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NPTEL ONLINE CERTIFICTAION COURSE

Course
On
Analog Communication

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Lecture 13: Power Spectral Density

Okay so we have already discussed about for energy signal time auto correlation function and we have also proven that whenever we take the Fourier transform of time autocorrelation function that becomes the energy spectral density we will just try to see we have also discussed in one of the previous class the modulation so let us try to see what happens to means whenever we modulate a signal what happens to the energy of it and the corresponding energy spectral density okay.

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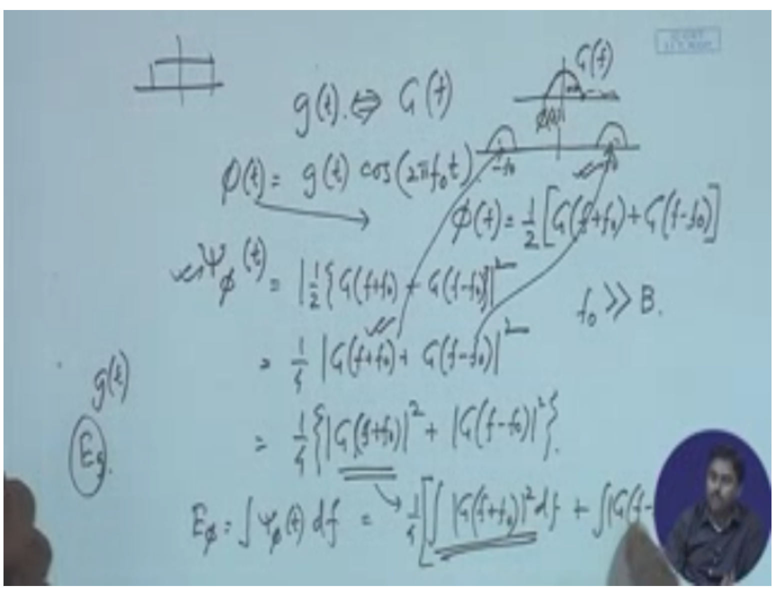

So let us see we have a signal gt which is again a time limited and we want to generate a modulated signal or a frequency shifted signal which is nothing but gt multiplied by cos $2\pi\ F_0T$ so we have seen already that if we multiply by cos and sign it gets frequency shifted in plus and minus as well as and the corresponding signal is called the modulated signal okay so I want to now evaluate the path spectral density of this πt right.

So basically what I wish to do is ψϕt which is the spectral density of this one okay so first of all if you wish to do that there is one technique we have already told that if I can get the Fourier transform of this one that is something we have already evaluated if gt we know the Fourier transform is capital gf then immediately.

We know that πF should be half GF+f0 +GF-F0 this is something we have proven in one of the previous class right so that is the frequency shifting property actually of Fourier transform so if I know this so basically I know the Fourier transform of this one all I have to do is take mode n^2 so that should be modulus of half Gf+F0+ GF-$F0^2$ so that means ¼ right so if I just try to put suppose gf was something like this of course.

What we are saying that the signal is you later on see that if a signal is time-limited then it will not be band width-limited okay so it will have frequency component up to infinity whenever a signal is time limited but what we are saying that probably it has an effective bandwidth that means the other component that it has Hardy on what whatever we have discussed if gt was something like this a time limited signal.

Then gf we have seen it has all the other lobes side lobes okay so we are saying that those are in significant okay so the signal is having effective bandwidth up to this as long as the signal looks like this then if I modulate so that πF will look like at F0 there will be this one at minus F0 there will be this one okay if F0 is sufficiently large compared to its bandwidth effective bandwidth then what will happen.

These two things will not have any overlap in frequency domain okay as long as that is happening so this is actually characterized by this one and this is characterized by this one okay so as long as these two are non-overlapping their square the cross product should be 0 because they are non overlapping so I can always write if that condition is true that F0 is much, much bigger than the bandwidth of the signal okay, effective bandwidth of the signal.

Then I can write this squares must be just $\sum 2^2$ right so that is what we get now if I just talk about suppose gt was having a measurable quantity which is Eg let us say that is the measurable quantity it might be interpreted as minus infinity plus infinity integrated G^2t in time or mod G^2F or mod GF^2 integrated from minus infinity plus infinity okay.

So anyway that is the Eg right so if that is Eg this if I integrate over frequency band so suppose I want to I wish to now calculate the energy of this signal the modulated signal so eπ that must be integration of this energy spectral density over the entire frequency okay so that integration should be one-fourth integration of this one plus integration of right now this integration is just equivalent Eg because it is nothing but the same gf shifted right the same gf shifted if I just integrate mod gf^2 that must give me same thing.

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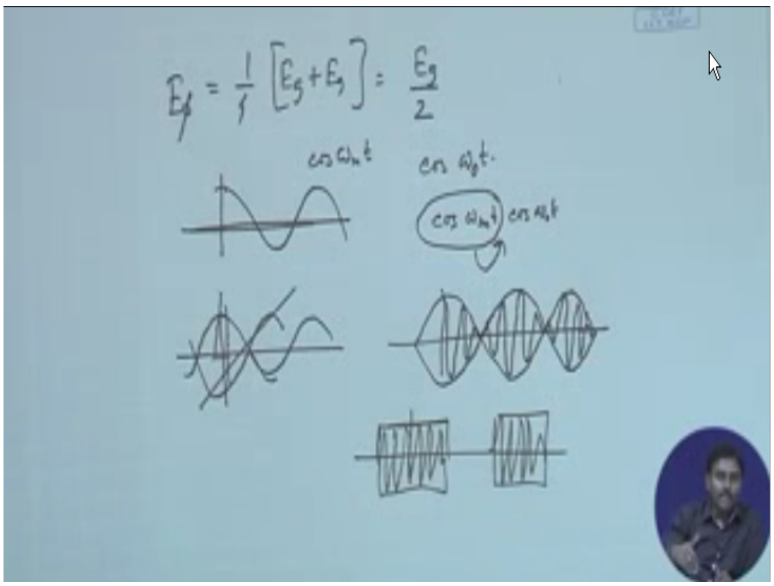

So this must be Eg this must be also Eg so what do I get effectively 1/4 Eg + Eg so that means my energy becomes Eg/2 so that must be the Eϕ so always remember this is going to be very essential for our analysis of any modulation always remember whenever we modulate that means our signal is multiplied by a cost of unit amplitude always the energy gets half due to the process of modulation.

That is a very peculiar thing you might be asking why this is happening I am having a signal gt I am multiplying that signal gt by another co sinusoidal and by the process of this multiplication I am losing energy you know what actually is happening it is very easy to understand so where the energy is getting lost it is because suppose I have a signal let us say that signal is just tone okay so it is just a cos ω empty okay.

So whenever I multiply this signal with another higher frequency signal okay let us say that is cos ω 0T once I multiply what happens basically it will be looking like this cos ω MT into cos ω 0T so this particular term becomes actually the amplitude of this cos ω0 T because cos ω 0 T has higher frequency so means if you see this one varying this will be varying at a very higher rate while this one is varying this will vary at a very low rate.

So therefore actually you can interpret it as if this is varying and the amplitude is slowly varying with respect to this one okay so eventually what will happen it is a cos ω0T only the amplitude will have this envelope so it will look like this let us try it one more time it will look like something like this okay so what is happening now you can see earlier it was a cos ωMT whatever energy it was having or it might be just a square wave whatever it is whatever energy it was having.

Now what is happening that particular part suppose if it was just a pulse train then after modulation it will look like this inside that there will be this sinusoidal again this inside that there will be cosine. So what is happening earlier it was almost keeping that same level of amplitude so the energy if you integrate okay it will be much more whereas here there is oscillation so the energy is getting degraded due to this modulation due to this oscillation okay which is also true.

If you just see the RMS value of cos that is always 1 by root 2 okay so the power if you wish to calculate that is 1/2 so basically whenever you multiply it getting multiplied by 1/2 so whatever it is the reason means there might be multiple reason that you can state but what happens always remember that whenever you multiply with a co sinusoidal or a sinusoidal your energy will be degraded it becomes half of the original signal energy.

That always happens okay so this is just to demonstrate the utility of or how to utilize our energy spectral density whenever we do some modulation or some modification of original signal what happens and how do you evaluate again the energy spectral density so it is just to give an example there might be multiple such example and we can take all of them as homework okay so what now we wish to do is we wish to do the similar thing for a power signal okay.

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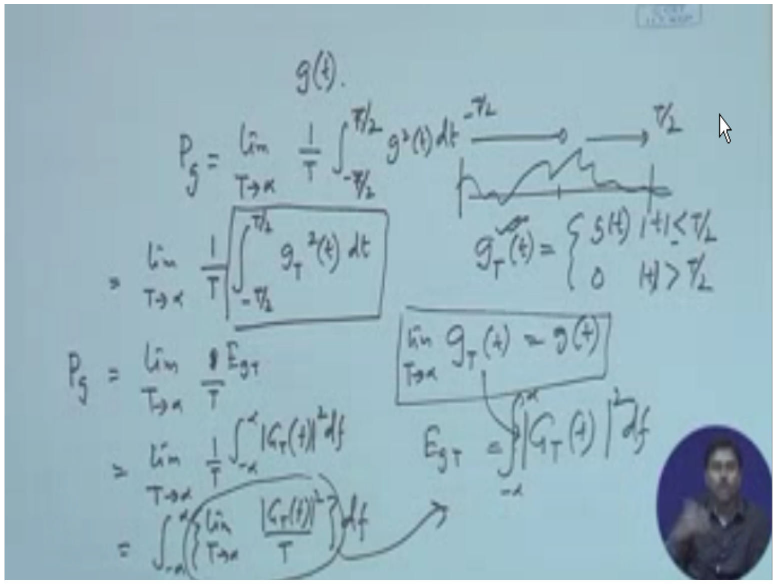

So now let us say we have a power signal gt it might be periodic it might not be periodic whatever it is power signal means first of all it is stretching from minus infinity to plus infinity it has energy that is infinite because it is stretching from minus infinity plus infinity but it has a finite power that means over a means if you just calculate the energy for the entire time duration from minus a few thousand and divide by T you will always get a finite value.

So it has a finite power but infinite energy so that characterizes for power signal or periodic signal or conversation okay so if I have a power signals okay now we have already stated that the power must be calculated like this limit T tends to infinity 1/ T integration minus infinity sorry minus T/2 +T /2 e^2TDT that means I am almost trying to evaluate the energy but divided by type so what I am doing I am taking a valid T or finite T calculating the power.

So calculating the energy dividing by T and then all we are doing is stretching that limit to infinity okay so that was our definition of power earlier we have done that ok if this is real signal if this is a complex signal then this should be mod DT square or VT into D strategy so that same thing so this is all good but now we should do suppose I have a power signal first thing I will do I will truncate that signal so I have suppose a power signal which is stretching from minus infinity plus infinity.

I first define a T and I define a signal gt which is defined as GT if T is or I should say mod T is less than or equal to T/2 so if this is suppose that minus T /2 and that plus T/ 2 so truncated with centering at 0 and we are saying that G TT will be exactly equal to GT so it just carries the, the property of GT from minus T /2 to plus T/ 2 rest of the places it is 0 so that is truncation so that means the whole signal up to T I am just taking rest of the things I am making 0 okay.

So that is my new definition of signal right so what we are trying to do again now you can see means in Fourier transform we keep on doing that earlier we have derived Fourier series from Fourier series we have actually taken it to Fourier transform because Fourier series was defined for power signal or I should say periodic signal then from there we wanted to derive something for time limited signal or the energy signal so we have again use the truncation property here also we are almost doing that earlier we are defined energy spectral density so energy spectral density was defined for a energy signal which is time-limited right.

So now we have got a time means unbounded signal so it is defined from minus infinity to plus infinity so what we are trying to do we wish to employ the same thing again so we are now have truncating the signal and then will put the same trick T tends to infinity right so this truncation if I put T tends to infinity GTT goes to GT so if I just say GTT limit T tends to infinity by definition it should be equal to DT so this particular thing I know but eventually what has happened the GTT that I have defined.

That is actually now energy signal it does it is truncate it so it must have a finite energy as long as how the portion are not going to infinity or how the portion are not blowing up so, so as long as the signal is characterized by that which was also the characteristics of GT so therefore I must always get a energy signal by doing that truncation okay so by definition now my GTT is an energy signal right.

So now this Eg I can define as limit T tends to infinity 1 by t minus T/ 2 to plus T/ 2 see this integration is defined from minus divided to plus T/2 so instead of G^2t I can write also GTT square that will give me same thing these two are equivalent according to my definition because GTT is exactly equal to GT from minus divided 2 to plus T /2π integration was always being done from minus T /2 to plus T/ 2.

So therefore GTT if I integrate or GT if I integrate both will give the same result so that must be Eg okay so if this is the case now you can see for this GTT this particular part because it is a time-limited signal that must be the energy right this is the energy of that signal so I can immediately write let me take in details to infinity 1 by T this is I can write this as EGT that means energy of that truncated signal.

I can write this so Pg is nothing but this thing okay only thing is that was defined for GT now Pg is defined for a truncated signal where I have the energy now the good part is this energy because it is a truncated signal so it is time limited I can actually now relate this to energy spectral density okay so I can immediately write this as limit T tends to infinity okay 1/T I can put this Eg T as if I have this GTT that signal if I take a Fourier transform of that that should be GT and if I take mod of that and if I do our integration over full frequency minus infinity that must be my EGT right this is well-defined.

Because I have now evaluating it for a energy signal so I should be able to write this so immediately I can write instead of this minus infinity plus infinity modulus of GTF^2 DF right so this is something I can always do fine if this is the case now this T has nothing to do with this F right I can take all these things inside so I can write minus infinity plus infinity limit T tends to infinity mod GTF whole square divided by T this whole thing into DF right.

So what I have got eventually the power is now related to this where I have this particular part this I can call as the power spectral density because what is happening now similarly like the energy spectral density I can this is defined as of course is the energy spectral density so it should be a spectral density so this if I integrate over entire if I actually get back my power so that must be defined as power spectral density of my signal.

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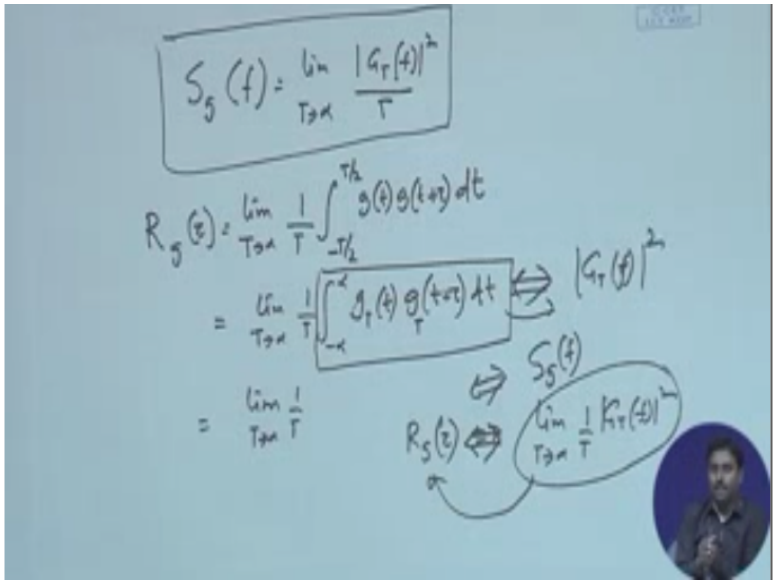

So I can now write that suppose power spectral density is defined as GF so that I can write as limit T tends to infinity mod GTF whole square divided by T right good so this is actually the definition of a possible density so any signal GT if you are willing to define the power spectral density what you do you truncate that over at P you evaluate the Fourier transform get modular square that means the energy spectral density of that and then you put that T tends to infinity that energy spectral density divided by T.

So that must be my energy spectral density by definition we have seen that okay and we have also proven that if I integrate this over F the entire f domain I will be getting my power back right and that is why it is called power spectral density okay again it should be related to autocorrelation function so what I wish to do now is I wish to define the autocorrelation function for a power signal so that is our G.

So what will happen because it is a power signal so just multiplying the signal and it is delayed version and then integrating will give me always infinity because that is very obvious if I just put equal to zero immediately I get G^2 integrated over minus infinity to plus infinity that is already I know already that energy is infinity so there is no point in evaluating that I should be evaluating it in terms of almost similar to power so what I will be doing again I will be doing similar thing I will be putting limit T tends to infinity I take1 by T and then I will integrate minus T/2 to plus T/2 GT okay.

That is now that definition of time autocorrelation function for a power signal okay because for power signal if I do not do this things I will always get infinite value okay so I have avoided this but remember this is just a definition okay why we are defined in this way because we will now see that if I do a Fourier transform of this again similar to the previous one.

We will get back our prospect density so this is what we will be trying to prove now okay so let us try to see how do we define that okay so this is my RG okay I can put this as limit t tends to infinity 1/T so this integration what I can do now is instead of minus T/ 2 to plus T/2 I can put this integration from minus infinity to plus infinity provided I put the truncated signal over here ok instead of GT because it is defined from minus infinity okay.

This integration allows me to do from minus T/2 to plus T/2 so if I just truncate that then I am still limited within that t only okay so therefore I can always take this stretch this integration from minus infinity plus infinity as long as I am applying this so this is good no problem in that now let us see what is this, this is actually for the truncated signal time autocorrelation function which is the energy signals we already know that okay.

So if this is the energy signal then Fourier transform of this one must be the energy spectral density so this entire thing has a Fourier transform pair or I can say they are paired by only this part not limit T tends to infinity and one by T not that part only this part has a Fourier transform pair which is GTF whole square I already know that therefore this RG must have a Fourier transform because this T has nothing to do okay with it.

That with Fourier transform Fourier transform will not have that capital T okay so our G must have a Fourier transform pair which is nothing but this will be as it is there only this part will be added so I will say it is a Fourier transform pair remember it is not equal it is just limit T tends to infinity 1 by T GT f mod square so this part is the Fourier transform this one now what is this we have already proven that that is NCF so therefore our G tau and our defined as Gs our Fourier transform pair.

So we know that for a power signal now if we the way we have defined time autocorrelation function if we do a Fourier transform of that the time autocorrelation function will be always getting that our spectral density right so this is another very important result that we were trying to get so what we have eventually done in these two classes we could actually earlier we have done Fourier transform but Fourier transform was little wait because there was no measurable quantity now from that same part we could actually get a measurable quantity.

And we could relate that in the frequency domain time domain we had that measurable quantity we knew what is the energy or power for corresponding energy signal power signal but in the frequency domain we never had that relationship what we have done first we have proven that okay in the frequency domain also we can get corresponding energy spectral density and power spectral density which is nothing but if it is a energy signal.

Then all you have to do is take the Fourier transform of that signal and then mod that square okay and we could also prove that if you integrate over frequency you get the overall energy for the power signals we just have to do little bit extra we need to truncate that signal do calculate the energy spectrum and then whatever truncation value we have taken we have to divide by that truncation and take limit that truncation parameter T tends to infinity that gives me the spectral density.

We could again prove that if you integrate that we will be getting overall our PG of that signal but then we have proven the tools for calculating energy spectral density and power spectral density one was if we for an energy signal if we do auto correlation function the way we have defined it if we do autocorrelation function we know that once we calculate autocorrelation function immediately if you take Fourier transform we will be getting your energy spectral density.

That is another way of doing it without doing the Fourier transform of the signal directly it is the Fourier transform of the autocorrelation function of course you will be asking why I am exerting myself to first calculate autocorrelation function then take that Auto Collision function anyway again you have to do Fourier transform will clarify that in the next class but that is another way of doing it okay.

So you first evaluate that autocorrelation function the way it has been defined for energy and power signal both similar things only the autocorrelation function definition is little bit different so if you do autocorrelation function of the energy hen you will get corresponding energy spectral density if you do Fourier transform if you do autocorrelation of the path signal.

Then you will get automatically if you do Fourier transform will be getting corresponding or spectral density now the big question is why I need that second path but remember probably that is the most essential part and most important part to evaluate any kind of realistic signal next we will give one example without talking about random process you will probably be able to appreciate why this is so much required.