NPTEL

NPTEL Online Certification Course

Course
On
Analog Communication

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Lecture 12: Energy Spectral Density

So in the last class what we have discussed is how from last few class how come four year series to four year transform that means we have a energy signal from there you know we have hot signal from there we get two the series that is we have already export and then if instead of energy signal hot signal we have a non periodic signal.

That is energy signal which is time bounded for that how do we actually evaluate the components so what we have seen there that whenever hence we have put a trick over there is instead of doing the analyses completely in the new fashion we have actually borrowed the idea from series what we did we have taken a time period bigger than the existences of the signal.

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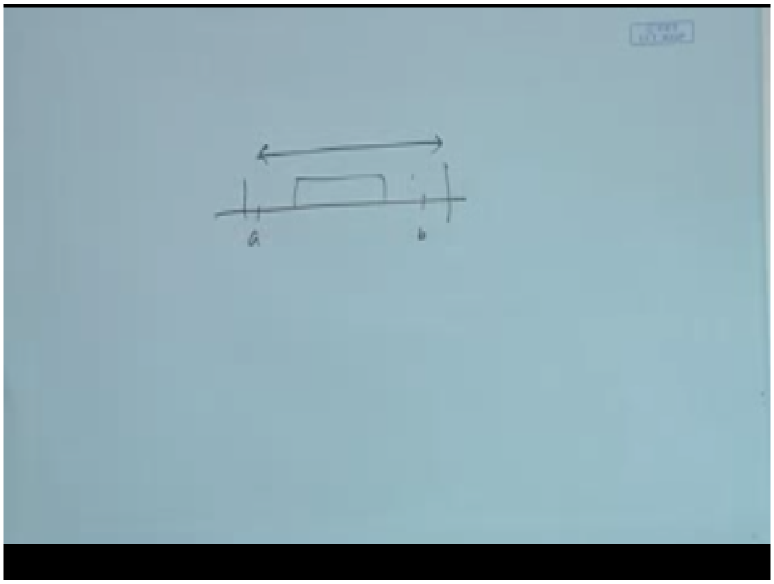

That means if the signal is defined from let say a to b we have taken the time period as the time period which includes the and then we started repeating this as it is So what was happening the targeted signal was repeated over the period t if I say this is t and then we said immediately if I start repeating it up to time processing and then immediately it becomes a again odd signal and so we can get periodic series accordingly.

And then you started sketching this tool so immediately you could see that you are getting of the transform case so we have given in the transform what exactly it means so for a signal GT we get a transform GF what is the meaning of that say that at any frequency f are the value of GF is not actually the spectrum component as it was for a discrete to a series case so we have told that generally gives 0.

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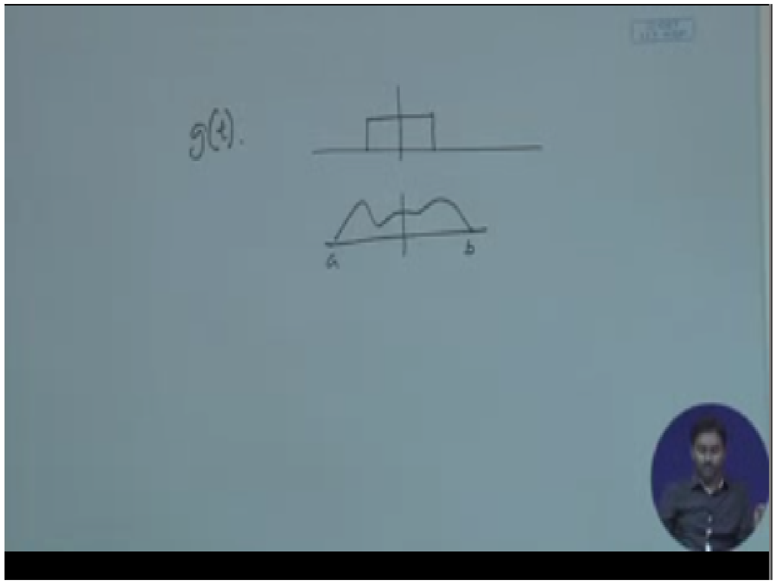

But if I just multiple GF with some ∇f so within this ∇f how much component is there can actually be evaluated so that is why GF is called as respected density because GF ∇F is almost similar to that $D_n = G(F)\ \nabla F/\nabla F = G(f)$ So this is actually the spectrum divided by the principle so it becomes the spectrum density so this is something we have already discussed okay so we have given some introduction about the clear what was that now what we wish to do is now from here to measurement .

We have already define that energy or power is kind of measurement for us it is almost likely vector also which is the distance of the vector which is measurement we have shown that similarly if we just integrate either g^2t or gt^2 we get energy and corresponding if it's the power signal then we can calculate evaluate power just dividing by time stretching the time up to infinity.

So okay so will now try to get into the energy or power of a particular signal so our target is similar to what we have done we also have to evaluated the power of the density that something we have already talked about and we have given the equivalence means power calculation theorem which is called problem so similar theorem will be trying to drive for a time limited series or energy series.

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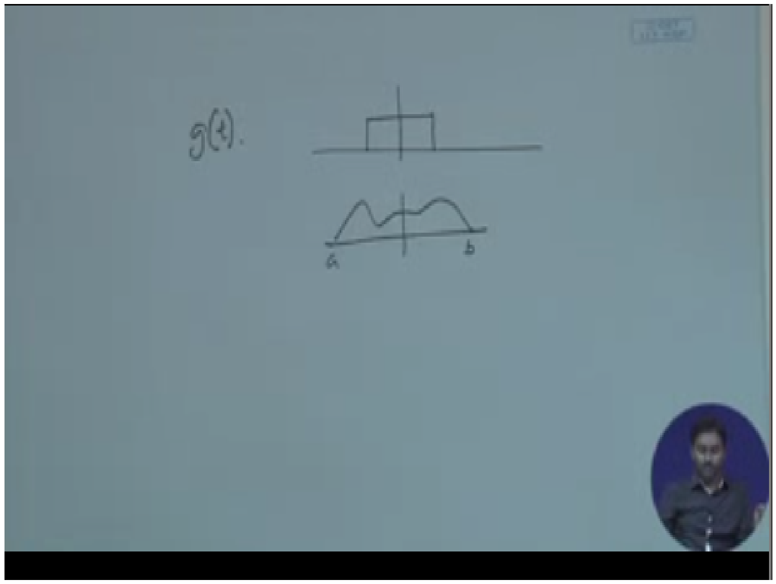

So if I have a signal GT which is time bounded so the signal can be anything and I'm just taking this signal it might be of any nature so it might be of this nature define between some finite amount of time okay so if I just take that GT I wish to now evaluate the energy of this signal so you might be saying okay I wish to evaluate this you might be asking this is a definition already going to define this.

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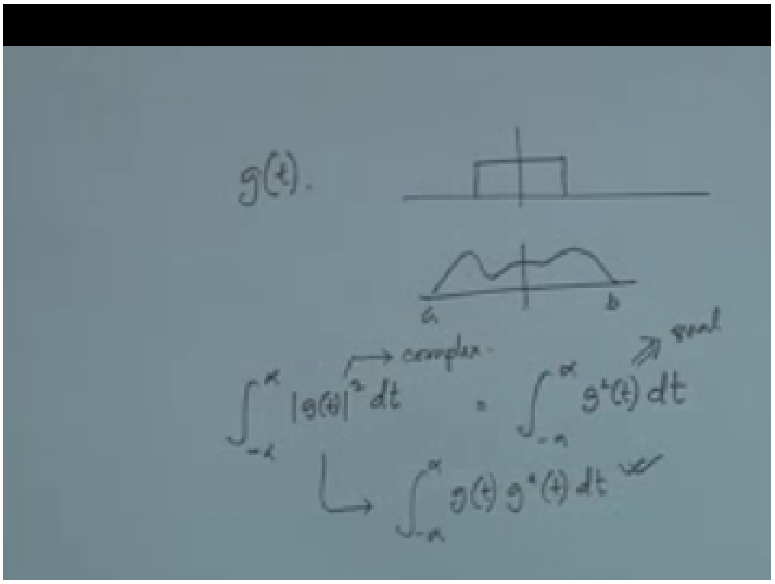

It is very easy to find this particular thing so it is all about integrating gt^2 or if it s real signal then it is just integrating this so this is how our hence this happens the signal is real this happens when this signal is complex right or we can write gt2 as right so what we will do we probably take the more general definition which is the complex signal definition because complex signal already if its real then we start the GT and we get g^2 for that case.

I wish to evaluate this you might be asking this is definition already you have defined this you have Fourier series and Fourier transform we have already defined that this should be the energy right so this is something that was already derived so what we are targeting now like what we have done in possible theorem for Fourier series we wish to also define energy from the sequence this is what we have seen from the time okay.

So GT as a equitant to a transform because GT is time limited signal so it must have a corresponding Fourier transform which is well defined okay so as long as GT as some as we discussed already GF is already defined so I want to see in the frequency domain can we evaluate the power of all energy signal so for this signal because it’s a time limited so it should be a energy signal so we will be interested in energy so let’s try to see if we can define so what we can do is something like this.

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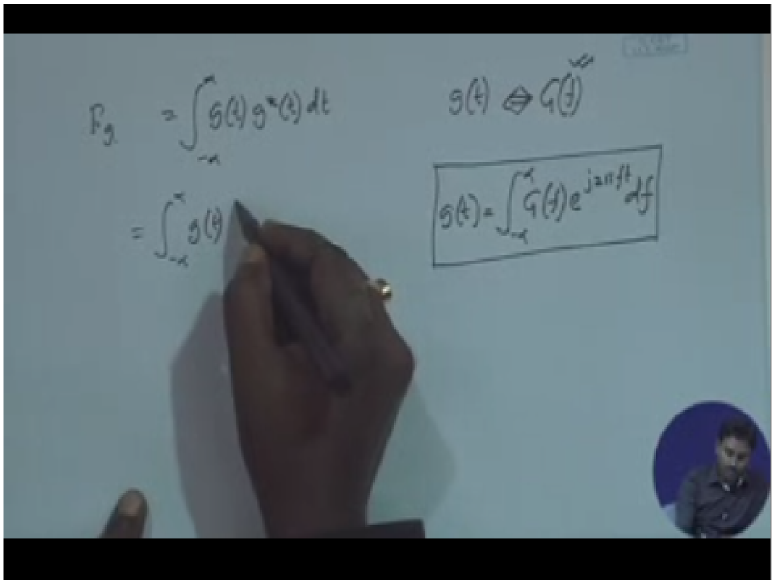

We have already talked about the inverse Fourier transform okay so in that inverse Fourier series you have said GT it is represented as long as GF is known is -∞ to∞ $G(f)e^{j2\pi ft}df$ so that is the theorem we have proven already in Fourier transform so this G start will be replaced by this so it will be one integration of course because we have to take complex congestion because this is GT we need to get g*t will be g*f so I can replace it by that so that will be so no problem in that whatever we have derived.

So far we are just using those equations because we want to take it to frequency domain that why we are applying to get inverse Fourier transform I'm actually representing it through inverse Fourier transform to frequency domain signal but so far I have already succeeded in replacing one of them okay we have to see what happens to this sheet so see this integration limit okay.

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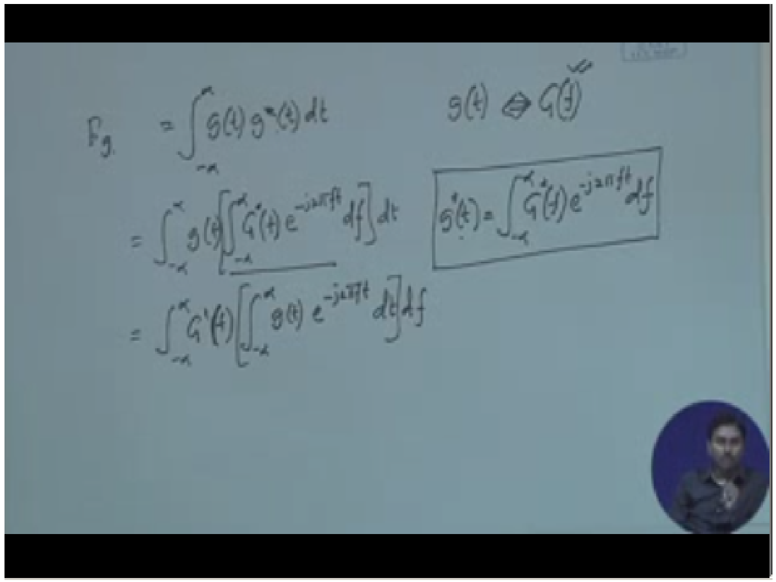

So they are depending on each other I can always spilt these two integration I can take the frequency integration outside so whatever comes which involve all the frequency I can take that outside so g^2f inside the time integration should be there where GT is already there this is also a thing which is depend on time which is taken on time integration so GT integration is done first and then no problem I have just rearranged first and I know how to do that and the limits are depended on each other okay and whichever function are depend on both so I have done all things correctly.

Now let us see what happens over here can you identify this thing it is actually the transform of GT we have already defined that and okay and we have decided and this is Fourier transform of GT therefore this must me GF so that is DF so that is basically what has happened we have started the definition of energy from time roman and eventually to get into frequency domain with same energy so what happens frequency domain is nothing but mode gf^2gf.

So basically in frequency domain we can see also what is happening GF might be complex we have seen that already it might have a face component because we are measuring the real quantity so basically we taking the formula says that I must take the formulas of GF which is real again and whatever I get back so in the frequency domain whatever pattern I'm getting .

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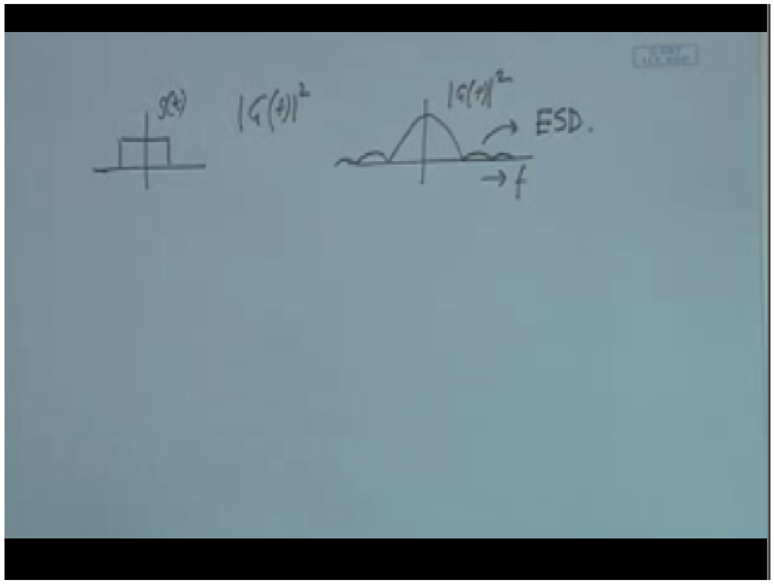

This mode GF^2 if I plot okay so suppose this frequency and I'm plotting this I have a corresponding GF looks like this and I'm plotting more GF^2 So you will later on see if GF is this box function it becomes okay so sinx/x square of that okay so it look like this so n if GT is this I plot this is actually mode GF^2 this I call as energy spectral density or ESD why it is called so because first of all if you integrate it all the frequency component.

If you are actually trying to evaluate something which is I still do not know what it is so if I integrate that I get the overall good and if I suppose try to see what are the frequency component it as or the measurable component it as we try to that what I will be doing I will be passing this through a band path filter or narrow band pass filter or let's say narrow band pass filter so if the filter if you correctly plot okay.

So if I have pass this what will happen I will have the multiplication of these two right so immediately what I will take on the frequency is mode GF will exist because it will just pass and in all other frequency will be 0 and then if I just try to see that suppose the band is over here so what I will get is just the same almost similar happens over here and if I just now integrate what should I except now in those frequency band how much power it has okay so in that DF because of overall power is if I integrate is now I am passing it to filter will be passing those all band it has exactly.

As it is so if I now passing through filter now I wish to calculate the energy that must be those frequency component because those are passed through this filter so that frequency component energy is becoming just the integration of this mode so this is the reason why this is called energy frequency density because what is happening for a particular frequency band energy is there that is character by this energy mode.

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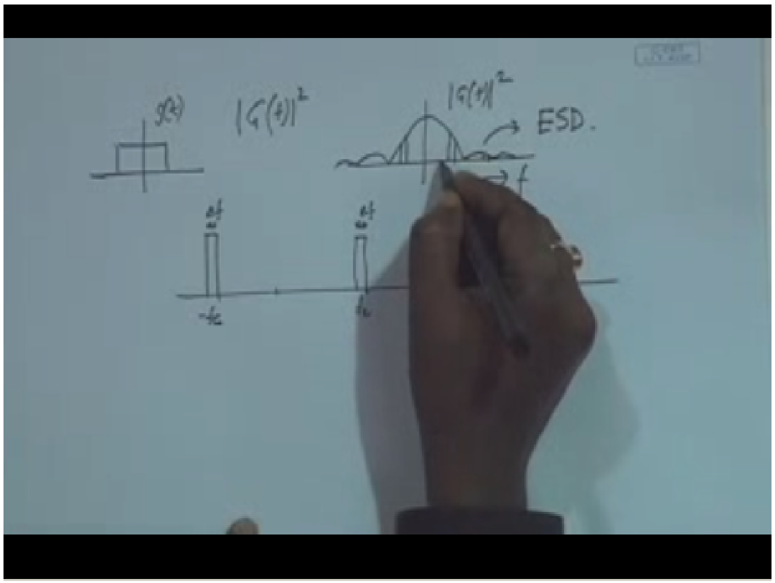

Just target the particular frequency at that frequency just if you take all the frequency you will not get any energy because if you integrate will be 0 right because I don’t have any space to integrate okay it’s just a on a point If I leave some DF I will be deviating carrying some amount of energy and that energy characters this and at that frequency DF is sufficient around that how much energy part unit band it has because it is integration of that

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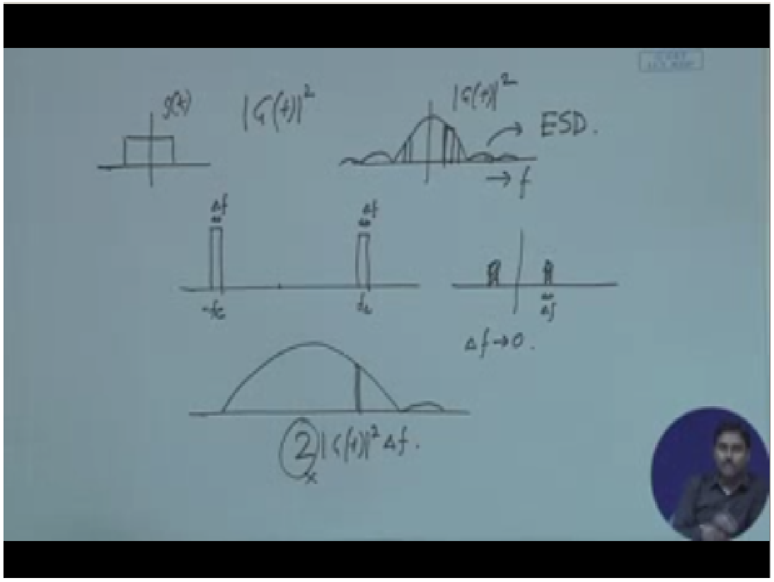

And then the value of input energy is almost flat if this is drawn like this and you can see that it is almost plane if I make by Δf then the overall energy will be almost similar concept because I will getting the same thing both the sides okay if I just ignore this 2 it will be always there which ever band I choose it will always be there so what is happening I divide this and I wait to see unit band because I'm dividing it it's not GF^2 it is actually giving me the energy for a band so that is why it's called energy spectral density because mode GF^2 gives me idea that at any frequency choose a unit and how much energy is heating there in that signal.

So it actually character the what are the signals you have told that this energy is a measurement so it character the overall signal it just says if frequency how much energy or spectral density I should not say spectrum should be spectral density how much spectral density has in that target okay because it character that so that is another way of characterizing the whole okay so that is the importance or significance of this so now some measurement in the frequency domain.

Now we have to separate domain for representing a signal where there is a time domain another one is a frequency domain now I have bought the frequency domain presentation of a measurable point in it okay all energy spectral density see there is something here now we will introduce another thing okay.

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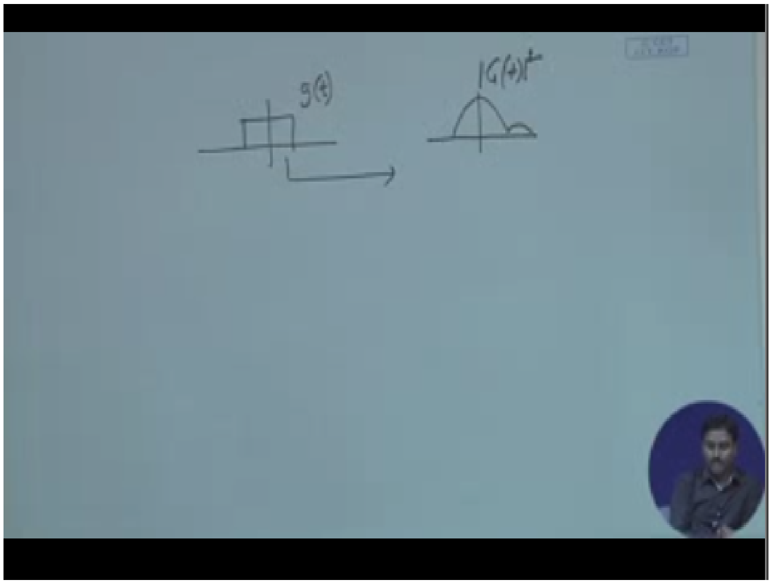

So that is if I have a signal GT how I will get the spectral density the technique is very simple first I will do the Fourier transform so whenever I do a Fourier transform I will be getting and first I will be getting GF and that becomes the energy spectrum this is one way of energy spectral density but I would say most of the time the helpful time not this one try to see the what is the another actually evaluate so for that we try to evaluate so suppose I have a time bounded signal GT I will write this everything okay so this GT if I multiple with either advance from that same signal let us say its advance by τ and if I integrate this so I take a signal.

So it might be this signal if I give this signal and then probably this means actually so it will be shifted backwards so if I just shift it by the some so this is actually τ and then I will multiple GT and this is GT we are saying we are defining this late on big implication so suppose if we are integrating it function of τ for different τ the integration will be different you can see already if I put a τ which is sufficient in large than this particular duration of this signal whereas here it is all 0 it as a whole so it depends on τ so it is a function of τ so this is time auto correlation function first of all let us see some property of it will later on link this two something of all interest and then we will appreciate while we are defining this but let us try to characterize this particular signal so first thing we wish to proof.

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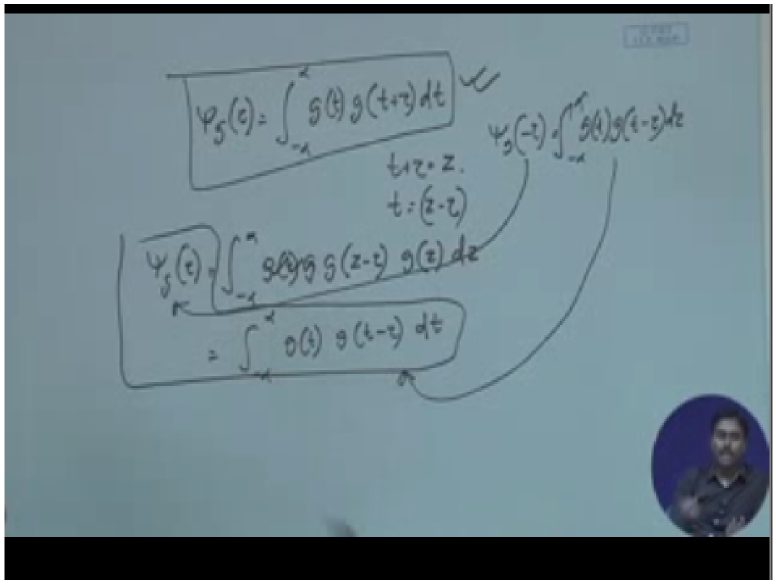

So let us say the definition was okay so now let us substitute as z okay so what will happen to t it should be taken by –τ right and DT will be τ is constant so now we have to do the substitution so t will become tz-τ and t+τ will become dz so z is just a variable just think about this what is happening if this is actually if this equal to this because the negative of τ if I replace by – τ/+ τ so it is even symmetric function right okay next the most important thing so now what I wish to do is I wish to Fourier transform of this one you will be very clear after sometime why im doing this okay so right now we will read this.

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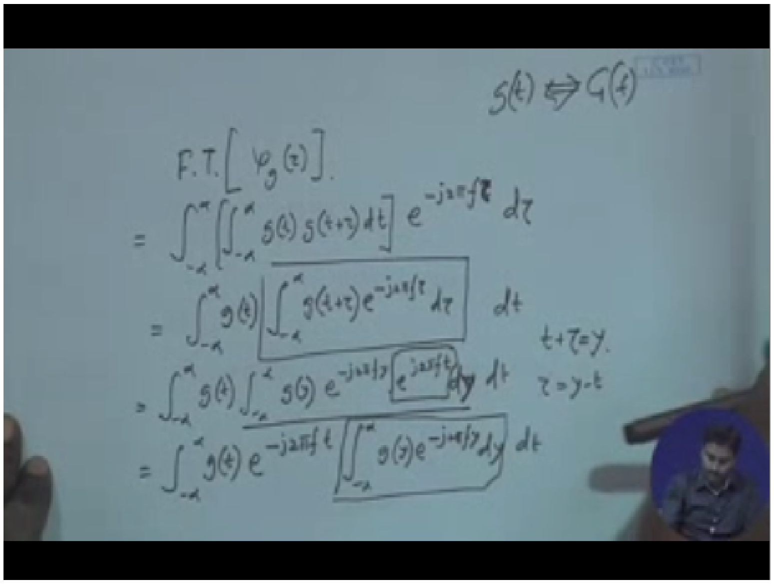

Now we will do Fourier transform of this particular signal right 5g τ of course when I'm saying my Fourier transform is actually transformation from my Fourier transform is actually from τ domain some other domain search that longer that domain the Fourier transform is τ and the independent variable is τ so Fourier transform should be on τ Fourier transform means I have to put the function.

So which is nothing but signal that has to be Fourier transform we will try to see if we can evaluate this I will do a change of this integration so I can keep the t integration out so t is the integration for constant so I should not bother about t I have just replayed τ if the particular now let's see this is the constant depending on but I have constant that should be replayed so now I have brought g(t) or any variable.

So that must be so I will be getting it this terms GT is the function it as started with auto correlated function because I knew that auto correlation function I will be getting back by density so this is that is why we were telling that of defining energy spectrum density first instead on taking we will first take the signal and you will take the auto correlation immediately you get back.

The auto correlation later on we will see it is actually of course we have done that only on the time auto collision this as theorem as when we study about the random process we will see that if it is actually the famous theorem actually we have done that for only for time correlation this is as higher we have random process will see that related to that but right now we should be satisfied with that if you take signal.

And a single define signal instead of taking directly from the transform it is just you take first of defining energy spectrum instead of talking transform and this is another way of defining energy spectral density it is just you take first instead of taking directly Fourier transform we will see why this is required you take first to the signal take the auto correlation of that okay and then you do auto correlation transform.

And immediately you will get back your auto correlation later on we will see this is actually the famous winner relationship of course we are done that for time correlation as theorem as higher education when we study about random process we will see that related to it so this is right now we should be satisfied with if you just take a signal single signal don't think about randomness in it a single defined signal if I take a time auto correlation I will do a Fourier transform on that
And I get back my energy spectral density the way we have defined what is energy spectral density so now with time auto correlation function.

We are getting back energy spectrum density what we will do next is this was done for a energy spectrum we will do the same thing on that signal and then we will try to interrupt why this is so much important okay why we have to take this D2 apart where we have take evaluate this time auto correlation function and then try to take Fourier transform give a simple example where you will see that Fourier transform will try to give a simple example where you will see that Fourier transform is not good enough just directly taking you to transform.