Power System Analysis
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Lecture - 44
Optimal system operation (Contd.)

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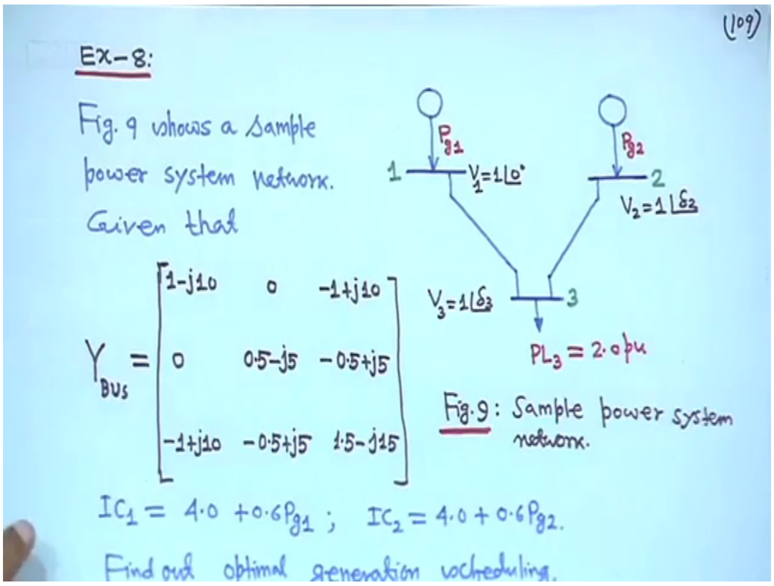

Then next we will take an example of like that right. So, just I just will see that how things are happening. Suppose you have a 3 bus problem 2 generators right here it is power because P g 1 and here power P g 2. So, voltage here it is given V 1 that is 1 angle 0 and this bus 1 this is bus 2 V 2 it is 1 angle delta 2 right and V 3 is equal to 1 angle. Delta 3 this is bus 3 and PL 3 is given 2 per unit, because here everything where dealing with real power only. So, no queue is required and line 1 to 2 is actually not connected.

So, and V 3 also 1 angle delta 3 right. So, Y bus matrix is given actually for this network Y bus matrix is given. So, it is 1 minus j 10 second one is 0, Y 1 2 because 1 to 2 actually not connected right and this is minus 1 plus j 10. So, from this thing you can make out that line charging admittance not consider because you we will some of all the elements are this row, it will become 0 because 1 minus 1 will be minus j 10 plus j 10 will be 0. So, for any row if you sum up all the elements whatever you will be left that is actually your charging admittance, but in this case charging admittance is not consider.

Similarly, say this is 0 this is 0.5 minus j 5, this is minus 0.5 plus j 5. And this is again this one and this one same Y 1 3 Y 3 1. So, minus 1 plus j 10 minus 0.5 plus j 5 and this is 1.5 minus j 15. So, this why Y bus matrix is given and for these two units let incremental cost characteristic is given that is IC 1 is equal to 4 plus 0.6 P g 1 and here it is IC 2 is equal to 4 plus 0.6 P g 2 these are given. You have to find out the optimal generation scheduling. So, what will be the optimal generation scheduling that is by you have to find out P g 1 and P g 2. So, that means, and that bus 3 no generator is there, there is a load at 2 you have this thing. So, just I will show you the step how one can manage it.

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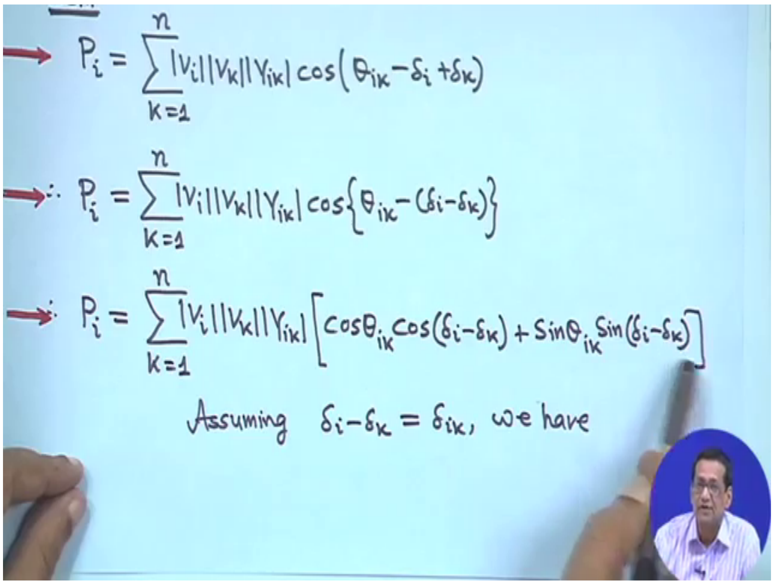

So, when your P i is equal to the power injection from the load flow studies the general formulas we know that P i is equal to k is equal to 1 to n that magnitude V i magnitude V k magnitude Y ik cosine theta ik minus delta i plus delta k. Therefore, P i is equal to we can write this equation we can write like this that k is equal to 1 to n the magnitude V i magnitude V k magnitude Y ik cosine in bracket theta ik minus delta i minus delta k these one we have taking minus common and in bracket we are writing minus delta i minus delta k. That means P i is equal to k is equal to 1 to n then magnitude V i magnitude V k magnitude Y ik and in bracket we are writing cosine theta ik that mean this equation that cos A minus B cos A cos B plus sin A sin B. So, we are writing cos theta ik cos delta i minus delta k plus sin theta ik sin delta i minus delta k. Now, you assume that delta i minus delta k let us write is equal to delta ik that means, delta ik is equal to delta i minus delta k this you assumed. Then these equation can be written like this.

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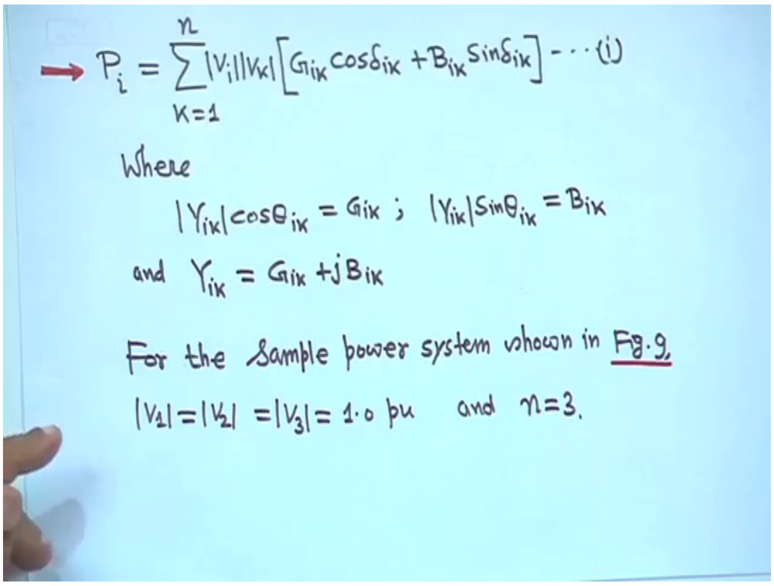

That P i is equal to k is equal to 1 to n V i V k G ik your what you call if you put here that your it will be cosine delta ik and this will be sin delta ik. Therefore, Y ik magnitude when you multiply by magnitude Y ik. So, it will be your magnitude Y ik cos i theta ik will be your B ik and your Y ik your sorry G ik and this is your what you call that and Y ik sin theta ik will be your B ik. That means these equation that when you do so it will be then k is equal to 1 to n V i V k it will be G ik cosine delta ik plus B ik sin delta ik. That means, your Y ik cosine theta ik G ik and B ik your is equal to that Y ik sin theta ik Y ik magnitude. So, where i have written here where Y ik cosine theta ik G ik and Y ik sin theta ik is equal to B ik; and Y ik is equal to G ik plus j B ik.

Now for the sample power system as it is given magnitude V 1, magnitude V 2, magnitude V 3 you are all are 1 per unit that means, in this diagram you have magnitude what is 1 angle 0, so magnitude is 1. Similarly it is 1 angle 0 magnitude bus 2 also voltage magnitude is 1 bus 3 also 1 angle delta 3. So, voltage magnitude is also one. So, all the buses we assume voltage magnitude actually it is 1. And it is a 3 bus problem, so it is n is equal to 3.

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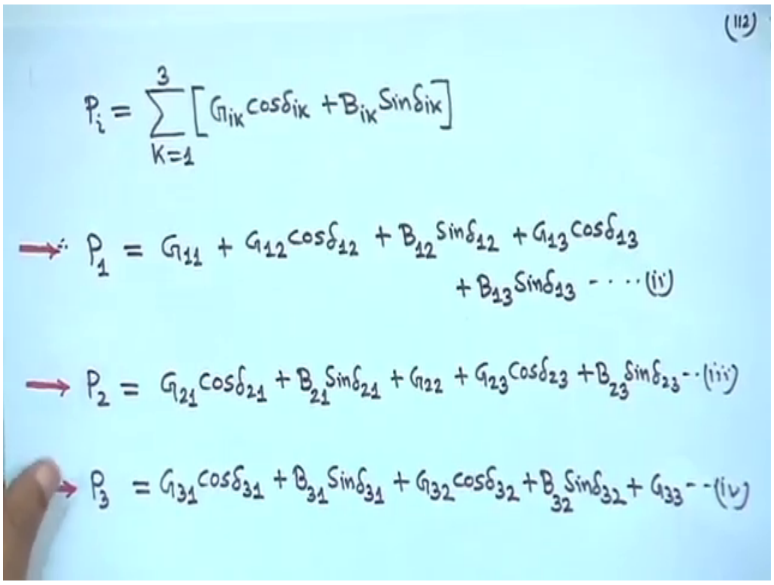

Therefore, put k is equal to 3 that is because n is equal to 3. Therefore, P i is equal to k is equal to 1 to 3 then it is G ik cosine delta ik plus B ik sin delta ik. Now, for i is equal to 1, you expand this equation for i is equal to 1, you will get P 1. So, it will be G 1 1 plus G 1 2 cosine delta 1 2 plus B 1 2 sin delta 1 2 plus G 1 3 cosine delta 1 3 plus B 1 3 sin delta 1 3 this is say equation 2, because this 1 actually we have mark this one as equation 1 here. So, similarly for i is equal to 2 and expand right. So, P 2 will be is equal to G 2 1 cosine delta 2 1 plus your B 2 1 sin delta 2 1 plus G 2 2 plus G 2 3 cosine delta 2 3 plus B 2 3 sin delta 2 3 this is equation 3. So, next is your put i is equal to your what you call i is equal to 3, you will get your G 3 1 cosine delta 3 1 plus B 3 1 sin delta 3 1 plus G 3 2 cosine delta 3 2 plus B 3 2 sin delta 3 2 plus G 3 3. So, if you explain these or this thing, so naturally one case will happen that everywhere you will find there are 5 terms, but according to, here two terms are there, there is no six term 5 terms because for anywhere you will find that this one sin delta 1 minus delta 1, so it will become 0. Sin 2 sin delta 2 minus delta 2 will be 0, so that is why everywhere in the explain it is 5 terms.

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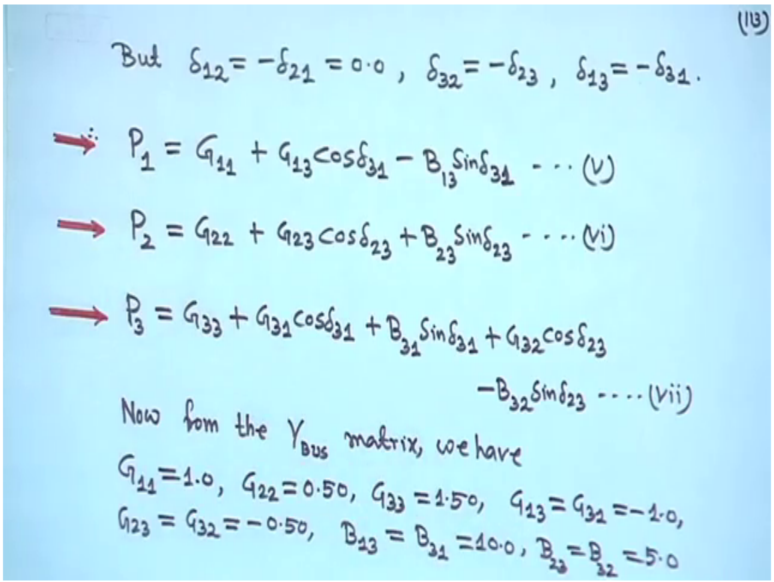

Now, this something this one the delta 1 2 and delta 2 just hold on that delta 1 2 and delta 2 1 is equal to 0 because where line 1 2 actually not connected. And then we can write delta 3 2 is equal to minus delta 2 3 and delta 1 3 is equal to minus delta 3 1 because delta 3 2 is equal to delta 3 minus delta 2, so that one you can write minus bracket delta 2 minus delta 3. So, that is minus delta 2 3 similarly here also delta 1 3 is equal to minus delta 3 1 because delta 1 3 is equal to delta 1 minus delta 3, so minus if you take common delta 3 minus delta 1 is equal to minus delta 3 1.

So, this equation that equation for P 1, P 2, P 3 all this things delta you substitute delta 1 2 is equal to delta 2 1 0 another thing substitute. If you do so, you will get P 1 is equal to G 1 1 plus G 1 3 cosine delta 3 1 minus B 1 3 sin delta 3 1. Similarly, P 2 is equal to G 2 two plus G 2 3 cosine delta 2 3 plus B 2 3 sin delta 2 3. And P 3 is equal to G 3 3 plus G 3 1 cosine delta 3 1 plus B 3 1 sin delta 3 1 plus G 3 2 cosine delta 2 3 minus B 3 2 sin delta 2 3 this is equation 7.

Now, from the Y bus matrix we have this all these Y bus you have this thing you have the data. This is actually this is your G 1 1 1 your what you and B your 1 1 is your minus 10, but all the this sorry it is G 1 1 is 1 then G 2 2 is your 0.5 and G 3 3 1.5. Similarly, if you say that your what you call that B 1 3 B 3 1 all are given B 2 3 B 3 2 all are require. So, whatever they are G and B from this equation you will get it right from this equation you will get it Y is equal to G plus j B; from here you will get it. So, use all the parameters are they then you substitute then you substitute all this G and B parameters you substitute.

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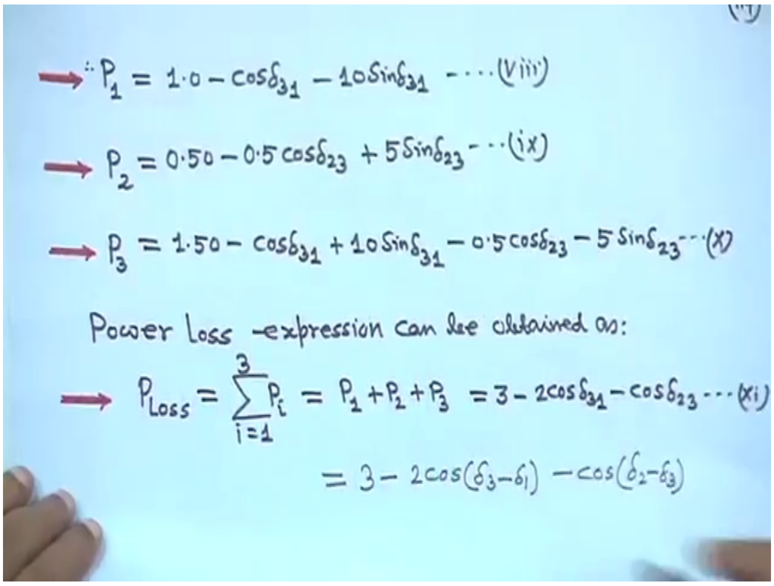

If you do so just hold on so then P 1 will be 1 minus cosine delta 3 minus 10 sin delta 3 1 this is not equation 8. P 2 will be 0.5 minus 0.5 cosine delta 2 3 plus 5 sin delta 2 3 this is equation 9. Similarly, P 3 is equal to 1.5 minus cosine delta 3 1 plus 10 sin delta 3 1 minus 0.5 cosine delta 2 3 minus 5 sin delta 2 3, this is equation 10. Now, power loss expression can be obtained as we know the sum of injected power is the power loss that means P loss is equal to sigma i is equal to 1 to 3 P i that is equal to P 1 plus P 2 plus P 3 is equal to that P 1 plus P 2 plus 3 you sum it up, this three you sum it up, you will get 3 minus 2 cos delta 3 1 minus cos delta 2 3, this we will get. That means, this one this equation also can be written like these three minus 2 cos this one written as delta 3 minus delta 1 minus cos delta 2 minus delta 3 this is the equation is a last one.

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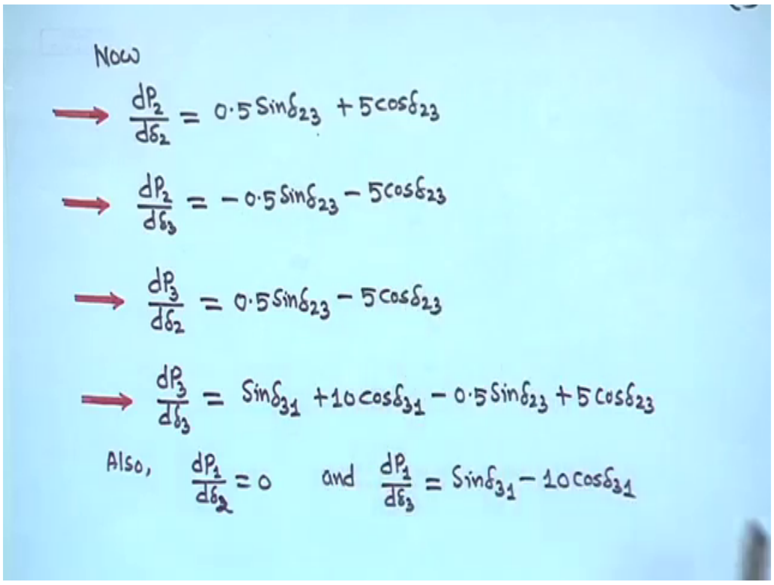

Next is that you take derivative of this thing this delta P 2 and there is delta P 3 take all the derivative, then what you will what you will get that if you take the derivative that delta P 2 delta P dP 2 the delta 2 take the derivative dP 2 delta 2. So, this is actually P 2 written it is written cosine delta 2 minus delta 3 it is written 5 sin delta 2 minus delta 3.

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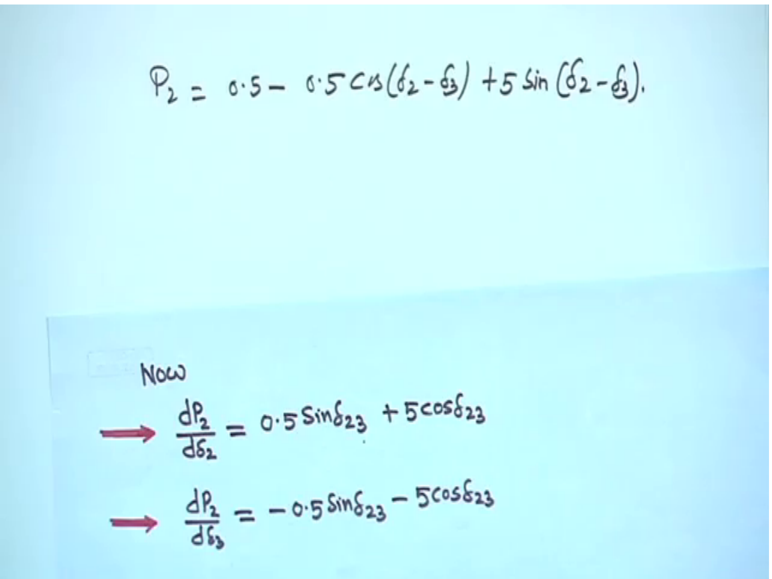

So, actually this equation, this P 2 equation actually like these; similarly for P 3. P 2 actually it is 0.5 right minus 0.5 in cosine it is delta 2 3 right delta 2 minus delta 3 plus 5 is sin this is also delta 2 minus delta 3 these where and take the derivative. Therefore, your dP 2 upon d delta 2 you will get 0.5 sin delta 2 3 plus 5 cos delta 2 3 similarly dP 2 d delta 3 will get minus 0.5 sin delta 2 3 minus 5 cos your delta 2 3.

Similar way for P 3 expression you write cosine delta 3 minus delta 1 plus 10 sin delta 3 minus delta 1 and so on. These you write sorry this one you take dP 2 by delta 3 similarly you find out dP 2. Similar way, dP 3 upon d delta 2 and dP 3 upon d delta 3 you take the derivative. Similarly we make it and take the right delta 3 1 you write cosine delta 3 minus delta 1 here plus 10 sin delta 3 minus delta 1 and so on. This way you make it and get all the dP 2 d delta 2 dP 2 d delta 3 dP 3 d delta 2 and dP 3 d delta 3. So, this is the expression you got. And also dP 1 d delta 2 is 0 because P 1 right your what you call that P 1 is not function of delta 2 nowhere delta 2 is acquiring in equation 8 no delta 2 that is why dP 1 delta 2 is equal to 0.

And dP 1 your d delta 3 dP 1 d delta 3 that is here function of delta 3 you can take the derivative dP 1 d delta 3, so that is coming sin delta 3 1 minus 10 cosine delta 3 1. So, take this derivative; only thing is that you will make it all your delta 3 minus delta 1 then here also sin delta 3 minus delta 1 this you take the derivative. Once you do so, then this equation that equation 76 whatever you have derive it will be reduce in this form.

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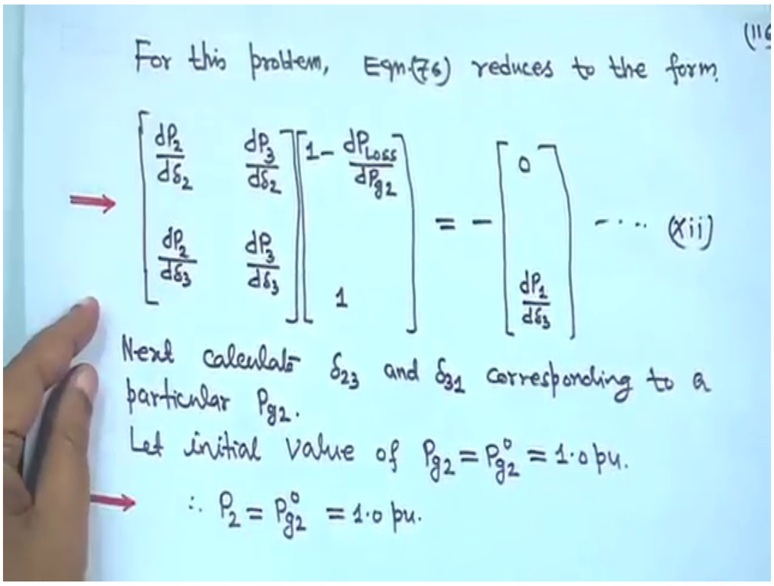

So, this matrix will be dP 2 d delta 2 dP 3 d delta 2 dP 2 d delta 3 dP 3 d delta 3, and it will be your only one bus bus 1 is slack bus; and one bus is a load bus and one bus another bus is generator bus, so that is bus 2. So, is equal to 1 minus dP loss dP G 2 and this will be 1 is equal to minus this is a 0 and this will be dP 1 d delta 3. So, your because dP 1 d delta 2 is equal to 0, dP 1 d delta 2 is equal to 0, but dP 1 d delta 3 exist that is why we have instead of writing dP 1 your d delta 2. Again here actually dP 1 d delta 2 is equal to this 0 we are put in here this is equations of 12.

The next is you have to calculate delta 2 3 and delta 3 1 corresponding to particular P g 2, delta 2 3 means delta 2 minus delta 3 and this is delta 3 minus delta 1 corresponding to a particular value of P g 2. So, you assume because bus 1 is a slack bus, bus 2 is a generator bus and bus 3 is a load bus only. So, you assume initially is a P g 2 is equal to P g 2 0 is equal to 1 per unit, let us assume this. Therefore, P 2 power injection at bus 2 is equal to actually P g 2 minus P i 2 there is no load at bus 2. So, it is actually P 2 is equal to P g 2 0 is equal to 1 per unit that means, from equation 9 actually that is your P 2 expression just. So, just let me let me find out right; that means, your this one. So, from equation 9 that means, in these equation your these equation right that is P 2 is equal to P g 2 0 these equation so that means, we are writing this one that 5 sin delta 2 3 minus 0.5 cosine delta 2 3 is equal to 0.5 this is a non-linear equation.

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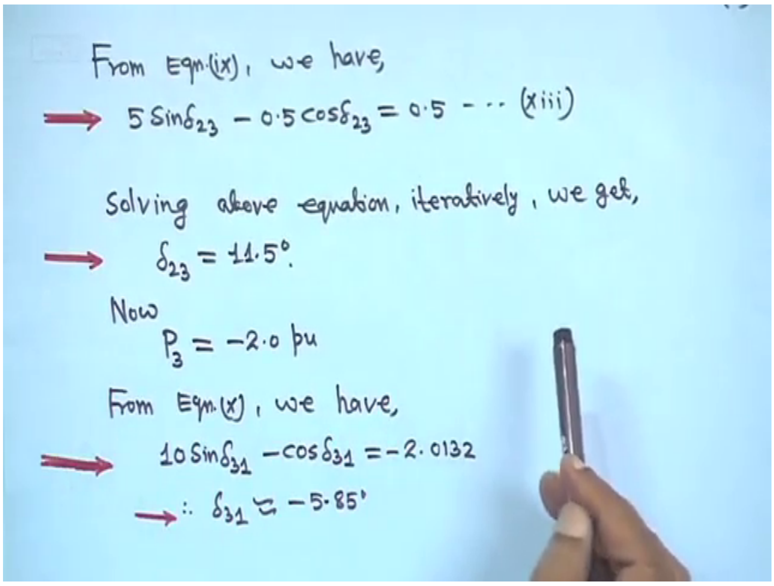

So, this is solving above equation iteratively we get delta 2 3 11.5 degree actually I have made some trial and error. So, some from very intuition some value you choose such that this equation satisfy left hand side and right hand side satisfy. So, delta 2 3 actually 11.5 degree approximately I have got. Now, at bus 3 only load is there, but generously is not there. So, power injection is minus 2.0. So, you go to equation 10, there you can write it that is equation let me see or equation 10 this is equation 10. So, this is actually equation 10 right and your and delta 2 3, you have got 11.5 degree delta 2 3 you have got 11.5 degree. So, in this equation 10 this cosine delta 2 3 sin delta 2 3 all you get and simplify this equation. Then you will get and P 3 is equal to a minus 0.20 at the bus 3 right then you will get 10 sin delta 3 1 minus cosine delta 3 1 is equal to minus 2.0132. So, again by (Refer Time: 17:09) only I have got delta 3 1 approximately minus 5.85 degree. So, this delta 2 3 and delta 3 1, I have got it.

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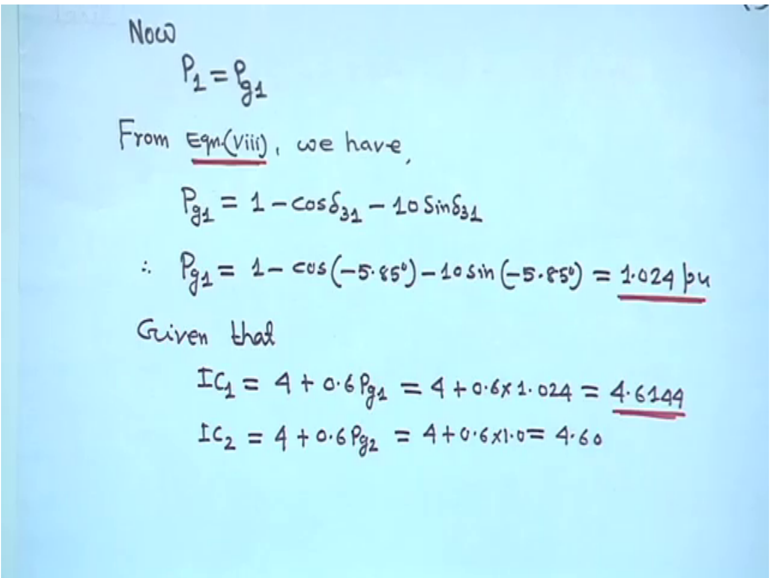

So, your this thing at slack bus that P 1 is equal to P g 1 that is a bus 1, you consider diagram I have shown you P 1 is equal to P g 1 that means, this one. Just see where the where the diagram has gone just hold on here. So, here it is your P 1 is equal to P g 1. So, from equation 8, then we has P 1 is equal to P g 1, so actually is P 1 is equal to this expression was given. So, that is actually writing P g 1 right is equal to 1 minus cosine delta 3 1 minus 10 sin delta 3 1. So, P g 1 after putting delta 3 1 a values, we are getting actually 1.024 per unit right and it is given the cost characteristic it is given that your IC 1 is equal to that incremental cost is given that IC 1 is equal to 4 plus 0.6 P g 1 is equal to 4 plus 0.6 into 1.024 that is 4.6144.

Similarly, IC 2 is equal to 4 plus 0.6 P g 2 is equal to 4 plus 0.6 into 1 that is 4.60, this you are getting. Next is that your solving equation 12 that mean this equation that your just hold on right just hold on these equation. So, then all this bus all the delta values you have got so in this expression that is equation to all dP 1 d delta 3 compute all these things are there delta values you have got it you put it.

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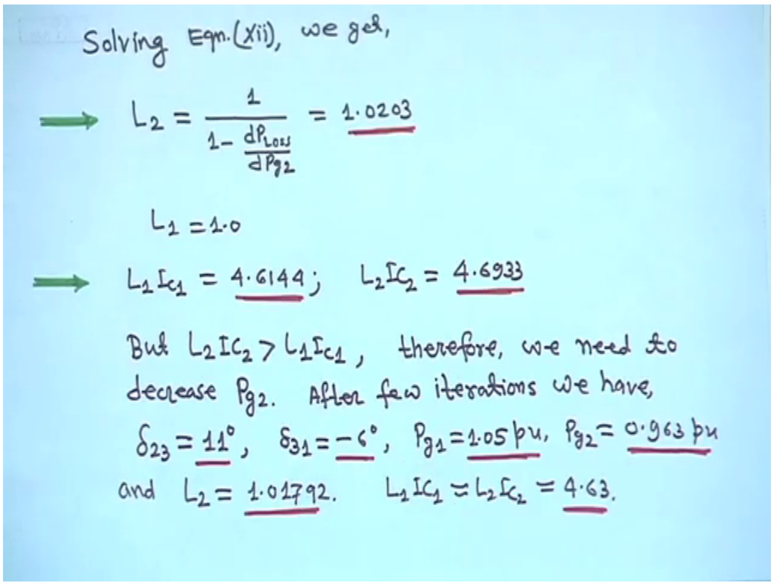

So, if you do so if you compute I am giving the final answer only, final answer only. So, in that case you will get that L 2 is equal to that first you will get L minus dP loss upon dP g 2 and L 2 that is your penalty term L 2 is equal to 1 upon 1 minus dP loss upon dP that is 1.203 these will get. But for L 1 it will be 1 right you have you have seen this 1 therefore, that L 1 IC 1 is equal to 4.6144. So, this one actually this is your this is your IC 1 this is your IC 1 computed. So, L 1 is 1. So, L 1 1 into 4.6144. So, that is this one right and L 2 you got 1.0203 that means, that this is your IC 2 4.60 multiply by 1.0203 you will get 4.6933.

But for optimal solution both should be equal, but here it is not equal, but because L 2 IC 2 much greater than your L 1 IC 1 therefore, we need to decrease P g 2 because it is more than that. So, there mean this is an iterative process in the classroom purpose we cannot do that iteration right, but after few iteration result will become actually delta 2 3 11 degree, delta 3 1 will be minus 6 degree. And P g 1 will be 1.05 per unit and that is the slack bus power whatever will be the. And P g 2 actually 0.963 per unit and L 2 will become 1.01792. And at the time L 1 IC 1 approximately equal to L 2 IC 2 will become four 0.63, this is the state.

Let me tell you this is actually to show you how things work, but it is not like that that in the classroom, we will do so many iterations and these that this is only steps are shown that how one can solve this. So, you may have to or do few calculation only, but as far as classroom is concerned and this iterative process or in the examiner also we cannot give like this. Similarly, what you call this is just to show you the your what you call few step. So, this is actually optimal economic load dispatch whatever I mean possible a learning in the classroom only that much we have talk. So, next show for you have assume the transmission loss formula. So, now, we will try to derive that your transmission loss formula.

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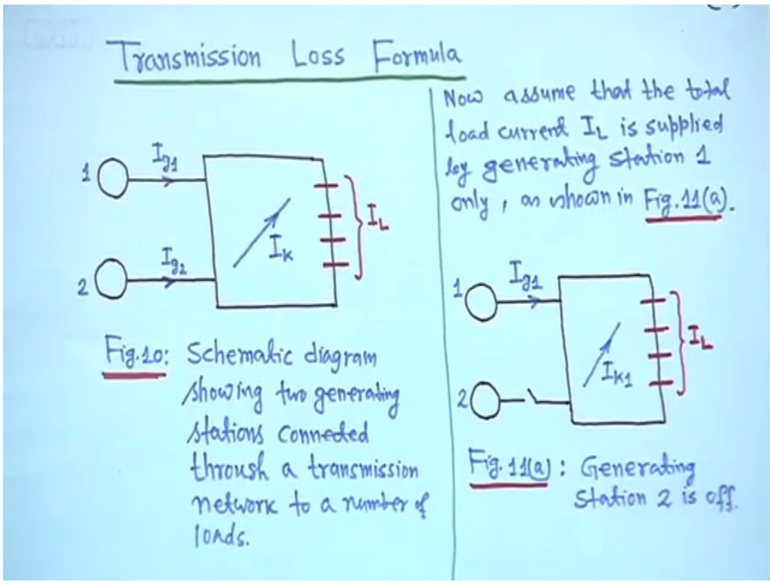

So, so next come to the transmission loss formula. So, consider now, this is actually transmission loss formula. So, you have to derive this just hold on, just 1 minute. So, now this suppose you assume this is actually your what you call schematic diagram. So, in two generating unit this is general look at this side only this is your generator 1 and this is your generator 2. And it is connected to transmission line transmission lines there are so many bus bar. So, schematically we are representing load current total load current say I L right and 1 particular branch it is say k branch right. So, through your current to the branch is your I k right this is; that means, to a just first we are assuming only 2 generators after that we will generalize that. And but to get this transmission loss formula we have to make few you have to few assumptions that will come later.

Now that means, it is a transmission network it has a number of buses right load current total load current is I L and one particular branch k having current I K and two generators basically you are what you call current from generator 1 generator 2 is I g 1 and I g 2. Now, you assume that the total load current supplied by the generating station one. So, we assume the total load current supplied by this one right that means, generate 2 is off. So, at that time that current through branch i or k say it is I K 1 right because generator 1 is supplying the current, so k branch current will change right the total current il supplying where the generator only. So, this is generating station 2 is off. So, it is off it is open. So, in that case what will happen so on the current through this is I K 1.

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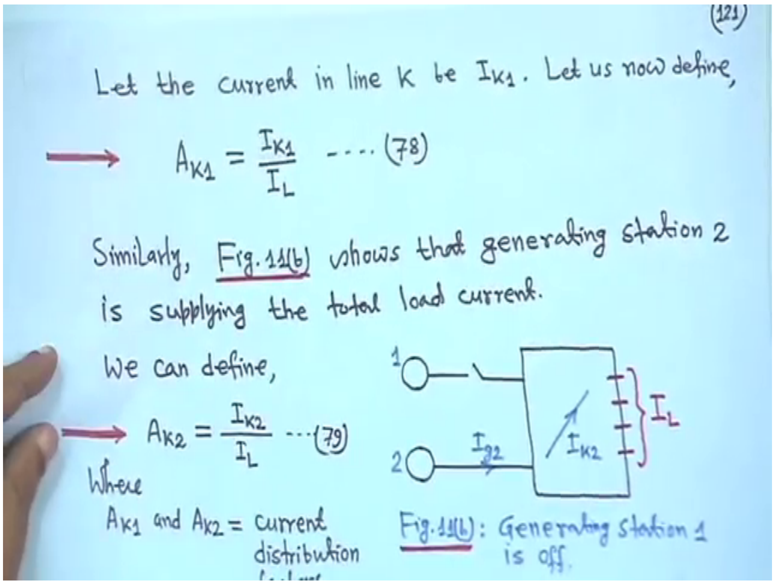

So, in that case what will happen that let the current in the line k have written be I K 1 right and let us now define A K 1 is equal to I K 1 upon il right you just define a term I K 1 upon I L. Later we will see that A K 1 actually it is a real quantity and i will apply a simple super position right. So, this is equation 78. So, similarly in P or B in B or B here you assume that generator 2 is supplying all the load currents, but generator 1 is off that means, current through this branch k is I K 2 and right and because here also branch current k this one will change. So, generating station 1 is off this is figure B. So, in that case you define another term A K 2 is equal to I K 2 upon I L right this is another term this is equation 79. So, this A K 1 and A K 2 these two terms are define because you have to make some approximation. So, A K 1 and A K 2 generally it make it as a current distribution factors.

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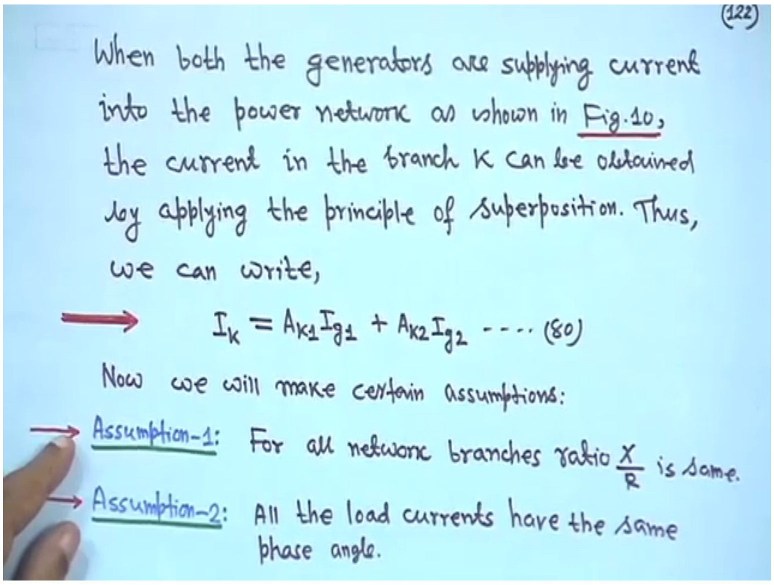

So, once you are what you call this making such kind of arrangement for analysis right now for in both the generators are supplying current into the power network as shown in figure 10 that means, here it is figure 10. So, this thing you have what you call that this both the generator when supplying power I g 1, I g 2 together. So, at the time current through branch I K this 1 actually you apply super position. So, in that case what will happen that I K you can write A K 1 into I g 1 plus A K 2 into I g 2. So, this is actually eighty this A K 1 A K 2 are the current distribution factor.

Now, we will make certain assumptions. So, assumptions are that for all network branches ratio X by R is same this is actually when a broad assumptions for transmission line X by R ratio is not same it is varies actually. Because you use different type of conductors, but we will assume that X by R ratio is same. Second thing is all the load currents have the same phase angle this is also not true, but we will assume that for approximation. So, this based on these two assumptions who will make your what you call that will derive that last formula.

So, to for this assumption, we will do something that why you are making this assumption suppose consider bus-i look at this one of the network as shown in this is figure 12. This is bus-i is voltage is voltage magnitude V i angle delta i and current through this load actually I L i. So, as load is equal to P L i plus j Q L i.

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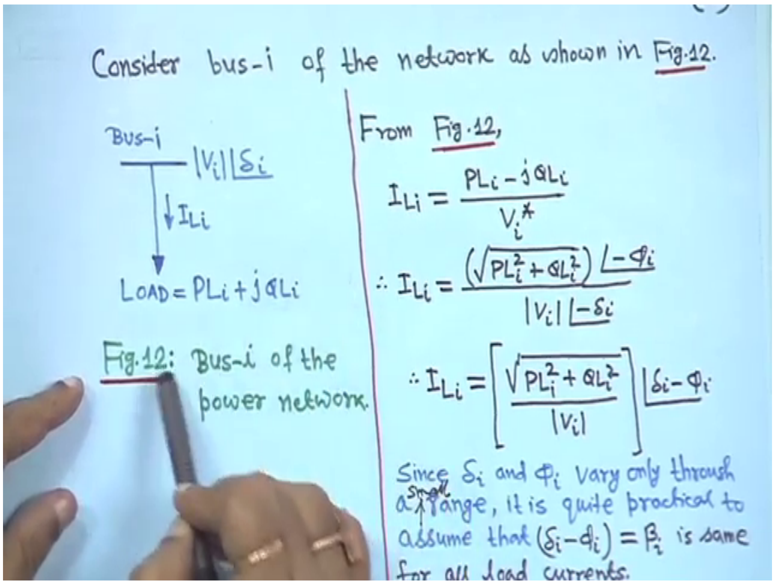

So, we know from figure 12, you can write now in general P minus j Q is equal to V conjugate i, I is equal to P minus jq upon V conjugate we know that. So, this; that means, this load current I Li we can write that is your P Li minus j Q Li divided by V i conjugate. That means, if this one next I Li can be written as the root over P Li square plus Q Li square and angle is minus phi i, phi i is equal to you are your what you call this actually your this thing tan inverse Q Li upon P Li.

So, this angle load angle you can write like this minus phi i right and this 1 voltage magnitude V i this conjugate its angle is delta i; that means, conjugate. So, it is minus delta i. Therefore, I Li is equal to root over P Li square plus Q Li square upon magnitude V i and this is minus delta i. So, it will go to their it is positive. So, it is angle delta i minus phi i. Now, this is the your what you call your load angle load current angle at bus i, but what will do this delta i minus phi i will assume they are same for all the load current that means, since delta i and phi i vary only through a small range in reality may not be not, but you will assume that. It is quite practical to assume that delta i minus phi i is equal to say beta i is same or right for all load currents that means, for all the load currents, we will assume this is same right for the all load currents. If it is so then what you call right that phase angle of I K 1 and I L or I K 2 and I L all will remain same. That means, A K 1 is equal to I K 1 upon I L or A K 2 is equal to your I K 2 upon I L they will become real quantity. So, that means, these angle will assume they are same for all the your what you call that all the load currents.

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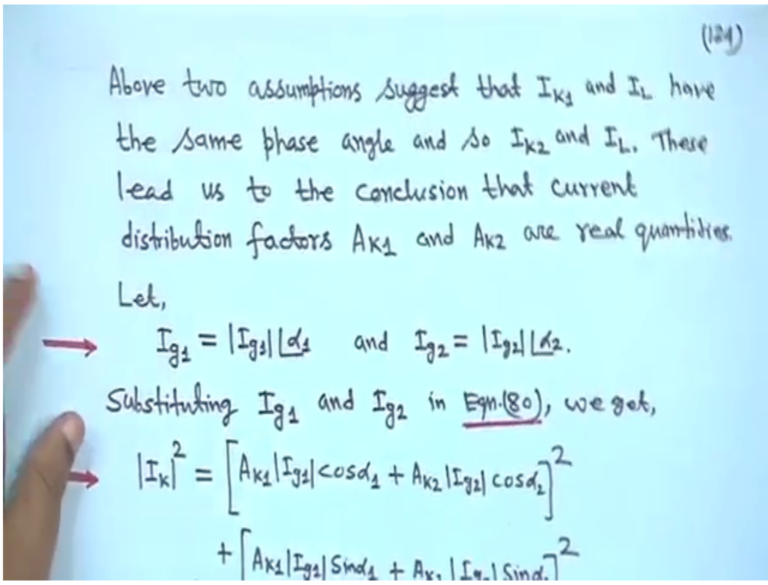

That means, A K 1 and A K 2 both are real numbers right real quantity. So, that means, that is why written here above two assumptions suggest that I K 1 and I L have the same phase angle. So, I K 2 and I L these lead us to the conclusion that current distribution factors A K 1 and A K 2 are real quantities.

Thank you again will come back.