Power System Analysis
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Lecture - 33
Load Flow Studies (Contd.)

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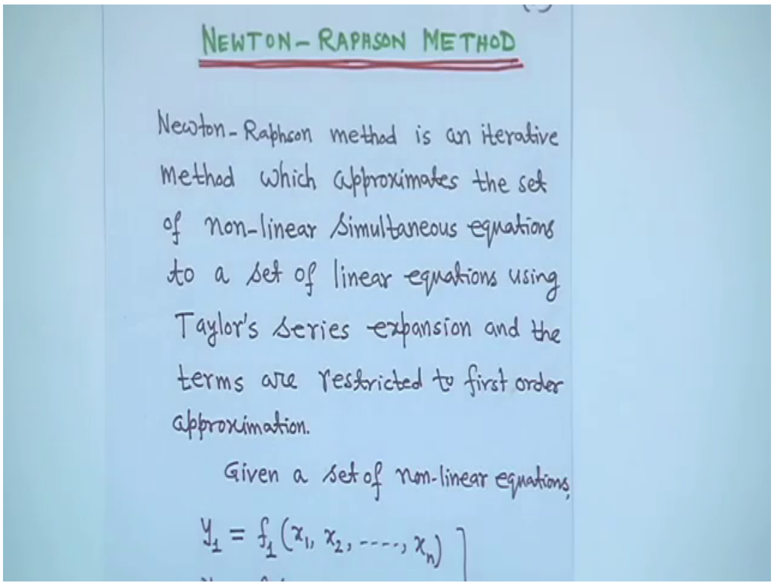

Next is Newton-Raphson method: considerlly have seen now say Newton-Raphson method in sort inner method right. So, Newton-Raphson method also is an iterative method in your in your mathematics course you have studied Newton-Raphson method right solving that you know what to call different problems right.

So, which approximates the set of non-linear simultaneous equations to a set of linear equations right using Taylor's series expansion and the terms are restricted to first order approximation we will only consider up to the Taylor's series expansion only the first order terms right will not go for second or higher order right. So, for example, you take given a set of non-linear equations.

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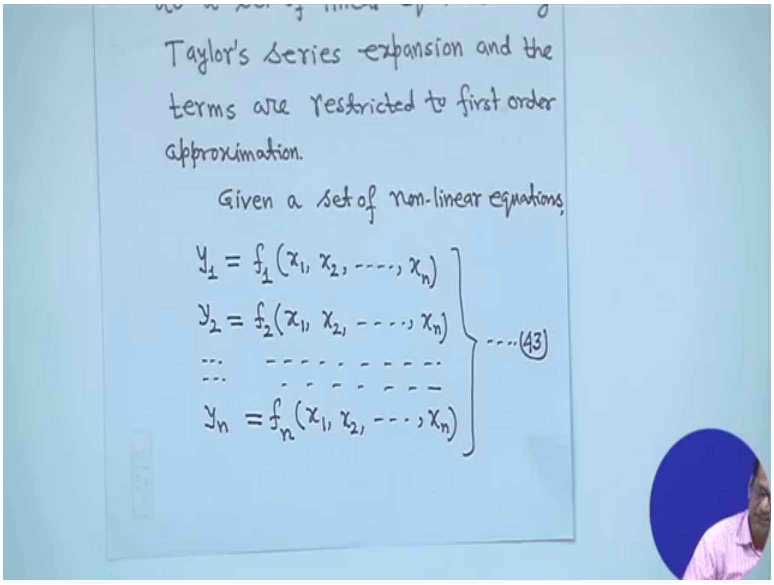

So, suppose it is given y 1 is equal to f 1 function of x 1 x 2 up to x n similarly y 2 is equal to f 2 x 1 x 2 up to x n. Similarly up to yn is equal to fn x 1 x 2 up to x n you take a set of non-linear equation say this is equation 43 first we will see that iterative process then we will go for load flow right. So, this is your non-linear equation. Now just hold on.

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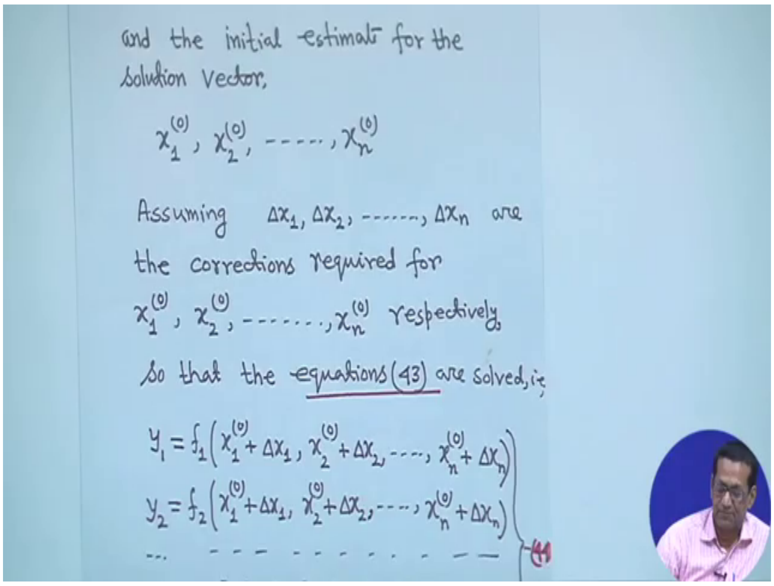

Now, suppose the initial estimates for the solution vector is given these are the initial values given x 10, x 20 up to x n0 for all x values initial values are given right. Now what to do assuming delta x 1 delta x 2 up to delta x n right are the corrections required for x 10, x 20 x n0 right respectively every iteration you need to update the x value.

So, these are the correction required delta x 1 delta x 2 delta x n right. So, so that the equation 43; that means, previous equation; that means, these equations; this equation; equation 43; this equation right.

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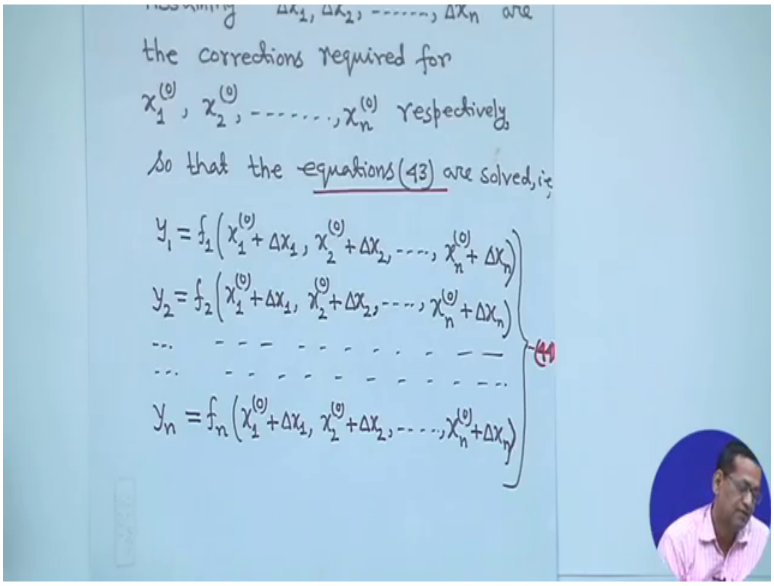

You can make it like this that y 1 is equal to f 1 it is x 10 plus delta x 1 comma x 20 plus delta x 2 comma up to x n0 plus delta x n right because every a you have to update this one right.

Similarly, y 2 is equal to f 2 x 10 plus delta x 1 x 20 plus delta x 2 up to x n0 plus delta x n for y n fn that x 10 plus delta x 1 x 20 plus delta x 2 x n0 plus delta x n this is equation 44 right. Now this equation when you put like this; this equation you have to expand in Taylor series right and we will consider only the first your first order first order this thing your second order had derivatives will not consider right.

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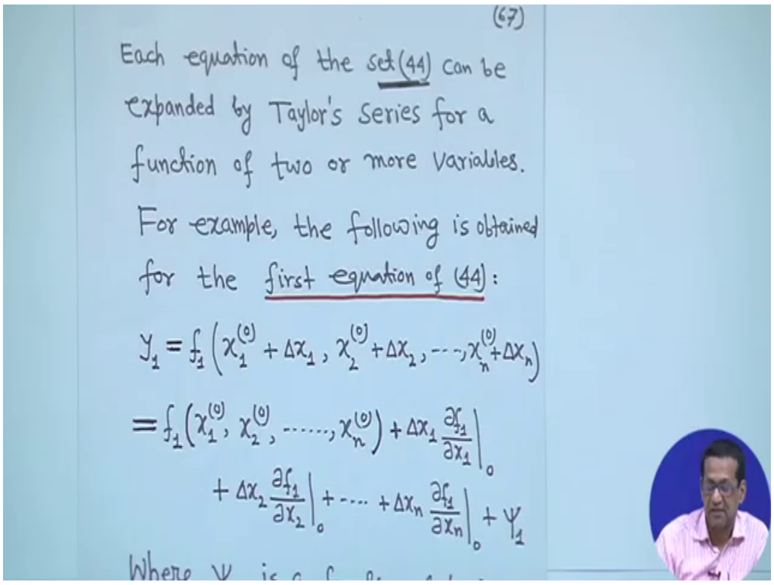

So, the equation the equation of the set 44 because you have n number non-linear equation can be expanded by Taylor’s series for a function of 2 or more variable right for example, the following is obtained of the first equation of 44 I mean if you take that first equation; that means, this one if just first equation you take which is set of non-linear equation right.

Consider only the 44 let me this one sorry this one this first equation of the 44 this is 44 first equation of this one right. So, if you do. So, so then you can write this y 1 is equal to f 1 x 10 plus delta x 1 x 20 plus delta x 2 up to delta x n0 plus delta x n right. So, you expand it in Taylor series.

So, it will become f 1 x 10 comma x 20 up to x n0 then plus delta x 1 del f 1 upon del x 1 put that whatever the I mean the here you have to take the derivative and put the all the initial value that is why one you are what to call the suffix 0 0 is shown here right that mean del f 1 upon del x 1 whatever it comes you have to put that values x values right.

Similarly, plus del x 2 del f 1 upon del x 2 same thing up to del x n del f 1 upon del x n plus psi 1 psi 1 is the higher order terms will not consider right.

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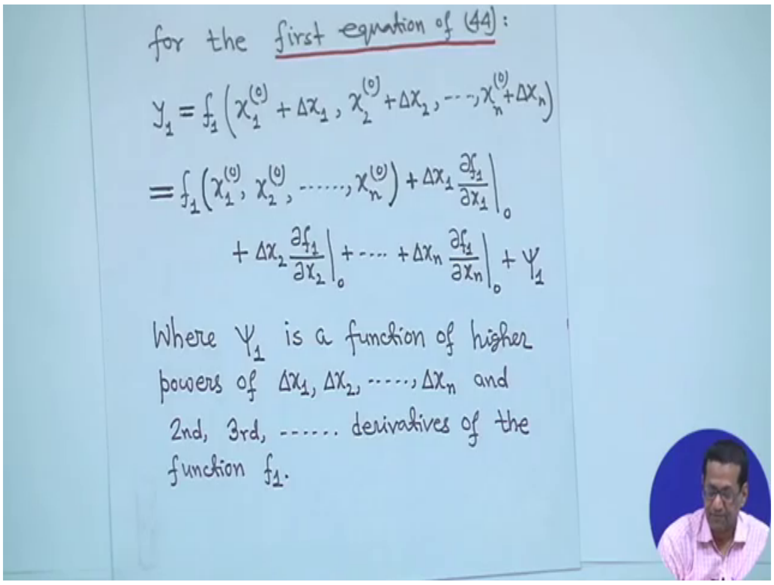

Where psi one is a function of higher power of delta x 1 delta x 2 and delta x n and second third derivatives of the function f 1 and will not consider that we will take only up to this right only this term. So, this we first we expand in Taylor series right.

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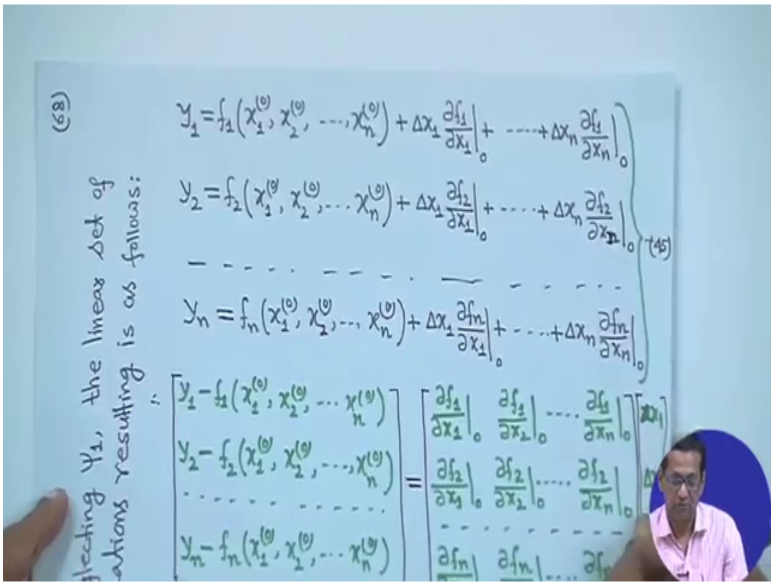

So, next is I will I will rotate this one. So, neglect neglecting psi 1 the linear set of equations resulting is as follows. So, I will rotate this right. So, y; that means, y 1 is equal to you can write f 1 x 10 comma x 20 up to x n0 plus del x 1 del del f 1 upon del x 1 plus del x n up to del f 1 upon del x n this all this values actually del f 1 upon del x 1 we have to substitute the value of initial values and evaluate that is why this these are these were these were shown right.

So, similarly y 2 is equal to same thing f 2 x 10 x 20 up to x n0 plus delta x 1 del f 2 upon del x 1 up to del x n upon del f 2 x n right. So, similarly that y n is equal to fn x 10 x 20 x n0 plus delta x 1 del fn upon del x 1 up to dot dot dot up to plus delta x n del fn upon del x n this is equation 45 right. Now you can write y 1 minus f 1 is equal to this 1 y 2 minus f 2 is equal to this 1 y n minus fn is equal to this 1.

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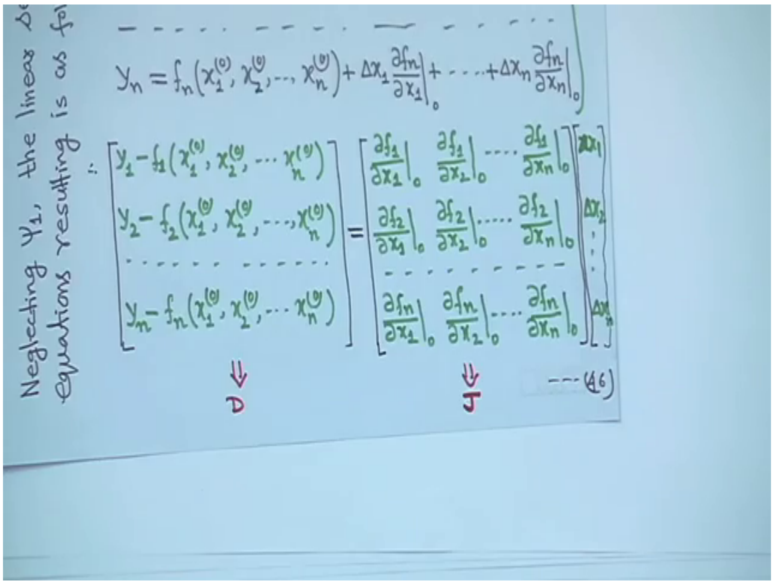

So, same thing we are writing here that y 1 minus f 1 x 10 x 20 up to x n0 then y 2 minus f 2 x 10 x 20 the function of all the initial values f 2 then y n minus fn x 10 x 20 up to x n0 is equal to this is equal to this from here this side this side right. So, it is del f 1 upon del x one, but all this things again and again noted understandable that after take the derivative at put that values right initial values right.

Del f 2 upon del f 1 upon del x 2. So, all the every iteration actually these matters will change right. So, anyway up to del f 1 upon del x n and similarly del f 2 upon del x 1 del f 2 upon del x 2 del f 2 upon del x n del x n here it is del fn upon del x 1 del fn upon del x 2 del fn upon del x n the delta x 1 delta x 2 up to delta x n right this equation is 46.

This one this mismatch this one let us define as a D right and this one this one you define as a J right. So, J actually is called the Jacobean for the function fi right.

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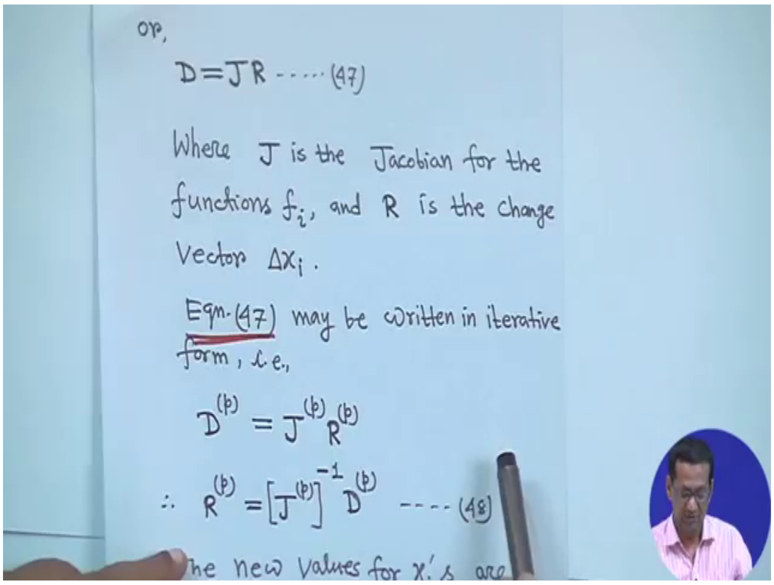

Therefore this equation if it is D if it is D and this is J and this residual vector delta x 1 delta x 2 this can be called as capital R right.

Delta x 1 delta x 2 delta x n this can you call a capital R right; that means, D is equal to your can be written as J R right if this is D this is J and this vector if it is R not shown here, but I am telling you can hear it delta x 1 this one be R then this one equation can be written as D is equal to R R this is equation 47 equation 47 right where J is the Jacobean for the function fi and R is the change vector that is delta xi right.

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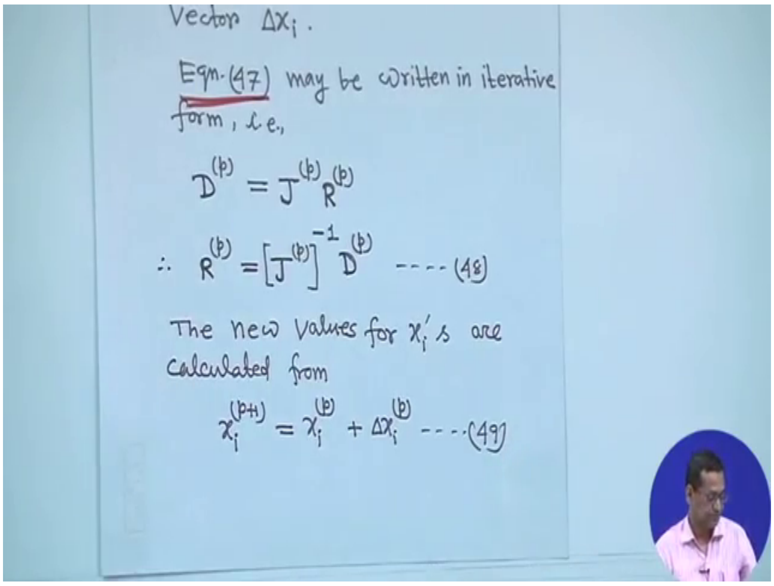

So, equation 47 maybe written in iterative form that is suppose any iteration count your p then D p is equal to J p into r p p is the iteration count right so; that means, R p will be is equal to J p inverse into D p right this is equation 48 right. So, the new value of xi s are calculated from at I mean the xi p plus 1 is equal to xi p plus del xi p where R p is the vector del your del x 1 del x 2 del x 3 up to del x n p.

So, all if you know this one if you know this one that all del x can be computed right this one can be computed. So, this is R p is that your this thing actually R is like this right.

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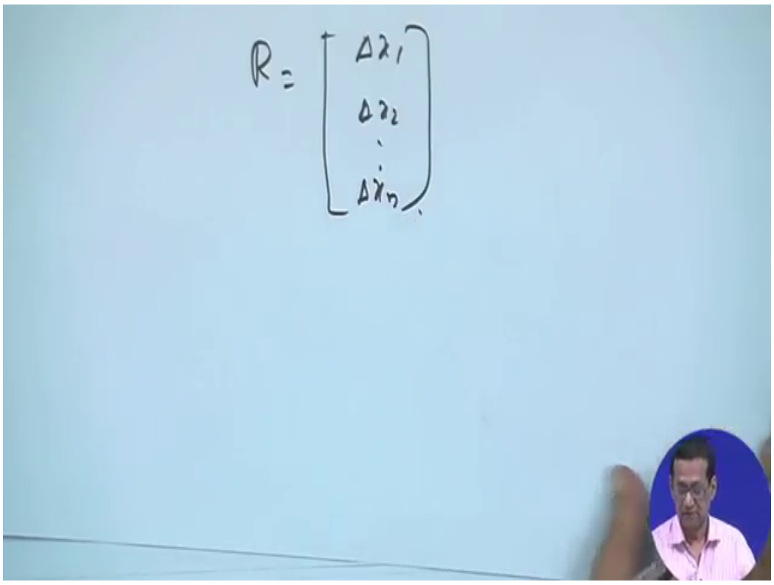

R is like this del x 1 del x 2 this one right. So, all the del x value can be computed from equation 48 right.

Therefore this x can be updated xi p plus 1 is equal to xi p and from here you will get all delta x values for delta xi p this is equation 49 right. So, the process is repeated until 2 successive values for each xi differ only by specified tolerance because you have to go for you have to go for convergence characteristic right.

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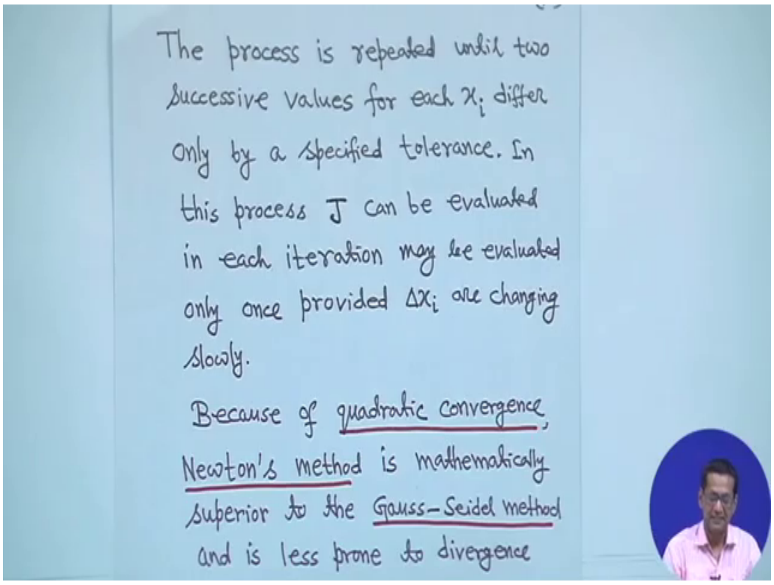

So, same way later we will see for Newton-Raphson method and you have to see that solves Sinex convex right. So, in this process J can be evaluated in each iteration maybe may be evaluated only ones provided del del xi changing slowly if changing delta x is very small then changing the value of x will very small. So, delta Jacobean matrix J instead of evaluating every iteration very fast iteration you compute after that written it constant throughout the iterative process right.

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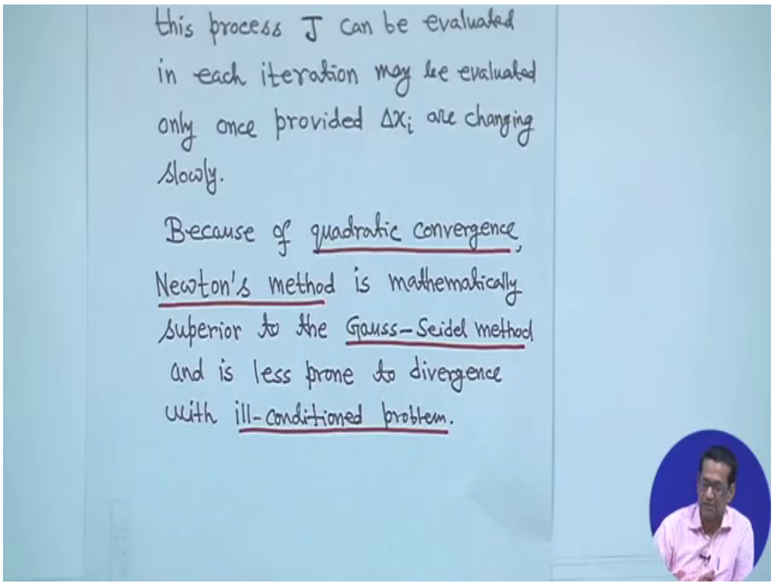

So, because of quadratic convergence Newton’s method is mathematically superior to the Gauss Seidel method. So, Newton-Raphson method is much superior than gauss Seidel method and is less prone to divergence with ill conditioned problem. So, if network is ill condition then there the; what you call then gauss Seidel may fail to converge right. So, what is ill conditioning later will see first let us discuss about your; what you call that Newton-Raphson method right.

So next, that load flow using Newton-Raphson method. So, what will little bit of mathematics you saw that is your this thing now load flow using your Newton-Raphson method right.

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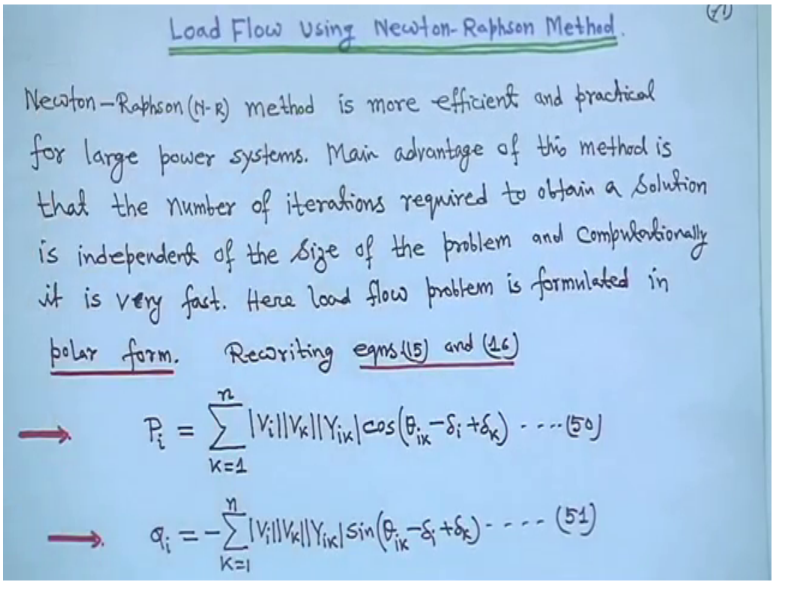

So, in this case you are what you call more efficient and practical for large power systems right main advantage of this method is that the number of iterations required to obtain a solution is independent of the size of the your problem and computationally it is very fast.

So, basically what happen for Newton-Raphson method that generally you will find it takes hardly 3 to 4 iterations to converge even 2 to 3 iterations it converge right whatever maybe the dimension of the problem dimension of the problem means that number of bus suppose it consider 500 or 1000 bus per problem right. So, you will find numbers of iterations more or less are independent right.

So, we have what we are doing is now that rewriting equation 15 and 16 same equation we are rewriting, but numbering again 50 and 51 here right. So, pi is equal to the power that your injected power and bus I earlier we have derived this k is equal to 1 to n V i V k y i k. So, hence will not alter magnitude because these are all magnitude bar it is there cosine theta i k minus delta I plus delta k.

So, and q i is equal to minus k is equal to 1 to n V i V k y i k sin theta i k minus delta I plus delta k this is equation 51 right. So, it is a set of your; what you call non-linear equations and but here also you have to consider p bus sorry p q bus and P V bus right so equation 50 and 51; that means, these 2 equation these 2 equations right that constitute a set of non-linear algebraic equations in terms of independent variables.

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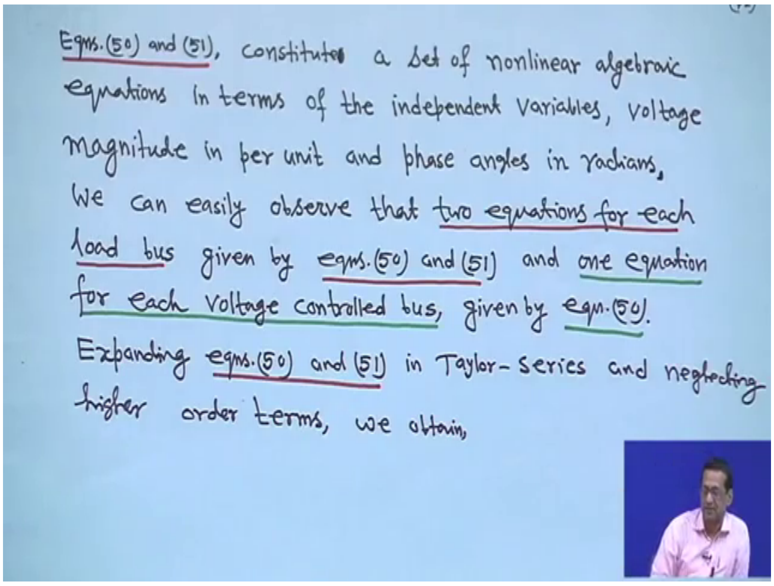

So, voltage magnitude in per unit and phase angles in radians right. So, we can easily observe that 2 equations for each load bus is given by equation 50 and 51; that means, there are 2 equations for each load bus given p i and q i for equation 50 and 51 right. So, and one equation for each voltage control bus given equation 50 and if it is a P V bus your voltage magnitude will remain constant right. So, at that time p is known so one equation right given by equation 50.

So, you have to now expand equation 50 and 51 in Taylor series and neglect higher term; that means, this equations right because all these things right we have to we have to solve iteratively. So, there and these are the non-linear equation just now we saw that non-linear equation how to expand in Taylor series. So, pi and qi we have to expand in Taylor series and you have to see that how things are happening right.

So, in this case what will happen for this case also will assume that bus one is a slack bus. So, will see i is equal to 2 to n right bus 1 is a slack bus. So, in that case what will happen that if you expand this thing and neglect higher terms in Taylor series then you are mathematical thing will be something like this right.

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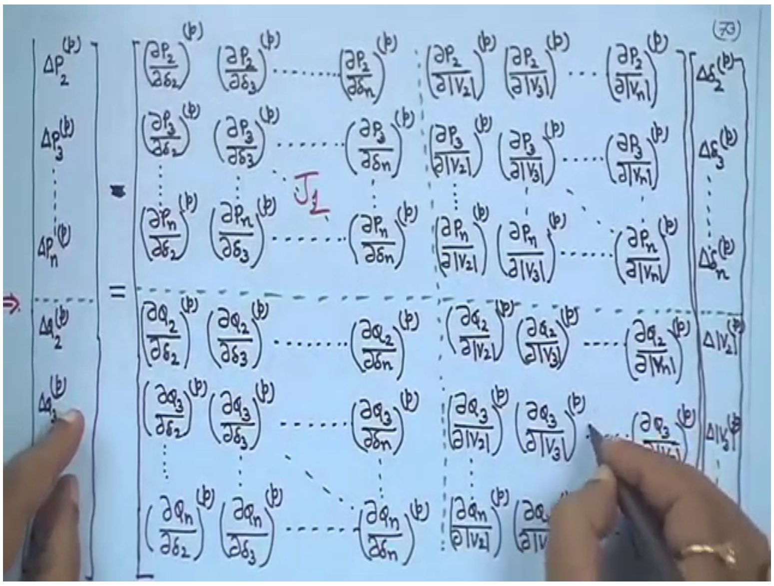

So, in this case that bus one is a slack bus and p is the iteration count. So, it will be delta p 2 p delta p 3 p up to delta p n p this is p delta p part then there is a mismatch and delta q 2 p delta q 3 p up to delta q n p is equal to is equal to right look at this matrix right.

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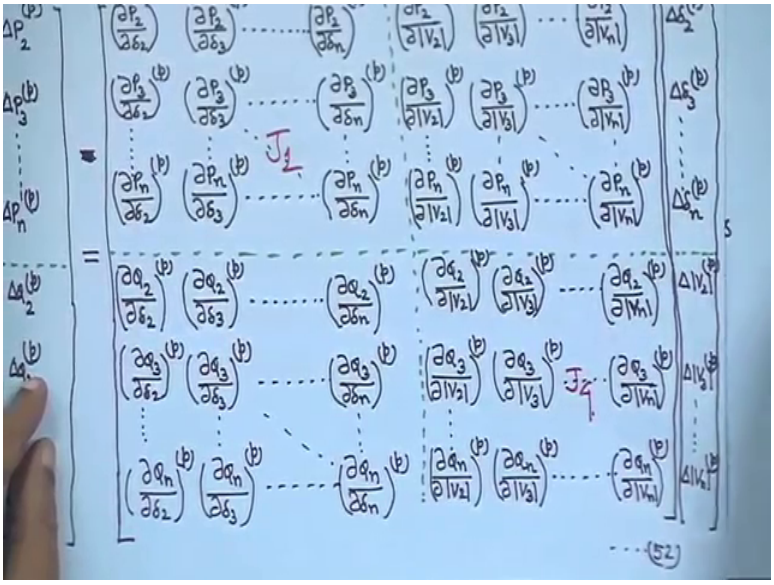

So, this is delta p 2 up p every look everywhere p means iteration count. So, I am not telling again and again p, but you can see it here everywhere p means iteration count I will just tell this partial derivatives.

Delta p 2 upon delta delta 2 delta p 2 upon delta delta 3 delta p 2 upon delta delta n right this is with respect to delta and this side is delta delta 2 delta delta 3 delta delta n the next one is delta V 2 delta V 3 delta V n these are all changes in magnitude right and then delta p 2 upon delta V 2 delta p 2 upon delta V 3 delta p 2 delta V n these all are magnitude, but not telling again and again under stable right.

Similarly, delta p 3 delta delta 2 delta p 3 delta delta 3 delta p 3 delta delta n; that means, when you take for delta p 2 delta p 2 all the derivative are taking with respect to delta 2 delta 3 delta n with these right and when you and so it is delta p 2 upon delta delta 2 delta p 2 upon delta delta and so on and this part will be delta p 2 delta V 2 delta p 2 delta V 3 and so on.

Similarly, here delta p 3 delta p 3 upon delta delta 2 delta p 3 upon delta delta 3 and delta p 3 upon delta n similarly here delta p 3 delta V 2 delta p 3 delta p 3 delta V 3 delta p 3 delta V n right this way you come up to nth term similarly for q also delta q 2 upon delta delta 2 delta q 2 delta delta 3 delta q 2 delta n and here delta q 2 delta V 2 voltage magnitude off course delta q 2 delta V 3 delta q 2 delta V n right this way you can construct.

So, this is actually this is actually partition here it is one here assuming that all the busses are p q buses for this one right later will see P V bus. So, and this is delta delta 2 delta delta 3 delta delta n and this is delta V 2 p delta V 3 p delta V n p these are all magnitude. So, this is equation actually 52 right.

So, this equation this part this one actually this equation is n minus 1 into n minus 1 because bus one is just slack bus this partition this matrix is n minus 1 into n minus 1 this is also n minus 1 into n minus 1 this is also n minus 1 into n minus 1; that means, as a whole dimension of a matrix is 2 into n minus 1 into 2 into n minus 1 right.

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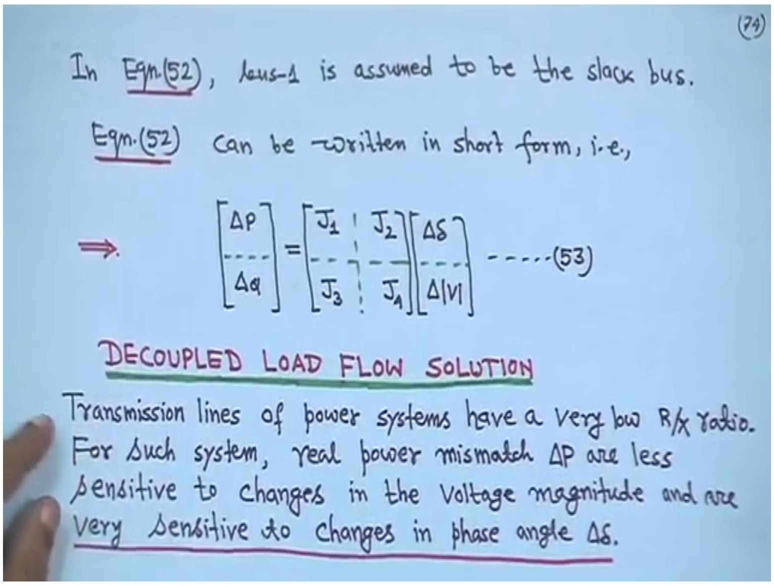

So; that means, this equation; equation 52 bus 1 is assumed to be the slack bus. So, this equation can be written in general that delta p delta q is equal to J 1 J 2 J 3 J 4 delta delta delta V right. So, delta p is equal to not writing here and delta p is equal to delta p 2 delta 3 delta n this vector and delta q is equal to delta q 2 delta q 3 delta q n right and J 1 is equal to this matrix right that and delta J 2 is equal to this matrix J 3 is equal to this one and J 4 is equal to 1 right.

Now, generally what happened in transmission line power system right they have a very low R by x ratio right where R I less and reactance is high right. So, process system real power mismatch delta p are less sensitive to changes in the voltage magnitude; that means, if you take the derivative delta p 2 delta V 2 this changes R your real power changes this is your not very voltage sensitive right.

There for real power mismatch delta p are less sensitive to changes in the voltage magnitude and very sensitive to changes in the phase angle I mean there are these are these are significant right very sensitive in changes in phase angle, but not very sensitive with that this thing changes in this one voltage magnitude right with respect to that right. So, in that case what will do this; this term this term will neglect this matrix will neglect right only this one will consider.

Similarly, just a similarly the reactive power mismatch delta q the reactive power mismatch delta q is less sensitive changes in angles. So, this part that reactive power changes is quite less sensitive. So, the changes in angle, but very sensitive to the changes in voltage magnitude; that means, this part and this part will drop right this matrix will not consider, but will consider this 1 and this 1 right.

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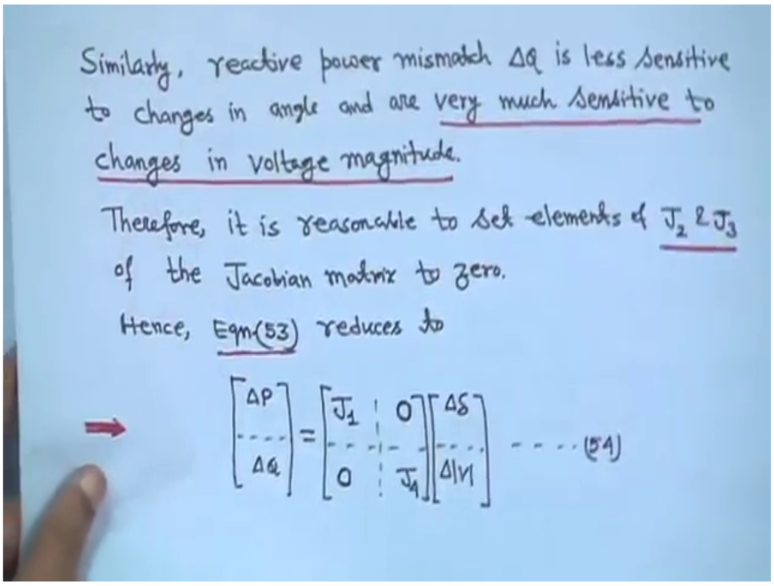

So; that means, that this is my J 1 this is J 2 this is J 3 this is J four; that means, that J 2 and J 3 the sub set to 0. So, these 2 things will not considered, but J 1 and J 4 will be considered the delta delta delta V right because this is your V 1 decoupled right and that one and if you consider this one if you consider this one this is coupled this is coupled right.

So; that means, you can write delta p is equal to J 1 delta delta and delta q is equal to J 4 delta V magnitude delta V understandable right. So, this is equation 54 so; that means, delta p is equal to J 1 delta delta this is equation 55 and delta q is equal to J 4 delta V this is equation 56 right therefore, for voltage control buses the voltage magnitude are known right.

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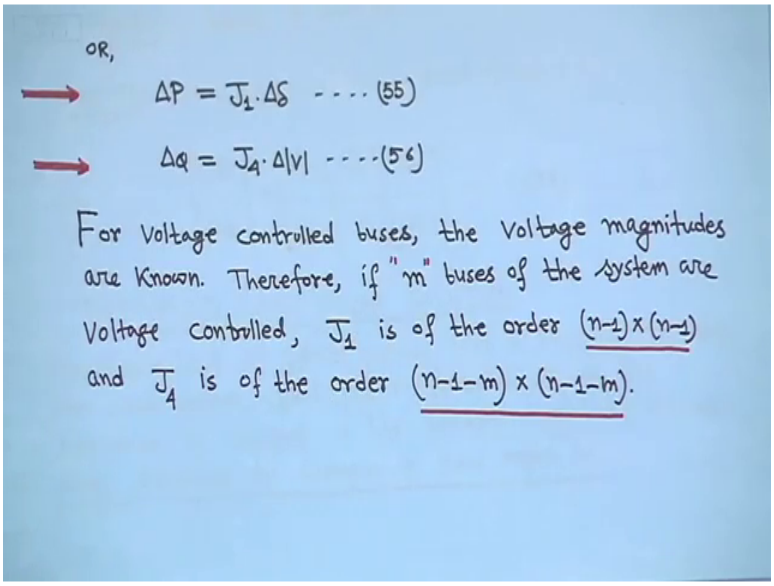

That means if an voltage control bus voltage magnitudes are known therefore, m buses of the system are voltage control bus J 1 will be always if the order of a n minus 1 into n minus 1 because this J 1 matrix this is your; this is your J 1 matrix this is this matrix actually this matrix your J 1 this is your J 1 matrix and this matrix actually your J 4 right.

So, if it is P V bus if you will be consider that in P V bus q and delta this is p and voltage magnitude known q and delta unknown right. So, this will remain as a n minus 1 into n minus 1 matrix this one right, but when you have a P V bus this thing. So, in this case what will happens suppose in general you have a n bus system and you have n m number of P V buses right then this J 4 order of the J 4 will become n minus 1 minus m into n minus 1 minus m because voltage magnitude is known right.

So, in that here those wherever voltage wherever the buses where voltage magnitudes are known for P V bus those delta V changes will not appear here right similarly here in this. That means, that J 4 that J 4 is the order of n minus 1 minus m into n minus 1 minus m right suppose you have a 10 bus problem then J 1 will be 9 into 9 matrix and suppose in 10 bus problem you have 3 your what you call 3 P V buses then it will be 10 minus 1 minus 3. That means, it will be 6 into 6 J 4 will be 6 into 6, but this one will be 9 into 9.

So, later will see when we consider P V bus will take one example right. Now, what you call that the diagonal elements of J 1. So, this is the J 1 matrix. So, diagonal elements is delta p 2 upon delta delta 2 delta p 3 upon delta delta 3 delta p n upon delta delta n in general delta pi upon delta delta I right. So, this way we have to first diagonal then off diagonal elements right first diagonal then off diagonal elements we have to obtain right.

Therefore, this is your just hold on this is your; this is your expression of pi right. So, this is your expression of pi right this is expression of pi.

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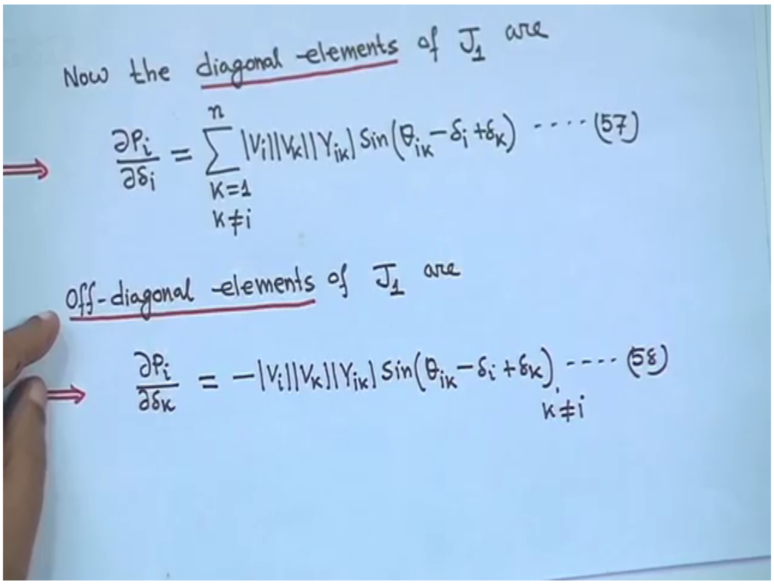

So, in this here you have to take the derivative now right. So, what you will do that your first is you take del pi upon delta delta I if you take del pi upon delta delta I diagonal element is k is equal to 1 to and k not is equal to i.

If take the derivative one it will be V i V k y i k right cosine if you take derivative minus and with respect to minus delta is another minus will come out. So, ultimately it will be your plus. So, it will be V i V k y i k sin theta i k minus delta I plus delta k, but this summation symbol will be there now question is that for your clarification what we will do that why this summation thing comes right.

So, what you can do is just for your just for your this thing just hold on just for your understanding right for example, you take you just hold on you take this one right.

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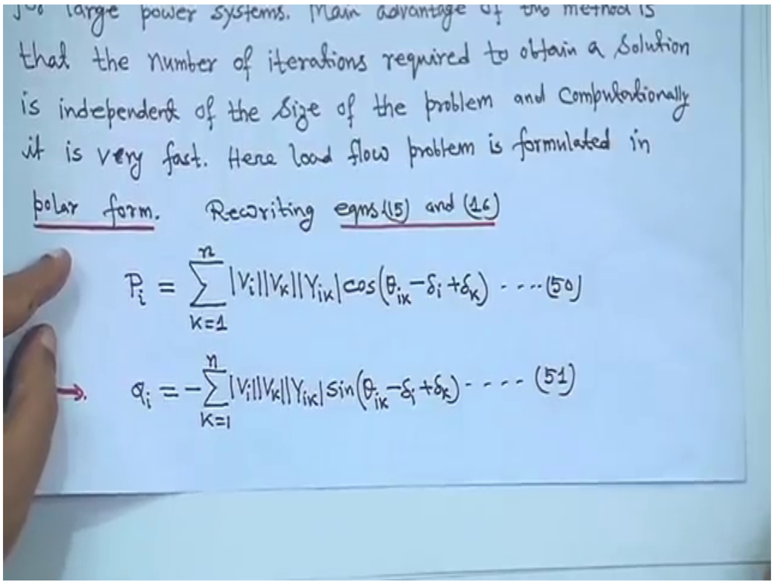

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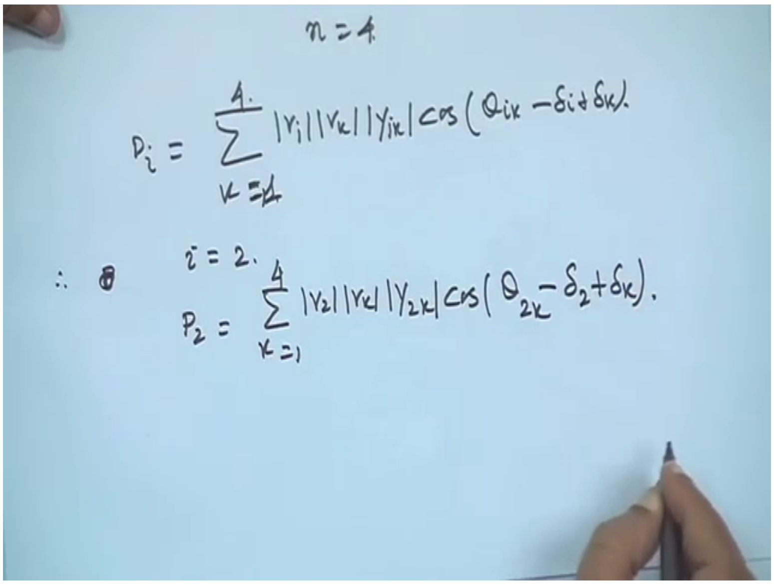

You take this equation this equation right let us let us for just for the purpose of understanding suppose n is equal to suppose n is equal to say 4 right and this equation this pi equation we can write like this that p i; p i is equal to say k is equal to 1 to 4 right.

Then V i then V k then y i k then cosine theta ik minus delta I plus delta k and your what to call this is the thing right. So, k is equal to 1 top on now you expand right if you expand then your say this is your pi now you have to expand your something say I is equal to say I is equal to 2 because bus one is a slack bus say I is equal to 2.

That means my p 2 will b is equal to k is equal to 1 to 4 right then V 2 then V k then y 2 k then cosine then theta I is equal 2 2 k minus delta 2 plus delta k right now you know what you do you expand you just I will show 1 then you will know. So, now, you expand this right if you expand this then your p 2 will become right.

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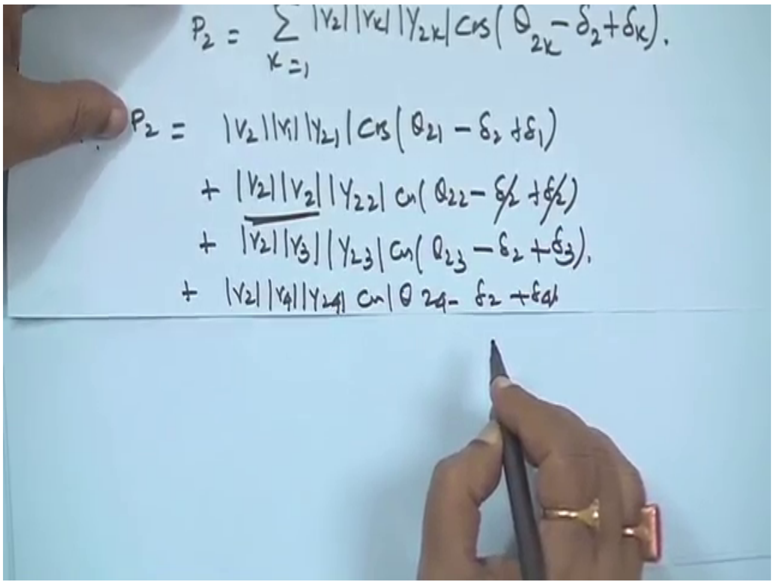

So, it will become V 2 and then V 1 then y 2 1 right then cosine theta 2 1 minus delta 2 plus delta one right then plus your V 2 then k is equal to 2. So, V 2 it will be V 2 square actually right V 2 into V 2 V 2 square anyway then y 2 2 cosine theta 2 2 minus delta 2 plus delta 2. So, this delta 2 delta 2 will be cancelled right.

Then plus your V 2 then your V 3 then Y 2 3 then cosine theta 2 3 minus delta 2 plus delta 3 this is the third when we have taken one more four. So, plus in k is equal to 4 V 2 then your V 4 then y 2 4 right then cosine theta 2 4 minus delta 2 plus delta 4 right this is the this is the fourth term we have taken, but this is actually V 2 square right.

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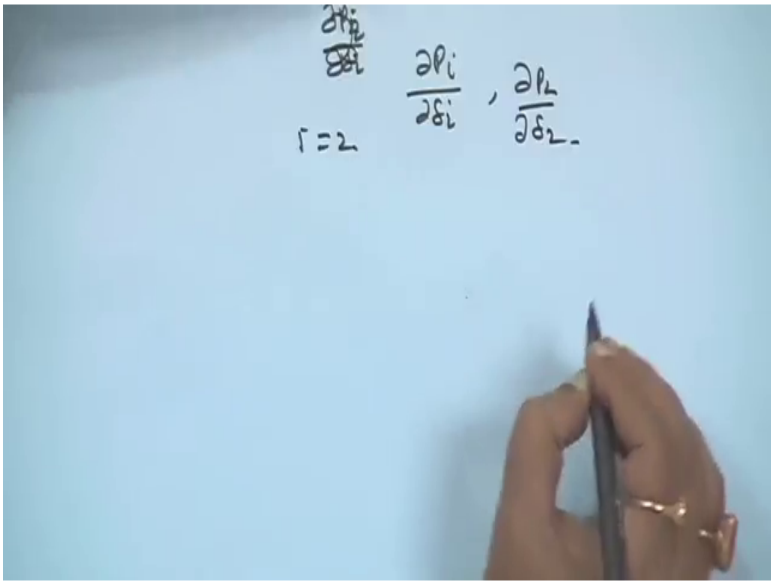

So, this delta 2 delta 2 will be cancelled now if you take derivative ray say we have taking the derivative in general delta p 2 upon delta sorry delta pi upon delta delta delta pi upon delta delta I we are taking the derivative right say it is diagonal. So, I is equal to 2; that means, delta p 2 upon delta delta 2 to take the derivative of this equation with respect to your delta 2 right.

If you take the derivative with respect to delta 2 you will find delta 2 is here. So, if you take if you take this term with derivative exist similarly here it is independent of your this thing second term that V 2 square cosine theta 2 2 right.

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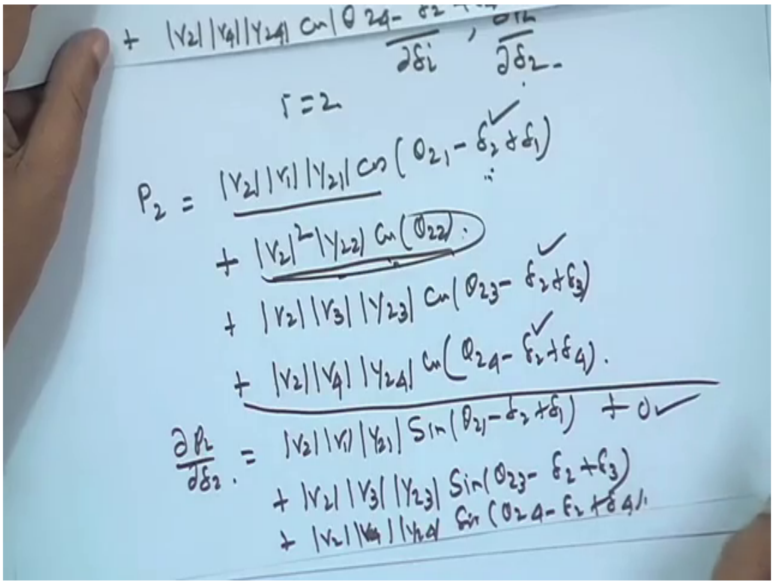

So, if I rewrite once again if I rewrite once again right it will be p 2 will be is equal to V 2 V 1 y 2 1 cosine theta 2 1 minus delta 2 plus delta 1 then plus this is this is V 2 into V 2. So, it is V 2 square then y 2 2 then cosine theta 2 2 because delta 2 delta 2 will be cancel next term is V 2 then V there then y 2 3 then cosine theta 2 3 minus delta 2 plus delta 3 right.

Then last term is V 2 then V 4 then y 2 4 then cosine your theta 2 4 minus delta 2 plus delta 4 right with these term when you take the derivative delta p 2 upon when you take delta p 2 upon delta delta 2 this one will be there and the derivative exist delta 2 here this term no delta 2 it will be 0, but here also delta 2 is there here also delta 2 is there here also delta 2 is there; that means, if you take it is V 2 V 1 y 2 1 right.

If you take minus because of cosine minus and because of minus delta 2 another minus will come out; so, it will be plus. So, sin theta 2 1 minus delta 2 plus delta 1 right this is the first term second term is 0 it is not there loop with respect to 2, but this 2 2 terms not there right plus V 2 V 3 then y 2 3 then here also sin theta 2 3 minus delta 2 plus delta 3 right.

And last term V 2 then your V 4 then your y 2 4 right then sin theta 2 4 minus delta 2 plus delta 4 right. So, all the 3 terms are there only second term is not there right because this is that i th term I is equal to 2. So, it is independent of delta 2. So, this will vanish. So, this is 0 right; that means, 3 terms are there that is why when you take the derivative of this one derivative of this one that diagonal one right diagonal one it is V i V k y i k sin because everywhere sin is there everywhere sin is there right.

So, magnitude V i V k y i k sin theta i k minus delta I plus delta k k is equal to 1 to and k not is equal to I because when I is equal to 2 this term is independent of delta i. So, it will become 0. So, that is why that k not is equal to I. Although in the equation everything k is equal to 1 to n only right, but this delta pi that diagonal element is sum of all this things, but k not is equal to i, but with a sin multiplication this is equation 57.

Thank you, again we are coming.