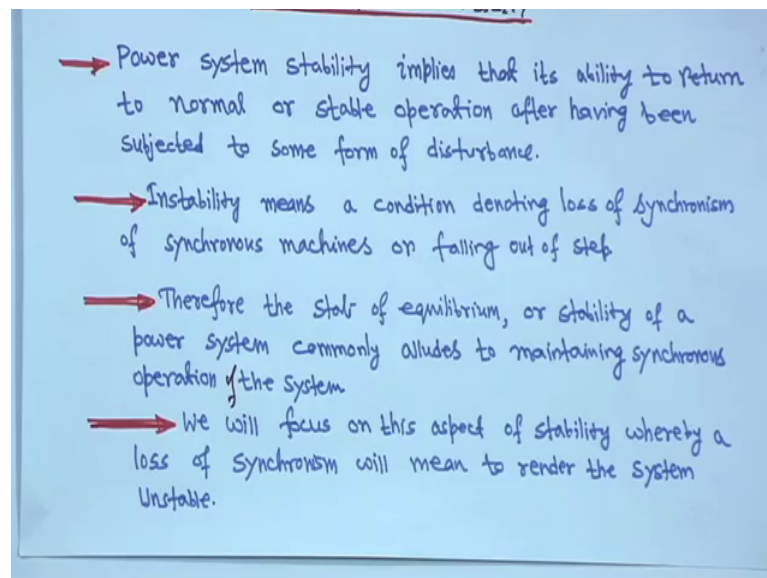


Power System Analysis
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Lecture - 56
Power System Stability (Contd.)

So, for this course this is the last topic, and the title of this is Power System Stability.

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And here mainly will see the transient stability for single machine infinite bus system. But before that will see something more regarding some description of your steady state stability, dynamic or a small signal stability, and some kind of mathematical derivation and understanding.

And power system stability actually that implies that its ability to maintain, to return to normal or stable operation after having been subject to some form of disturbance. That means, you give some disturbance and if it come backs to its after some kind of your what you call it will deviate from stay or what you call from its original steady state point, but after some time if it returns very to the stable point or very close to that; that means, we call that system is stable.

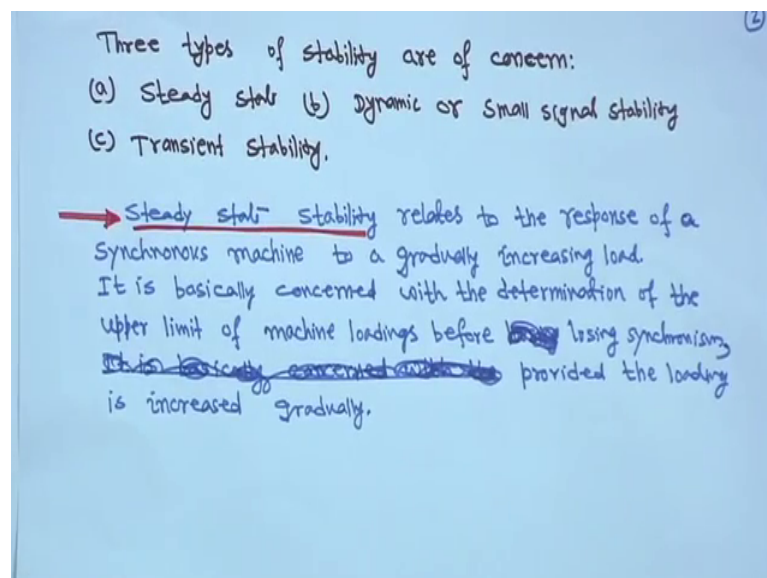
So, actually in a power system you will find that there is always a disturbance, because some loads are switch or switching on some loads are switched off, but systems are

always stable. I mean this kind of disturbance is very small, what you call this verse for this is I will not say small disturbances, but this kind of disturbance always is on in power system. But our system remains stable, so that is why that power. And your power system stability that is why implies that is ability to return to normal or stay or stable operation after having been subjected to some form of disturbance. So, what kind of disturbance that will see later; but in power system the disturbance is always on, I mean slide because some lights are switched on fans are switched off something like and some loads are switched on switched off it is a continuous process.

So, an instability means a condition denoting loss of synchronism of synchronous machine or falling out of step; that is instability. Actually stability itself is a huge course or huge thing, but will restrict to as far as this course is concerned at your at undergraduate level. Therefore, the state of equilibrium or stability of a power system commonly alludes to maintaining synchronous operation of the system, right. That means, you have to maintain the stability of power system.

So, we will focus on this aspect of stability that whereby a loss of synchronism will mean to render the system unstable. So, synchronous machine will never lose synchronism and will or not fall out of step and system will remain stable.

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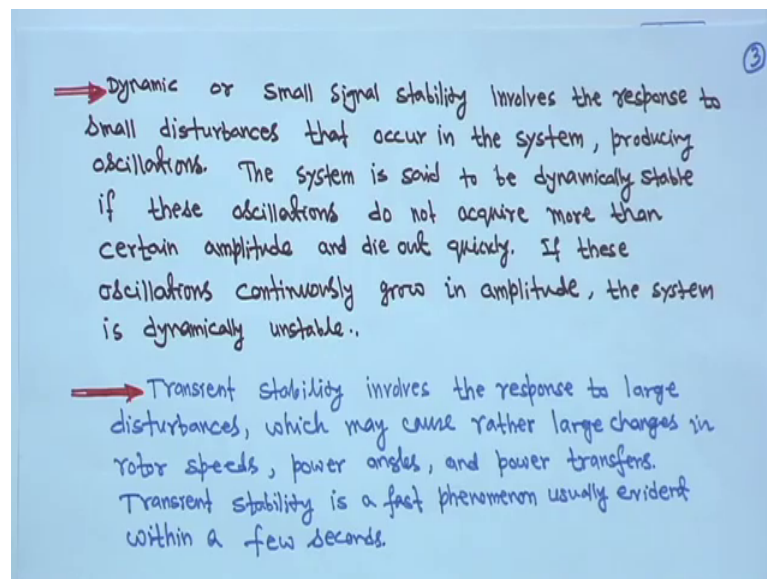


So, when we define stability of the power system there will be three types of stability actually, of course generally are of concern one is steady state one another one dynamic

are called small signal stability and last one is that transient stability. So, of course many other stabilities are there in power system that is beyond the scope for example, voltage stability. So, steady state stability relates to the response of a synchronous machine of a gradually increasing load that you I mean you increase the load gradually, and you can find out that data up determines the upper limit of the machine before it losing synchronism.

That means stability that steady state stability leads to a response of a synchronous machine to a gradually increasing load it is basically concerned with the determination of the upper limit of the machine loadings before losing synchronism. Provided the loading is increased gradually. So, your gradually your loading is getting increased, but at up to it will sustain up to a maximum limit after that it may fall out of your step; that is it may lose synchronism.

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And dynamic or small signal stability actually it involves a response to small disturbances that occur in the system producing oscillations. That means, because of small disturbance there is a continuous oscillation. So, the system is said to be dynamically stable if this oscillation do not occur more than certain amplitude and die out quickly, but if there oscillations continuously grow in amplitude the system is dynamically unstable.

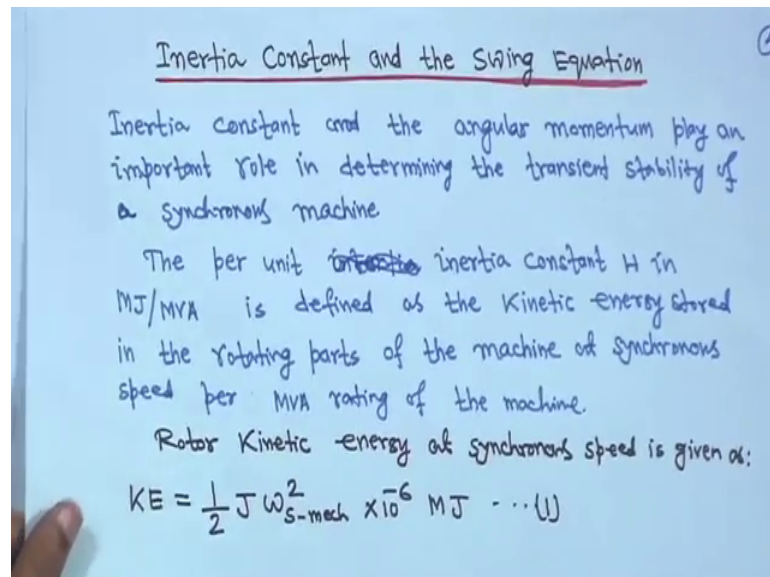
Actually, this small signal stability it involves huge analysis will not study this one; it involves huge analysis for synchronous machine and to damp out this what you call this oscillation if there is a small disturbance say some step increase in the mechanical torque some pong and there will be continuous oscillation and to damp out those oscillations but in your in that omega that is in that case commonly use the thing is that power system stabilizer; so lead lag stabilizer.

So, those things will not study for small signal stability analysis, but for this case you need that what you call that detailed analysis of synchronous machine particular dynamics. And another one is- the transient stability: it involves the response to large disturbances that you know which may cause rather large changes in rotor speed power angles and power transfer. But transient is a past phenomenon and usually evident within a few second. But among this transient stability of course is very severe, because it is due to a large disturbance means suppose there is a fault three phase fault or line to ground fault, in that case what you call it will be large change in the rotor speed power angle and power transfer.

For example, a double circuit line there is a fault in one line. So, naturally power transfer capability will change and those kind of what you call that large disturbances its cause transient your that that phenomena is called transient stability. So, in that case what you call it is basically within a few second. That means, with such fault is occurred you have to see the fault is cleared in very quickly maybe 4 cycles or 6 cycles the fault has to be cleared.

So, these are the three different type of faults: one is I told you that steady state sorry not fault steady state stability then three types of stability, then small signal or dynamic stability, and then transient stability. But our object, our concern for this course will be transient stability and generally single machine infinite bus system.

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So, inertia constant and the swing equation: the swing equation that describe the dynamics of the rotor of synchronous machine; so inertia constant and the swing equation. So, inertia constant and the angular momentum it is played an important role in determining the transient stability of a synchronous machine. So, the per unit inertia constant H ; actually it is we are calling per unit inertia constant H , but later we will see its dimension is mega joule MVA per MVA or it is can be given a unit in second also- is defined as the kinetic energy stored in the rotating parts of the machine at synchronous speed per MVA rating of the machine. That is we define the inertia constant

Now the rotor kinetic energy at synchronous speed is given as that kinetic energy $K E$ you define half $J \omega$ s it is mechanical speed right synchronous with mechanical coming s square into 10 to the power minus 6 omega joule. So, J is moment of inertia actually in kg meter square and omega s square is mechanical speed, but I will come to that.

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Where,

$J = \text{moment of inertia of rotor (kg-m}^2\text{)}$

$\omega_{s\text{-mech}} = \text{synchronous speed in mechanical-rad/sec.}$

But $\omega_{s\text{-elec}} = \left(\frac{P}{2}\right) \omega_{s\text{-mech}} = \text{Rotor speed in electrical-rad/sec.} \dots (2)$

Where $P = \text{number of poles of machine.}$

From eqn. (1) & (2), we get,

$$KE = \frac{1}{2} \left[J \left(\frac{2}{P} \right)^2 \omega_{s\text{-elec}}^2 \times 10^6 \right] \cdot \omega_{s\text{-elec}}$$

$$\therefore KE = \frac{1}{2} M \omega_{s\text{-elec}}^2 \dots (3)$$

And where J is equal to moment of inertia of rotor kg meter square, omega s mechanical is synchronous speed in mechanical radian per second.

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$\theta_e = \frac{P}{2} \theta_m$

$J/\text{sec} = W$

$J = W\text{-sec.}$

$MJ = MW\text{-sec.}$

MJ/MVA

$= \frac{MW\text{-sec}}{MVA}$

And you know the relationships that we know relationship that is your theta electrical is equal to P by 2 theta mechanical- these we know. So, same philosophy that omega s electrical is equal to P by 2 omega s mechanical that is rotor speed in electrical radian per second, but if P is equal to 2 the two pole machine then omega s electric is equal to omega s mechanical, where P is equal to number of poles of machine.

Now from equation 1 and 2; that means this equation if you replace ω_s mechanical by ω_s electrical then it will become kinetic energy it will half J into 2 by P square whole square into ω_s electrical into 10 to the power minus 6 that is over into your ω_s this ω_s electrical. That means, kinetic energy also you can write that half M into ω_s electrical. That means this portion we are defining as M , this is say equation 3

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Where,

$$M = J \left(\frac{2}{P} \right)^2 \cdot \omega_{s\text{-elect}}^{-6} \times 10^{-6} = \text{moment of inertia in MJ-sec/elect-radian}^{-1} \quad (4)$$

We shall define the inertia constant H , such that

$$GH = KE = \frac{1}{2} M \omega_{s\text{-elect}} \quad \text{MJ} \quad \dots (5)$$

Where,

- G = three-phase MVA rating (base) of machine
- H = inertia constant in MJ/MVA or MW-sec/MVA or sec.

Where M is equal to J into 2 by P whole square ω_s electrical 10 to the power minus 6 , there is moment of inertia in mega joule second power electrical radian. This is that unit of this what you call the same is moment of inertia mega joules second for electrical radian. This is equation 4.

Now, we shall define inertia constant H such that- you define inertia constant H such that G into H is equal to kinetic energy is equal to half M ω_s electrical radian electrical; that not radian ω_s electrical say this is mega joule. So, we are defining GH is equal to actually kinetic energy half M ω_s electrical this is in terms of mega joule; if wherever possible I have written the unit there will be no confusion then. Where G is equal to three-phase MVA rating that is the machine it is a base value, and H is equal to inertia constant in mega joule or MVA or megawatt second or second.

Actually, if it is your mega joule per MVA say you know that what you call joule per second is equal to what; that means, joule is equal to I am writing it simply short form

joule is equal to your watt second. That means mega joule is equal to megawatt second. That mean this one can be written as megawatt second slash MVA. So, MVA megawatt by MVA it is a dimensionless quantity that means H can be defined that in second also. So, if you read some book you will find inertia constant H is given as a second actually it is megawatt second power MVA. So, it is actually MVA megawatt MVA is dimensionless. So, it is in given in second.

So, that is why each inertia constant in mega joule MVA I told you it will or it is megawatt second per MVA or it is second. Next is that from equation 5, from this one.

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From eqn(5), we can write,

$$M = \frac{2GH}{\omega_{s_elect}} = \frac{2GH}{2\pi f}$$

$\therefore M = \frac{GH}{\pi f}$ MJ-sec/elect-radian (6)

or

$$M = \frac{GH}{180f}$$
 MJ-sec/elect-degree --- (7)

\rightarrow M is also called the inertia constant.

Assuming G as base, the inertia constant in per unit is

$$M(pu) = \frac{H}{\pi f}$$
 (8) [sec²/elect-radian]

So, you can write that M is equal to 2 G H up on omega s electrical is equal to you can write two G H up on 2 pi f omega s. Actually omega s is equal to 2 pi f. So, writing omega s electrical is equal to 2 pi f. Now M is equal to whatever I showed you here I have made it made it here also that mega joule is equal to megawatt second we will come to that later on.

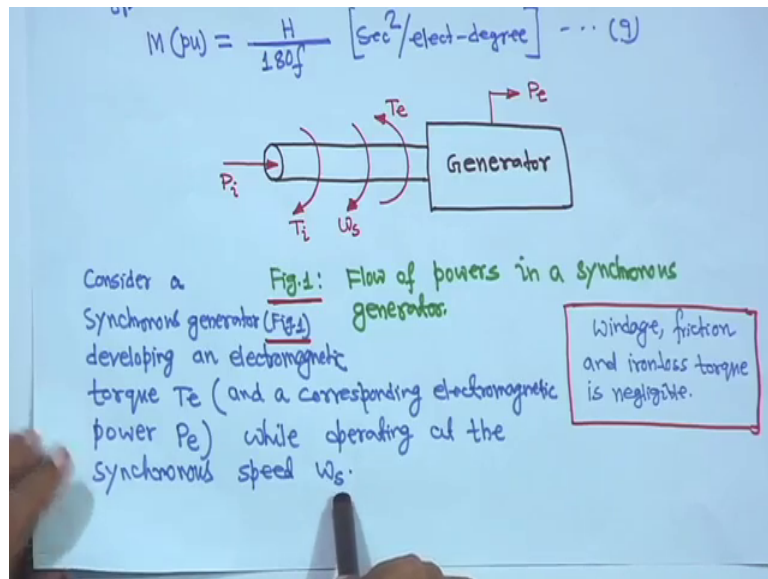
So, M is equal to G H up on pi f 2 2 will be cancel, it is mega joule second per electrical radian that is your M. Now, or if you want in degree then instead of radian pi you can put at 180 right. So, M is equal to J is up on 180 f that is mega joule second per electrical degree it is in radian if it is a pi; if it is 180 it is electrical degree this is equation 7. M is also called the inertia constant. That means, assuming G as a base the inertia constant in per unit is M pu is equal to M per unit is equal to H up on pi f.

That means M is equal to $G H$ up on say πf . If you divide both sides that G , actually G is that I have told you that three phase MVA rating that is the base of the machine, so if you divide by G on both side that is M by G is equal to your $H \pi f$. That means, this one we can put M per unit is equal to H by π . So, that is why that your M if your what you call that M is also called the inertia constant and assuming G as a base I showed you the inertia constant in per M pu which H up on πf that is second square by electrical radian. Because this is mega joule second and mega joule is equal to actually your megawatt second.

So, if you replace this your what you call the dimension wise if you replace like this- that just hold on; dimension or if you replace that mega joule second then electrical degree. About mega joule is equal to megawatts second, if you put is megawatt second it will make about second square; that will be your megawatt second square per electrical a degree now as this is the original unit. But if both side when you divide by its machine base G in that case the unit of this thing coming second square per electrical your radian or what you call the way you wants this is per electric radian and this will be if you want this one it will be second square per electrical degree. I have written here second square per electrical degree, because mega joule is equal to megawatt second, so if you put here it will be megawatt second square per electrical degree on both side if you divide by G it will be a per unit and this side.

And ultimate megawatt per MVA dimensionless, so ultimately unit will be your second square either second square per electrical degree or if you want a radian it will be second square or electrical radian. So, that is why M per unit is H up on πf and its unit is second square per electrical radian. This way it can be defined. So, there should not be any confusion for particular the unit. Little bit you practice then absolutely there is no problem.

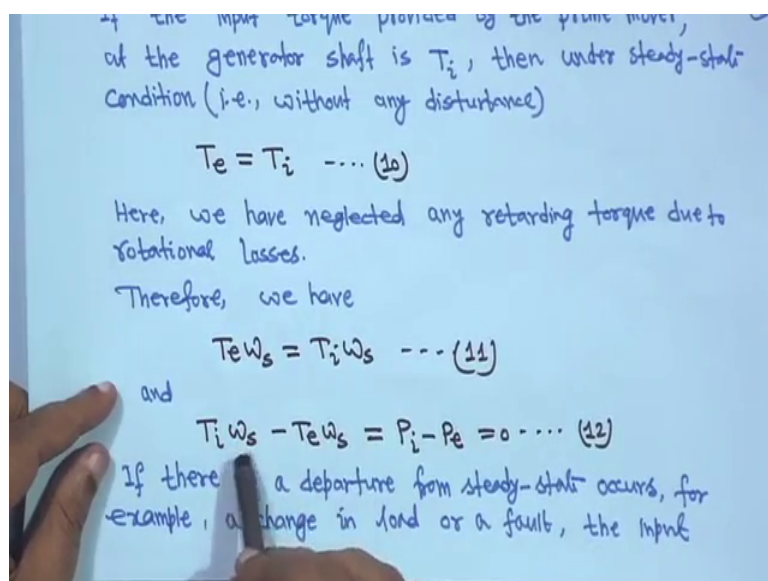
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Therefore, if we write in degree then M pu is equal to H up on $180 f$ second square or electrical degree. This is equation 9. Now will come to your; slowly and slowly will come to this swing equation other thing. Now, figure one actually flow of powers in a since this is a generator. So, consider a synchronous generator as shown in figure one; developing an electromagnetic torque T_e and the corresponding electromagnetic power P_e this is P while operating at the synchronous speed ω_s .

So, before telling this what we will do- we will assume windage friction and iron loss torque these are all negligible; all these things, any such thing is negligible we will not consider for our study. This all will be neglected.

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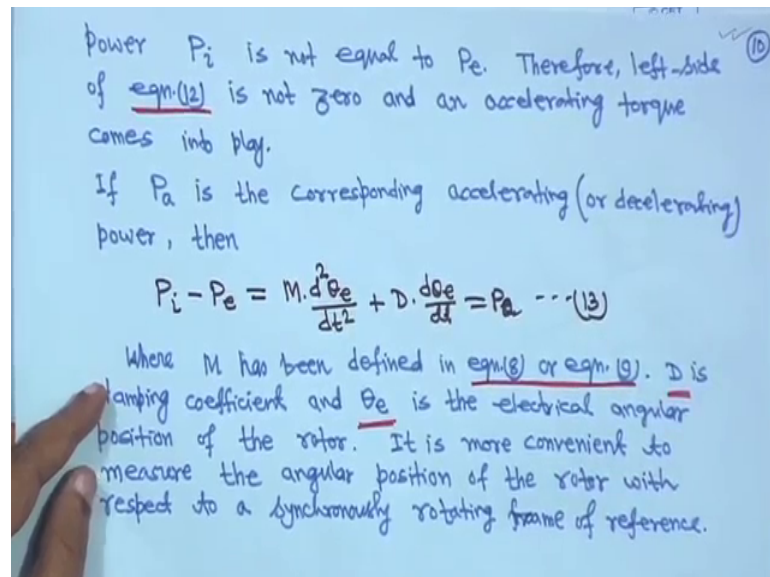


Now if the input torque provided by the prime mover; I mean if the input torque provided this is your input torque provided by the prime mover at the generating shaft that is T_i , this is the input torque, this is the input torque, then under steady state condition that is without any disturbance that T_e must be is equal to T_i . We are assuming all losses everything is neglected. So, T_e will be is equal to then T_i . So, that is without energy steady state condition. Now if you multiply both sides of this equation by ω_s the synchronous speed, so that is why here I am writing also here you have neglected any retarding torque due to rotational losses.

So, if you multiply both sides this equation by ω_s it will be $T \omega_s$ equal to $T_i \omega_s$. That means, this equation actually you know that your power is equal to torque into angular speed that you know; that means, that $T_i \omega_s$ putting this fast minus $T \omega_s$ is equal to we can write it is power this is P_i actually, P_i is equal to $T_i \omega_s$ and this is P is equal to $T \omega_s$ is equal to P_i minus P is equal to 0. This is 0 when system is at your steady state condition. That is this is equation 12.

So, now if there is a departure from the steady state occurs for example, a change in load or a fault this is the perfect condition at steady state say. And if there is a departure from steady state occurs for example, a change in load or a fault the input power P_i actually is not equal to your P_e . That means, that input power this P_i is not equal to P_e . I have taken here input power P_i some article book they are considering these is P_m , but I have taken its at input power P_i .

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So, the input power P_i is not equal to that electric power P_e , therefore the left side of equation 12 is not 0. That means, this P_i minus P_e actually it is not 0. And then acceleration torque comes to play. So, if your P_a is the corresponding accelerating power accelerating or decelerating, then this P_i minus P_e can be written as M into d^2 theta a dt square plus D into d theta e up on d theta e is equal to P_a ; P_a actually is equal to P_i minus P_e that accelerating or decelerating power- will come to that later on.

So, M we have seen M has been defined in equation 8 or equation 9, D is the damping coefficient and θ_e is the electrical angular position of the rotor. First we write like these way, although later we will assume that d is equal to 0 for this your study. So, it is more convenient to measure the angular; rather than this one more convenient to measure the angular position of the rotor with respect to a synchronously rotating your frame of reference: a rotating reference frame synchronously rotating reference frame. So, that is better more convenient to measure.

Even for stability studies we do so. That means, if we measure the angular position with respect to synchronously rotating your reference frame.

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Let

$$\delta = \theta_e - \omega_s t \quad \dots (14)$$

$$\therefore \frac{d^2 \delta}{dt^2} = \frac{d^2 \theta_e}{dt^2} \quad \therefore \frac{d^2 \theta_e}{dt^2} = \frac{d^2 \delta}{dt^2} \quad \dots (15)$$

Where δ is the power angle of the synchronous machine. Neglecting ^{damping} (i.e. $D=0$) and substituting eqn.(15) in eqn.(13), we get.

$$M \cdot \frac{d^2 \delta}{dt^2} = P_i - P_e \quad \dots (16)$$

Using eqns.(16) and (6), we get

That means your delta we define is equal to say theta e minus omega s into T. With respect to synchronously rotating reference frame; that is omega s into T. And if you take the double derivative of this equation 14 you will get d square delta dt square is equal to d square theta e dt square or other way you can write d square theta e dt square is equal to d square delta dt square. This is say equation 15.

And where delta is the power angle of the synchronous machine sometimes we call torque angle also right. So, neglecting damping your that is d is equal to 0 and substituting equation 15 in equation 13 then what we will get in this equation you put D is equal to 0. And this d square theta e dt square you replace by d square delta dt square. That means, your this equation can be written as M d square delta dt square is equal to Pi minus Pe. So, using equation 16 and equation 6; so if you equation 6 means that M is equal to in equation 6 M is equal to G H up on pi f- M is equal to J H up on pi f that is in equation 6.

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$$\frac{GH}{\pi f} \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ MW} \dots (17)$$

Dividing throughout by G, the MVA rating of the machine,

$$M(\text{pu}) \cdot \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \dots (18)$$

Where $M(\text{pu}) = \frac{H}{\pi f} \dots (19)$

or

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \dots (20)$$

Eqn.(20) is called swing equation. It describes the rotor dynamics for a synchronous machine.

So, put it here, $d^2\delta/dt^2$ is equal to $P_i - P_e$ that is this is megawatt. So, we are not taking the real unit of M not per not divided by G here. So, if you divide this throughout by G the MVA rating of the machine; that means, what will happen that divide by G. So, if you divide by that actually MVA rating of the machine means it is being shown base so you can write M pu into $d^2\delta/dt^2$. And if you divide by G the machine base megawatt will convert it to per unit. So, M per unit $d^2\delta/dt^2$ is equal to $P_i - P_e$ that is equation 18.

So, where M per unit we have seen $H/\pi f$ we have seen it before. Therefore, $H/\pi f$ this in into $d^2\delta/dt^2$ is equal to $P_i - P_e$; wherever is possible I have written wherever megawatt wherever per unit here it is per unit. So, this equation actually is called swing equation it describes the rotor dynamics for a synchronous machine. This is swing equation and it describes the rotor dynamics of the synchronous machine.

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Multi-Machine System.

In a multi-machine system, a common system base must be selected.

$$G_{\text{machine}} = \text{machine rating (base)}$$

$$G_{\text{system}} = \text{system base.}$$

Eqn. (20), can be rewritten as:

$$\frac{H_{\text{machine}}}{\pi f} \frac{d^2 \delta}{dt^2} = (P_i - P_e)$$

$$\begin{aligned} G_{\text{machine}} \cdot H_{\text{machine}} \cdot \frac{d^2 \delta}{dt^2} \\ = (P_i - P_e) \cdot G_{\text{machine}} \cdot \text{MW} \end{aligned}$$

$$\therefore \left(\frac{G_{\text{machine}}}{G_{\text{system}}} \right) \left(\frac{H_{\text{machine}}}{\pi f} \right) \frac{d^2 \delta}{dt^2} = (P_i - P_e) \cdot \frac{G_{\text{machine}}}{G_{\text{system}}} \dots (2)$$

So, once it is done then what we will do let us come to the multi machine system. Look multi machine system transient stability we will not study in this class; that is basically for postgraduate student. And multi machine system transient stability we cannot solve numerical in the classroom, because one has to follow the iterative technique. So, why I have written multi machine system means; we want to just find out the equivalent inertia constant that H h e q. So, that is why a multi machine system, but we will not study the transient stability analysis of multi machine system.

So, in a multi machine system a common system base must be selected. For example, earlier we are doing G and H , but what we will do instead of G will write G machine is equal to machine rating that is base value and this system is equal to system base. So, equation 20 then can be written as; that means, this equation instead of H I am putting H machine, but H and H machine it is same. Just the multi machine system, so you have to make it machine one machine two like this, but H machine H both are same. So, this equation 20 we can write that is H machine into divided by πf into d square δ by dt square is equal to P_i minus P_e . So, that is this equation.

Now if you multiply both side by say G machine that is the machine rating that is the machine rating, then G machine this equation I have written here for your understanding and because this is in per unit, but if I multiply the G machine the machine rating base, so G machine into H machine into d square δ dt square is equal to P_i minus P_e into G machine at the time it is converted to megawatt there. So, I have written here megawatt right.

So, now that whole thing, this whole equation now you divide by the system base. Suppose you have taken say G system at different base. So, divided by G system this whole equation you divided by G system that is system base, this equation. If you do so it will become G machine upon G system, G machine up on G system into H machine d square delta dt square H machine your here I have missed one thing that is your what you call that pi f term I have missed, right

So, this pi f one missing here, so now I have written. So, into d square delta dt square is equal to that H machine up on pi f d square dt square is equal to Pi minus Pe into G machine by G system. This is equation 21. So, everything is same, but we have a different base now.

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$$\therefore \left(\frac{H_{\text{system}}}{\pi f}\right) \frac{d^2 \delta}{dt^2} = (P_i - P_e) \text{ pu on system base} \dots (22)$$

where

$$H_{\text{system}} = \left(\frac{G_{\text{machine}}}{G_{\text{system}}}\right) \cdot H_{\text{machine}} \dots (23)$$

= machine inertia constant in system base.

Machines Swinging in Unison (Coherently)

Let us consider the swing equations of two machines on a common base, i.e.,

So, H system by pi f into then d square by delta square- that means, this thing what you call whatever here you have in terms that pi f is there, but other things G machine into H machine up on G system will write that one as H system. So, we are rating something H system up on pi f into d square delta T square is equal to Pi minus Pe per unit on system base. That means, Pi minus Pe and G machine up on G systems, so we are writing it is per unit on system base. This is per unit on system base, because G machine up on G system.

So, each system is equal to G machine H machine up on G system. So, H system is equal to G machine H machine divided by G system that is system base that is 23. That means,

this is machine in inertia constant in system base. So, this is require actually when you try to find out that equivalent inertia of the your machine for a multi machine system. And for a multi machine system that when a group of generators they operate in parallel they swing in unison; that mean they are in coherent group; that means, their increase or decrease of the speed will be same. That means, there will be in coherent group, sometimes we call coherency.

So, we will come back to that.