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Course
On
Analog Communication

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Lecture 11: Fourier Transform (Contd.)

Okay so we have discussed about this frequency shifting property right.

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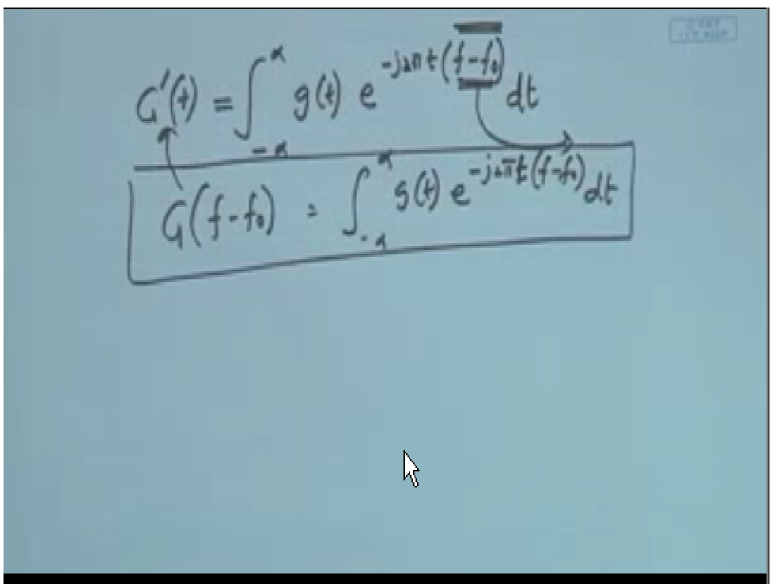

So that was the end result that we have got in the last class so basically what we have seen that if I have a particular signal GT.

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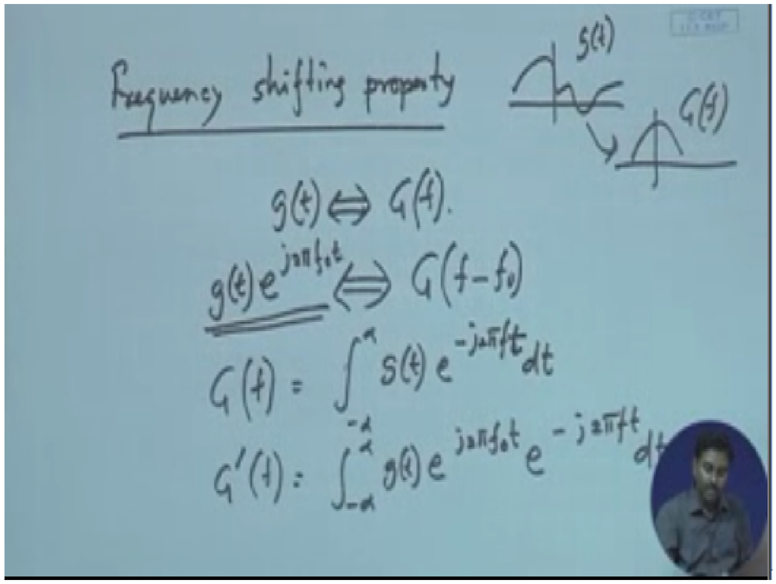

And I know the Fourier pair of that if I multiply that signal with e to the power $J2\pi F_0$ T I get our frequency shifting or frequency translation by an amount F_0T whichever exponential I am multiplying by okay so basically what happens the shape remains the same it just gets translated so now let us try to see what is the implication of this thing okay the biggest implication is suppose I have a signal G (t).

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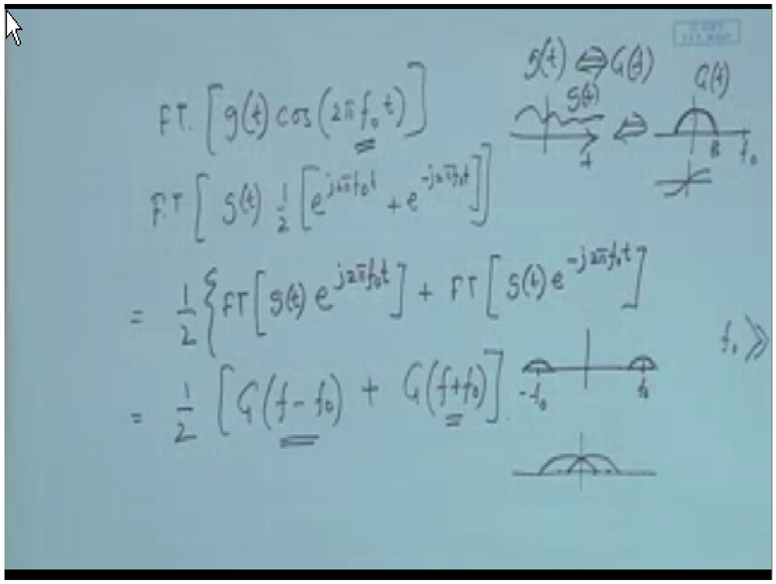

Now let us say I will be multiplying this signal with cos this is the very typical actually this is called modulation okay so let us say I will be multiplying by a cos co sinusoidal signal 2 π F_0 P some doing this and this G(t) is my actual signal problem and this has a Fourier pair already known which is gf so suppose I have a signal GT in time domain and I have a corresponding Fourier pair which looks like this stage here you see that because it is a real signal I have constructed the spectrum in such a way that it is even symmetric.

So this is the amplitude spectrum I have drawn correspondingly there will be a phase spectrum probably which is odd symmetric something like that okay. so I have got this G(f) now I know that this C(T) goes to GF okay in frequency domain now if I multiply by G(T) with cos ω CT what will happen to this g(f)how that will look like how the spectrum of this composite signal will look like so I have to just take a Fourier transform of this okay.

Now what I will do this cause I can write as ½ $e^{J2\pi F0T}$+ e $^{-J2\pi F0T}$ right I can write this now because Fourier transform means this will get multiplied GT into this DT into this and then addition so linear combination Fourier transform gets distributed so I can always write half it should be a Fourier transform of GT x $e^{J2\pi F0T}$ + Fourier transform of GT x $e^{-J2\pi F0T}$ okay now go back to our previous results so half if GT has a Fourier transform of G F then if I multiply with this it should be G F- F0 and similarly if I multiply by minus it should be –(F0).

So that should be plus so G(f)+F0 this is what I gained so basically what is happening first of all this gf strength is becoming half and it is getting translated same G pattern is just getting translated to plus f0 so this is centered at plus f0 this is centered at -f0 so suppose my f0 is something like this so what will happen this whole thing will be little bit reduced and it will be centered around+ f0 and -f0 remember the shape of this spectrum which actually specifies the signal quality that remains the same.

The shape does not change the spectrum shape does not change I can again bring it back to 0 and I will get the same thing so this is in particular a very interesting property of Fourier transform so what is happening any signal if I know the Fourier transform of that and as long as the signal is somewhat I should say bend emitted that means now the concept of and limited things will be coming so what I know about the signal is if I see the spectrum of the signal the spectrum or spectral density I should call that gf beyond some value it either completely vanishes beyond some value being suppose either it completely vanishes or diminishes or beyond a certain level which is insignificant okay.

So then we say that this particular signal because in time domain we cannot see this particular signal does not contend any higher frequency component beyond some value B so beyond B it does not have those component spectral component okay and then we call this signal as band emitted signals that means it is bend emitted up to B so whatever spectral component it has or frequency component it has it is limited up to bandwidth B or value spectral value B so beyond B does not have any spectral component or I should say any corresponding sinusoidal component.

So if I represent the signal like Fourier series or even in Fourier transform so I do not get any significant spectral component beyond this particular sinusoidal or exponential whichever way I am representing okay so once the signal is band limited if I try to multiply that that signal with a real Co sinusoidal signal of frequency F0 where I have deliberately put this f0 to be greater than this B or sometimes much greater than this B then what will happen this signal will get translated to that f0 okay.

So if F 0 is not greater than B then what might happen so suppose f0 is somewhere over here I mean the signal is or somewhere over here so this signal will be something like this and it will be centered again at -f0 it will look like this so these two things will overlap so that is something I do not want okay so why I do not want that will be clear later on but right now we are saying that this is what if happens this condition is true there is a signal that particular spectrum part is still as it is it might be a little bit reduced but whenever they are reduce their relative part is equivalently reduced.

So that means the spectral feature or characteristics which define the signal that remains the same so that relative strength at every point are relatively equivalent compared to this one okay so the signal pattern remains almost the same so what is happening I am just actually translating them to a higher frequency it remains the relative shape remains the same I am just translating them to a higher frequency and this particular part is called modulation why now we can define what is modulation and what is the advantage of doing modulation.

Suppose, I have a channel we have talked about channel in the first few process and it is a shared channel and everybody wish to use that same channel so suppose I want to transmit something on the channel so I put some voice signal to my transducer it converts it into a electrical domain signal it becomes a voice signal and then it is being through antenna it is being radiated into the air and by corresponding recipient wants to receive this as long as I am the only one talking to one particular guy this is all good.

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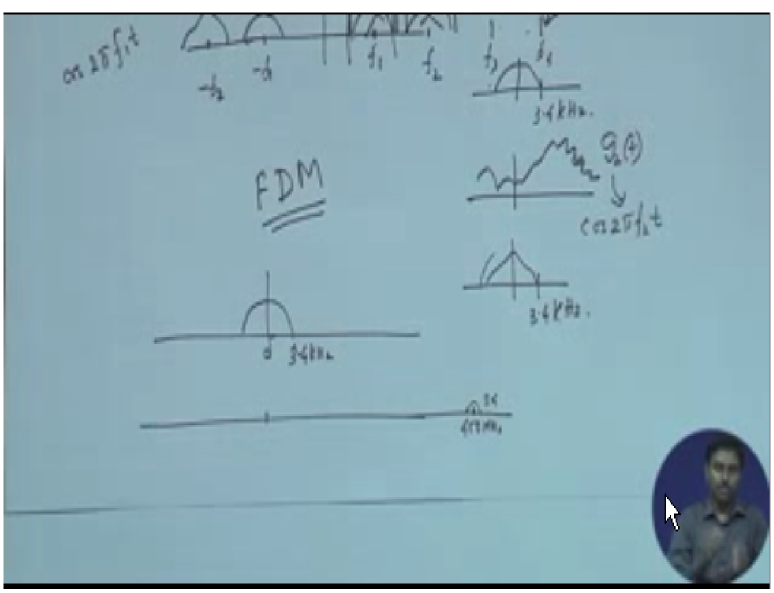

But suppose two fellows wish to communicate now after suppose this is by speech signals it has a random variation and he corresponding spectrum looks like this okay it will it will be a band emitted spectrum in speech we say that beyond three point four kilo Hertz there is nothing all are insignificant so it is bend emitted up to 3.4 kilos so whatever spectrum component we will have it will be up to 3.4 kilo Hertz okay so another guys so this is my g_1 (T) which is my signal another guy wish to also communicate he also has generated a separate speech signal.

A different kind of speed signal so this we call as g_2T which also looks like similar spectrum because it is speed so it will still have same kind of frequency component but it might look little bit the spectrum might look little different but it will be almost up to 3.14 now if I just super impose this G 1 and G2 and put through antenna at the same time what will happen these two spectrum will coexist and then neither in time domain because time domain the signal will be just added.

If it is additive channel and in frequency domain also the frequency component will be just added then it is very hard to separate them out because neither they are separated in time domain nor they are separated infrequency domain so I will have no device or no mechanism to separate them out so if my receivers wish to now listen he will get a jumbled signal or added signal g1 with g2 and that will completely deliver him a completely different speech.

It will be it will be just a distorted speech so he will not neither he will be able to listen neither the other guy wish to receive g2T will be able to receive so that is where if I wish to multiplex multiple users data into the same common channels I need something else the device that was designed by the understanding of frequency shifting property is something like this let us multiply this G1T with some cos 2πF0T or F1 T.

Let us say then what will happen this particular frequency component will go around f1and -f1 and it will sit nicely over there and when we are client transmit G to T with us separate multiplication with again a cos term but this time the cos frequency will be different let us do it at F2 T well F2 is predominantly different from F1 and this particular signal will nicely be sitting over here and at -F2 the good part is now in the frequency domain they have separate location and at the receiver side what I can do is I can just filter them out.

So there is a device called filter which most of you are familiar that just specifically takes some of the frequency or it just passes some of the frequency component and it suppresses or the frequency components so if this is the composite signal coming to the filter and my filter is tuned at f1 so what will happen is a band pass filter so it will just take this amount and it will reject this whereas the other guy can tune his filter at f2 and he can reject this so both of them will get their original signal as if it is transmitted on the air single.

As long as we choose this f1 f2 carefully and as many we wish we can actually multiplex many we can have corresponding f3 corresponding F4and soon we can actually multiplex multiple users simultaneously transmitting over the channels and their separate existence is still remaining because of this frequency translation property what is happening whenever we multiply with cos 2 πF1 T or F0 T or F2 T or F3 T we know that the spectral component or the relative spectrum which is the characteristics of the signal remains the same it just gets translated to our different frequency band and all.

We have to do at the receiver we have to carefully choose the frequency band put a filter and take extract our own signal and reject all of the signals this is facilitated by this communication technique or that is why this particular part is called modulation so what you are trying to do whenever you are trying to use the media which is being used by all others you translate or you multiply by co sinusoidal signal to translate the frequency spectrum into a higher frequency and then actually transmit it in the common media.

Everybody will be doing that as long as they are using separate bands to transmit they will have separate identity in time domain it might not have separate identity but because we have done this frequency domain they will always have their own separate identity that is fantastic so this particular thing is called frequency division multiplexing so what we are trying to do is because now you can see all these things are so important.

Because we have understanding of Fourier series and transform and because of that we could get another representation of signal and from the Fourier transform only we could derive that there is a property called frequency shifting property by which we can actually keep the signal intact but translate it into a higher frequency and because we have some device called filters which can separate out some portion of frequency.

So we can actually give separate entity for separate signal in the frequency domain so the entire manipulation is done in the frequency domain if we would not have understood Fourier transform would not have understood the frequency the presentation of a signal we could not have produce these things okay if we were not able to understand the mathematics behind that frequency shifting property we could not have produced this thing.

So it is very important means now you can appreciate it is very important for communication any simple technique that has been applied in communication it is deeply rooted in Fourier series or Fourier transform is absolutely necessary that you understand those concepts very clearly okay so now we have understood some portion of it we have talked about this modulation why this modulation is required we have told that one particular thing is this multiplexing.

So this is now probably clear you have stated earlier but now it is all mathematically clear that what we are trying to do and why that is important there is another aspect of this frequency shifting property or this modulation that is we have already talked about that that whenever you put multiply with cause this one you can see the frequency component that it is having whenever we are transmitting it is around f0 some +/- so earlier it was having if you just see this one it was having frequency from 0 to some 3.4kilo Hertz.

Now suppose I put them at it is a very high frequency let us have some 400 megahertz so what will happen it will just go center around 400 megahertz and it will just be around that 400 megahertz 400+ 3.4 and 400 -3.4 okay so it will be around that 400 megahertz only so the frequency component it predominantly had that has been transmitted in the air will be around that 400 megahertz whenever it is 400 megahertz the frequency component.

The corresponding wavelength will be very short okay 3.4 kilo Hertz one by that you will see the corresponding wavelength is quite big whereas 400 megahertz you should do one by that you will see because of that 10^6 it will be very small and we have also discussed that whenever you are putting a transmitting antenna or receiving antenna to actually transfer the energy properly we need to have the antenna size or to capture the energy properly through the antenna we need to have an antenna size which is comparable to the wavelength of the correspond frequency that may translate or transferring okay.

So it is very much essential that the frequency is higher to facilitate the antenna size to be smaller so the with modulation we are also achieving that particular part because what is happening here if you see there are very small frequency component which will have huge wavelength because it is inversely proportional and the antenna size will become very huge where else but by just this simple technique by multiplying the signal with a cos it serves two purposes one is it translate it to a very high frequency immediately corresponding wavelength will be very smaller.

So I can devise very small antenna for my transmitter and receiver and on top of that we are now creating in the frequency domain multiple such places where we can start multiplexing multiple signals simultaneously and transmit them through the air media simultaneously and just use a filter to take my own signal and reject other signal so that is actually the story behind modulation you will see that later on okay.

But right now we are happy with this frequency shifting property which will help us to do modulation okay the next property that we are trying to provide is.

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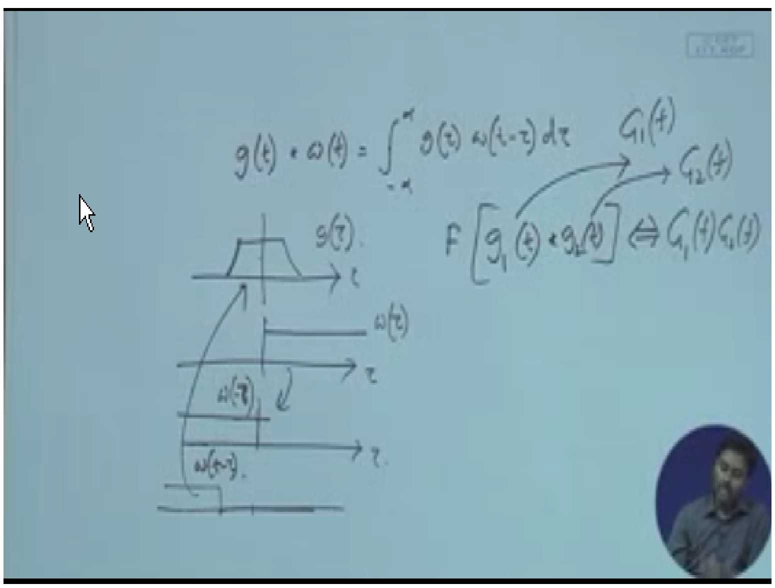

Something called convolution I am not going to prove that but I will just state what do we mean by convolution so this convolution is given by this * symbol so that means the signal is convoluted with another signal GT is convoluted with another signal WT what does that means convolution actually means this so what is happening so you take this signal GT now is just do not worry about τ that the dummy variable of in this integration okay.

So take the signal GT and for ω you can see it is already -τ so basically you reverse this so you take ω - τ so like u -T I was taking earlier so take ω - τ and then time shifted by steel amount okay so do this and then keep varying this τ multiply this and integrate so that is called the convolution so basically any signal you take and the other signal you invert it first in time domain and then time shift and keep varying this time shift and you integrate it multiplication and integration .

So basically one particular signal you will take that will be suppose this is my GT let us say this is like this and suppose the other one ωT is something like this now what you do first you flip this ω T so from there it would be ω - T or- τ so this is ω τ this is G τ okay so time I am defining respect to τ and what you do for a particular so then you time shift it at τ =T you put it okay so this will be just at okay.

So whatever that value of T wherever that T will be it will look like that so at so this will be ω t-τ so what different value of P this will this signal will be I mean going to 0 at different location and then you multiply these two things and integrate it so basically you will be keep on shifting this for different value of T and you will be getting a different value of those integrations so this is called convolution and there is a well known result in Fourier transform so I will give this as homework you have to prove that if I have two signal G 1 T& G 2 T if I convolute them and try to take a Fourier transform of them if I individually know the Fourier transform of G 1 T which is suppose D 1F and G 2 T which is G 2 F we can always prove that this should be G 1 F x G to F.

So in time if we convolute infrequency it becomes multiplication and if in frequency if we convolute in time it will be multiplication that comes from another property of Fourier transform which is called a duality so the duality property if you wish to know that is also very simple you can just very quickly give them.

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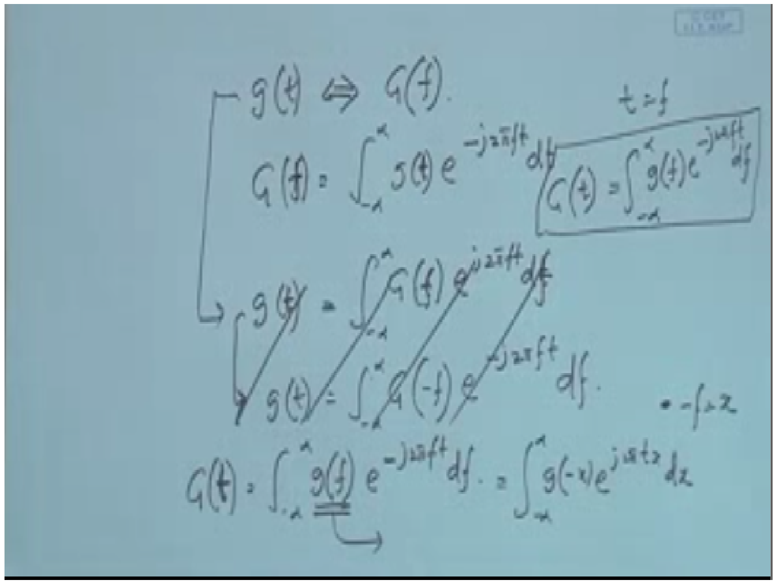

So we know that GT I have a Fourier transform which is called G F okay so in that case what we can write is this G F is nothing but -∞ GT it will be power - j25 Ft dt side now so this is something we already know we also know because it is a Fourier transform so we can also represent this G T as -∞ +∞ gf e $^{+J2\pi F T}$ DT sorry DF right we know this now what we can do instead of F if we just put -F so then from this equation we can get GT I can write -∞ so f is replaced by –F.

So what will happen this is G -F and here we will just get the power -J 2πFT right so this is something we will get okay now all you have to do is okay so what we have done we have just replaced this okay so we can do one more thing instead of doing it over here I can I can also so do it over here so basically what we are trying to do I have got this right now you just replace T by F so T= F if I just put that so what will happen this GF will become GT and I get -∞,+∞ this becomes G F e $^{-J2\pi F}$ becomes replaced by T and P becomes replaced by F.

So I get F T and T becomes G s right so this is something I get okay so I get GE T is equal to -integral to +∞ G F e $^{-J2\pi F T}$ DF okay so now you can see that if I have this as frequency domain okay so this becomes T right so this is almost becoming inverse Fourier transform but I only have one problem this there is a minus over here inverse Fourier transform is having plus so what I can do is instead of this if I can put -F right.

So immediately what will happen if F is replaced by minus F or I can write minus F means minus F as some X so immediately what will happen so this will become X so this is becoming G- X because its F F is - XC - X it devolve minus this would become plus because - F becomes X so J2π T X and D this would be D X- and + so that will just change the limit so this will still remain the same okay so G T becomes this now you can replace X / F immediately.

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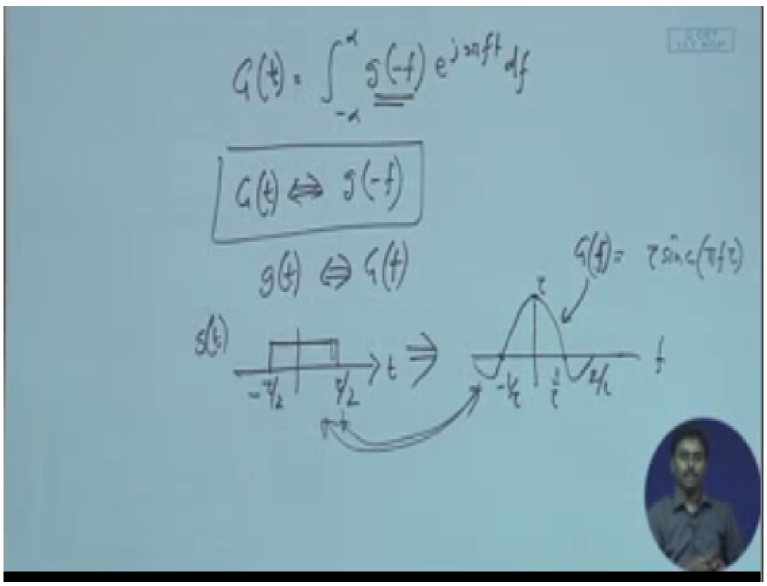

What you get our G T is becoming just -∞, + ∞ G -F $e^{J2\pi FT}$ DF right so what is happening now I can see this G T and G -F are Fourier transform to each other so G -F if I take inverse Fourier transform I get G T so that becomes a Fourier there so basically we have started with a four-year player GT which has a Fourier transform of G F if I just this whatever time domain function I have got that if I represent in frequency domain if I just take negative of that that will have whatever frequency domain function.

I have if I just represent it in time domain I will be getting that so that is called the duality of Fourier transform so that means suppose I have already got a Fourier transform let us say I do a Fourier transform of some this kind of pulse okay which is defined from - T/ 2 to +T /2 take that as homework if you do a Fourier transform easily you can just put the fundamental Fourier transform this things you will see that it will be a sin c function it is defined by this it is Fourier transform will be GF should be τ sin C π F τ sin C means sin of this argument divided by this argument.

So sin π F τ / π F τ so DF gf becomes this which will be defined as at equal to 0 it looks like this at means τ T =0 it is having a value of τ at 1/τ it goes to 0 again at 2 / τ it goes to 0 and so on at-1 /τ it goes to 0 and so on okay so if I already have got a GT which is looking like this and if I already have got a GF which looks like this now if I give a sin c function in time domain that means this becomes GT and if I wish to actually get a Fourier transform this would be just similar like this quality formula tells me that.

So if the frequency domain 1 okay frequency domain 1 is now becoming my time domain function so now the sin function which was the frequency domain representation that becomes my time function this will be just same time domain function will be the Fourier transform now this will be frequency domain okay so this will be now this is in time domain this is in frequency domain if I take this in time domain it will just be frequency domain the time domain1 becomes the frequency domain 1 only thing is that it must be having this -F but because this function is symmetric over T or F .

So it will remain the same so they become ups Fourier conjugate of each others that means a square box function if I take if I do a Fourier transform I get a sin c function if I take a sin c function in time domain if I do our Fourier transform I get a box function in frequency domain so if I know 1 I will be always knowing the conjugate of that so that is the duality property of Fourier transform.

so with all these basics what we will try to do is we will try to now go into the measurement part which we have ignored means we have done that for fluid series but for Fourier transform the measurement power part that means the energy or power we have still not devised that so now we will try to see for a signal the very important property is the measurement of the signal which is energy so we will try to see how to evaluate the energy of a signal so that will be our next target thank you.