Power System Analysis
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Lecture - 05
Resistance & Inductance (Contd.)

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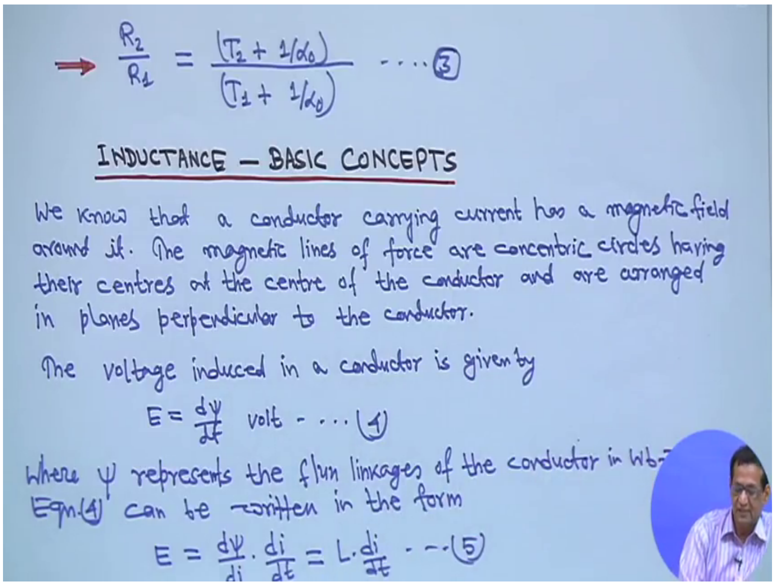

So, now we will go for the inductance of transmission line the basic concepts, right. So, we know that a conductor when carrying current has a magnetic field around it for example, suppose if we apply right hand rule, suppose if this is your conductor and if you have the conductor in the like this then if this is the direction of the current flow then this is that anti clock if you see that anticlockwise this is the direction of (Refer Time: 00:52) So, whenever a conductor carrying current it has a magnetic field around it right. So, the magnetic lines of force are concentric circles right having their say having their centers at the centre of the conductor if this is the conductor if this is the conductor then they are concentric circles right and there centre at the centre of the conductors. If I rapid like this and if this is the direction of the current the term the upwards then you have the magnetic field right, but they are concentric circles actually and there centre will be at the centre of the conductor, right.

So, the voltage in that conductor right induced in the conductor can be written as e is equal to their; your D psi by D t volt right everywhere wherever possible I have written the units right. So, it is e is equal to D psi by D t volt this is actually equation number 4 right where psi represent the flux linkages of the conductor in Weber tones this is Weber tones right. So, this equation 4 it can be written in this form right in this form it can be written. So, it is e is equal to D psi by D t it would make it like this e is equal to D psi upon of D I into D I upon D t is equal to you write down L into D I upon D t this is equation 5 where L is equal to D psi by D I all right so; that means, L it actually is a constant of proportionality it is basically inductance right. So, for a linear when L is equal to D psi L is equal to D psi by D I is the inductance in Henrys.

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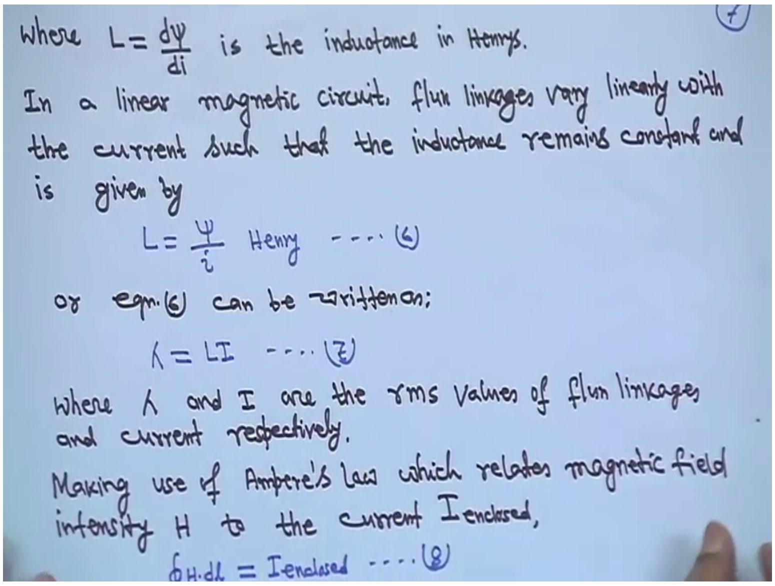

In a linear magnetic circuit right flux linkages vary linearly with the current right such that the inductance always remains constant. So, if the magnetic circuit is linear then instead of a and flux linkages vary linearly with the current therefore, inductance always we will remain constant; that means, this L is equal to D psi by D I we can write L is equal to psi by I in Henry right. So, as it is linear magnetic circuit. So, we can write like this that L is equal to psi by i, right. So, in that terms will remain constant. So, it is in Henry. So, in this is given equation number 6, right.

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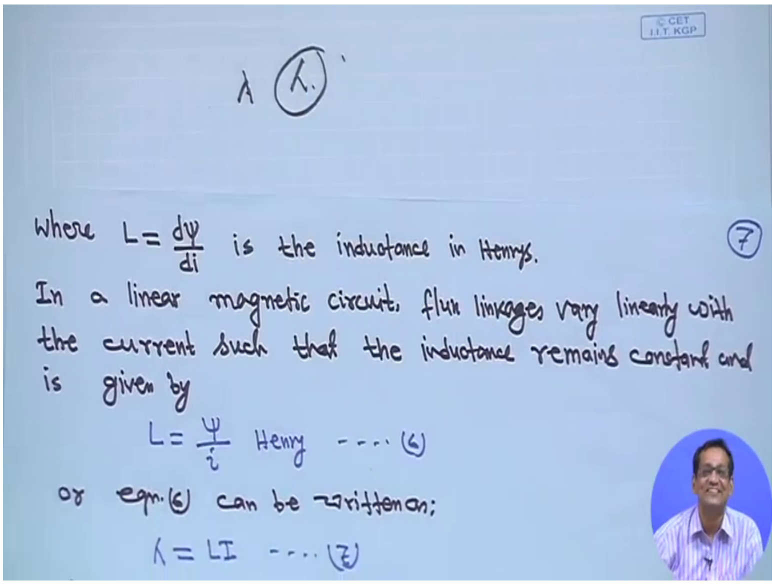

So, actually one more thing I would like to tell that certain look actually lambda symbol is like this right, but my habit has become to write lambda like this. So, throughout this I have to use this, so, over all, there should not be any symbolic change or anything. So, this high actually we use lambda c this thing as a flux linkages. So, it provide equation 6 we can write that lambda is equal to we will actually lambda is equal to psi right flux linkages, but we representing it by lambda now right and lambda and psi are the RMS values of the flux linkages and current respectively these are the RMS values and this is equation 7 right.

So, making use of amperes law if you that is that making use of amperes law which relates magnetic field intensity H and to the current enclosed that is I suffix is enclosed current I enclosed right and relates magnetic field intensity is the loop integral this integral one this is called loop integral a H into D L is equal to I enclosed right so; that means, this is your amperes law that you know from you know physics right. So, this is equation number 8 right at the same time because later this equation will be used this equation will be used right.

Now, the flux density right that is in Weber per meter square right is given by B is equal to mu into H that you know, right.

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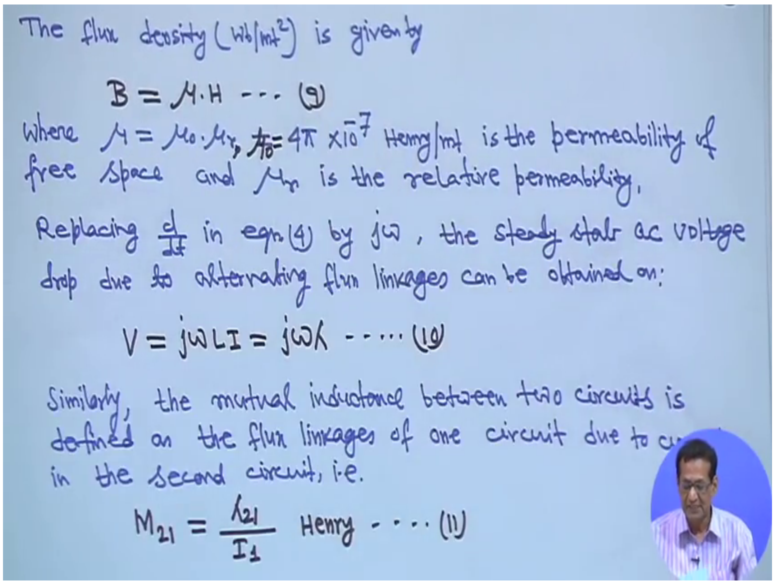

So, this is equation 9 where mu is equal to mu 0 in to mu R and mu 0 is equal to 4 pi into 10 to the power minus 7 Henry per meter is the permeability of the free space and mu R is the relative permeability right. So, what we will can do it. So, replacing d; D by D t in equation 4; that means, this equation this equation 4; that means, this equation right you replace D by D t right by j omega and psi instead of psi I said psi and lambda both are same you take. So, in that case that in equation 4 if you replace D by D t by j omega the steady state a c voltage drop due to alternating flux linkages can be obtained as that V is equal to j omega L I that is j omega lambda; lambda is equal to L i.

Psi is equal to lambda; lambda is equal to L i. So, these voltages drop in V j omega L i. So, j omega L is nothing, but that your reactance you know you can I is the RMS value of the current so; that means, j omega and L I is equal to your flux linkage lambda. So, B is equal to j omega lambda this is actually equation 10; right. Now similarly if you want to find out the mutual inductance between 2 circuits it is defined as the flux linkage of one circuit due to current in the second circuit right. So, it is it may be given as generally it is m a that is m 2 one later we will see in the further when you will take the your conductor if to find out the inductance of the single phase to a line and other things conductors inductance where that I will see that mutual inductance also.

So, in that case that we can write m 2 one is equal to lambda 2 1 upon I 1 right; that means, this is that mutual inducts it is Henry it is your equation number 11 is given. So, so mutual flux linkage between 2 circuits is defined as the flux linkage of one circuit due to current in the second circuit right so; that means, this is the flux linkage this thing due to the current of the second circuit due to the current in the. So, we are what we call due to current in the there what is this between 2 circuits is defined as the flux linkages of one circuit due to the current in the second circuit. So, that is flu if the lambda 2 one upon I one that is your flux linkage of one circuit due to the current in the second circuit due to the current into the first circuit that is first circuit current is I 1.

Now, therefore, if you find out this because of this mutual inductance or it will be the voltage drop sub voltage drop in circuit 2.

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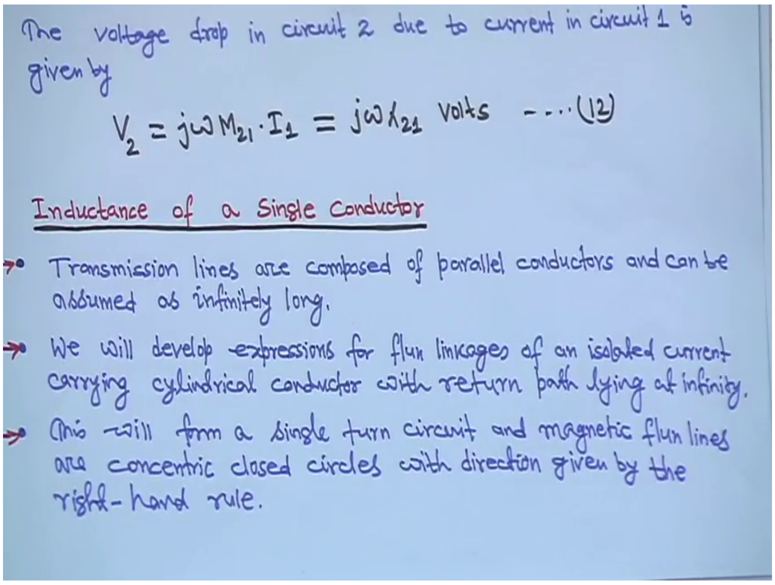

Then due to the current in circuit 1 is given by V 2 is equal to j omega same as before into m 2 1 I 1 right. So, j omega we will remain as it is, but lambda 2, 1 is equal to m 2 1 into I 1 volts is right. So, this is the voltage drop in the second circuit. So, this way we can determine that mutual inductance, but details we will see later right now inductance of a single conductor. So, transmission lines are that is why every step I have written it here such that we should not miss anything. So, transmission lines are composed of parallel conductors and can be assumed as infinite long because if you if you have seen the over a transmission line we can assume there parallel conductors right and it can be we assume that there infinite long.

So, in based on that we will develop expressions for flux linkages of an isolated current carrying cylindrical conduct. So, we will assume that conductors such of cylindrical type right solid conductors right with return path lying at infinity. That means, we will see that what you call isolated current carrying cylindrical conductor and return path is lying at infinity means this will completed single ton circuit right based on these assumption and magnetic and your what you call and magnetic flux lines are concentric enclosed cir concentric closed circles with direction given by the right hand rule. I will told you at the beginning that if this is that if this is the your vertical thing that conductor and if you rub the conductor like this and if this is the direction of the current then this will be the flux lines, right.

So, with direction given by the right hand rule, so, next is I mean.

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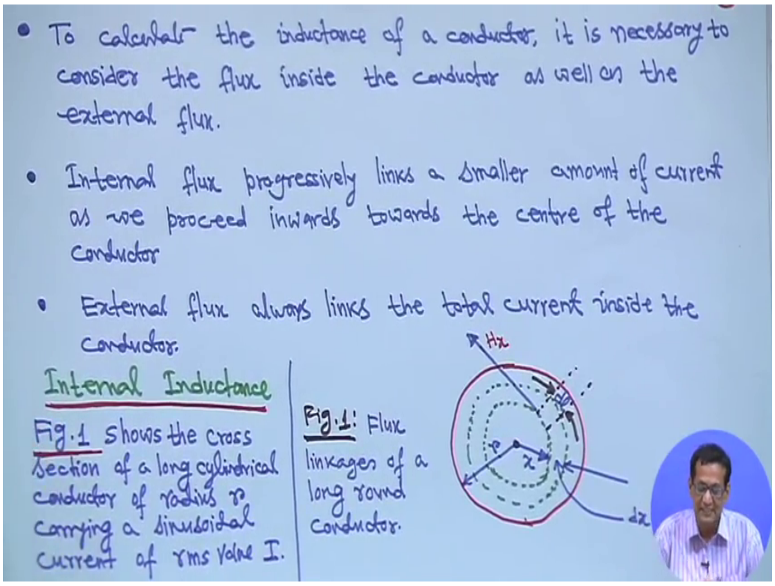

Next is to calculate the inductance of a conductor it is a necessary to consider the flux inside the conductor as well as the external flux both internal as well as external both the flux linkages we have to considered to determine the inductance of a single conductor right current carrying conductor.

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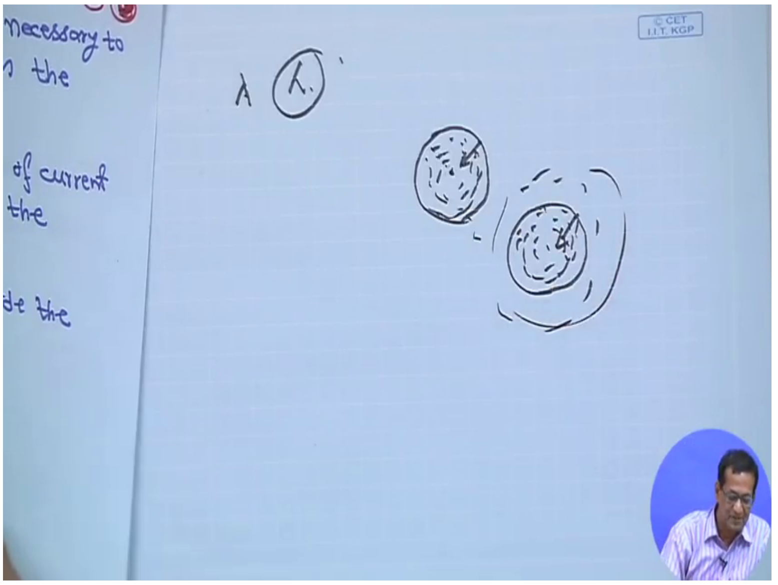

Now, internal flux progressively links smaller amount of current as we proceed inwards towards the center of the conductor for example, for example, say they this is the cross sectional view of the conductor right and it is carrying current. So, now, now when you proceed and if and we assume that uniform current density we assume uniform can you proceed suppose this is the center from this center from this surface actually if you move towards the center right then what we will happen it progressive links as smaller amount of current because we are assume in the current density say it is uniform, right.

We are assuming that therefore, as we move like this then we; that means, your internal flux progress progressively links as much suppose if you make all internal flux line all internal I mean I making it like this; suppose you make all intern flux line like this flux line like this like this as you move from this one towards the centre then it progressively links smaller amount of current right. That means, it is written here as we proceed inwards towards the centre of the conductor right. So, an external flux always link the total current inside the conductor external flux means this is out that is outside the conductor if this is the conductor then outside the conductor. So, it will always link the total current in the conductor, right.

So, based on this we will try to find out what will be the internal your what you call that in inductance first. So, suppose this is this is your figure 1 and this is your conductor right, it is a long cylindrical conductor and this is the cross sectional view and its radius is actually R and you consider at a distance x right a small no small your infinite a (Refer Time: 12:22) small that lay in D x this is your D x and this is the arc lengths is very small one it is actually D L right and from here that e electric field intensity at a distance x because from here it is x here it is x it is a circular path. So, from here say you assume that is your what you call that electric field it is your magnetic field intensity is H x right. So, this is a cross sectional view of conductor this is a radius R and at a distance H we consider small very small this thing your with D x right and this is that your length D L this is the your what we call the arc length D L, right.

So, in this case what we will what we will do that the MMF round a concentric close circular path of radius x this is a concentric closed circular path of radius x right internal to the co conductor.

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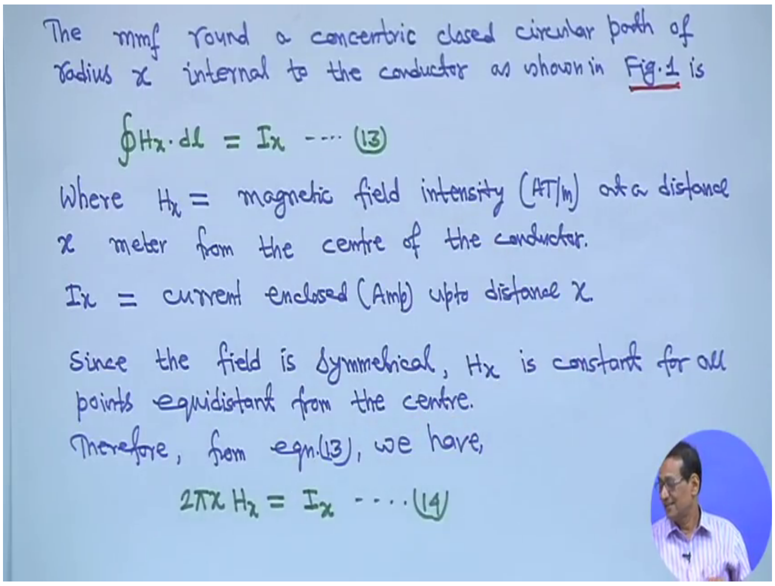

As shown in figure one, so, this is your figure one this is figure one right and therefore, the loop integral you can write in that H x into D L is equal to I x suppose at a distance x the current is say I x right this why you defined and then where H x is equal to magnetic field intensity that is ampere transfer meter at a distance x meter from the center of the conductor. So, here also it is x here also it is here also it is x right that the distance x from the center of the conductor right and I x is equal to current enclosed up to distance x. So, I x is equal to current enclosed up to this distance x right is ampere, right.

Since the field is symmetrical as the field is totally symmetrical that the H x H x is constant that every point right, so, H x is constant for all points equidistant from the centre. So, field is symmetrical. So, it is a circular path. So, everywhere H x we will remain constant. So, if it is. So, then H x if H x remain constant then this D L integration of this D L actually it will be the total circumference that will be your 2 pi x right that is your 2 pi x and H x is constant everywhere. So, into H x this integral actually is equal to in H x is constant that everywhere. So, H x will remain as it is and D L it is circuit this is the total arc length, but you can if you take the total circumference integral of that loop integral then it will be 2 pi x into H x is equal to this current your what you call is equal to I x right. So, there this is equation 14.

Now, so, we will we will neglect skin effect we will neglect this and assuming uniform current density.

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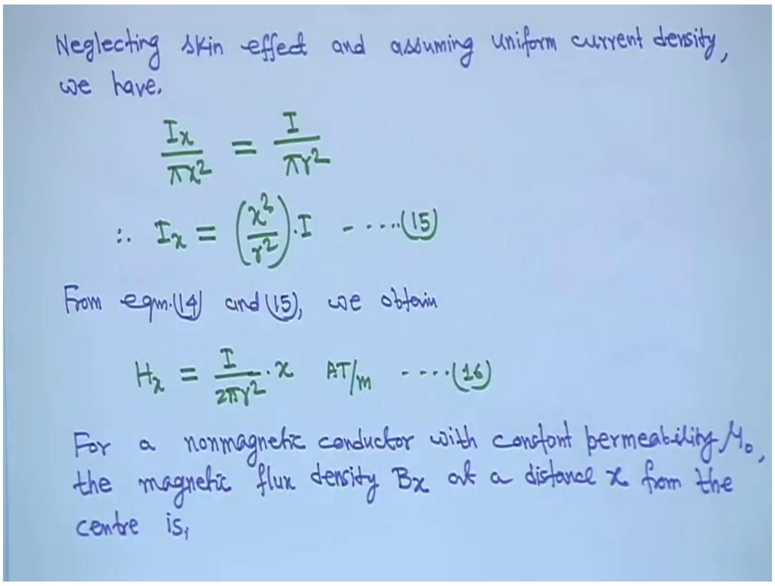

That means, at that if you take the cross section we will take the uniform current density right. So, at a distance x I just hold on at a distance x right. So, this area of this (Refer Time: 15:40) pi x square and at a distance R area of the circle is pi R square. So, at a distance x current is I x. So, current we will assume that current density is uniform therefore, I x upon pi x square is equal to I upon pi R square right therefore, I x is equal pi will be cancel on both side I x is equal to x square upon x square into I this is equation 15 right. So, after this from equation 14 and 15 we obtain. So, in equation 14 here it is.

So, equation 14; in equation 14 you substitute I x is equal to this one and small simply (Refer Time: 16:26) this H x will become that I upon 2 pi R square into x ampere ton per meter right.

So, this is equation sixteen. So, for a nonmagnetic conductor with constant permeability mu 0 the magnetic flux density B x at a distance x from the centre is right therefore, again the again the for a look non magnetic conductor with constant permeability new 0 right. So, at a distance x what will be my B x there flux density right. So, in this case what you can do is that D x is equal to actually B x is equal to your new 0 into H x and H x is equal to I upon H x is equal to 2 I upon 2 pi as square into x.

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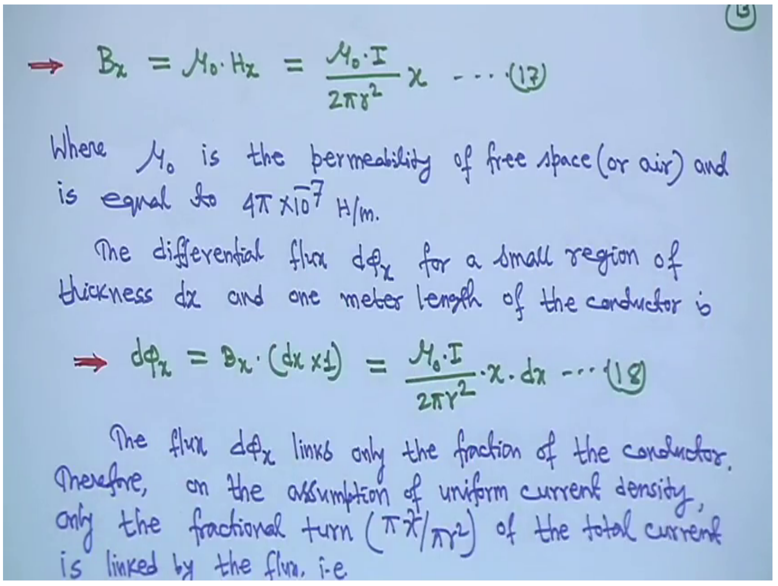

So, new 0 into I upon 2 pi R square into x right. So, this is equation seventeen right. So, I mu 0 is the permeability of the free space or air right and is equal to 4 pi into 10 to the power minus 7 Henry per meter. So, now, the differential flux D phi x for a small region of thickness D x; that means, this one this diagram again right.

So, differential flux D x for a small region of thickness this is thickness is D x this is D x right and one meter length of the conductor is. So, the small difference is D x D phi x is equal to B x into D x into and it is something like this I hope you will be able suppose you fold it suppose this is a this is your conductor this is a conductor suppose this is this length is this length is 1 meter and you have this thickness is D x right this thickness if you fold it then this thickness is D x and this length is 1 meter. So, area will be this portion D x into 1 square meter right I think you have got it so; that means; that means, that D phi x is equal to B x that is a flux density (Refer Time: 18:37) up a meter square into this area D x into 1 right.

So, that is why the differential flux D phi x for a small reason of thickness D x and 1 meter length your considering of the conductress D phi x is equal to B x into D x into one that is B x is equal to mu 0 I upon 2 pi a square into x right into your what you call D x. So, this is equation eighteen. So, the flux D phi x actually links only the fraction of the conductor because it is at a distance x right it is at a distance x only. So, it links that your fraction of the your conductor right therefore, on the assumption of uniform current density therefore, on the; we have assume the current density uniform right only the fractional turn we call reality this there is no turn because it is constructional we have constructor. But only the fractional turn of the total current is linked by the flux because at a distance the area is at a distance x area it phi x square and conductor radius is R. So, its radius is phi R square. So, we call this ratio pi x square upon pi R square pi R square as a fractional turn of the total current is linked by this flux, right.

This way we imagine. So, in that case then what we will happen then flux linkage will be this is the fractional turn.

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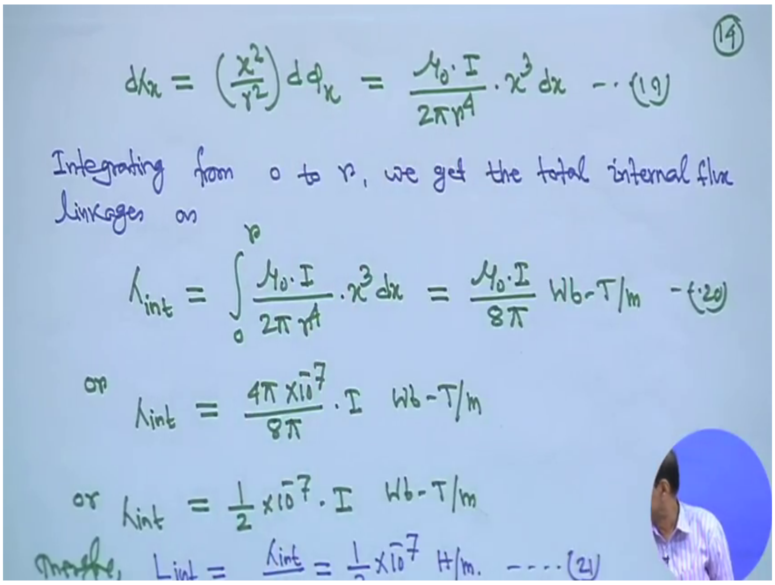

So, flux linkage will be a D d lambda x into this is actually fractional turn right into D phi x right, so, x square upon R square pi pi cancel actually x square upon R square into

D phi x. So, D phi x is equal to you have seen that mu 0 by R they have D phi x is equal to here it is mu 0 I upon 2 pi square into x into D x substitute then you will get the D phi x is equal to mu 0 into I divided by 2 pi R to the power 4 into x cube D x this is equation number nineteen right now there is the conduct radius is R. So, integrate from 0 to R right then you will get the total internal flux linkages.

So, lambda internal is equal to 0 to R mu 0 I 2 pi R to the power 4 into x cube D x. So, you integrate and this will be mu 0 into I upon 8 pi Weber tons per meter after integrating right. So, you will find that this is the independent of R right there is no R because it will come know R to the power 4 upon 4. So, R to the power 4 four we cancel right. So, this is equation 20 there is lambda internal actually is equal to we can mu 0 is equal to 4 pi into 10 to the power minus 7 Henry per meter square. So, you substitute here 4 pi into 10 to the power minus 7 divided by 8 pi into I Weber tons per meter square right or lambda internal is equal to half into 10 to the power minus 7 into I Weber ton per meter right.

So, lambda internal this is the flux linkage per meter therefore, inductance internal is equal to lambda upon I.

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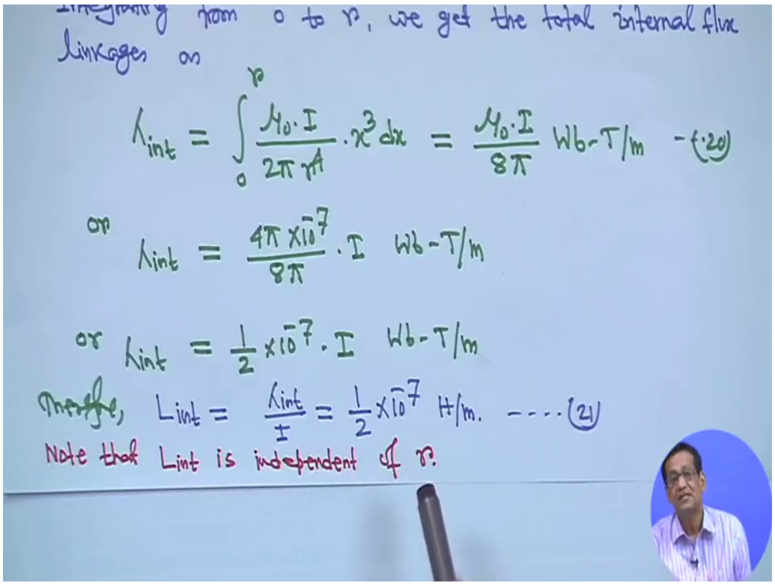

So, L internal is equal to lambda internal upon I is equal to half into 10 to the power minus 7 Henry per meter; that means, this internal induct internal inductance of a current carrying conductor alternating current carrying conductor right is half into 10 to the power minus 7 Henry per meter and it is independent of the radius or diameter of the conductor. That means, whatever may be the diameter of the conductor right or radius of this thing radius of the conductor whatever may be radius or diameter this L internal is always a constant value that is half into 10 to the power minus 7 Henry per meter square this is I have marked one equation number 21.

So, that is that that is the internal inductance of a current carrying conductor half into 10 to the minus 7 right, next one is that inductance due to external flux linkage.

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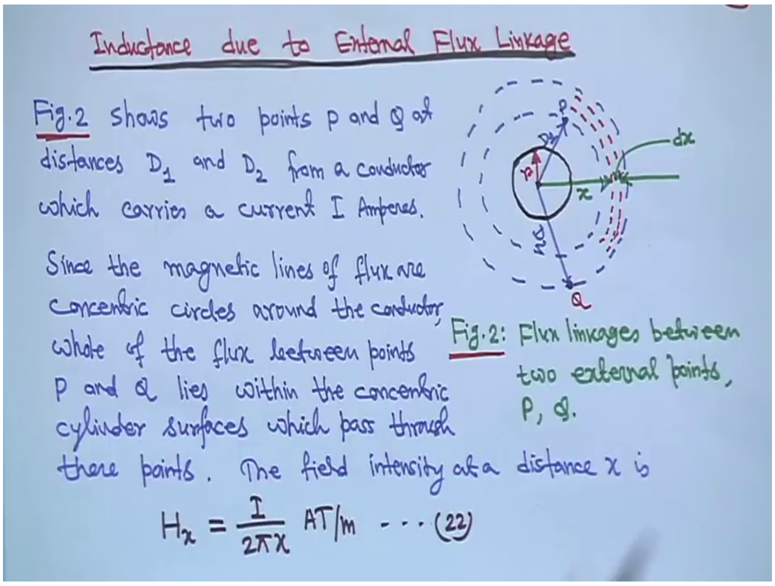

So, this is the cross sectional view of the conductors right. So, what we have taken we have taken to this radius of this conductor is R we have we have taken 2 external point one is at p that is from the centre of the conductor the distance is D 1 this; this from the center of the conductor to this point p distance is D 1 and another point we have taken q and the distance is D 2 right and we have taken this; this thing right D 2 and these 2 points are external these are the flux line we have taken right. So, that this is figure 2 flux linkages between 2 external points p q write p and q. So, that is why we have written here figure 2 shows 2 points p and q at distances D 1 and D 2 from a conductor which carries a current I ampere.

So, current is carrying by this conductor is I right since the magnetic lines of flux are concentric circle this all this line (Refer Time: 24:12) concentric circle surround this say your around the conductor whole of the flux between points p and q right within the concentric cylinder surfaces right which pass through the pass through this points right.

So, all are concentric I did not between this point is taken q here p here. So, in between p and q q point whatever the flux lines there concentric right, so, the field intensity at a distance x right. So, we have taken at its some distance x external to the conductor upon this x is external to the conductor. So, the field intensity at a distance x can be given as H x is equal to I upon 2 pi x we have seen that any point this magnetic field intensity H x we will remain constant because is a concentric circle right and at a distance x the circumference is 2 pi x.

So, I is equal to an conductor is carrying current I. So, H x is equal to I upon 2 pi x is equal to ampere transfer meter wherever possible I have given the; your units right this is equation 22 now and the flux density just you have seen at a distance x B x is equal to mu 0 into H x. So, mu 0 H x is I upon 2 pi x this is Weber per meter square this is equation this is equation 23, right.

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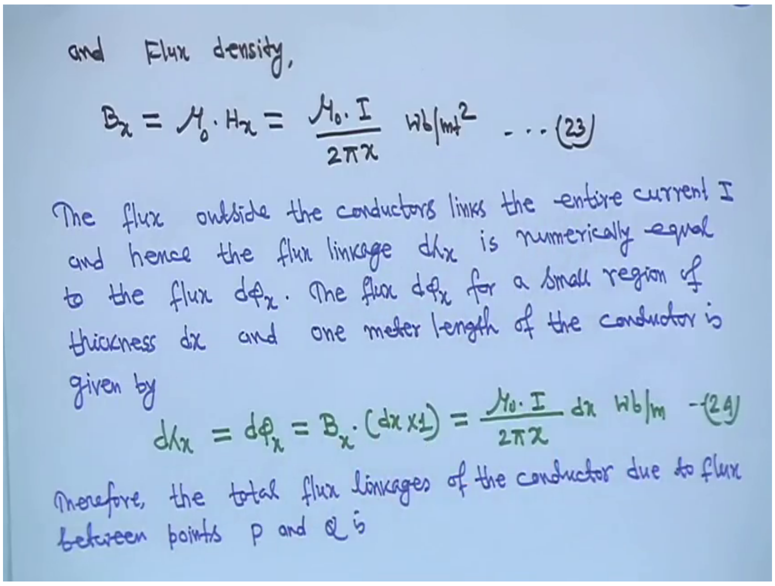

So, the flux outside the conductors linked the entire current I you are the beginning I told right and hence the flux linkage D lambda x is numerically equal to the flux D phi x here no question of fractional turn will come because it is external to the conductor. So, numerically D lambda x and differential flux D phi x both are same right.

Therefore the flux D phi x for a small region of thickness D x that small region of thickness D x the flux differential flux D phi x here right and we consider 1 meter length of the co conductor it is given by D lambda x is equal to D phi x both are same is equal to B x into D x into 1 because same concept of this area right this is your if you imagine like this; this cylinder this thickness is D x and this length is 1 meter and this thickness is D x right then if you fold it then this thickness in this 1 D x into 1, the cross sectional area right. So, in this case same as be for D lambda x is equal to D phi x is equal to B x into D x into 1 is equal to you substitute B x again mu 0 I upon 2 pi x D x this is Weber per Weber per meter, right. So, this is equation 24 therefore, the total flux linkage of the conductor due to flux between points p and q you have to obtain.

So, distance of point p from the center of the conductor is D 1 and point q is D 2. So, what we have to do is you have to integrate this.

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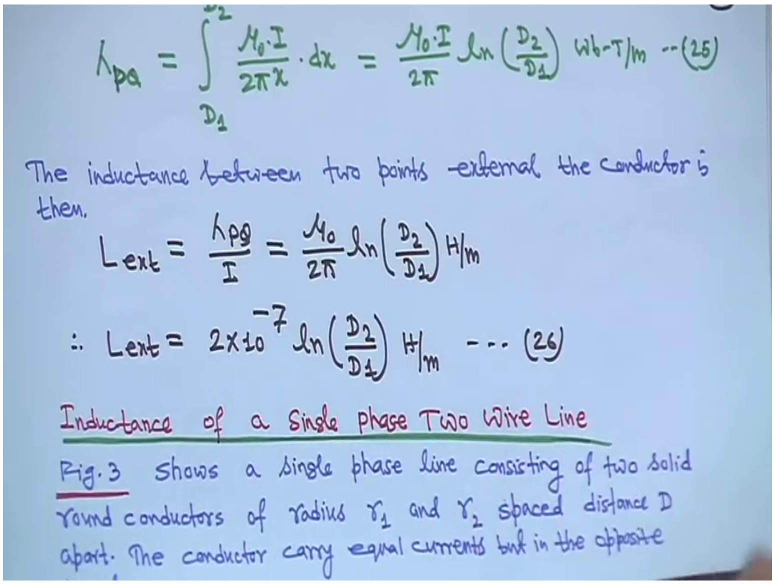

That means lambda t q right between this 2 point integration limit is D 1 to D 2 mu 0 into I upon 2 pi x into D x is equal to mu 0 I upon 2 pi and if you integrate it this will be natural law L n D 2 upon D R D 1 right this is equation 25 right therefore, the inductance between 2 points that is p and q external to the conductor is then given by L external is equal to lambda p q upon I. So, lambda p q is this much divided by I means this I this I will be cancel. So, it will be mu 0 upon 2 pi natural law L n D 2 upon the D 1 Henry per meter right and if you substitute mu 0 is equal to 4 pi into 10 to the power minus 7 Henry per meter if you substitute mu 0 then 2 pi 2 pi will be cancel right.

So, ultimately it will be L x your it will be 4 pi divided by 4 pi into 20 to the power 7 divided by 2 pi; that means, L external be 2 into 10 to the power minus 7 L n D to upon

D 1 Henry per meter this is your equation 26 right so; that means, what is in this case we can see that L external case it depends on that your what we call later we will see that from centre of the conductor to some distance D how things will happen, but if you have 2 points D 1 and D 2. So, you will external is function of this your both the distance D 1 and D 2 right. So, it is dependent on their distance D 1 and D 2 later we will see how we will simplify further right. So, next one is that inductance of a single phase 2 wire line right for example, before this thing that suppose you have taken that your single phase 2 wire lines.

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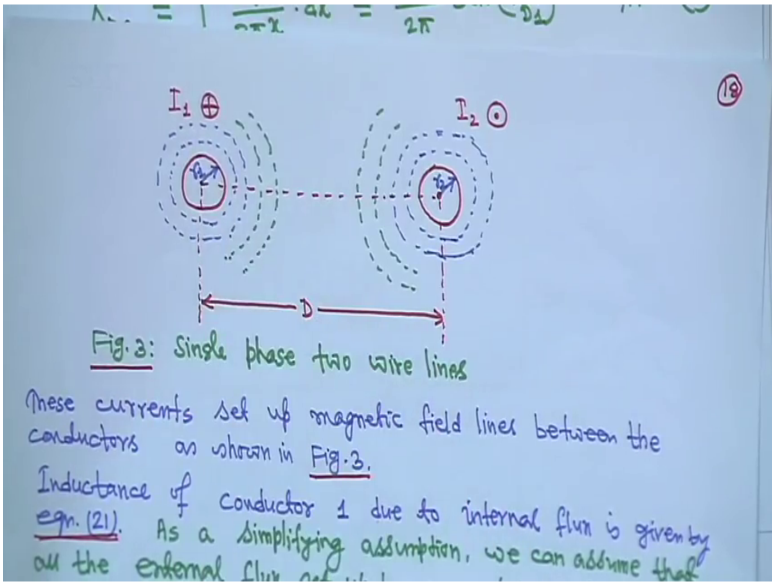

So, this is one conductor that red, red one this is another conductor we assume its radius is R 1 is radius is R 2 it is it is your current I 1 and plus means the current entering into the page of the conductor right that is why plus thing is given this is return path dot is showing; that means, current living the page right and distance between these 2 conductor is D right. So, this is single phase 2 wire system. So, one is carrying your current I 1 that is going living into this (Refer Time: 30:07) and this is return path actually right and we have to find out that inductance of a single phase 2 wire line.