Power System Analysis
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Lecture - 38
Optimal system operation (Contd.)

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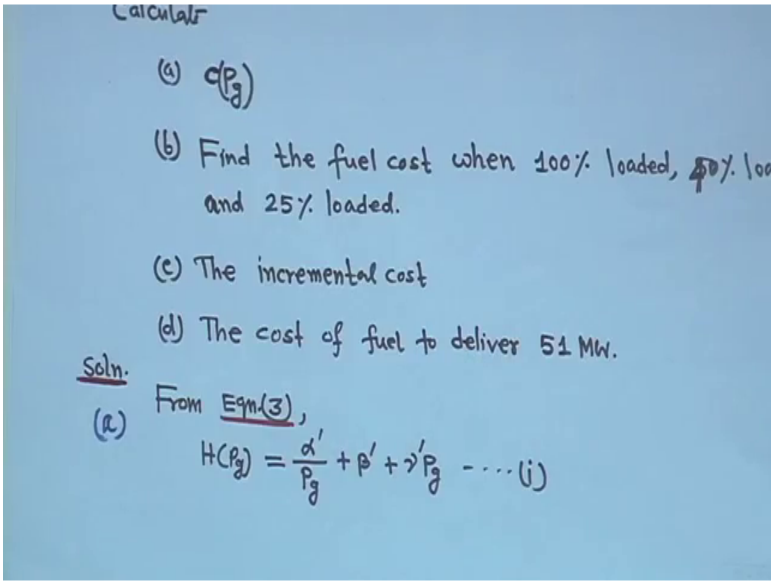

The next stage, your from equation, equation 3 that H P g is equal to alpha dash upon P g plus beta dash plus gamma dash P g this is say equation 1 right. So, that 3 measurement data actually given to you right.

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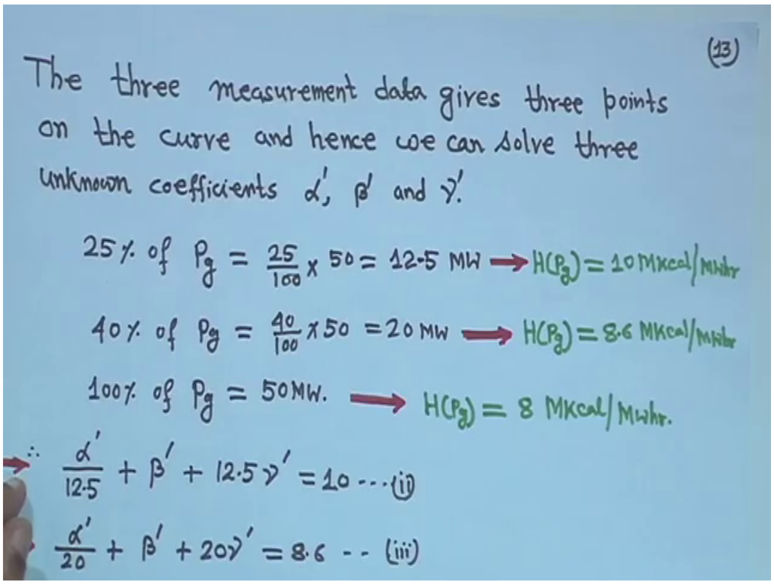

So, the 3 measurement data gives 3 points on the curve and hence we can solve 3 unknown coefficient alpha dash, beta dash and gamma dash right. 25 percent of P g that is equivalent to that it is 50 megawatt rating. So, it is 12.5 megawatt and correspondingly H P g is given data given that is your 10 mega kilo calorie per megawatt hour. This is given because these are.

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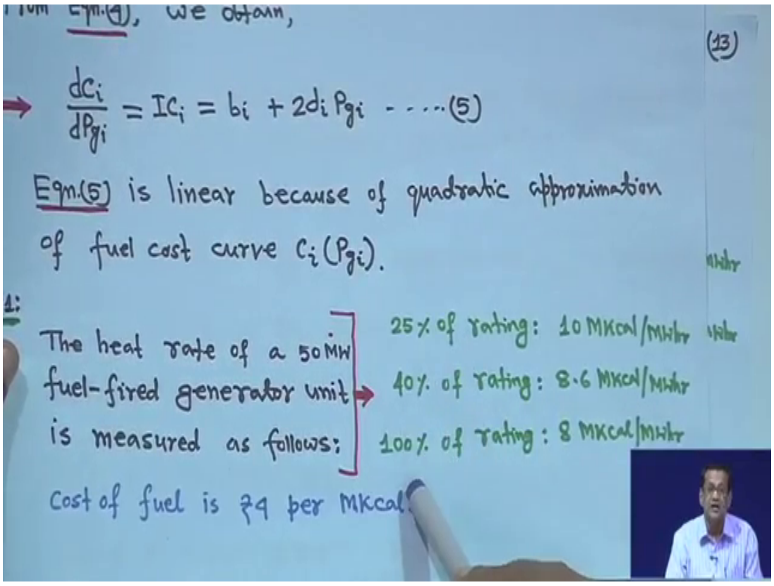

This is the data given here data given at 25 percent 40 percent 100 percent it is given here right.

Similarly, 40 percent of P g that is 20 megawatt right; 40 0.4 into 50, so that is h P g corresponding that 8.6 mega kilo calorie per megawatt hour and 100 percent of P g means 150 megawatt. So, that is H P g is equal to 8 mega kilo calorie per megawatt also in that equation 3 you put this value right. P g value and h value. So, this is alpha dash by 12.5. I mean this equation in this equation you will substitute h value and P g values for different rating right. Different power generation data right.

So, it is alpha dash upon 12.5 plus beta dash plus 12.5 gamma dash equal to 10 this is equation 2. Alpha dash by 20 plus beta dash plus 20 gamma dash that is 86 that is 3 and this one alpha dash by 50 plus beta dash plus 50 gamma dash is equal to 8 right.

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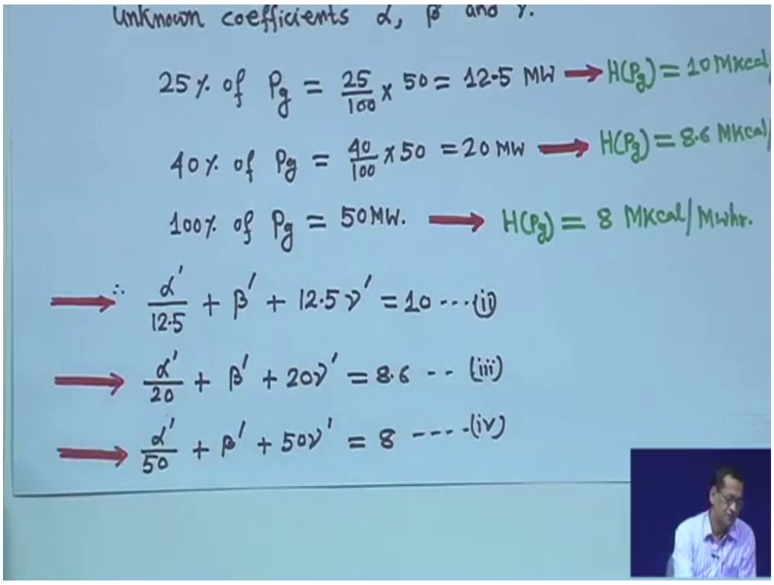

So, these 3 equation linear equations. So, you can solve these linear equations, your what you call? Using your calculator right. If you solve this then alpha dash beta dash gamma dash value will get right.

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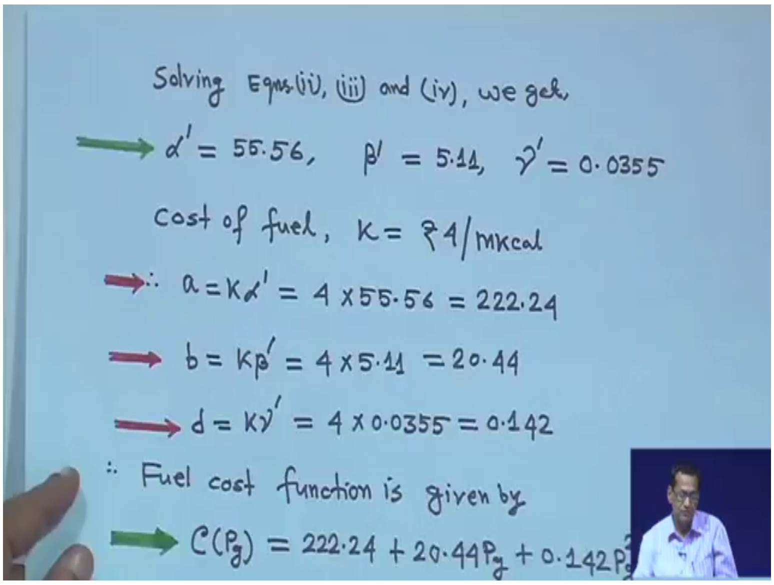

That means you solving equation 2 3 and 4 the 3 linear equation. 3 linear equation you will get alpha dash is equal to 55.56 right. Beta dash is equal to 5.11 and gamma dash is equal to 0.0355 therefore, cost of fuel K is taken as a 4 right. Therefore, is equal to we have seen K alpha dash that is multiply this 1 by 4, so 4 into 55.56. So, 220; 222.24.

Similarly, b is equal to K beta dash that is 4 into 5.11 that is 20.44 right. And d is equal to K gamma dash. So, 4 into 0.0355 is equal to 0.142. So, fuel cost function therefore, is given by that you substitute in that expression of a plus b P g plus your what you call d P g square. So, it is 222.24 plus 20.44 P g plus 0.142 P g square this is that function of C P g are obtained right.

Now, you have to find out that what is C P g at your 25 percent rating. At 25 percent rating means 50 megawatt it is generator.

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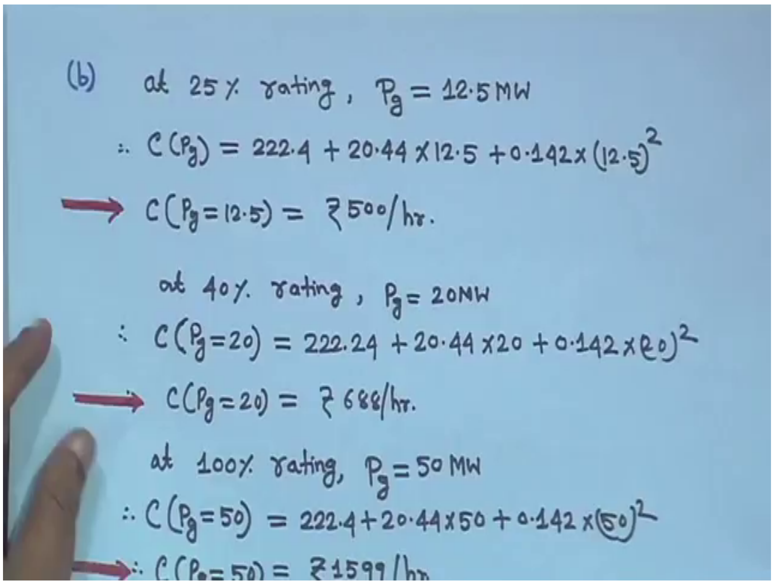

So, it is P g is equal to 12.5 megawatt therefore, C P g will be this p g value you substitute here you substitute here right. So, you will get C P g is equal to rupees say 500 per hour right. Just you will give a you will give this will give you a feeling that how is how you can go for solving such things right.

Similarly, at 40 percent rating again P g 20 megawatt put in that expression right.

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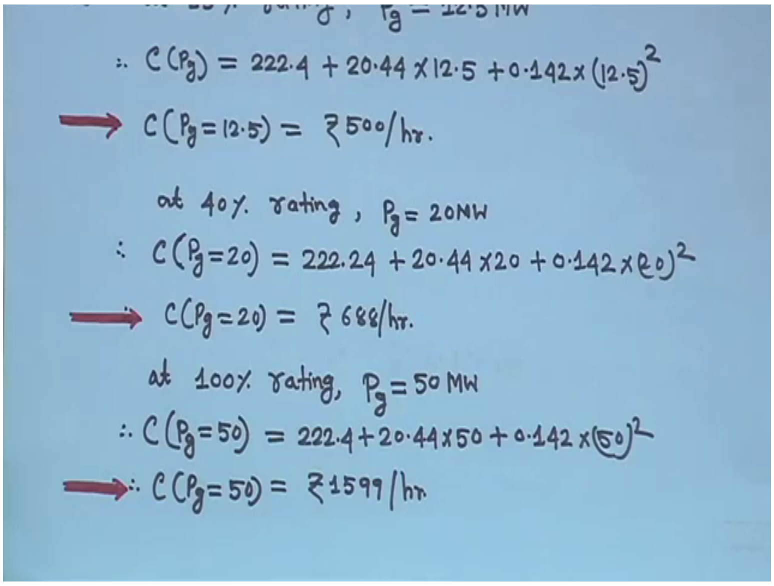

You will get it is rupees 688 per hour and at 100 percent rating when P g is equal to 50 megawatt C P g, P g is equal to 50 we will get rupees 50 and 1599 per hour this is C P g right.

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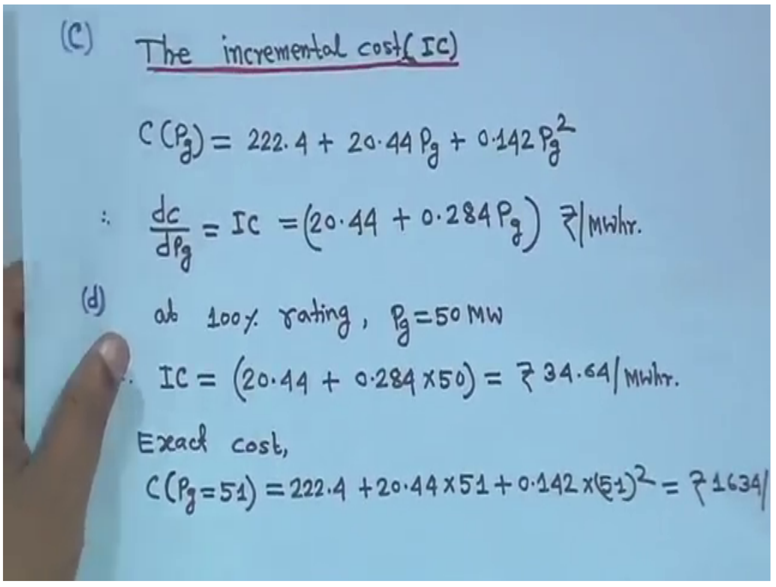

Now, this incremental cost when you will take you have to take the derivative of c with respect to P g right. With respect to pg; that means, incremental cost C P g will be 222.4 plus 20.44 P g plus 0.142 P g squared this we have obtained. Now take the derivative dc by d P g, dc d P g is equal to incremental cost IC that is 20.44 plus 0.284 P g rupees per megawatt hour right. At now at 100 percent rating P g is equal to 50 megawatt there were IC will be rupees 34.64 per megawatt hour right. An exact cost, exact cost for C P g 50 one if you substitute here in that expression whatever you have got for C P g that is your rupees 1634 per hour right. So, this is a simple example you have taken that how to make the how to get this here some ideas, some ideas you will get right. Regarding that economic operation right, these some ideas.

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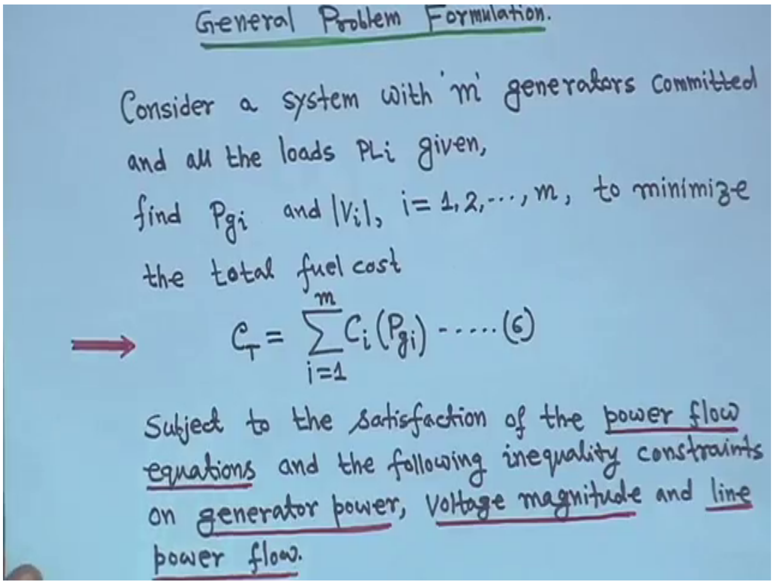

Next is that general problem formulation. So, slowly and slowly we will enter into the detail right. Now consider a system we have m number of generators and committed and all the loads P L i given actually you have you have suppose you have n bus n bus problem right. In a power network and out of which suppose you have your all the buses your you do not have your generators you have m number of buses generator right. So; that means, m, m 1 is equal to i will matter less than n say right. So, you have m generators committed and all the loads P L i given. So, you have your object will be fine P g i and voltage magnitude V i right. For i is equal to 1 to m to minimize the total fuel cost.

Now, in our here in this thing we will not go to solve where that what is will be our V i because, for that you need detail load flow and this then your large problem then only we can manage it right. That is without computer we cannot we cannot do it actually right. So, whatever this ah, whatever we will come as far as the classroom exercise that a we will do right. So, so the total fuel cost can be given you have m number of generators therefore, i is equal to 1 to m C i P g i this is equation 6 right.

Now, question is that your subject to the satisfaction of the power flow equation that is the load flow equations right; that means, load flow equations that is that is called equality constant in power system right. And the following in inequality constant like generator power, you have a minimum maximum limit of generator power voltage magnitude also you have a minimum and maximum value and line power flow right. Line because every line conductor a transmission line conductor has it is maximum power carrying capability or maximum current carrying capability they call thermal limit right. So, you have to, you have to put the bar right; that means, limit has to be put right. Just whatever you want you cannot do that; that means, so many, so many constraints are there right. So, those this is equation 6 right. This is equation 6.

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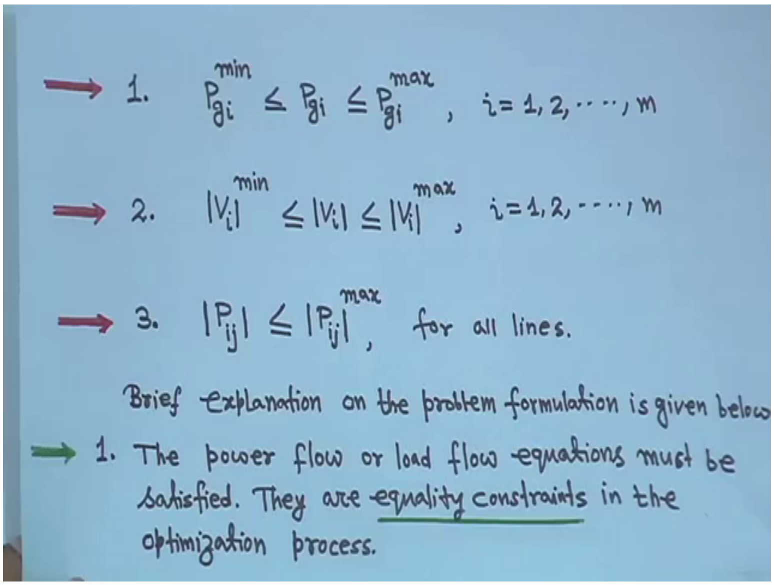

So that means, that first constraint is that P g i generation must be greater than equal to P g i min and less than equal to P g by P g i max for i is equal to 1 to m because you have m number of generators right. And voltage magnitude V i must be greater than is equal to V i min less than equal to V i max i is equal to 1 to m again right. And the power flow that P ij mod P ij less than equal to say mod P ij max, I mean it has to be less than equal to that. So, this is for all of all the lines right. So, so many constraints are there right.

So, we will take judiciously that how to solve this. So, when you are solving economic load dispatch problem then you have to see that all the constraints also are not violated right. It has to be within the constraint right. And within the constraint all you have to optimize the thing. So, brief explanation on the problem formulation is given below. The power flow or load flow equation must be satisfied, that is true. You have to solve load flow right. Right for large problem even if you take a large problem you have to solve the load flow without load flow you cannot move right.

So, therefore, they are equality constraints in the optimization process because you are solving load flow. So, it is the equality constraints right. So, that it first is load flow the second one is your the lower limit on P g i is due to boiler.

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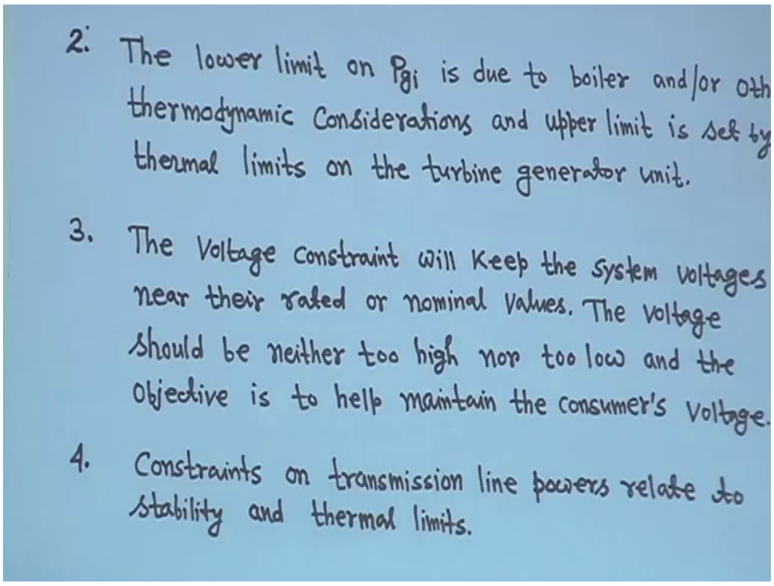

And or other thermodynamic consideration right, and the upper limit is set by thermal limit is on the turbine generating units right. So, the voltage constraint will keep the system voltages near the near the rated or nominal values right. The voltage should be neither too high nor too low right. And the objective is to help maintain that consumer's voltage you have to maintain the voltage right. Such that at the consumer ain the voltage should be within the limit, right I mean, within the permissible limit right.

So, that is why voltage constraint also must be you have to be careful right. And fourth one is constraint on transmission line powers relate to stability and thermal limit is that also you have to consider right. That your line through the transmission line your power flow limit that also you have to consider right.

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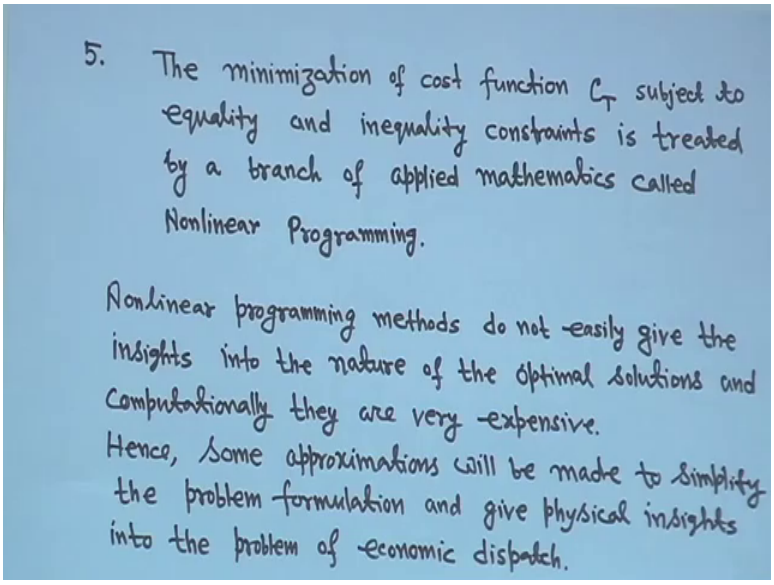

And these are the, these are the several constraints several constraints right. The minimization of cost function C T subject to equality and in inequality constraints is treated by branch of applied mathematics called non-linear programming.

So, using non-linear several different type of non-linear programming, one can solve this problem right, but nowadays that soft computing techniques are available right. Although here I have written something that computationally that this non-linear programming technique very expensive. Expensive in the sense that of course, you know it will computationally it may take more time, but soft computing technique if you apply that will give you a global optimum value. In that case what will happen it takes more time than non-linear programming, but question is that. So, this kind of non-linear programming method there is a strong possibility that it will start to local optima; that means, either local minima or local maxima right.

But whereas, this thing for soft computing technique actually write your generally it gives the your what you call that global optimum value. So, that is why nowadays people are solving this kind of problem using different type of soft computing techniques right. So, anyway so, we will follow, we will follow, we will not follow here in soft computing technique that is a different thing right. What we will see using this your what to call classical optimization method right.

So, so, but before that to solve as some approximation will be made to simply the prom problem for that is why I have written non-linear programming methods do not easily give the insight into the nature of the optimal solution and computationally they are very expensive right. So, I mean it takes time, but cop counting soft computer technique though gives your global optimum, but it takes much more time right. As, but in some approximation, but ah your what you call and your there are many comparisons will be there between this classical optimization method and soft computing methods those are not discussed over here right, but if you send mail to me I can tell you that what are those differences right.

Hence some approximation will be made to simplify the problem formulation and it gives the physical insight into the problem of economic dispatch right.

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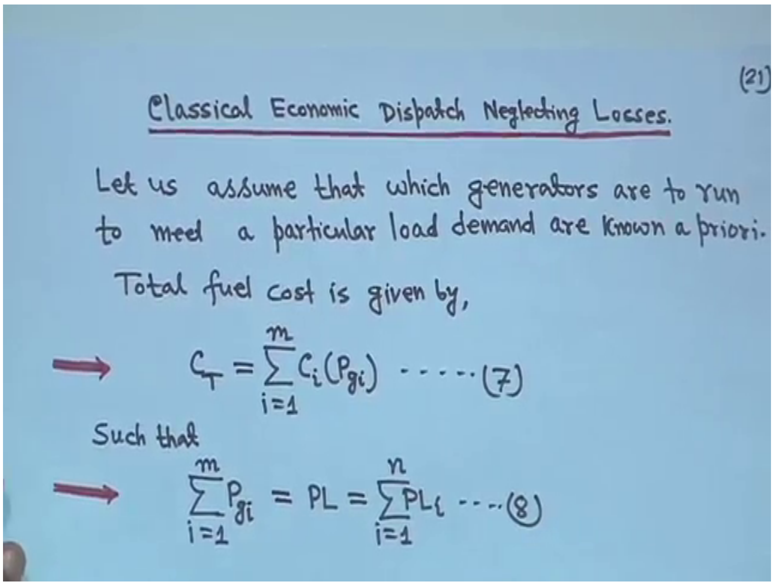

So, what we will do we will first go for a classical economic dispatch neglecting losses right. Because we know the generation is equal to load plus losses right. So, first we will consider that without power loss without, I mean neglecting power losses right. So, this is classical economic dispatch neglecting losses right.

Let us assume that we generators are to run to meet a particular load demand are known a priori right. You know suppose you have So many generators and you assume that which generators will run that you know that did that that is known to you right. So, suppose you have m number of generators. So, C T will be the total the C T will be i is equal to 1 to m C i P g i this is equation 7 right. Subject to that generation total generation i is equal to 1 to m P g i that we P g 1 plus P g 2 up to P g m is equal to the PL is equal to total load. Loss is not, loss we will see later at that time it will at the time it will become PL plus p loss that we will see later right. Is equal to i is equal to 1 to n P L i the power system you have n number of buses. So, i is equal to 1 to n that is sigma P L i this is equation 8 right.

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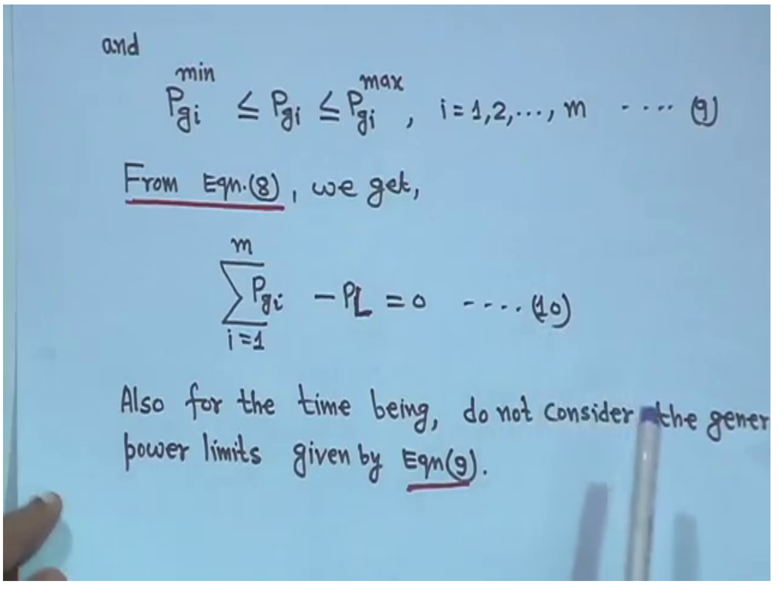

And your this P g i, this P g i must be within this limit that P g i min and P g i max for i equal to 1 to m this is equation 8 right; that means, equation 8 can be written sigma your y is equal to 1 to m P g i the total generation minus PL that is total load PL is equal to sigma i equal 1 to m P L i right. That is equal to 0 this is equation 10.

Now, also for the time being do not consider generated power limit is given by equation 9. For the time being will not consider this generation power limit right. For the time being will not consider it right.

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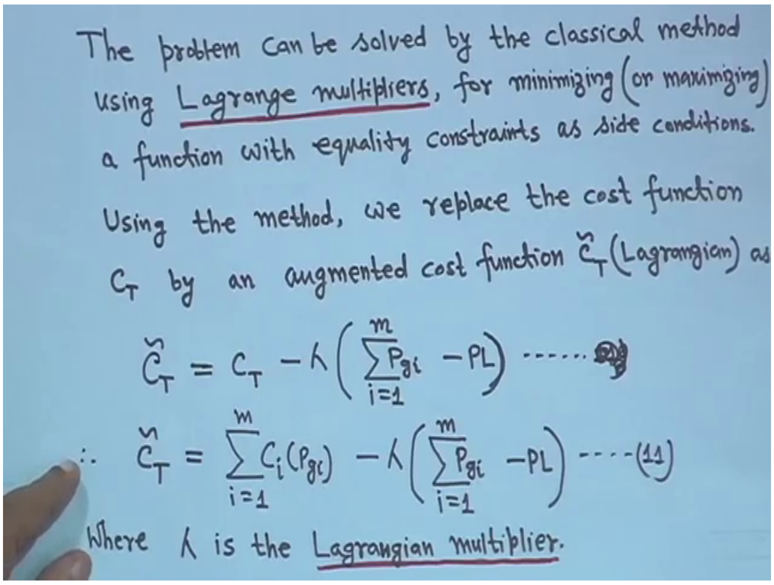

Therefore, the problem can be solved by the classical method using Lagrange multipliers right. So, for minimizing or maximizing or in general for optimizing right. So, a function with equality constraint as side cons conditions right.

So, using the method we replace the cost function C T by an augmented cost function C T tilde that is Lagrangian as. So, C T tilde we can write that is augmented cost function C T minus lambda in brackets sigma i is equal to 1 to m P g i minus PL right. So, that is your this constraint, this your equality constraint, this one is your equality constraint right. This one we are bringing into this augmented cost function the C T tilde is equal to C T minus lambda in bracket i is equal to 1 to m sigma P g i minus PL, when solution will convert sigma P g i will be sigma Pl. So, this term will vanish I mean it will be 0 right. So, at the time C T tilde will be C T right, but multi minus lambda multiplied right.

Now; that means, C T tilde equal to i is equal to 1 to m C i P g i right. Minus because C T is equal to sigma i is equal to 1 to m cig minus lambda c in bracket i is equal to 1 to m P g i minus PL this is equation 11. And where lambda is the Lagrangian multiplier actually, lambdas actually, lambda it has become my habit actually lambda is this one. This is the lambda, this is the lambda, but these are become my habit. So, please forgive me for that right.

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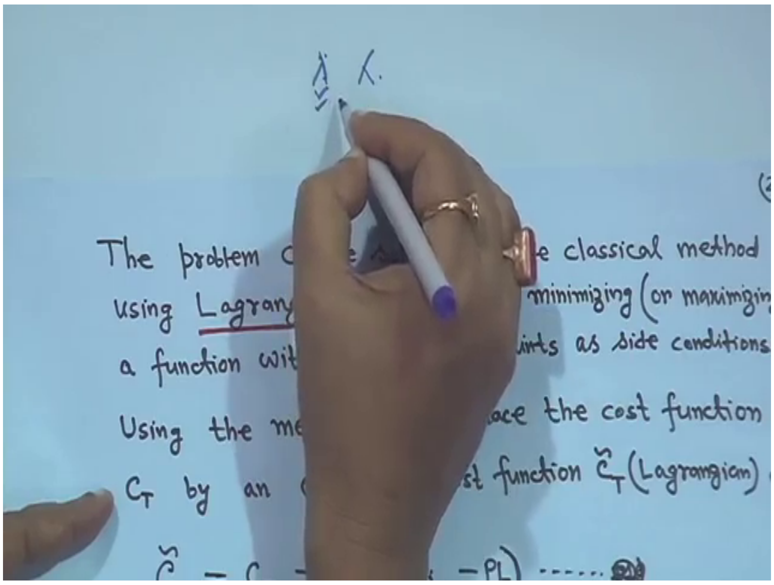

So, this is actually lambda is the Lagrange multiplier right. Next what you have to do is that that minimization can be achieved by the condition dc tilde that d P g i equal to 0 right.

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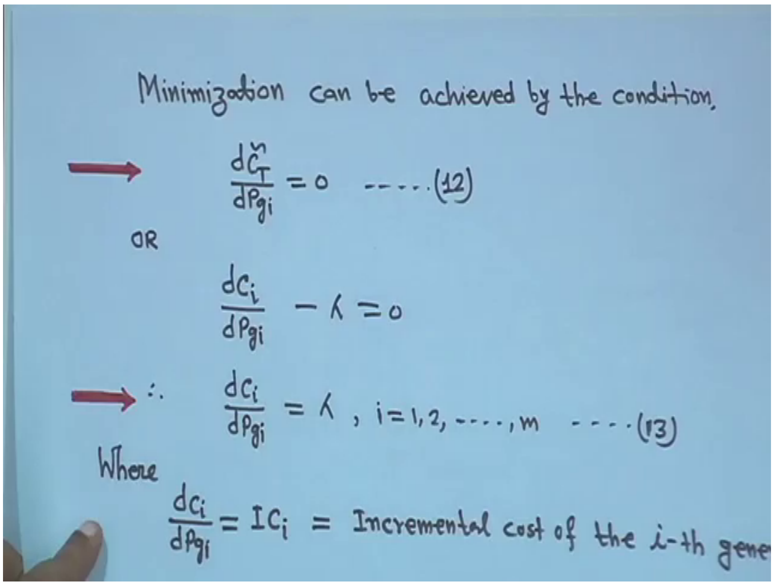

This condition you can put there for d C i d P g i minus lambda is equal to 0 right. If you take the derivative of this one right. With respect I mean dC T your derivative of this one delta p your P g i right. Then it will be your d C i d P g i and here also if you take with respect to i th one then only minus this is constant and you are taking derivatives to the i th one it will be minus lambda, right into one so; that means, this is minus lambda is equal to zero; that means, this d C i d P g i equal to lambda for i equal to 1 to m. This is equals to equation 13. The d C i d P g i is equal to I C i is called incremental cost of the i th generator right.

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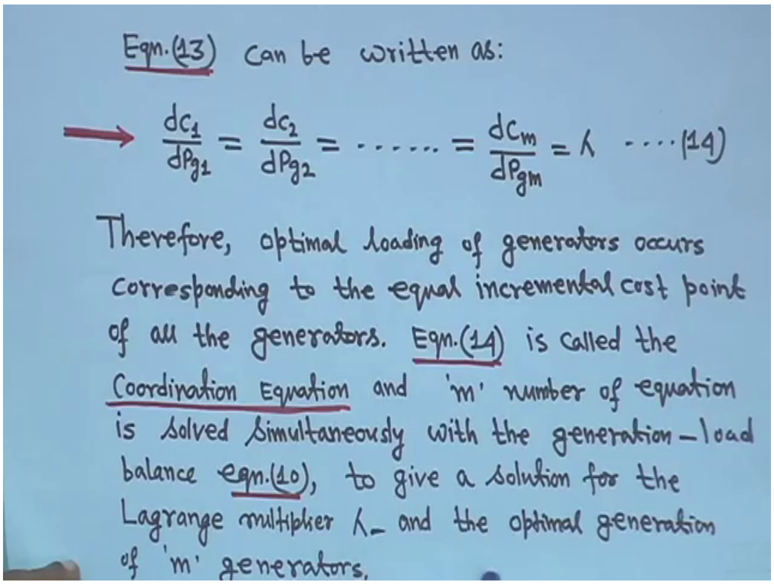

So, I C i right; that means, that means, that means, equation 13 can be written as d C 1 upon d P g 1 is equal to d C 2 upon d P g 2 up to this a m th term d C m upon d P g m is equal to lambda; that means, what that incremental cost for the of your what you call optimum operation for all the all the generating unit is they are equal lambda right. Therefore, optimal loading of generator occurs correspond to the equal incremental cost they are all equal incremental cost right. Point of all the generators that is equation 14, is called the coordination equation this equation is called coordination equation right. And m number of equation is solved simultaneously right. With the generation load balance that is equation 10 right. To give a solution for the Lagrange multiplier lambda and the optimal generation of m generators right.

That means they I mean system will operate your optimally whatever cost fuel cost curve you have C 1 C 2 right, but their incremental cost curve incremental your cost value that d C 1 d P g 1 d C 2 they have all have to be same right. So, this is the condition when you are not considering the losses right. So, this is actually how these and this equation is called coordination equation one has to solve right.

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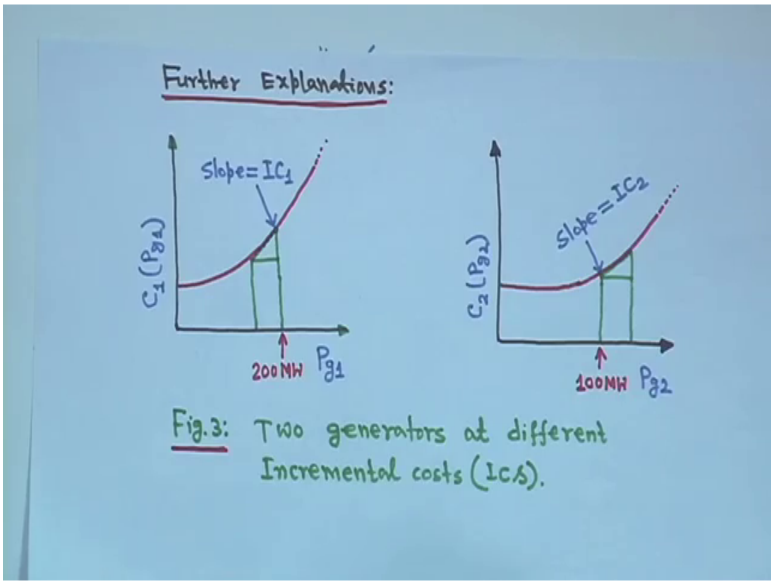

For example for example, say for example, you have a, you have a to a cost characteristic of 2 generating units. One is like this C 1 P g 1 this side P g 1 another is c 2 P g 2 this one, these. So, this is another type of characteristic this is another type of characteristic right. Suppose if you increase there right. For example, this is a slope you call IC 1 for in unit one there is a slope somewhere IC or is equal to IC 2 right. So, if you suppose some load has change right. Some load has changed. So, naturally this generator has to increase or decrease right. Ultimately your for that you if you change it I mean this is 200 megawatt, this is 100 megawatt right. So, if you try to find if you just your what you call if you try to adjust P g 1 and P g 2 for example, equally say right. Although in reality optimal will be different right.

So, in that case you will find that change, changes that what you call your delta changes here and there because, some load has changed right. You will find the slope IC 1 and slope IC 2 will be different because this cost characteristic is different. For a change in your what you call that low if change the load generation will change. So, for that way the slope will be different if it moves from this point to this point it comes or if you move from this to this point. You will find it from the drop also something has been drawn So that you will find slopes are different right. So, 2 generators are different incremental cost ICs right, but they have to operate at equalizes right.

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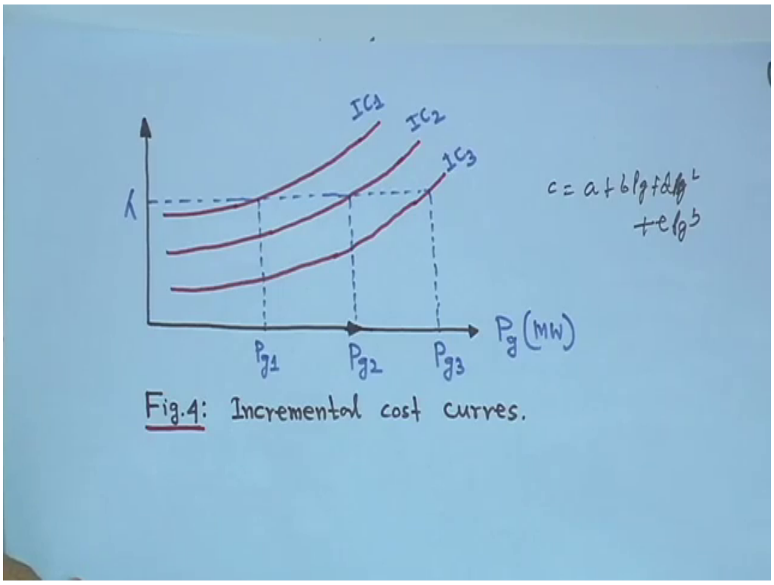

So that means, that means, if you look into this suppose if I have 3 generating units, if I have 3 generating units right. Then this is say this side is say lambda and this side is 3 this is this is a IC 1 incremental cost curve for this one. Actually whatever we have taken that quadratic object quadratic cost function C i write A i plus B i P g i plus D i P g i square when you take the derivative right. It will be linear, but if you take cubic it will be not linear it will become slightly quadratic right.

So, instead of drawing that this linear one I have made little bit of curve we linear right. You can make it linear also no problem right, but this one what will happen although they have 3 different ICs right. Your IC 1 IC 2 IC 3 right; 3 different IC right and P g 1 P g 2 P g 3, but they all have to operate at equal lambda because we solved d C 1 upon d P g 1 is equal to d C 2 upon d P g 2 up to d cm upon d P g m is equal to lambda. So, they all have to operate at equivalent equal lambda.

So, you say this is the lambda value. So, at this case if you see generation for this one P g 1 this is P g 2 this is P g 3 right. There that generation P g 1 P g 2 P g 3 actually completely depends on their cost characteristic that and the slope also right, their slope also. So, that is the incremental cost curve and this is P g megawatt although it is linear, but once again repeating that your this is your little bit curve linear are taken right. So, this is your what you call that some ideas about the incremental cost right.

Next is that algorithm that how to solve this one solve this problem.

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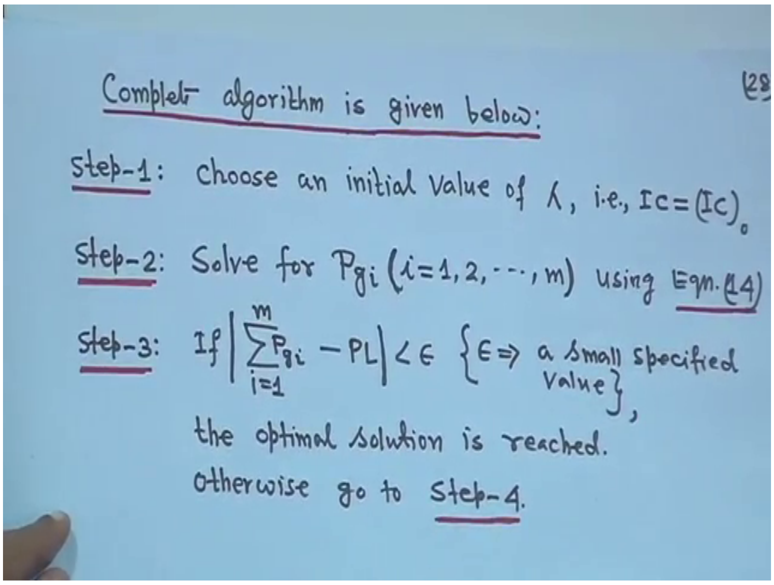

So, in this case the complete algorithm is given below for example, step one you have to choose an initial value of lambda. Later, later we will see the gradient method how we will go for iterative way, but for this one choose an initial value of lambda that is IC is equal to IC 0 suffix 0 is here right. That is lambda that is that is IC 0 right. Step to solve for P g i using equation 14 right. So, using that is your this coordination equation; that means, these equation that is your coordination equation this equation you have to solve this one right, this one you have to solve.

Then your what you call then you have to check the absolute value right. That i is equal to 1 to m P g i minus PL it was a minus PL whether it is less than epsilon or not. Epsilon is a small specified value that you convergence characteristic you put it because generation has too much load loss is ignored for this case right. So, optimal solution if it is reached, the optima if it is less than epsilon optimal solution is reached otherwise you have to go to step 4 right. I mean next step you have to go.

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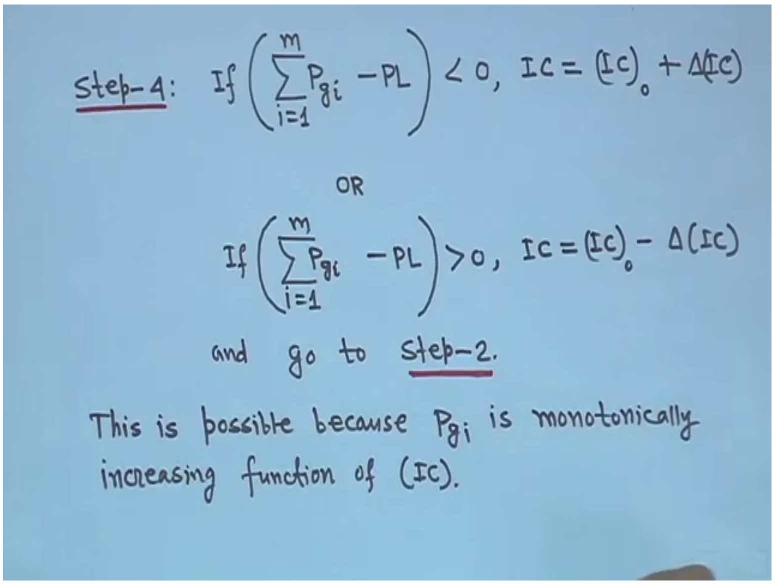

So, in that case, in that case what will happen that now you check if sigma i is equal to 1 I mean total generation minus load if that difference is negative. Then incremental IC 0 you increase it later iteratively we will see it, but IC is equal to IC 0 plus delta IC that you this thing what you call you increase it. Or if it is positive if the total generation minus load the difference is positive then IC is equal to IC 0 minus delta IC here in this case, you increase the generation in this case you decrease the generation right. And go to step 2 again from the step 2 you have to repeat this is possible because P g i is monotonically in your increasing function of IC right.

So, that we have seen the previous one that with the increase on pc that IC that is increasing right. So, that way this condition 2 conditions are considered right. Therefore, this is the thing, so this is the idea that how to solve it later when we will take numerical. So, we will see that right, at least 7 I think I cannot recall number of 8 numericals, we will solve for this particular topic right.

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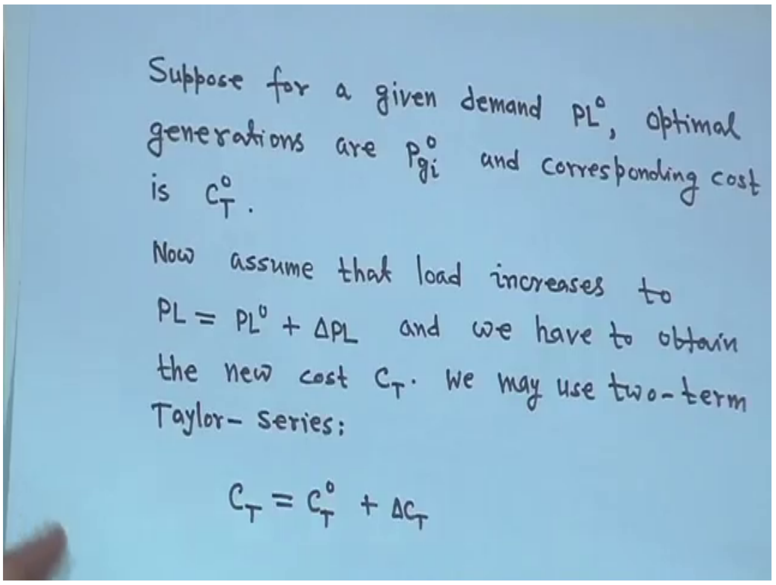

Now, suppose for a given PL 0 right. So, this is your, suppose for a given demand PL zero; that means, some demand is given PL 0 right. So, optimal generations are P g i 0 and corresponding cost is C T 0 suppose initially it was total load was PL 0 this PL 0 is a superscript right. And similar and at that time total generation was P g i 0 loss is ignored and corresponding cost is C T 0 right.

Now, assume that load increases to make PL is equal to PL 0 plus PL 0 plus delta PL suppose load has increased. Load has changed by delta PL right. So, PL is now PL 0 plus delta PL and we have to obtain the new cos C T we may use 2 term Taylor series C T is equal to C T 0 plus a delta C T right. C T 0 is equal to a C T is equal to C T 0 plus delta C T right.

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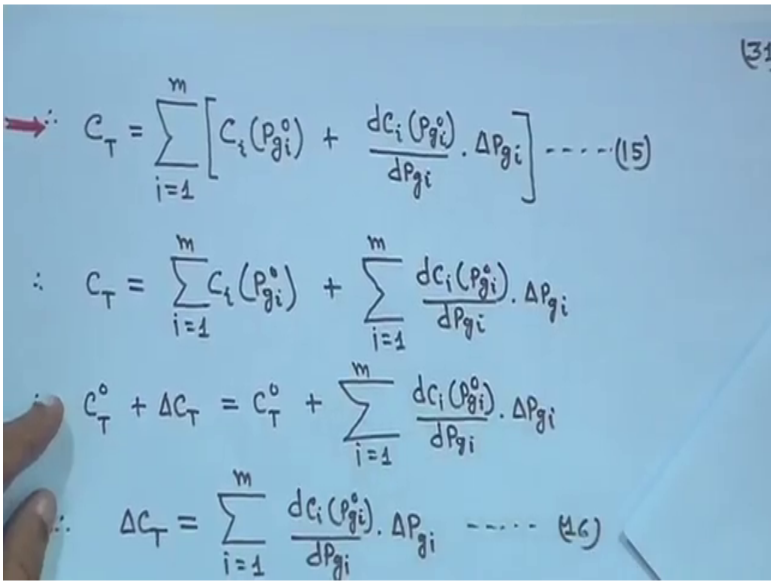

That means, that means, your C T is equal to right. If it is, so then C T is equal to your sigma i is equal to 1 to m that is C i P g i 0 plus d C i d P d C i upper this is C i P g i actually C i is a function of P g i 0. So, d C i d P g i into delta P g i this is equation 15 right.

Because C T is taken as your what you call C T 0 plus delta C T right. We have to obtain a new we may use to term in Taylor series; that means, this C T we are actually writing that i is equal to 1 to m C i P g i plus d C i P g i 0 d P g I into delta P g i this is equation 15. Or C T is equal to i is equal to 1 to m this term C i P g i plus right. i is equal to 1 to m d C i P g i, i instead of again and again telling function of that understandable d C i d P g i into delta P g i right.

So, this term this is actually C T 0 therefore, C T than the C T is equal to also you know C T 0 plus delta C T. Because C T is equal to your C T 0 plus delta C T no equation number is given here C T 0 plus delta C T and here also; that means, C T we are writing C T 0 plus delta C T is equal to C T 0 because this term is C T 0 plus that sigma i is equal to 1 to m d d P g i into delta P g i. So, C T 0 C T 0 will be cancelled therefore, delta C T will be i is equal to 1 to m d d P g i into delta P g i this is equation 16 right; that means, delta C T will be this one.

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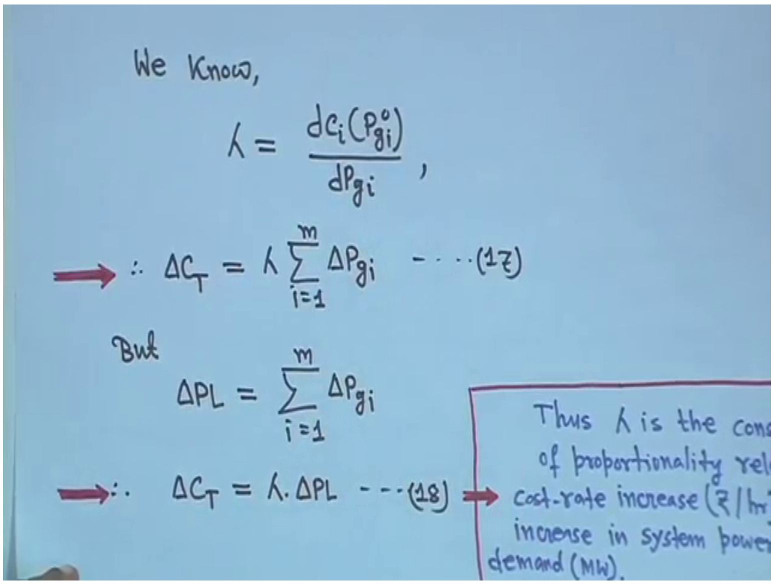

Now, we know lambda is equal to d, your d C i d P g i this you know that incremental cost lambda right. Therefore, in this equation therefore, in this equation you this one you replace it by lambda all right. This equation 16 you replace it by lambda. So, if you do So, then lambda then delta C T will be is equal to lambda i is equal to 1 to m your sigma i is equal to 1 to m delta P g i, this is equation 17 right.

Now, that as the as the for a small change in this, is that small total change in some load delta PL that is basically, nothing but summation of all the generation change that is sigma i is equal to 1 to m delta P g i right. So; that means, whatever load change generation has to accommodate that your generator has to generate that power loss we have in ignored here right. So, delta PL is equal to this much; that means, this one, this one is equal to delta PL you put it here therefore, delta C T will be is equal to lambda into delta PL right; that means what? What does it mean? This thus lambda is the constant of proportionality relating to cost rate increase that is rupees per hour to increase in system power demand right.

So, basically when there is a that is the that is another significance of lambda that load has changed right, but delta C T actually directly what you call directly proportional to that your total change in load delta PL and that proportionality constant actually is lambda right. That is why thus lambda is the constant of proportionality relating the cost rate increase that is whatever it is increase rupees per hour to increase the system power demand that is in megawatt. That is your, what you call this is that physical significance of lambda also right.

Thank you.