 or both 0. In both cases over lapping gives me positive value and there si another half probability that they will be actually, one will be overlapping with 0, so that they will create a negative value, or 0 will be overlapping with 1.

There will be creating a negative value, so because there will be infinite number of such pulses, so if I just all those things need to be added okay. So what is happening? Half of them will give me positive values because they are equally likely half of them will give negative value. It will get cancelled and I will get 0 all the time. So this becomes 0 all the time as long as my $\tau > T_b/2$. This is clear because the statically definition of the signal was earlier given. Though the signal was not deterministic but the time I could evaluate because of the signal already that has been given.

So from there I immediately get a time auto correlation function which is $R_G \tau$, now this is the deterministic signal, now I can do transform it is very easy, it becomes because it is a triangular one okay, so it happens to be this where it is actually T_b/ 4,s o you take this and transform will see that, that becomes sin ^{2}/f x /2 and that is the pattern. So immediately you can see that I had random pulse which is going to the receiver.

Now receiver has to know the frequency components because accordingly he can designed these things like he use to put some fit there. You might be asking why the filtering is required there might be signal also, so he as to separate them out not only that, even if there are no other signals you will later on see whenever we will be talking about noise, noise remains everywhere in entire band. If I just do not filter out my signal then if I put all pass filter I will get my signal.

But I also get my entire noise from the entire band, should be the signal, so I wish to reduce the noise then I should put up the filter so that out of noise are surpassed. So for that reason also I need to know where the signal is existed or which frequency band is existed, so for that I need to

first know even if the signal is random I need to know what is the shape of that signal that for a random signal, I cannot imply the step method of energy or prospective.

So I have to imply the second method that I have proven like through, so that chest tells us probably the 2ne method is much more important than the first method because we have already explained that a signal which is important to the receiver must be a random signal, and for the random signal it is very difficult direct Fourier transform. Rather auto correlation if you know some statistical property that is pretty much possible okay. So we will end today class over here, next class we will probably we will start discussing about modulation.