Power System Analysis
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Lecture - 53
Symmetrical Components (Contd.)

In the previous class we ended with that unbalanced or unsymmetrical fault analysis. So, different types of unbalanced fault that occur in a power system is one is called shunt type fault another series type of faults. So, shunt faults are of you know three types: that is single line to ground fault that is we call L-G fault, then line to line fault we call L-L fault.

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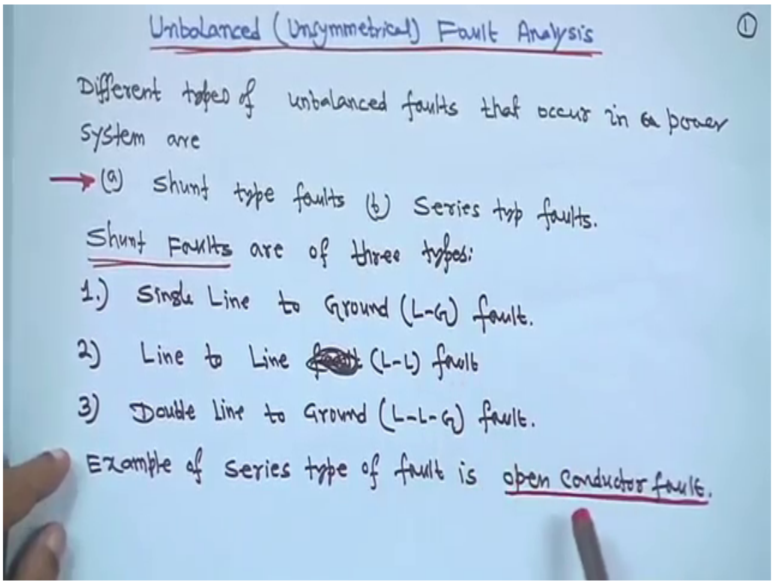

And double line to ground fault that is L-L-G fault. So, these are the fault very common in transmission system, but I mean it is not like that simultaneously if fault occurs in one phase say line to ground fault another two phase is line to line fault. These kinds of occurrence are very rare, but this three are very common. And the example of series type of fault is that open conductor that is open conductor fault. For example one phase suppose one conductor has broken or may be two conductors broken, but this are very rare faults but still will see, will go for this kind of analysis.

So, already we have studied the symmetrical component and positive negative and zero sequence components. And in to that most important is that zero sequence network for the fault studies that is more important. So, first what you will do; will study that your line to ground fault that is your L-G fault.

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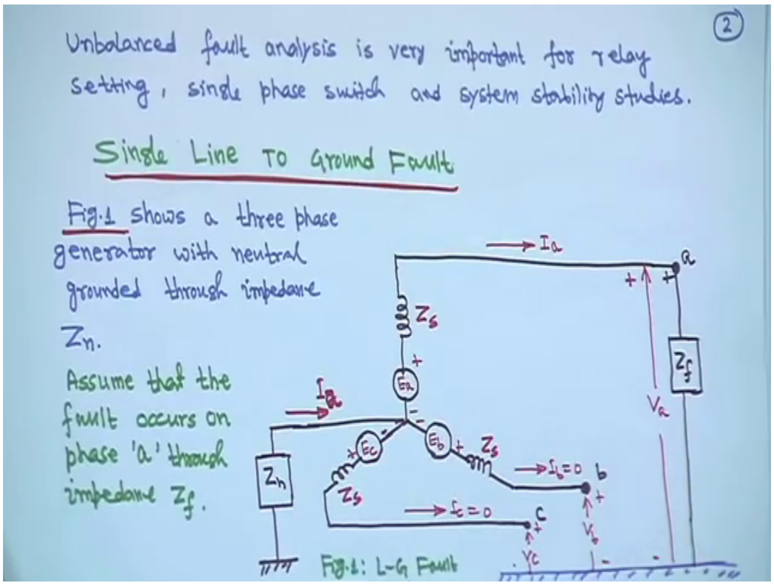

For example: suppose that your single line to ground fault. So, suppose this is figure one suppose at phase a that there is a single line to ground fault right. Therefore, your phase b and c the bound I mean that is the boundary condition right that is I b will be 0 and I c also will be 0, because we assume in that fault has occurred this thing. So, figure this one is shows a three phase generator with neutral grounded through impedance Z n, then neutral is grounded. And we are assuming that fault occurs at phase a through impedance Z f. So, fault impedance is there so it is your Z f. That means, this V a is equal to actually this your I a into Z f. And your this thing that current through phase b and phase c it is 0; I b is equal to 0 I c is equal to also 0 because fault has occurred here.

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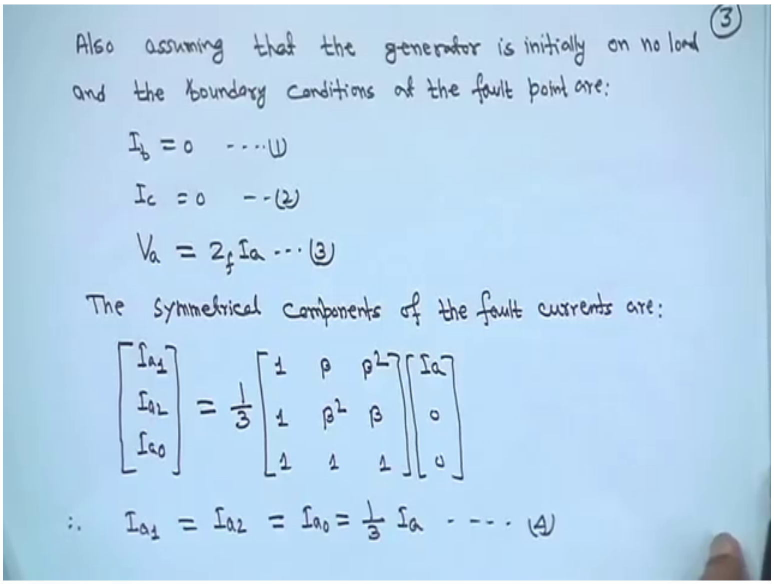

So, in this case assuming that the we have we will make some assumption that generator is initially on no load and the boundary conditions of the fault point are that I b is equal to 0 and I c also is equal to 0, because fault has occurred in phase a. And V a is equal to Z f I a. So, this is the fault current this I a is going through the fault, so V a is equal to Z f in to I a.

So, the symmetrical component of the fault currents are we know this from our symmetrical component analysis, we know this that I a1 is equal to I a2 is equal to I a0 is equal to one-third 1 beta beta square, 1 beta square beta and 1 1 1: this is I a, I b is equal to 0, I c is equal to 1.

Therefore this I a1 is equal to I a2 is equal to I a0 is equal to one-third I a, you will find I a1 also one-third I a, I a2 also one-third I a, and I a0 also one-third I a. That means, I a1 is equal to I a2 is equal to I a0 all are equal that is one-third of the I a. This is I b 0 is you a marked equation 1, I c 0 we have marked equation 2 and V a is equal to Z f in to I a we have marked equation 3. And if you express equation 3 that equation 3 that V a is equal to actually.

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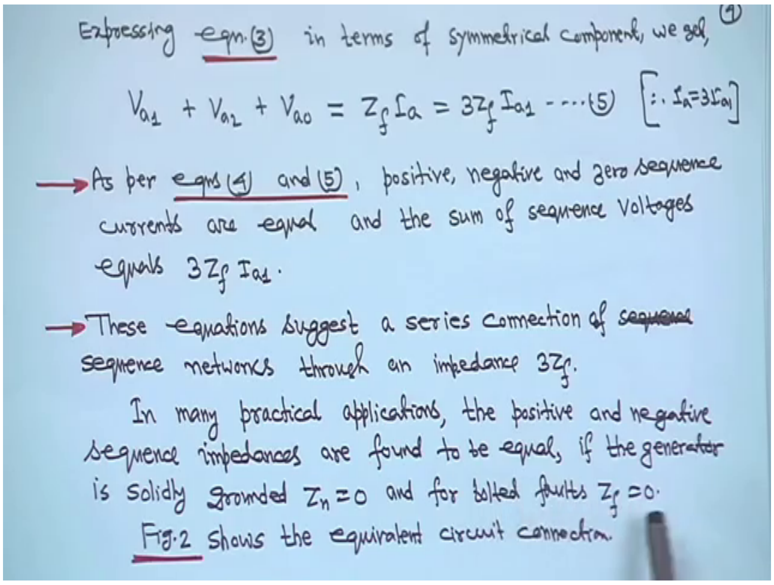

V a is equal to V a1 plus V a2 plus V a0. So V a is equal to Z f in to I a and is equal to Z f in to I a, but your I a is equal to your 3 I a1, because here you know I a1 is equal to I a2 is equal to I a0 one-third I a. So, I a is equal to 3 into I a1. So, here we are putting 3 Z f I a1. In bracket I have write in I a is equal to 3 I a1. That means, as per equation 4 and 5 then if you come to this equation 4 that all I a1 I a2 I a0 I, I mean they are all equal is equal to one-third I. Its close actually, it is a series connection of the positive negative and zero sequence network, because all component positive negative and zero sequence components they are same and they are equal to one-third I a.

That means, an equation and this equation 5 V a1 plus V a2 plus V a0 that is 3 Z f in to I a1. That means, this positive negative and zero sequence currents are equal and the sum of the sequence voltages is equal to 3 Z f I a1.

So, these equations suggest particularly this one equation 4, because all your sequence currents are equal. Therefore, the these equations suggest a series connection of sequence network; twice I have written sequence, sequence network through an impedance 3 Z f; because it is 3 Z f, this is 3 Z f in to I a1. In many practical applications the positive and negative sequence impedances are found to be equal. That also we have discussed before. If the generator is solidly grounded then Z n is equal to 0. That means, in this thing if your generator is solidly grounded then this Z n will be 0. And for bolted faults that Z f will be 0.

That means, figure two shows the equivalent circuit connection. So, how we have made this connection? I mean you will make it like this that first what we will do it is a serious connection.

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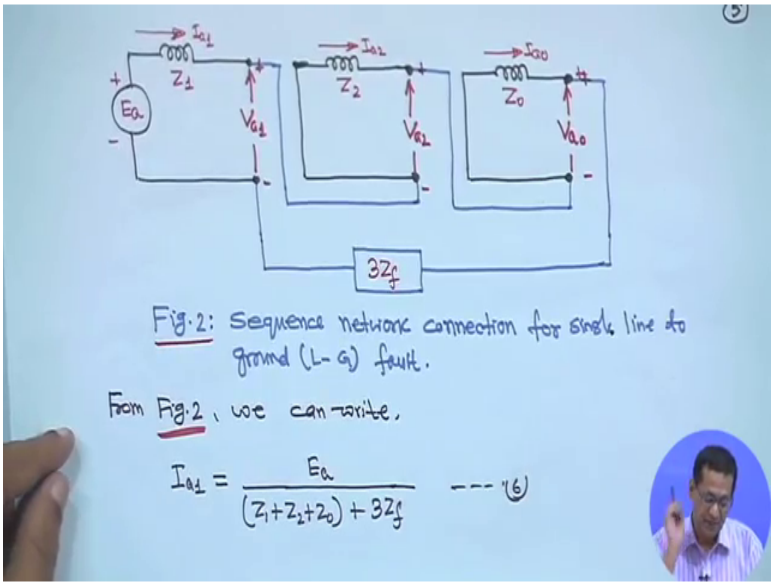

So, first the black ink I have made it, this is your positive sequence. First you make this is your positive sequence network, so E a will be there plus minus this is Z 1 positive sequence impedance and current is I a1. Next that black one there is other two there will be no voltage source, so this is Z 2 and this voltage is V a2. And for zero sequence this is Z 0 current is I a0 and this is your V a.

Now as it will be series connection and all I a1 I a2 and I a0 currents are same, is equal to one-third I a all currents as same so this series connection this is plus this blue one plus will be connected to the minus. Similarly here from negative sequence that from the plus terminal it will be connect to the minus and finally from zero sequence plus will be connected to the minus through 3 Z f, because your V a1 plus V a2 plus V a0 is equal to 3 Z f.

So, in this case you can see that this I a1 I a2 I a0 all are same, because this current is flowing like this, flowing like this, flowing like this, and flowing like this and returning to that here. And this is actually sequence connection for single line to ground fault. And if you apply what you call that KVL thing then will find I mean will come to that. So, V a1 plus V a2 plus your V a0 that will be equal to your 3 Z f in to I a0 because all are same actually I a1 I a2 I a0 all three are equal is equal to one-third I a. So, in this case now from this figure we can write that I a1 is equal to this is the voltage a divided by your Z 1 plus Z 2 plus Z 0 and plus 3 Z f, because it is a series circuit. If I make this one as something like this, it is a series circuit this is your Z 1, this is Z 2, this is Z 0, and this is say your 3 Z f and this is your voltage E a, this is Z 1, this is Z 2, this is Z 0, and this is 3 Z f. And current is here your I a1 is equal to I a2 is equal to I a0.

So, that is why as all three as same you are writing that I a1 is equal to E a up on Z 1 plus Z 2 plus Z 0 plus 3 Z f. So, this is that simple one. So, fault current I a then is given by because this is I a1, therefore fault current is then given by I a is equal to 3 I a1. So, I a1 is equal to this 3 in to then I a1 you substitute here. This is equation 7. Now under L-G fault condition the voltage of the line b to ground.

Now V b is equal to underline to ground fault we know this expression beta square V a1 plus beta V a2 plus V a0 you know beta is equal to e to the power j 120 degree. That we have seen also. Therefore, that V a1 from this diagram, from this positive sequence diagram Z 1 I a1 I am just telling for my this thing. From this diagram Z 1 I a1 plus V a1 minus E a is equal to 1. So, I am writing Z 1 I a1 plus V a1 minus E a is equal to 0 here.

That means, V a1 is equal to E a minus Z 1 I a1. So, here we are substituting V a1 instead of V a1 you are like beta square in to E a minus Z 1 I a1. Now plus beta now V a2 now in the negative sequence you apply again in this network KVL. So, it will be Z 2 I 2 plus V a2 is equal to 0. So, Z 2 I a2 plus V a2 is equal to 0. So, V a2 is equal to minus Z 2 I 2.

Similarly, in the zero sequence network Z 0 I 0 plus V a0 is equal to 0, therefore V a0 will be minus Z 0 I a0. So, here I am writing V a0 is equal to minus Z 0 I a0. So, here in the expression of V a1 we are substituting E a minus Z 1 I a1 in to beta square, then plus beta in to that V a2 is equal to minus Z 2 I a2 plus this minus Z 0 I a0. Or V b is equal to we can write that I a1 is equal to I a2 is equal to I a0 is equal to I a by 3 we know that. So, it will be E a minus Z 1 in to I a by 3 plus beta in to minus Z 2; instead of I a2 you write I a by 3 plus minus Z 0 I a0 is equal to I a by 3. This is equation 8.

Now from this equation 7 we are writing from equation 8 and 7, so from this equation what you will do you will this I a is equal to this expression 3 E a up on Z 1 plus Z 2 plus Z 0 plus 3 Z f this I a you put it here you put in this equation 8; you put in equation 8 and you please simplify such that I am giving you the final expression.

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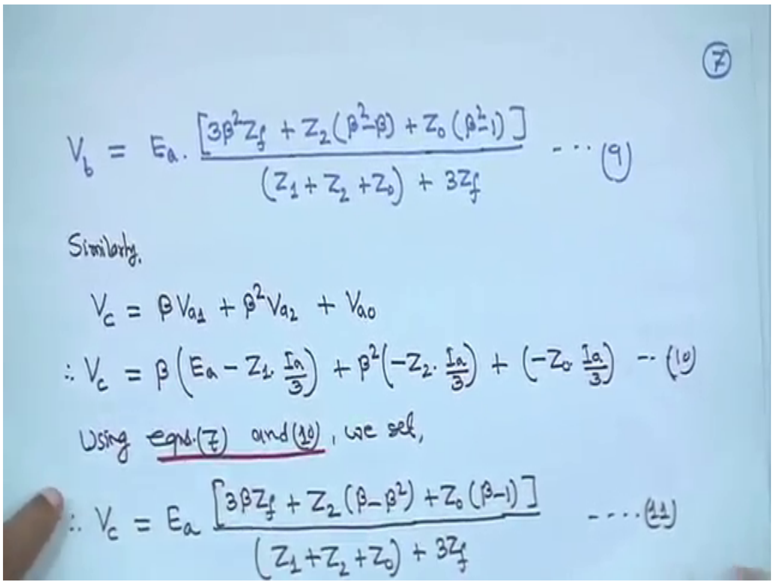

If you simplify you will get V b is equal to E a in to 3 beta square Z f plus Z 2 in to beta square minus beta plus Z 0 beta square minus 1 divided by Z 1 plus Z 2 plus Z 0 plus 3 Z f. This is equation 9. Similarly if you do in similar fashion for phase voltage your V c then V c will be beta V a1 plus beta square V a2 plus V a0. Then again same as before substitute V a1 is equal to E a minus Z 1 in to I a 3 because I a1 is equal to I a 3. Similarly plus beta square minus Z 2 in to I a2 I a2 is equal to I a by 3 plus in bracket minus Z 0 in to I a by 3; I a0 is equal to I a by 3 So, equation 10.

Here also you substitute expression of I a from your equation 7. If you substitute expression of I and simplify then you will get V c is equal to E a into 3 beta Z f plus Z 2 in to beta minus beta square plus Z 0 beta minus 1 divided by Z 1 plus Z 2 plus Z 0 plus 3 Z f. And if fault impedance is 0 then this term will be drop and this term will be dropped. From this equation 11, similarly fault impedance is 0 then from this equation also this term will be dropped if Z f is equal to 0 this will be dropped.

So, that is for your single line to ground fault, these are the expression.

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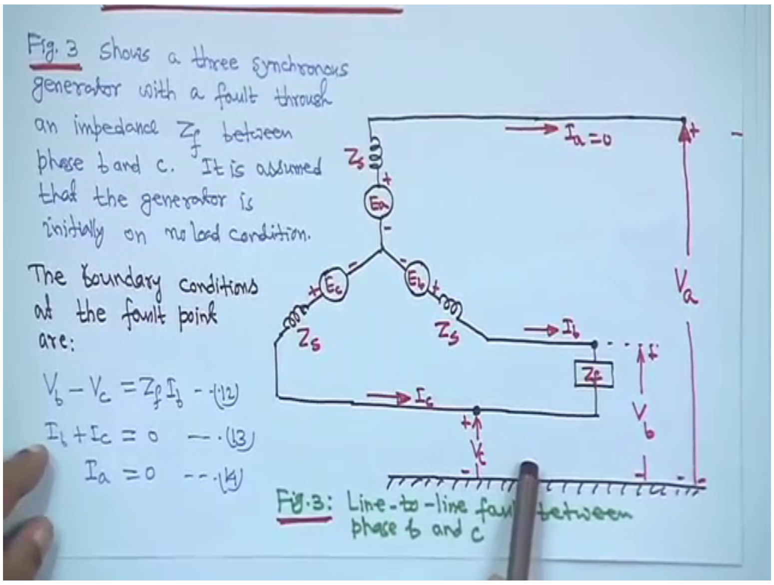

Similarly, now let is consider line to line fault that is L-L fault. So, line to line fault. So, suppose in phase b and c we are assuming there is a line to line fault. So, figure 3 actually this is figure 3; it shows a three synchronous generator with a fault through an impedance Z f. So, there is a fault, line to line through an impedance Z f when Z f is there, if it is not there it will be your Z f will be 0. So, between phase b and c it is assumed that the generator is initially on no load condition.

Now the boundary condition at the fault point are, so fault this is a line to line fault through impedance Z f that means, I a will be equal to 0. And second thing is that V b this is ground as reference. So, V b minus V c this what you call we can say this is Z f in to I b, but question is that this is I c this is I b; that means, I b plus I c if you is equal to 0. Because I c we have taken this direction, I b this direction at point that is basically it is going I b flowing this direction, this is this direction so opposite direction, so I b plus I c is equal to 0.

And V b minus V c it will be Z f in to I b, because if you apply your KVL like your this way, so it will be your V b minus V c is equal to; that means I mean in this way we put I b in to Z f plus V c minus V b is equal to 0. Therefore, V b minus V c is equal to I b in to Z f. That means, I b plus I c is equal to 0 this is another condition and I a is equal to 0. This are all the you call sometimes boundary condition at what you call fault point. That means, now with this with this fault condition will try to find out other things.

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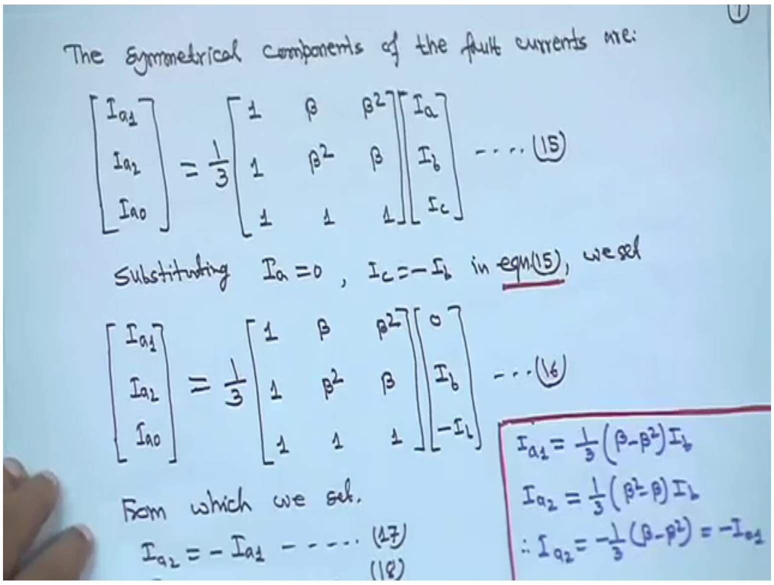

So, the symmetrical components of the fault currents are: that means, we know this I a1 I a2 I a0 is equal to one-third 1 beta beta square, 1 beta square beta and 1 1 1, this is I a I b I c. So, this is actually rewriting this equation from the previous topic your symmetrical component.

Now I a is equal to 0 and I c is equal to minus I b, this are the boundary condition for your line to line fault through impedance Z f. So, if you put this we are putting I a is equal to 0, I b will remain as I b and I c is equal to minus I b. So, this is actually minus I b. From which you will get that I a2 is equal to minus I a1, I mean from this equation another will get I a0 is equal to 1 that means no zero sequence current. So, if you take I a1 here, I have written here that I a1 is equal to from here one-third your beta minus beta square I b, and I a2 is equal to from this equation only one-third beta square minus beta I b.

That means I a2 is equal to minus 1 third beta minus beta square is equal to then minus I a1. That is why we are writing from this I a2 is equal to minus I a1 and I a0 is equal to 1. So, that is line to line fault there is no zero sequence current. And I when I a2 is equal to minus I a1 I mean it is just in opposition. So, will see later all this.

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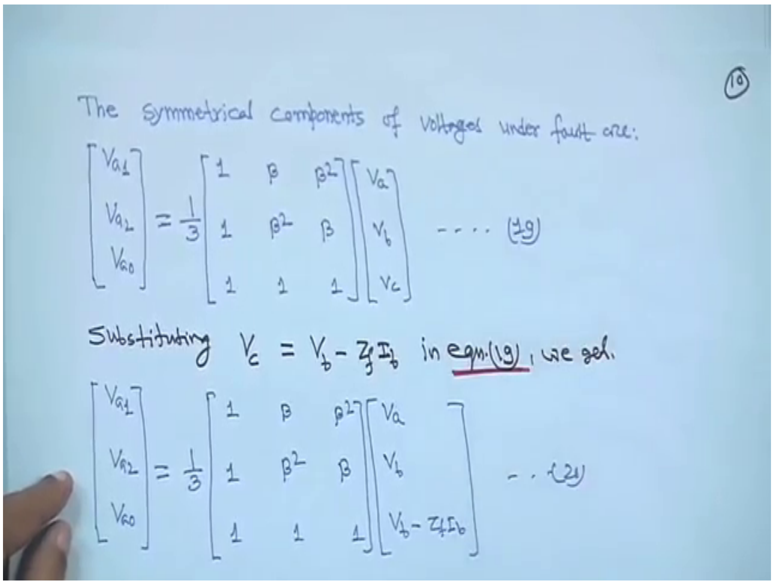

With this the symmetrical component of the, your what you call symmetrical component of the voltages under fault are that V a1 V a2 V a0 we know this repeating from the symmetrical component only that one-third 1 1 1, beta beta square 1, beta square beta 1, V a V b V c. Now we know V c is equal to from this equation 12 V c is equal to V b minus Z f in to I b. So, this V b minus z V a V b will be there, but this V c is equal to V b minus Z f I b we are substituting in equation 19. So, here it is V b minus Z f in to I b. This is equation 20.

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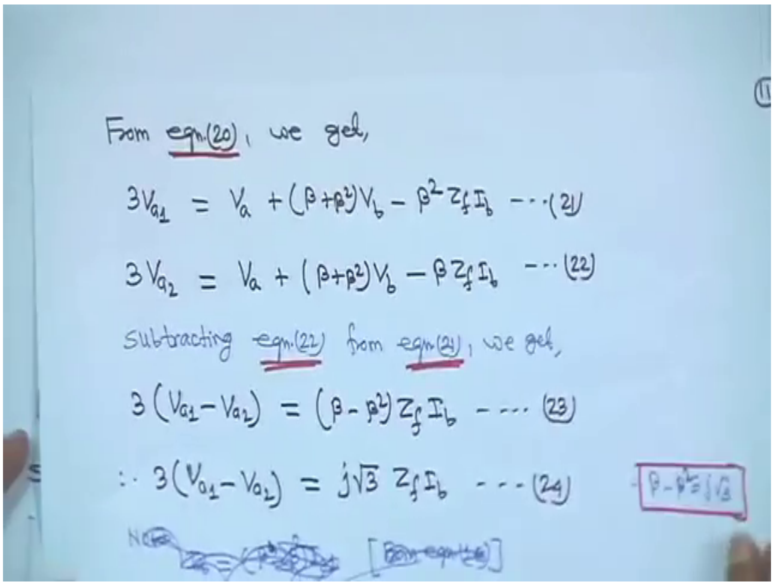

Now from equation 20, what you will get you make a first V a1 I mean this one V a1 if you make it will be one-third of everything, so cross multiplication I am writing 3 V a1 is equal to V a plus beta plus beta square V b minus beta square Z f I b. That means, from this equation you can write V a1 actually one-third in to V a plus beta V b plus beta square in to V b minus Z f I b. If you simplify and cross multiply by 3 this side then actually you will get 3 V a1 is equal to V a plus beta plus beta square V b minus beta square Z f in to I b. This is actually from this one, just you write one line, I am not writing understandable very simple thing.

Similarly 3 V a2: similarly we are making it that this V a2 is equal to one-third of this, so 3 V a2 from this equation will become V a plus beta plus beta square V b minus beta in to Z f I b, this is equation 22. Now subtracting equation 22 this one you subtract from this equation 21 you subtract. If you do so you will get 3 V a1 3 into V a1 minus V a2 is equal to beta minus beta square in to Z f I b. This is equation 23. If you subtract V a Va will be cancel, even beta plus beta square, beta plus beta square V b also will be cancel ultimately it will remain beta minus beta square Z f in to I b. This is equation 23.

Look we are not doing anything on your what you call on V a0. The reason is very simple that initially we have seen that zero sequence current I a0 is equal to 1. That means, in that equivalent circuit that zero sequence network will not appear for line to line fault, because I a0 is equal to 0 will come to that. That means this beta minus beta square if you please simplify beta is equal to e to the power j 120 degree, so beta minus beta square actually it will become j in to root 3. That means, this beta minus beta square we are putting here j in to root 3 into Z f in to I b.

Therefore, from equation 16, again I have to just hold on again I have to go back to equation 16. From equation your 16 this one we can write that one-third beta I b minus beta square I b; that means, from this equation 16 I a1 is equal to one-third 1 into 0 then beta I b minus beta square I b. So, that is why from equation 16 we are writing that I a1 is equal to one-third then beta I b minus beta square I b is equal to one-third beta minus beta square in to I b.

So, I a1 is equal to one-third and beta minus beta square is equal to j in to root 3; I gave you that beta minus beta square is equal to j in to root 3. So, here if beta minus beta square is equal to you put here j in to root 3. So, I a1 is equal to one-third in to j root 3 into I b is equal to j by root 3 I b. That means, I b is equal to root 3 up on j I a1 numerator and denominator multiply by j, so it will be j square minus 1 so it will be I b is equal to minus j root 3 into I a1. This is equation 25.

Now using equation 24 and your 25; that means in this equation that equation 24 in this equation you put the expression of I b from equation 25. So, in this equation you put I b is equal to minus j root 3 into I a1; that is why using equation 24 and 25 we get you if you put in this equation I v is equal to your minus j root 3 I a1 then you will get V a1 minus V a2 is equal to Z f in to I a1. This is equation 26.

So, equation 17 and 26, so I have to come back to equation 17; this equation 17; that means, I a2 is equal to minus I a1 this equation 17. Even 18 also it suggest because I a0 0 means zero sequence network will not come up here because I a0 is equal to 1. So, from equation 17 and from equation 26 that means, this equation, can be represented by connecting the positive and your negative sequence networks in opposition and the equivalent circuit is shown- I will show you.

So, from this equation only you have to judge that how this network will be connected. So, if you so, look how we can make it.

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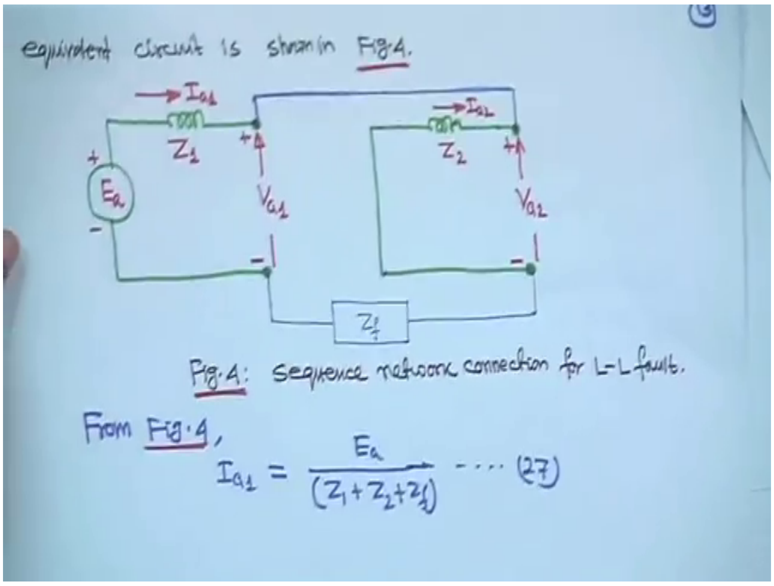

That in this case, this is our green color, this is our positive sequence network this is our negative sequence network, but we know your I a2 is equal to your minus I a1. That means, I a1 is equal to minus I a2 other way I a2 is equal to minus I a1; that means, I a1 is equal to minus I a2, that means there is no opposition. That means, this is plus minus V a1, plus minus it is V a2 negative sequence. So, as there in opposition plus will be connected to the plus.

And similarly minus is connected to the minus. And from this equation if you apply KVL then you will see V a1 minus V a2 is equal to this condition, Z f in to I a1. So, this is your V a1 this is your V a2. Apply now KVL here then it will be like this plus your, if you come like this then plus your V a2. Then this current is your what you call if you take this way I a1 then it will be I a1 Z f that is actually nothing but minus I a2 right minus V a1 is equal to 1. That means, you will get V a1 minus V a2 is equal to Z f I a1.

And if I make it for you just this thing; if you make it like this. So, this is V a2, so it is plus so it is V a2 right then this I a1 current let us take suppose this I a1 movingly of course, I a1 is equal to minus I a2, but will consider this I a1 only moving like this, so it is your I a1 Z f and then minus V a1 is equal to 0. That means, V a1 minus V a2 is equal to I a1 Z f.

So, that is why this equation V a1 minus V a2; this two suggest actually current direction that it will be opposition. And zero sequence network will not come because I have shown you that I a0 is equal to 1. So, this is that your line to line fault that equivalent circuit connection positive sequence and negative sequence. Through that impedance of course is there.

Now from this figure you can write I a1 is equal to E a up on Z 1 plus Z 2 plus Z f. I mean this direction if you take and apply your KVL like this then you will get I a1 is equal to E a up on Z 1 plus Z 2 plus Z f. So, this is equation 27. And as it is line to line fault at the beginning we have seen that I b is equal to your minus I c.

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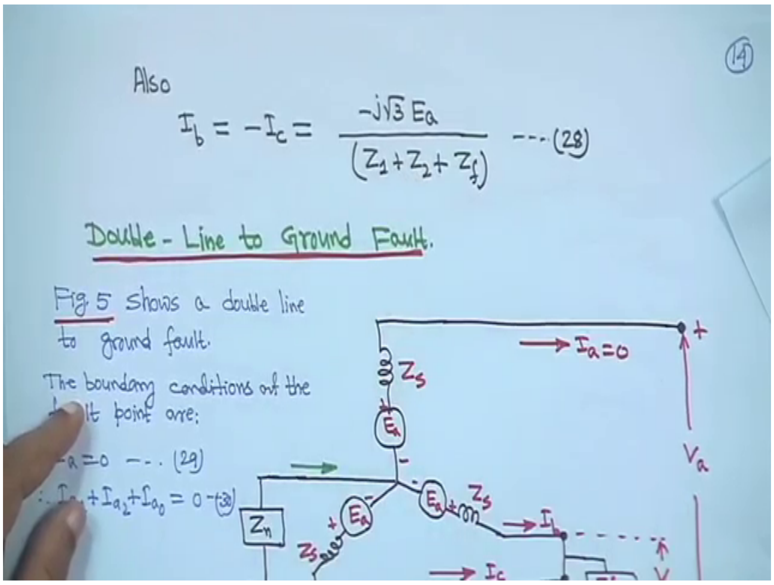

I b is equal to your minus I c it can be made minus j root 3 E a divided by Z 1 plus Z 2 plus Z f. So, this is the expression of because I a1 is equal to your E a up on Z 1 plus Z 2 plus Z f and I b this one, I b expression is this one minus j root 3 I a1. So, this I a1 you substitute in equation 25 I have not written here. So, in this expression you know I b is equal to minus I c that directly I am writing I b is equal to minus I c is equal to, but I b is equal to minus j root 3 into I a1. So, this I a1 you directly substitute. Then you will get this I a1 then you will get I b is equal to minus I c is equal to minus j root 3 E a divided by Z 1 plus Z 2 plus Z f this is equation 28. So, this is actually your line to line fault.

Now next one is that double line to ground fault

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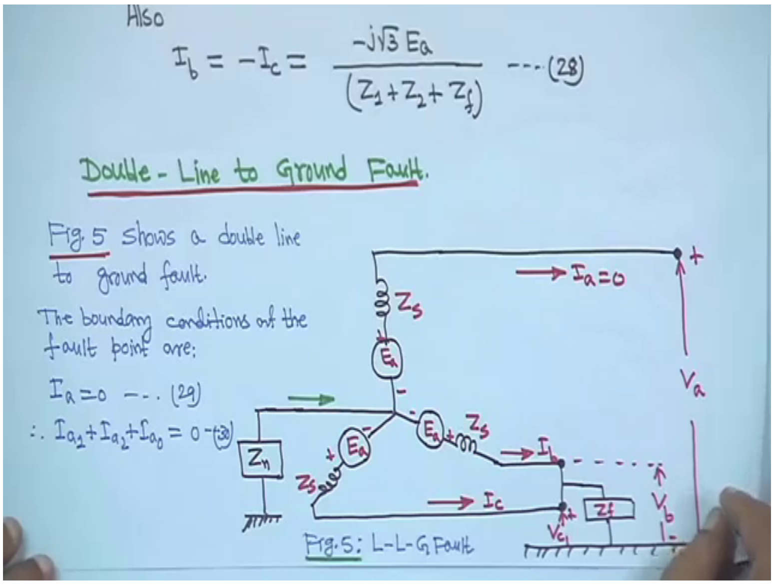

That is L-L-G fault. For example: suppose this two line your first let me tell you this is figure 5 this is double line to ground fault, we call L-L-G fault. So, figure 5 shows a double line to ground fault that is this phase b and phase c this two are short circuited then it is your double line to ground fault and from one fault impedance Z f is here. So, in this case that I a is equal to 0 because it is double line to ground fault I a is equal to 0.

That means if I a is equal to 0 means I a1 plus I a2 plus I a0 is equal to 0, I a is equal to I a1 plus I a is equal to this thing 0, then V b is equal to your what you call V c V b is equal to this total current this is this current is I b, this current is I b this is I c. So, in the fault current is going I b plus I c, I have not written here but this current is I c this is I b. So, in this case that current is I b plus I c this is the current going to a fault say during fault. That means, you can write and this two point are short circuited these two point actually together. Just here I have made it separately for your understanding this two point are short circuited together.

That means, V b is equal to your b then before showing you then V b will be is equal to V c will be is equal to your what you call Z f. The idea is something like this just two points that b and c phases are short circuited.

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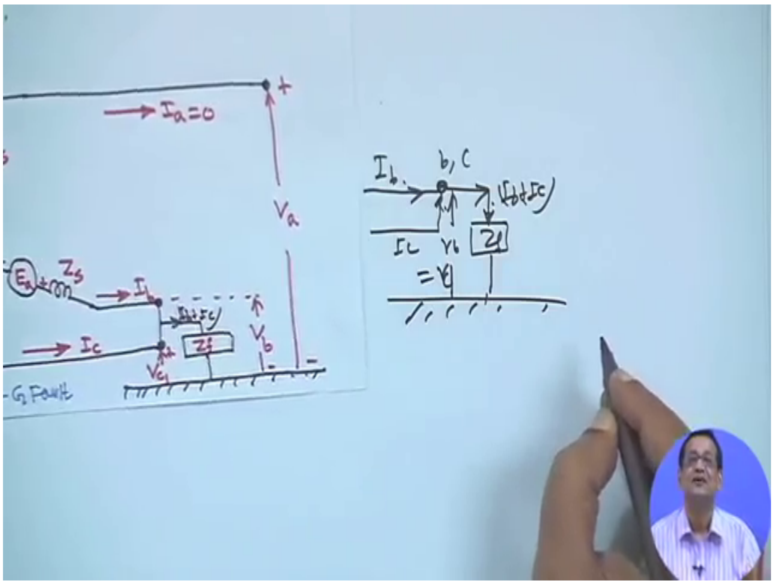

That means this is your b and c short, this is a common point right b and c together short circuited. So, in this case current is coming from your this thing for phase b I and from c it is your I c I b and I c and this is your what you call this is your fault impedance Z f right and this is this is ground. So current here, current actually going through this I b plus I c. As this two points are common, so b v and b c both are same this is b v this two point are common is equal to b c. Therefore, b v is equal to b c is equal to Z f in to I b plus I c because this two points are common.

So, in that case; I hope this simple thing I hope you are understood. So, in that case your we can say that b v is equal to b c is equal to I b plus I c in to Z f is equal to 3 Z f in to I a0 this is equation 31.

Thank you, again we will be back.