Power System Analysis

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Lecture - 09
Resistance & Inductance (Contd.)

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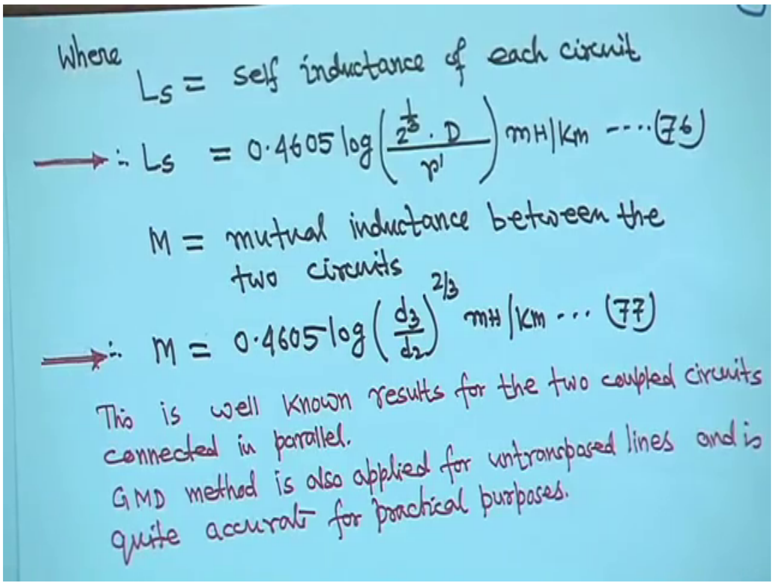

So, that is L s is equal to yourself inductance of each circuits right therefore, L s is equal to 0.4605 log 2 to the power one third into D upon r dash Milli Henry per kilometre, this is equation 76 and M is the mutual inductance between the 2 circuits that is M is equal to 0.4605 log d 3 upon d 2 to the power 2 third Milli Henry per kilometre.

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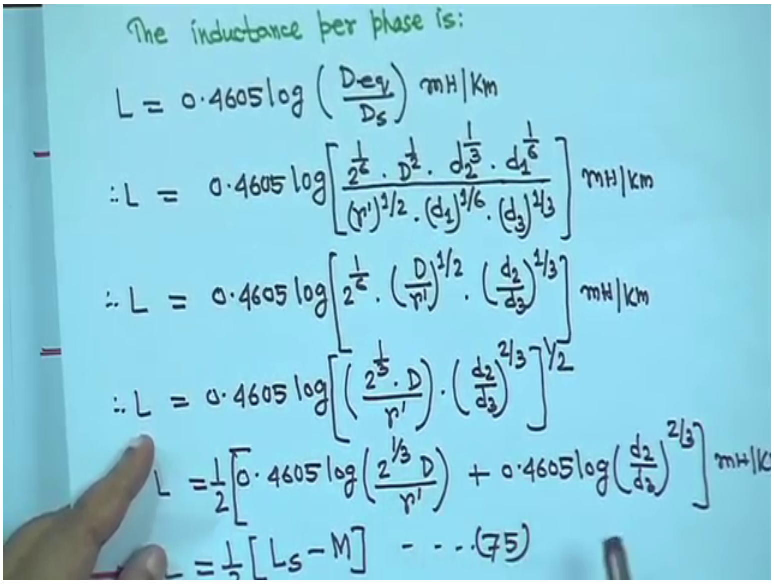

Now, this one this one we call self and this is that mutual inductance between the 2 circuits. If you look at that if you look at the configuration. So, we are calling this is that your what you call the first term the first term that this term we are calling as a self inductance right and this is mutual a between the 2 circuits right between the 2 circuits this is true, because look at the expression d 2 upon d 3. That means, here if you look mutually because of minus sign this minus this minus we have taken L s minus M this minus we have taken that is why it is in this expression it is d 3 upon t 2 to the power 2 third.

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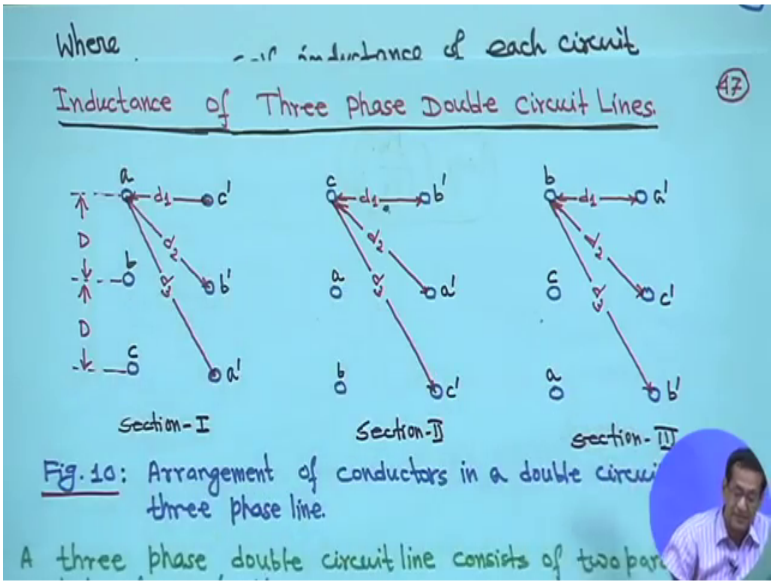

So, if you look at the configuration right. So, d 2 D 3 this as we take this is coming because of this your between the 2 circuit right, but when it is D out cube root of 2 into D upon r dash that terminology is coming, it is actually within that that is your D and r dash right that is why this part we are calling as a self this part we are calling as a self and this part we are calling as a mutual inductance.

So, this is well known result for the 2 couple circuit right connected in parallel GMD method also can be applied to un-transposed transmission line and it is quite it keeps quite accurate result.

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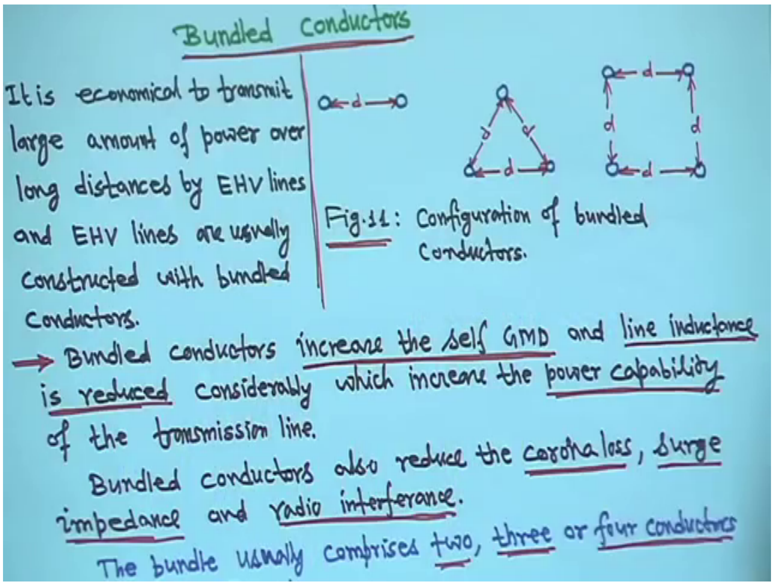

Now, we will come to bundled conductors, right. So, in this case that bundled conductors actually; that means it is economical to transmit large amount of power over long distance by extra high voltage transmission lines. And those things are basically constructed with bundled conductors. So, bundled conductors configuration maybe 2 conductors right or maybe 3 conductors for may be 4 conductors right even more also right.

So, so many conductors if it is there in parallel right. So, naturally that it is it has been I mean for extra EHV transmission lines generally they use bundled conductors such that it increase the self GMD a lines inductance is reduced right with increase the power capability of the transmission line, because if inductance are less then reactance will be less and the transmit power transmission capability will increase.

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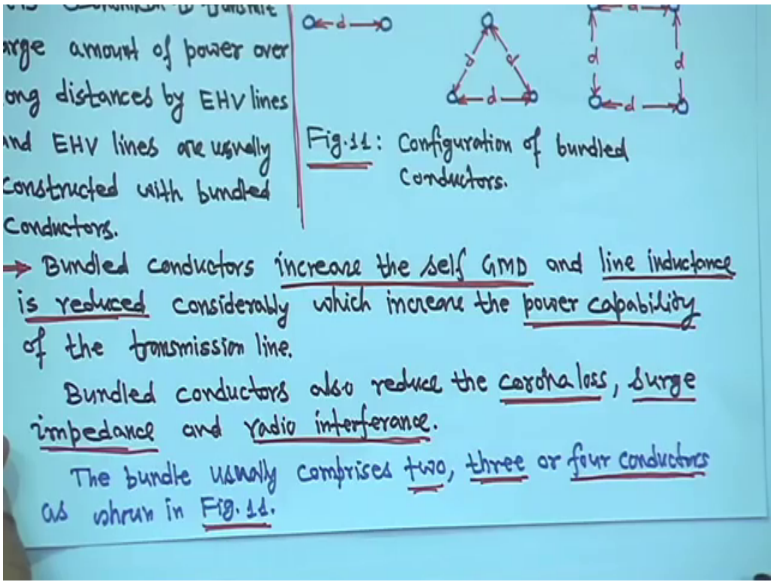

So, bundled conductors are also reduced the corona loss surge impedance and radio interference, right. So, generally the bundle; so, basically bundled conductors usually comprises 2 3 or 4 conductors as shown in figure 11 right, here I am showing 2 3 or 4, but it may be more also. So, if you if you try to find out what will be the self GMD for all this configuration.

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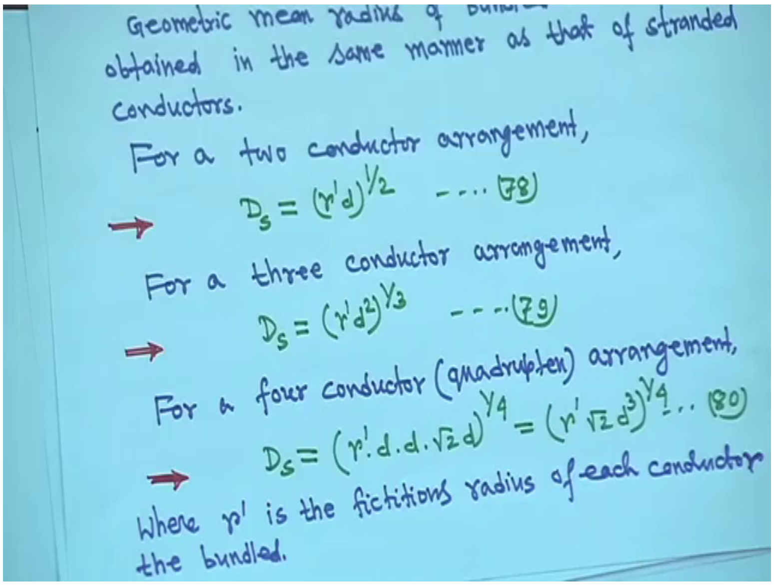

For example if you take this 2; this 2 conductor case this 2 conductor case. So, for a 2 conductor arrangement right D s will be just r dash d to the power half this is equation 78.

Similarly, if you consider this, this 3; your 3 conductors configuration right then in that case D s is equal to r dash d square then to the power D s is equal to r dash d square to the power one third, how things will happen that just I am making it for example, this is this is sub take any conductor take any conductor.

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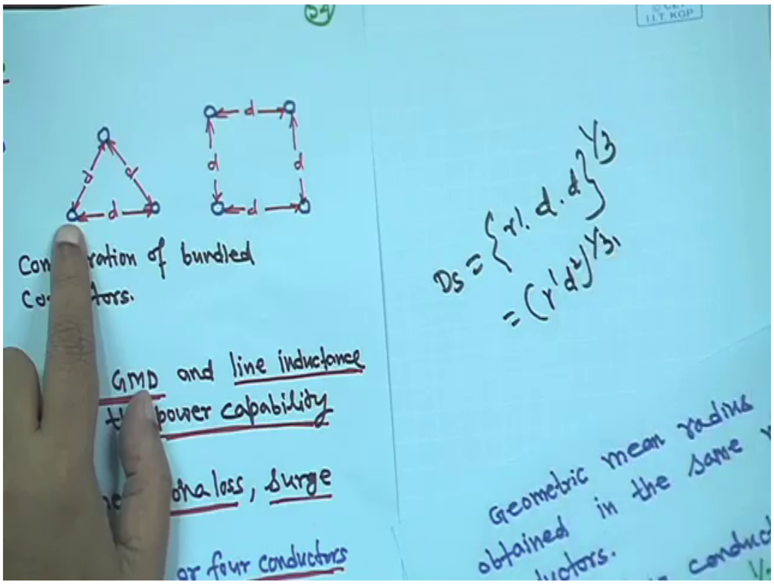

So, suppose this D s is take any conductor D s is first you will take its r dash right then from here from this conductor take the distance of it that is in d and this conductor again you take the d right assuming all the conductors have the same radius r

And distance all distance also the same and to the power one third. So, it will be r dash d square to the power one third that is what I am writing for 3 conductors d square one third right this is for 3 conductor case. Now for 4 conductor case right when you take the 4 conductor case I mean this one 4 conductor case this one; so, from any conductor right.

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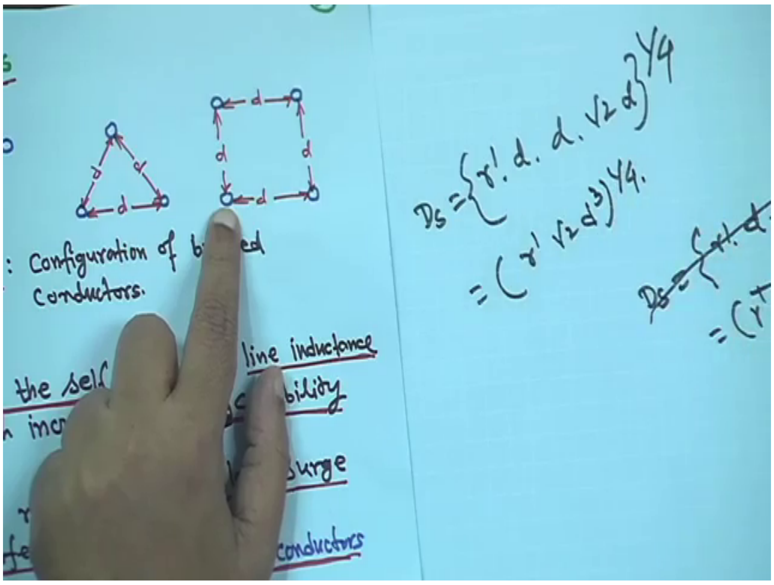

Say D s this is for 4 conductor case and this diagram D s is equal to first you will take the self one r dash then you take the distance from this conductor to here it will be d then this conductor 2 to here it will be d and then this conductor to that this is d, this is d. So, this distance diagonal distance will be root 2 D to the power 1 by 4.

That means this will be your r dash root 2 D cube to the power 1 by 4 right. So, that is why it is coming that for your conductor 4; 4 conductor case that is quadruplex r dash root 2 D cube to the power one upon 4 where r dash is the fictitious radius of each conductor in the bundle right. So, that your what you call that self you are what you call you have to find out that all D s right self GMD of this kind of configuration now till now for all this regarding your derivation whatever possibilities are there.

So, we have discussed all the, but so far we have not taken any numericals. So, next I will take several numericals to support those derivations right and different type of numericals we will take and just. So, far theory we have seen all the formula mathematical development all the things we have seen.

Now, we will consider all this thing you are what you call that your numericals and as many as possible I will show you such that different type of problem such that it will be helpful for you. So, your objective will be try to understand this right. So, first take the example one.

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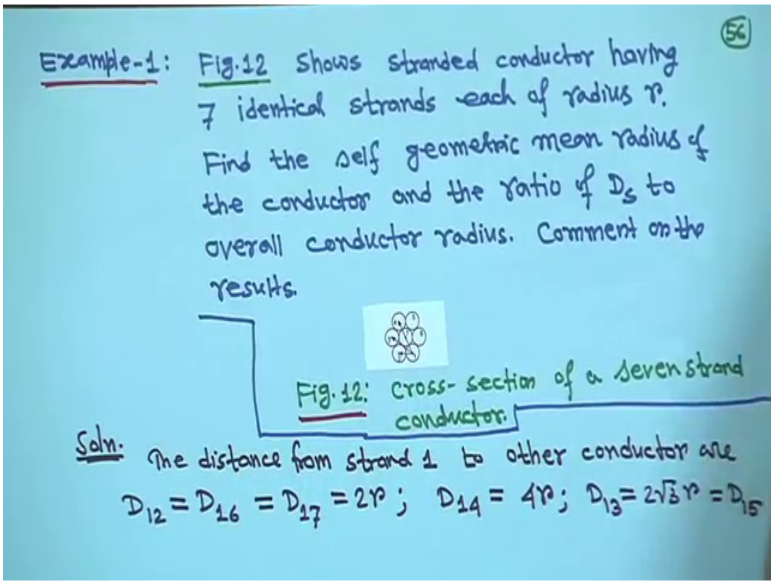

So, when we take the in this diagram which is small one I have taken I have taken from a book right rather than draw it. So, this is figure 12, actually it shows the stranded conductor having 7 identical strands one central strand and surrounding this there are 6 conductors right.

So, and each having radius r. So, and if all are of same radius then it is one and around it 6 conductors possible if it is radius are different then things are; it cannot be something this kind of configuration. So, you have to find out the self geometric mean radius of the conductor and the ratio of D s to overall conductor radius. Finally, comment on the results. So, this is the configuration this configuration little bits I think hope it will be readable to you.

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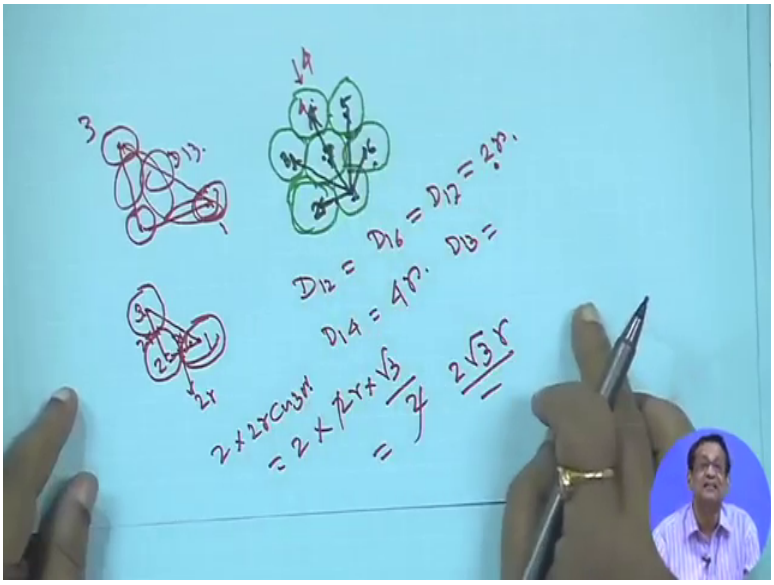

But first let us see that I am trying to make it little bigger hope that drawing will be alright right this is the this is central strand this number is given 7 right. So, around this there are 6 conductors are there right. So, here this number is one say this is 1, say this is 2, this is 3, this is 4, this is 5 and this is 6 centre one is 7, this is 7 right, 1, 2, 3, 4, 5, 6, right. So, first you have to see the distance from the strand 1. So, this is this distance means here know from here to here centre to centre this is one distance here to here, here to here, this is distance and of course, here to here right. And from here to centre also right.

So, first what you can what you can see is that one to 2 actually this is 1, this is 2. So, D 12 is equal to D 16, D 12 is equal to D 16 is equal to D 17 right. So, here it to here to here r, so, it is equal to 2 r right. So, this is 12, 16 and this is central conductor is 7 right, when you will listen this video see that hopefully you hopefully everything will be readable to you right. So, this is central is 7 you can hear my voice right. So, D 12 is equal to D 16 is equal to 2 r right then D 14, this is actually; this is 4, this is conductor 1 2 3 4 this is conductor 4.

So, D 14 is equal to 4 r because from here to here r this diameter is 2 r and again here to here is r. So, all together D 14 is equal to 4 r right, now D 13, D 13 1 2 3 right; 1 2 3. So, actually this is your one just separately I am trying to draw it this is this is another one, this is your 1, this is your 3 and this is your 2 right and this is your 2. So, this is you have to find out you have to obtain D 13 right. So, 1 2 3, it will be this sorry; this is actually all together right this is all together. So, I will how to make it just hold on hold on I am making it.

So, this is one this is one and this is one right and here to here we want to make it and here to here; here to here right. So, this is here to here it is your 1 to 1 to 2 is 2 r, this is 1, this is 2, this is 3, this is actually 2 r right. Similarly 2 to 3 also 2 r and this angle is 30 degree, right. So, here you take this projection that 2 r cos 30 plus 2 r cos 30; that means, 2 into 2 r cos 30 total 1 2 3 right hold on I will put it little bit up right. So, 2 into 2 r cos 30 that is 2 into 2 r right into root 3 by 2 2 2 cancel. So, it will be 2 root 3 r.

So, D 13 will be your 2 root 3 r right. So, so this angle is 30 degree from the symmetry you can make it this angle is 30 degree. That means, that is why D 13. Similarly for D 15, 13 and 15, same distance 1 to 5 same distance. So, this is D 13 is equal to your 2 root 3 r is equal to D 15 right. Similarly if this is for even you are taking conductor one if you take conductor 2 then meaning will same then D 24 will be is equal to D 26 if you take conductor this one D 35 is equal to your D 31, this way it things will be there; that means, you have to consider every conductor separately.

Therefore you have to because you have to obtain yourself GMD right. So, in that case what will happen first thing is look how we will make it first thing is that you have to this thing you have to see that there are 7 conductors there are 7 conductors right and you want to find out self GMD; that means, it is n square. So, 49 one upon; that means, you have to take 49 root right. So, such that 49 possibilities are here. So, how now first that there are 7 conductors there are 7 conductors if you take that fictitious radius of each one r dash. So, including central one 7 conductors are there. So, it should be r dash to the power 7, right.

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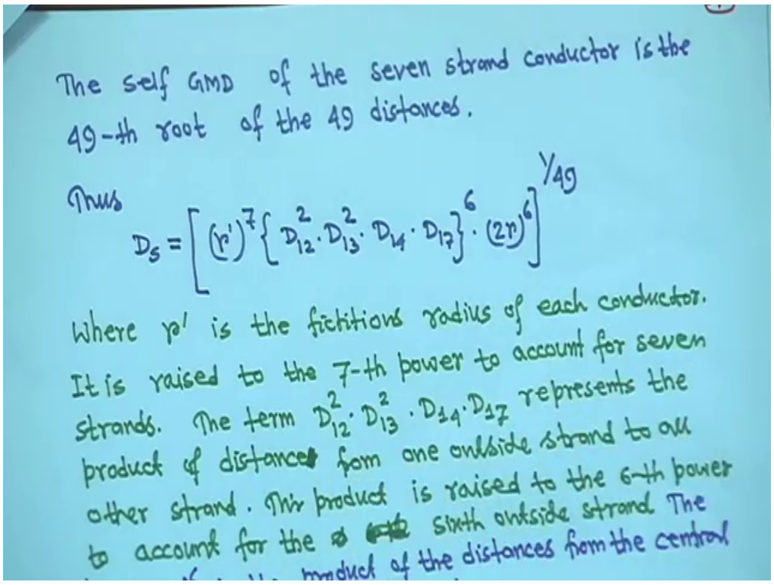

Next I am making D 12 square, it has to be repeated 6 times you can look every D 12 square I will explain each one then D 13 square into D 14 into D 17 to the power 6 right; that means, D 12, if you take this, if you take this; this diagram if you take this diagram D 12 is equal to D 16 twice, it is coming similarly if you consider this one D 21 is equal to D 23. Similarly if you consider this D 31; D 34 is equal to your; what you call D 3 2 this way you have to consider each thing. So, for every conductor you will be consider that D 12 is equal to D 16 distance remains same.

Similarly, if you consider this; this conductor also then you have to take this distance and this distance if you consider this one, this one, this one. So, every time 2 or twice it is coming and all D 12, D 16, D 32 all distance are same. So, for each conductor this is a the distance repeated twice because one is here D 12 and D 16 they are same D 12 is equal to D 16 that is why here it is made it D 12 square D 12 square, but you have 6 conductors surrounding the central conductor you have the 6 conductor.

So, that is you have the 6 conductors right surrounding the central conductor you have the 6 conductors that is why D 12 square or power should be to the power 6 because each conductor if you consider 2 distance are same that is why written D 12 square rather than complicating that thing D 12 into D 16 into D 2 3 into D 34 like this. So, simply we have written the all are same; so D 12 square.

Similarly, if you consider one 3 then D 13 square coming because 1 2 3 1 2 5 same distance if you take now 2; 2 to 4 2 to 6 same distance right if you take 3 3 to 5 3 to 1 same distance. So, this way twice it is coming and 6 times because surrounding conductors are 6; 6 times coming that is why D 13 square again to the power 6 whole I have written all to the power 6 because all are coming 6 times.

Next D 14 D 14 means from here to here similarly 2 to 5 similarly your 3 to 6 this way it is coming only once D 14 there is no other similar distance here one 4 2 5 3 6 right then your 4 1, this way it is coming 6 times that is why into D 14 then to the power 6 then D 17; D 17 means from this conductor to the what you call this your 1 to 7 that centre of the centre strand.

So, 17 27 37 47 57 67 all are same. So, into D 17 to the power 6 right; that means, D 12 square repeated 6 time then same distance D 13 also similarly 6 times that is square 6 times then D 14 once D 17 once 6 times into your D 2 r right D 2 r means this; this central conductor; now you consider because this is left out central conductor that r dash we have considered for every one; now for central conductor also you have to consider the distance.

So, from here to here 2 r here to here 2 r here to here 2 r here to here 2 r here to here 2 r here to here 2 r 6 times that is why into 2 r to the power 6. So, total to the power 1 49; now for how we will check it for that inside whatever will be there all together power if you add it as to be 49 if you say this is not matching. That means your computation is somewhere wrong.

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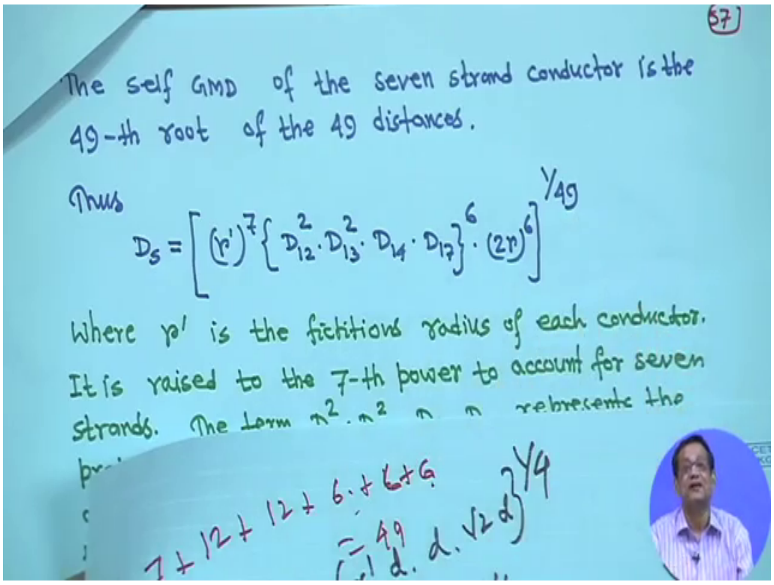

For example if you take this power this is 7 if you take this 2 6 into 2 12 just add 12 plus 2 into 6 12 plus a power is one means 6 right and here also one means 6 and here also 2 r to the power 6 right then plus 6 if you add 12 plus 7 19; 19 and 2 12 you are 31 then 6 6 6 that 18. So, total is 49 and power is one upon 49. So, that; that means, your things are correct right.

Otherwise you have to assume somewhere calculation something had gone wrong you have understood or I hope you have understood. So, although looks like things simple, but calculation possibilities you know that you are what you call that there is a possibilities of calculation error. So, this way you have to understand that how many possibilities are there and I have written one upon n square because 7 conductors are there and all this power just add roughly if power is different and this is different that dimension will not match because ultimately D s suppose it is metre it as to be metre. So, in terms of power those things have to match.

So, that now I am writing for you that r dash; r dash is the fictitious radius of each conductor it is raised to the seventh power to account for 7 strands that is why 7 strands are there then I explain the term D 12 square D 13 square D 14 and D 17 represent the product of distances from the outside strand to all other strands this product is raised to 6 power to account for the sixth outside strand this one also I explain that how I took.

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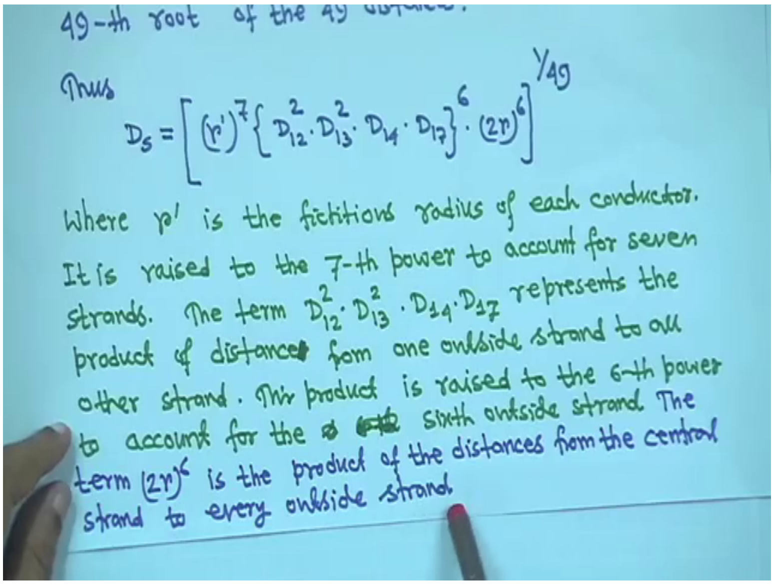

Then the term 2 r 6 also from the central strand I told you 6 times it will come if the product of the distances from the central strand to every outside strand this right if I have given I have explained also how to get it.

So hold on; now, if you substitute I mean if you substitute all this what you call just now I showed all this distance that D 12 D 1; your D 14 whatever you got all the all this values you substitute here right if you substitute and calculate here I did not bring those calculation because then it will kill more time this you can do as an exercise right after substituting all this right all this values you will substitute.

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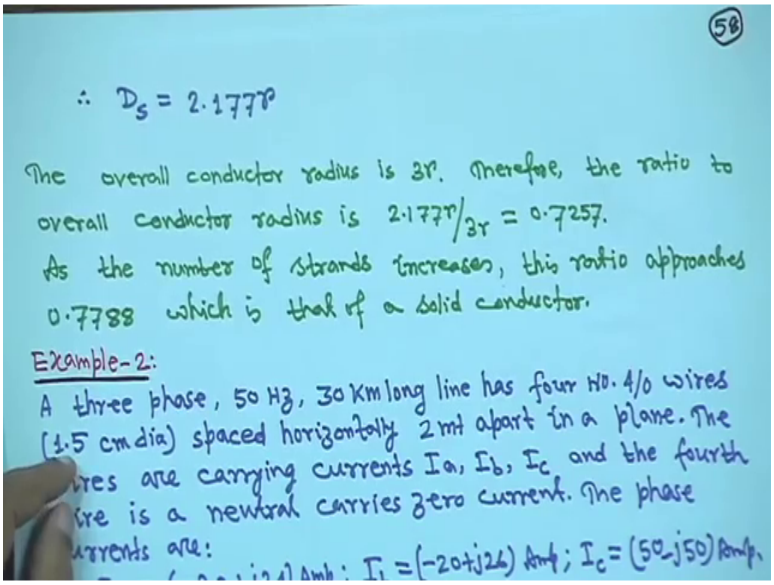

Then what will happen that your; you will get that D s is equal to 2.177 r; this is the final answer will come this you do please of your own right. So, the overall conductor radius is 3 r because from here this is 6 times. So, it is from here to here r right if you take overall; this things 10 this is 2 r this is 2 r this is 2 r. So, 6 r. So, radius is 6 r by 2. So, 3 r therefore, the overall conductor radius is 3 r I have written here 3 r.

Therefore the ratio to overall conductor ratio radius is 2.177 r divided by 3 r I equal to 0.7257; right I have the number of that I have made one thing that comment on the result right in the beginning therefore, as the number of strand increases this ratio actually approaches 0.7788, right which is that of solid conductor that is the comment right. So, this is example one hope you have understood.

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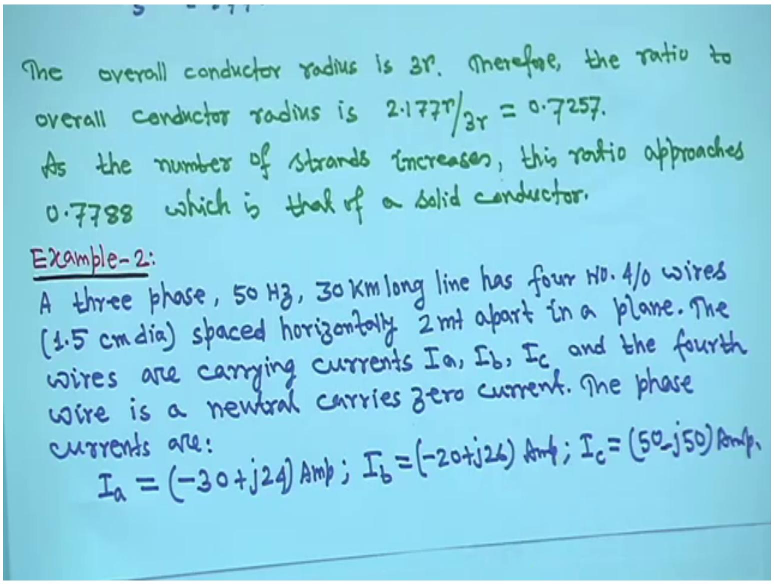

Next is example 2 now. So, first I am reading this problem right then we give the diagram. So, a 3 phase 50 hertz 30 kilometre long line has 4 number 4 by 0, this some kind of conductor conductors specification wire in bracket 1.5 centimetre diameter right spaced horizontally 2 metre apart in a plane the wires are carrying currents I a I b I c and the fourth wire is a neutral current is 0 neutral fourth wire is a neutral caries 0 current right the phase currents are I a is equal to minus 30 plus j 24 ampere I b is equal to minus 20 plus j 26 ampere I c is equal to 50 minus j 50 ampere; one thing is there in the problem neutral current may not be this, but if you add I a plus I b plus I c if it becomes 0 means current phase are neutrally 0.

Although it is mentioned in this numerical right next the given that line is un-transposed the line is un-transposed.

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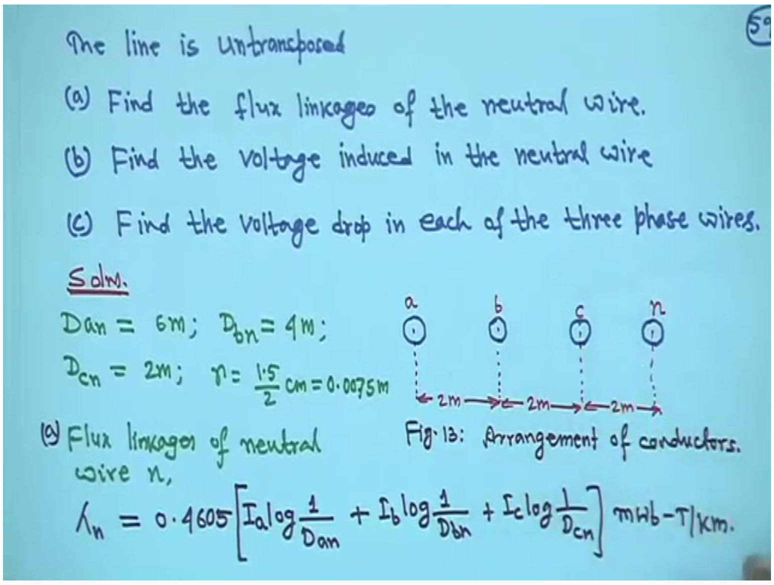

So, you have to find out a find the flux linkages of the neutral wire find the voltage induced in the neutral wire and find the voltage drop in each of the 3 phase wires right. So, first is the diagram it is on the horizontal plane. So, 3 phases a b c. So, I a I b I c this current carrying conductor and this is the neutral and distance between this conductors are phase are neutral everything remain 2 metre, 2 metre, 2 metre right. So, this is the arrangement of the conductors.

Now, first this calculation is very simple D a to n 2 plus 2 plus 2. So, 6 metre D b n b to n 2 plus 2, so, for 4 metre and D c n c 2 n is 2 metre and radius is given diameter is given 1.5 centimetre. So, radius is r is equal to 1.5 by 2 centimetre is equal to 0.0075 meter, this is this thing.

So, generally it transmits a line that between the 2 phases distances is quite or large compared to the radius. So, d minus r is equal to approximately equal to D because r is too small. Now, flux linkages of neutral wires n. So, many theories we have told from the earlier discuss also. So, directly you can write the flux linkages of neutral wire lambda n is equal to 0.4605 then your objective is to find the flux linkages with that neutral conductor. So, I a is equal to sorry lambda n is equal to 0.4605 I a log 1 upon D a n a to n distance plus I b log 1 upon D b n plus I c log 1 upon D c n Milli Weber tons per kilometre it is milli right; Milli Weber tons per kilometre.

Now, in this expression; this expression you substitute D a n D b n and D c n all have been given here you substitute.

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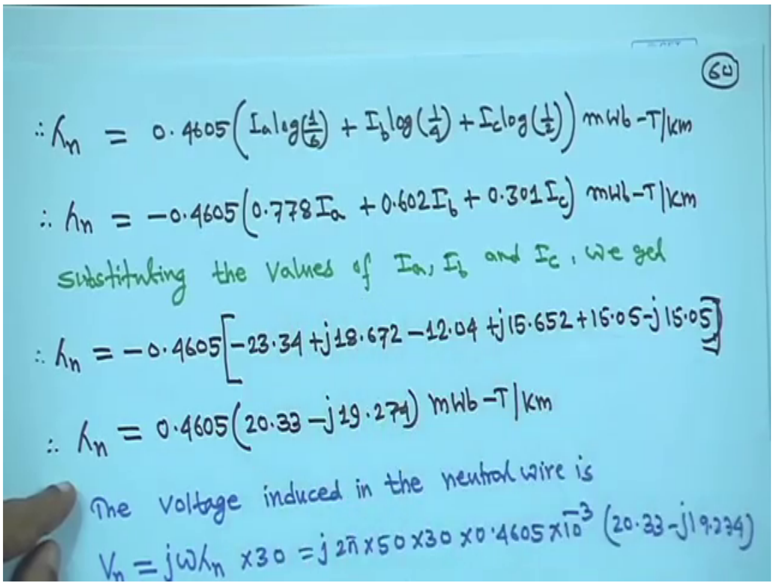

If you substitute you will get lambda n is equal to 0.4605 in bracket you write down I a log 1 upon 6 plus I b log 1 upon 4 plus I c log half milli 1 upon 2 Milli Weber tons per kilometre right now after this; this one log 1 upon 6 log 1 upon 4 log 1 upon 2 whatever value comes you put it will get you will get lambda n is equal to minus 0.4605 bracket 0.778 I a plus 0.602 I b plus 0.30 r 301 I c Milli Weber ton per kilometre.

Now, after this you substitute the values of I a I b and I c this I a I a I b I c all this values are given. So, what you do you substitute here I a I b I c if you substitute I a I b I c here I have written one step here skipping this one final one is lambda n is equal to 0.4 size 05 in bracket it will be 20.33 minus j 19.274 Milli Weber wave band tons per kilometre.

Now, flux linkages will neutral wire this is the expression is complex one.

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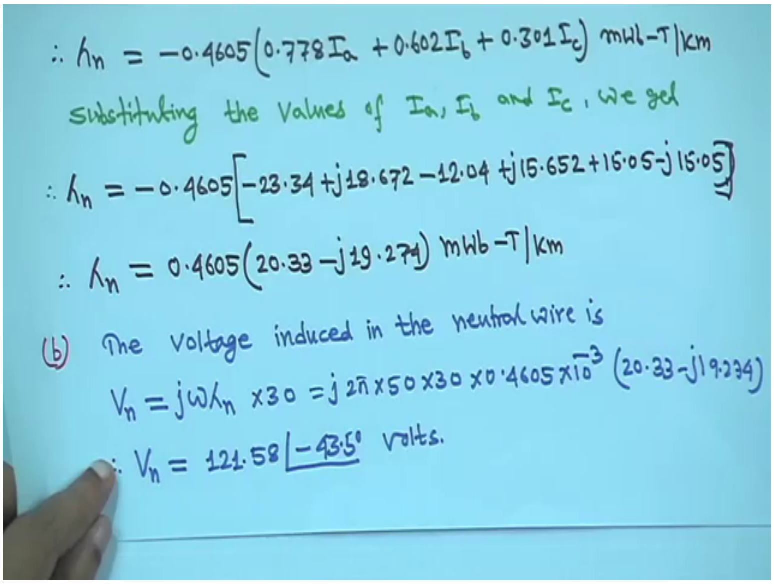

Now, second part is you have to find out the this is the flux linkages part b is the voltage induced in the neutral wire if v n is equal to j omega into lambda n right 30 because length of wire is given 30 kilometre; so j omega lambda n into 30. So, it is j 2 pi assuming that it is it is written there 50 r system, it is written there, if 3 phase 50 r 30 kilometre long, because here we are multiplying in 30 such that we want in volts, because it is given per kilometre that is why multiply by 30 j omega lambda n into 30 is equal to j 2 pi frequency is 50 r into 30 into 0.4605 it is Milli Weber. So, milli we are making 10 to the power minus 3 into this thing 20.33 minus j 19.234 such that it will be in volts right.

So, if you multiply then v n is equal to 121.58 angle minus 43.5 degree volts. So, this is the voltage induced on the neutral wire right now part c.

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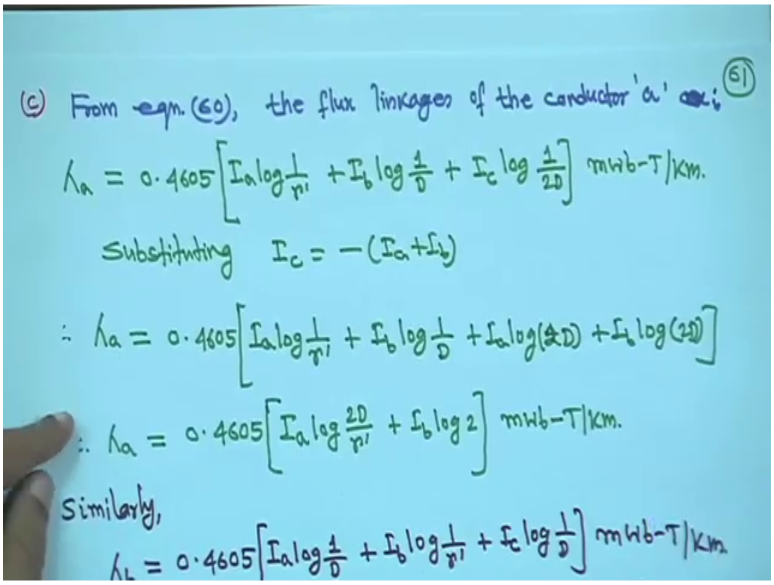

From equation 60; the flux linkages of the conductor a I have kept a under quotation right inverted comma that is right. So, from equation sixty the flux linkages of the conductor a we write lambda a is equal to 0.4605 I a log 1 upon r dash this is for conductor a right that is why we are writing I a log 1 upon r dash plus I b log 1 upon D plus I c log 1 upon 2 D.

So; that means, your instead of writing directly 2 metre 2 metre that from a 2 from first self 1 I a log 1 upon r then I b means a to be distance is d that is log 1 upon D then your a to c just hold on then your d is 2 metre. So, instead of making that that a to b I b that is your log 1 upon D and I c to this thing it is log 1 upon 2 D because this is 2 D right; so, Milli Weber tons per kilometre.

Now, it is neutral current is 0. So, I a plus I b plus I c is equal to your 0; that means, I c is equal to minus I a plus I b right therefore, what you will do you can do it right from this one you are what you call that I c is equal to you substitute minus I a plus I b; in this expression you substitute I c here only I am not showing I am putting you substitute and then simplify right if you do. So, you will get lambda is equal to 0.4605 I a log 1 upon r dash plus I b log 1 upon D plus I a log 2 D plus I b log 2 D.

Now, if you further simplify lambda; lambda a will be 0.4605 I a log 2 D upon r dash plus I b log 2 Milli Weber tons per kilometre.

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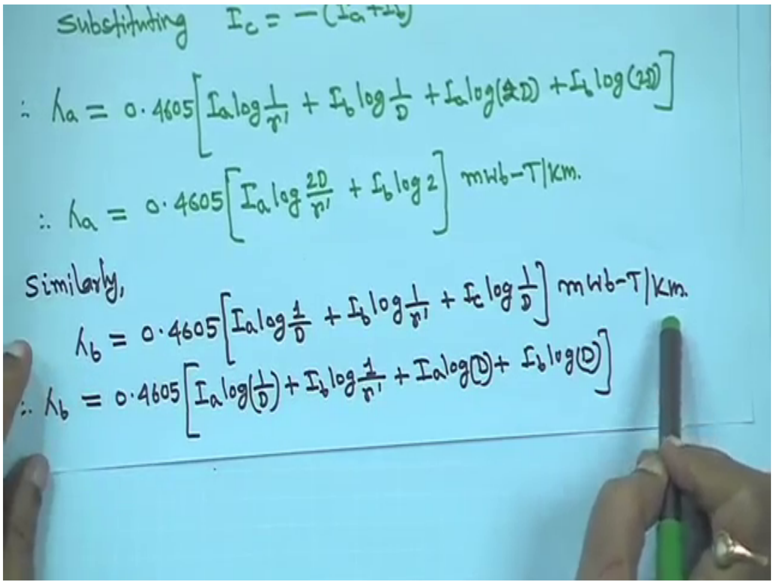

Similarly, you compute lambda b lambda b also 0.460 I a log 1 upon D further case it will be b. So, I b log 1 upon r dash plus I c log 1 upon D Milli Weber tons per kilometre right here also follow the same philosophy right here also you eliminate your what you call I c. In this case also you put I c is equal to minus I a plus I b here if you do.

So, you will you will get here right after simplification after simplification you will get here lambda b is equal to 0.4605 I b log d upon dash Milli Weber tons per kilometre.

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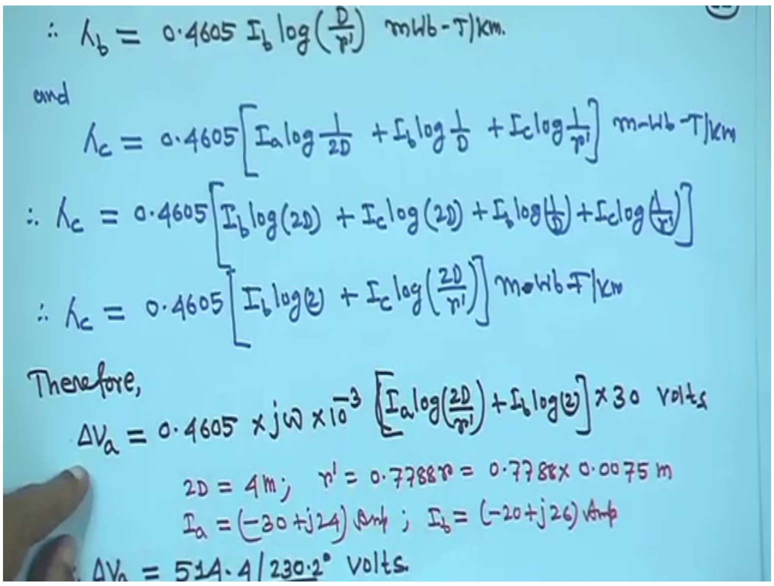

Similarly, lambda c also you make 0.4605 I a log 1 upon 2 D right because c 2 a distance is 2 D right then plus I b log 1 upon D plus I c log 1 upon r dash because for lambda c I c log Milli Weber tons per kilometre here also follow the same thing that I c is equal to minus I a plus I b we substitute in this expression here only then simplify right in this case your; no in this case in the case of lambda; lambda c that your I a log this I b; I b log 1 upon log 2 D plus I c log 2 D plus I b log 1 upon D.

So, here in this case what you have done we have eliminated actually your I b I b I c we have eliminated I a actually. So, in this case you eliminate your; what you call that I a part and just simplify. So, in the in the in this case not like not like I c is equal to minus I a plus I b you just simplify. So, whatever will come I a I b I c. So, I b and I c is there I a is actually eliminated. So, in that case lambda c will be 0.4605 I b log 2 plus I c log 2 upon r dash Milli Weber tons per kilometre. So, in this case what even I b I c is there. So, just eliminate substitute I a is equal to minus I minus I b plus I c then you do this.

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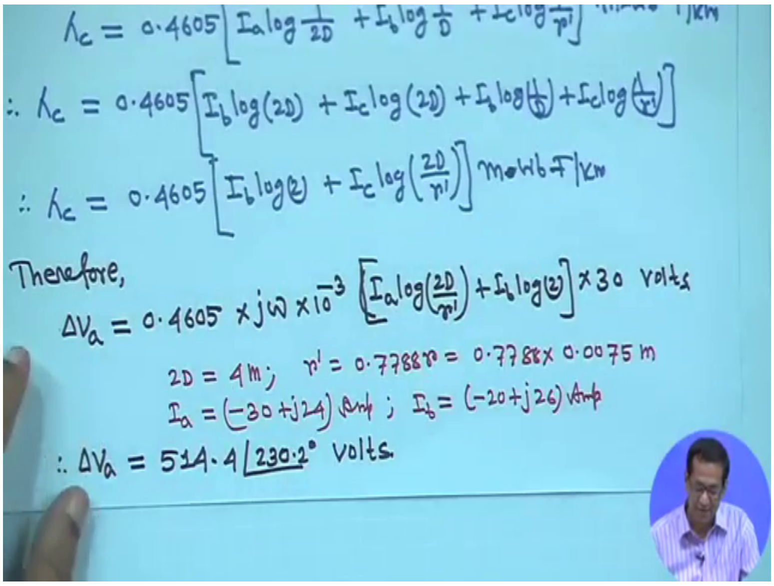

So, this is lambda a lambda b lambda c now voltage induced in that just voltage induced lambda v a will be 0.4605 into your j omega right into Milli Weber is there that is why 20 to the power minus 3 and 30 kilometre length is there into 30 and in bracket this expression for delta b a right. So, I a log 2 D upon r dash plus I b log 2 right. So, where is your where is our expression for lambda a right this is this is expression for lambda a I a log 2 D upon r dash plus I b log 2.

So, I a; I b then here you substitute the current I a and I b I a is equal to this much I b is equal to this much you substitute D is equal to 2 meter right and calculate. So, you will get delta b is equal to 514.4 angle 230 degree 0.2 volts; that means, in phase a this is the induced voltage. So, whenever it is written b c. So, accordingly I have eliminated.

Similarly, if you calculate for phase b and phase c right phase b and phase c delta b; b will be 360.8 angle 217.56.

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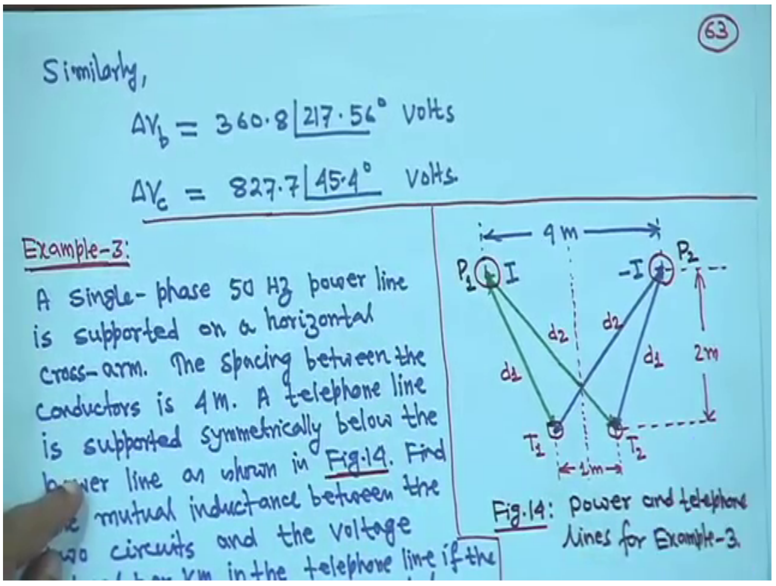

This you please calculate of your own, but this answers hopefully it is correct right and delta b c is 827.7 angle 45.4 degree volts. So, my suggestion is when you will listen to this if you have any problem or anything you just mail to me. And if you have any doubt or any sort of you know complicacy arises you just mail to me, definitely we will try to reply you quickly.