

**Power System Analysis**  
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**Lecture - 31**  
**Load Flow Studies (Contd.)**

So, let us take now an example on Gauss-Seidel method.

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respectively.

Example-2:

(a) Using the G-S method, determine the values of the voltage at buses 2 & 3. [perform only two iterations]

(b) Find the slack bus real and reactive power after second iteration.

Fig.7: 3-bus sample power system.

So, we will take look at as for as classroom exercise is concerned, we cannot consider you know large size of problem for that we need computer. So, we have taken a three bus power system, bus 1 is a slack bus and bus 2 and three it consider as a pq bus. And this things solution exact solution it will take more number of iterations, but your what will do will perform only two iterations such that things will be clear this is when both the bus is a pq time and another example will take on Gauss-Seidel for pv bus.

So, we have to find we have to use first one is using the Gauss-Seidel method determine the your values of the voltage at buses two and three; and second one is that find the slack bus real and reactive power after second iteration only data will be given data will be given.


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(c) Determine the line flows and line losses after second iteration. Neglect line charging admittance.

DATA

TABLE-2: Scheduled Generation and Loads and Assumed Bus Voltage for Sample power system. [BASE MVA = 100]

| Bus code<br>i    | Assumed<br>bus voltage | Generation |      | Load  |       |
|------------------|------------------------|------------|------|-------|-------|
|                  |                        | MW         | MVAR | MW    | MVAR  |
| 1<br>(slack bus) | $(1.05 + j0.0)$        | -          | -    | 0     | 0     |
| 2                | $(1.0 + j0.0)$         | 50         | 30   | 305.6 | 140.2 |
| 3                | $(1.0 + j0.0)$         | -          | -    | 138.4 | 45.2  |



And your part c will be your determine the line flows or line losses after second iteration and neglect line charging admittance. So, line charging admittance is not considered and data are given like this. Base MVA is 100. So, in this case three buses, bus 1 is a slack bus and slack bus voltage at mean consider as  $1.05 + j 0$  this is fixed. And starting values for bus 2 and this is assume bus voltage bus 2 and bus 3, you take  $1 + j 0$   $1 + j 0$ .

Although, at the time of developing the theory, we have said that slack voltage start is better that means, if it is  $1.05$ , other two also initial value should have been  $1.05 + j 0$ , but here we have the or taken that initial values  $1 + j 0$  and  $1 + j 0$ . And bus 1 is slack bus, so no parameter no generation no load should be there, by chance at slack bus if some load is there, so that is actually dummy load and you do not need anything during iterative process. But if load is there at the slack bus then what you can do you do not consider it will never come in your iterative process, but when you get the slack bus power that  $P_g 1$  and  $Q_g 1$  real and reactive power.

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power system. [BASE MVA = 100]

| Bus code<br>i    | Assumed<br>bus voltage | Generation |      | Load  |       |
|------------------|------------------------|------------|------|-------|-------|
|                  |                        | MW         | MVAR | MW    | MVAR  |
| 1<br>(slack bus) | $(1.05 + j0.0)$        | -          | -    | 0     | 0     |
| 2                | $(1.0 + j0.0)$         | 50         | 30   | 305.6 | 140.2 |
| 3                | $(1.0 + j0.0)$         | 0.0        | 0.0  | 138.6 | 45.2  |

TABLE-3: Line Impedances

| Bus code<br>i-k | Impedance,<br>$Z_{ik}$ |
|-----------------|------------------------|
| 1-2             | $0.02 + j0.04$         |
| 1-3             | $0.01 + j0.03$         |
| 2-3             | $0.0125 + j0.025$      |

Whatever you get and if they you can load is there at the slack bus, so that with that P g 1 whatever you got that load also has you have to get it. Later I will tell you this thing. And other wise if anything is given also this, this will be act as your dummy parameters, so iterative process. And load at bus to is given 50 megawatt, 30 megawatt base MVA is 100, and load is also given 305.6 megawatt, 140.2 megawatt.

And bus 3, it is load, but it no there is no generation there no generation connected, so 0 0, but load is a 138.6 and megawatt and 45.2 megawatt. And line impedances are also given that is 1 to 2, 1-3 and 2-3 all are given these are in per unit value. So, impedance is a per unit value. These voltages are also in per unit values, but these are given in real unit megawatt and megahertz you have to divide all this thing by 100 because base is 100 such that they can be transform into per unit.

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Solution:

Step-1: Initial Computations

→ Convert all the loads in per-unit values

⇒  $PL_2 = \frac{305.6}{100} = 3.056 \text{ pu}$ ;


⇒  ~~$QL_2 = \frac{305.6}{100}$~~   $QL_2 = \frac{140.2}{100} = 1.402 \text{ pu}$

⇒  $PL_3 = \frac{138.6}{100} = 1.386 \text{ pu}$ ;  $QL_3 = \frac{45.2}{100} = 0.452 \text{ pu}$

→ Convert all the generation in per-unit values

⇒  $P_{g2} = \frac{50}{100} = 0.50 \text{ pu}$ ;  $Q_{g2} = \frac{30}{100} = 0.30 \text{ pu}$

→ Compute net-injected power at bus 2 and 3



So, next is that; how will go for step by step calculation. So, in that case that your step one that initial your computation first you have to convert all the loads in per unit values. So, at bus to load is P L 2, so 305.6 by 100, so 3.056 per unit. Similarly, Q L 2 also 142.2 by 101.402 per unit. Similarly PL 3 138.6 by 100, so 1.386 per unit and QL 3 45.2 by 100 – 0.452 per unit; now, all the convert all the generation in per unit values there, at bus 2 it is given P g 250 megawatts 50 by 100 – 0.5 per unit and Q g 2 is 30 by 100, so 0.3 per unit. So, all these things given now compute net injected power at bus 2 and 3.

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⇒  ~~$QL_2 = \frac{305.6}{100}$~~   $QL_2 = \frac{140.2}{100} = 1.402 \text{ pu}$

⇒  $PL_3 = \frac{138.6}{100} = 1.386 \text{ pu}$ ;  $QL_3 = \frac{45.2}{100} = 0.452 \text{ pu}$

→ Convert all the generation in per-unit values

⇒  $P_{g2} = \frac{50}{100} = 0.50 \text{ pu}$ ;  $Q_{g2} = \frac{30}{100} = 0.30 \text{ pu}$


→ Compute net-injected power at bus 2 and 3.

⇒  $P_2 = P_{g2} - PL_2 = (0.5 - 3.056) = -2.556 \text{ pu}$

⇒  $Q_2 = Q_{g2} - QL_2 = (0.3 - 1.402) = -1.102 \text{ pu}$

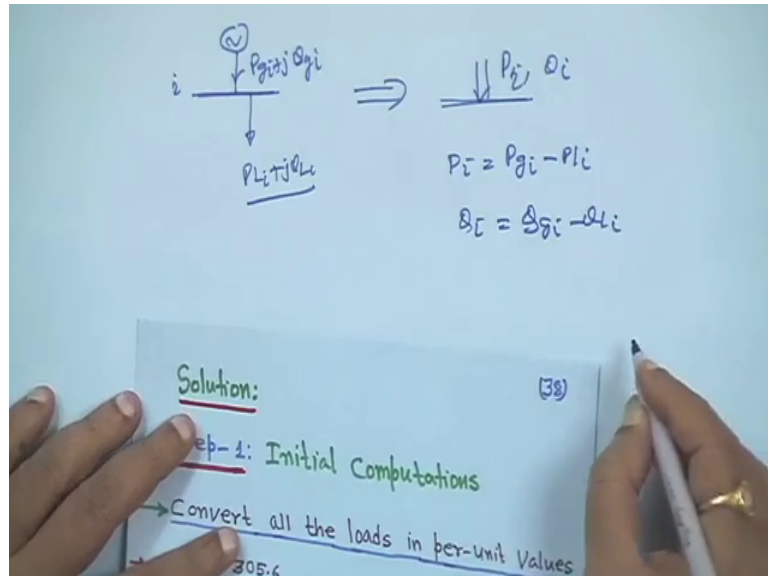
⇒  $P_3 = P_{g3} - PL_3 = 0 - 1.386 = -1.386 \text{ pu}$

⇒  $Q_3 = Q_{g3} - QL_3 = 0 - 0.452 = -0.452 \text{ pu}$



So, net injected power means this one suppose you have a suppose you have a generation in general I am talking.

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So, this is bus i and you have the load say you have P L i plus j Q L i for bus i and here also say P g i plus j Q g i. So, in this case in a in this case that net injected power will be. So, make it like this that P i and Q i will be the net injected power. So, P i will be actually P g i minus P L i; and Q i will be Q g i minus Q L i this is the net injected power at this bus you take that injection on because this is power being injected a load is going out. So, net injected will be P g i minus P L i. Therefore, your P 2 that is at bus 2 it will be P g 2 minus P L 2 I told you that pi is equal to P g i minus P L i. So, it is P 2 is equal to P g 2 minus P L 2, so 0.5 minus 3.056. So, it is minus 2.556 per unit.

Similarly, Q 2 is equal to Q g 2 minus Q L 2, so 0.3 minus 1.402. So, it is minus 1.102 per unit. And P 3 is equal to P g 3 minus P L 3, but at bus 3 there is no generation and bus 3 there is no generation then we zero P g 3 and Q g 3 zero. Therefore, it is 0 minus 1.386, so minus 1.386 per unit. And Q 3 is equal to Q g 3 minus Q L 3, so 0 minus 0.452 that is minus 0.452 unit. This way you convert this is the first step first step is these are all initial computation.

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Step-2: Formation of  $Y_{bus}$  Matrix.

$$Y_{12} = Y_{21} = \frac{1}{Z_{12}} = \frac{1}{(0.02 + j0.04)} = (10 - j20)$$
$$Y_{13} = Y_{31} = \frac{1}{Z_{13}} = \frac{1}{(0.01 + j0.03)} = (10 - j30)$$
$$Y_{23} = Y_{32} = \frac{1}{Z_{23}} = \frac{1}{(0.0125 + j0.025)} = (16 - j32)$$

Now  $Y_{11} = Y_{12} + Y_{13} + Y_{10}$

Charging admittance is neglected, i.e.,  $Y_{10} = 0$

$$\therefore Y_{11} = Y_{12} + Y_{13} = (10 - j20) + (10 - j30) = (20 - j50)$$

Now, second step is that your formation of your Y bus matrix. So, all Z i value, Z ik values are given. So, first you find out small y 1 2 is equal to small y 2 1 1 upon Z 1 2. So, this values is given 0.02 plus j 0.04, so that is 10 minus j 20. Similarly, y 1 3 equal to y 3 1 is equal to one upon z 1 3 is equal to 1 upon 0.01 plus j 0.03 that is ten minus j thirty similarly y 2 3 equal to y 3 2 small y small y do not confused between capital letter and small letter. When it is capital letter that I showed you that a that a your Y matrix and this is a this is only for the line admittance different line admittance that is 1 upon Z 2 3 that is 1 upon 0.0125 plus j 0.025, this 16 minus your j 32.

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Now  $Y_{11} = Y_{12} + Y_{13} + Y_{10}$

Charging admittance is neglected, i.e.,  $Y_{10} = 0$

$$\therefore Y_{11} = Y_{12} + Y_{13} = (10 - j20) + (10 - j30) = (20 - j50)$$
$$Y_{22} = Y_{21} + Y_{23} = Y_{12} + Y_{23} = (26 - j52)$$
$$Y_{33} = Y_{13} + Y_{23} = (26 - j62)$$
$$Y_{11} = 53.85 \angle -68.2^\circ$$
$$Y_{22} = 58.13 \angle -63.4^\circ$$
$$Y_{33} = 67.23 \angle -67.2^\circ$$

Now, charging admittance is neglected. So,  $y_{11}$  is equal to  $y_{12}$  plus  $y_{13}$  plus  $y_{10}$ . I mean charging admittance I mean if you find out  $y_{11}$ . So, it will be  $y_{12}$   $y_{13}$  plus one zero, but charging admittance is neglected. So,  $y_{10}$  is 0, here it is 0. Therefore,  $Y_{11}$  is equal to small  $y_{12}$  plus small  $y_{13}$  so  $10 \text{ minus } j 20$  plus  $10 \text{ minus } j 30$  is equal to  $20 \text{ minus } j 50$ . Similarly,  $Y_{22}$  will be  $y_{21}$  plus small  $y_{23}$ , no charging admittance because we have not consider that. So,  $y_{12}$  is equal to  $y_{21}$  is equal to  $y_{12}$ . So,  $y_{12}$  plus  $y_{23}$ . So, if you sum it up, it will be  $26 \text{ minus } j 52$ .

Next is these thing  $Y_{33}$  capital  $Y_{33}$  small  $y_{13}$  plus  $y_{23}$  it will some it up it will be  $26 \text{ minus } j 62$ . Now, this capital  $Y_{11}$  from here it is coming capital  $Y_{11}$  its magnitude will be 53.85 and angle minus 68.2 degree similarly capital  $Y_{22}$  will be 58.13 angle minus 63.4 degree. And similarly capital  $Y_{33}$  this one will become 67.23 minus angle 67.2 degree. So, this all diagonals diagonal elements of the Y matrix made it  $Y_{11}$ ,  $Y_{22}$ ,  $Y_{33}$ .

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$$Y_{12} = -Y_{21} = -(10 - j20) = 22.36 \angle 116.6^\circ$$

$$Y_{12} = Y_{21}$$

$$Y_{13} = Y_{31} = -Y_{31} = -(10 - j30) = 31.62 \angle 108.4^\circ$$

$$Y_{23} = Y_{32} = -Y_{23} = -(16 - j32) = 35.77 \angle 116.6^\circ$$

$$\therefore Y_{bus} = \begin{bmatrix} 53.85 \angle -68.2^\circ & 22.36 \angle 116.6^\circ & 31.62 \angle 108.4^\circ \\ 22.36 \angle 116.6^\circ & 58.13 \angle -63.4^\circ & 35.77 \angle 116.6^\circ \\ 31.62 \angle 108.4^\circ & 35.77 \angle 116.6^\circ & 67.23 \angle -67.2^\circ \end{bmatrix}$$

Next the off diagonal elements off diagonal elements capital  $Y_{12}$  is equal to minus your small  $y_{12}$ ; so minus  $10 \text{ minus } j 20$ . So, this will be  $22.36$  angle  $116.6$  degree, so that that we are capital  $Y_{12}$  is equal to capital  $Y_{21}$  it will symmetric. So, capital  $Y_{13}$  is equal to capital  $Y_{31}$  is equal to minus  $y_{13}$  is equal to minus  $10 \text{ minus } j 30$ , it will be  $31.62$  angle  $108.4$  degree. And capital  $Y_{23}$  is equal to capital  $Y_{32}$  is equal to minus small  $y_{23}$  is be minus  $16 \text{ minus } j 32$ . So, this will become  $35.77$  angle  $116.6$  degree.

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$$\therefore Y_{bus} = \begin{bmatrix} 53.85 \angle -68.2^\circ & 22.36 \angle 116.6^\circ & 31.62 \angle 108.4^\circ \\ 22.36 \angle 116.6^\circ & 58.13 \angle -63.4^\circ & 35.77 \angle 116.6^\circ \\ 31.62 \angle 108.4^\circ & 35.77 \angle 116.6^\circ & 67.23 \angle -67.2^\circ \end{bmatrix}$$

Step-3: Iterative Computation.

$$\Rightarrow V_2^{(p+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^{(p)})^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} - \dots - Y_{2j}V_j \right]$$

So, with that with that you form the y bus matrix y 1 1, y 1 2, y 1 3. So, y bus matrix is formed. So, after this step three iterative computation; so in that case you have to down first all the three equations for the Gauss-Seidel method, we have seen that that equations for the Gauss-Seidel method. So, all this things are given in general. So, for the bus 2 bus 2 here writing V 2 P plus 1 this already just hold on already we have discuss this.

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is used in the solution of the subsequent equations. Therefore, above set of equations can be written in iterative form, i.e.,

$$\Rightarrow V_2^{(p+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^{(p)})^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} - Y_{24}V_4^{(p)} \right]$$

$$\Rightarrow V_3^{(p+1)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^{(p)})^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)} - Y_{34}V_4^{(p)} \right]$$

$$\Rightarrow V_4^{(p+1)} = \frac{1}{Y_{44}} \left[ \frac{P_4 - jQ_4}{(V_4^{(p)})^*} - Y_{41}V_1 - Y_{42}V_2^{(p+1)} - Y_{43}V_3^{(p+1)} \right]$$

Let me find out here already we have discussed all this things. For example, for four bus system already we have discussed all this things for four bus system, but in this case we



have considering only your what you call that is the three bus problem. So, you have only two equation because bus 1 is a slack bus. So,  $V_2^p + 1$  P is a iteration count is equal to one upon capital Y2 2 and in bracket P 2 minus j Q 2 upon V 2 P then conjugate minus Y 2 1 V 1 minus Y 2 3 V 3 P this is equation 1. And as V 1 is a slack bus, so please do not put here any iteration count because toward these thing V 1 is a slack bus and his voltage is taking 1.05 plus j 0, so that will remain constraint. And all buses are P Q buses that Y 2 and any were this a administration matrix element.

So, this will remain constant and P 2 and Q 2 also will remain constant throughout the iterative process. So, only variable arrays that is your V 2; that means, is magnitude angle both this thing these two things because these are complex one, these are complex voltage. So, this is in general we one upon Y 2 2 then this is the for V 2

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$$\Rightarrow V_3^{(k+1)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{(V_3^{(k)})^*} - Y_{31}V_1 - Y_{32}V_2 \right] \quad \dots (ii)$$

Slack bus voltage  $V_1 = (1.05 + j0.0)$

Starting voltage  $V_2^{(0)} = (1 + j0); V_3^{(0)} = (1 + j0)$

Now

$$\frac{P_2 - jQ_2}{Y_{22}} = \frac{(-2.556 + j1.102)}{58.13 - j63.4} = 0.0478 | 220.1^\circ$$

$$\frac{Y_{21}}{Y_{11}} = \frac{22.36 | 116.6^\circ}{58.13 - j63.4} = 0.3846 | 180^\circ = -0.3846$$

Similarly for bus 3 say more you can write for bus 3 that  $V_3^p + 1$  Y 3 3 P 3 minus j Q 3 upon V 3 P conjugate P is a iteration count any were minus y three v one Y 3 1 V 1 minus Y 3 2 V 2 P plus 1. So, instead of instead of P, you write that immediate values whatever you are obtains. So, V 2 this one will compute first V 2 P plus 1. So, instead of writing taking V 2 P you take V 2 P plus 1 such that conversion will be faster that is will number of iteration will be lays. So, these two equation save got

Now, slack bus voltage V 1 is given 1.05 plus j 0 this is given. Now, starting values it is all data it has given starting values 1 plus j 0. So, V 2 0 is equal to 1 plus j 0 that is initial

values and  $V_3^{(0)}$  superscript you have taken in bracket 0. So,  $1 + j0$  these are the initial values. Now, what will do that except, what you call except  $V_3$  and  $V_2$  both the equation here I have  $V_3$  and  $V_2$  all are constant throughout the iterative process. So, for example, that how will compute for example.

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$$\Rightarrow V_2^{(p+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^{(p)})^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} \right]$$

.....(i)

$$V_2^{(p+1)} = \left\{ \frac{P_2 - jQ_2}{Y_{22}} \right\} \times \frac{1}{(V_2^{(p)})^*} - \left( \frac{Y_{21}}{Y_{22}} \right) V_1 - \left( \frac{Y_{23}}{Y_{22}} \right) V_3^{(p)}$$

These equation look I am writing here these equation these equation that you can write  $V_2^{(p+1)}$  equal to write you can write  $P_2 - jQ_2$  multiplied this  $Y_{22}$  into 1 upon  $V_2^{(p)}$  conjugate then minus  $Y_{21}$  upon  $Y_{22}$  all are capital into  $V_1$  then minus  $Y_{23}$  upon  $Y_{22}$   $V_3^{(p)}$ . Because this term this term actually constant throughout the iterate process this one also constant throughout iterate process. And this term also constant throughout the iterate process. That mean if you compute one this term and put it in the that equation then no need to compute this terms again and again.

So, that is why in your computation that first you compute this term  $P_2 - jQ_2$  upon  $Y_{22}$  and  $Y_{21}$  upon  $Y_{22}$  and  $Y_{23}$  upon  $Y_{22}$  all capital first you compute.

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Slack bus voltage  $V_1 = (1.05 + j0.0)$

Starting Voltage  $V_2^{(0)} = (1 + j0); V_3^{(0)} = (1 + j0)$

Now

$$\frac{P_2 - jQ_2}{Y_{22}} = \frac{(-2.556 + j1.102)}{58.13 \angle -63.4^\circ} = 0.0478 \angle 220.1^\circ$$

$$\frac{Y_{21}}{Y_{22}} = \frac{22.36 \angle 116.6^\circ}{58.13 \angle -63.4^\circ} = 0.3846 \angle 180^\circ = -0.3846$$

$$\frac{Y_{23}}{Y_{22}} = \frac{35.77 \angle 116.6^\circ}{58.13 \angle -63.4^\circ} = 0.6153 \angle 180^\circ = -0.6153$$

Therefore, your these thing your  $P_2 - jQ_2$  upon  $Y_{22}$ ,  $P_2$  and  $Q_2$  already, we have got that per unit values before I have shown. So, this is minus 2.556 plus  $j$  1.102 and  $Y_{22}$  also we have got 58.13 angle minus 63.4. So, this will come actually 0.078 angle 220.1 degree. Similarly, next is I told you that  $Y_{21}$  upon  $Y_{22}$  I mean all this all this things we have to compute  $Y_{21}$  upon  $Y_{22}$   $Y_{23}$  upon your  $Y_{22}$ , all you have compute. So, then  $Y_{21}$  upon  $Y_{22}$  will be substitute these two values 22.36 angle 116.6 degree and this is 58.13 angle minus 63.4 degree. So, that will become 0.3846 angle 180 degree is equal to minus 0.3846, it will come like this.

Similarly, capital  $Y_{23}$  capital  $Y_{22}$  that is 35.77 angle 116.6 degree and this is 58.13 angle minus 63.4 degree that is 0.6153 and again angle is 180 degree, so minus 0.6153. So, these are the parameter for this equation 2 was a equation  $V_2$ , this will not change for iterative process and they will remain constant. Similarly, for these equation also  $P_3 - jQ_3$  you have to compute  $P_3 - jQ_3$  divided by  $Y_{33}$  this you compute then  $Y_{31}$  upon  $Y_{33}$  you compute because you multiply this one by one upon  $Y_{33}$ . And similarly minus  $Y_{32}$   $Y_{33}$  when you multiply this you compute because these are the term it will remain constant.

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Therefore eqn(i) can be written as:

$$\Rightarrow V_2^{(p+1)} = \left[ \frac{0.0478 \angle 22.0^\circ}{(V_2^{(p)})^*} + 0.3846 V_1 + 0.6153 V_3^{(p)} \right] \quad \dots (ii)$$

Now,

$$\frac{P_3 - jQ_3}{Y_{33}} = \frac{(-1.386 + j0.452)}{67.23 \angle -67.2^\circ} = 0.0217 \angle 229.2^\circ$$

$$\frac{Y_{31}}{Y_{33}} = \frac{31.62 \angle 108.4^\circ}{67.23 \angle -67.2^\circ} = 0.47 \angle 175.6^\circ$$

So, that means if you do so then you are what you call first you write then V your equation one V 2 P plus 1 first you write all this you have computed that P 2 minus j Q 2 upon your Y 2 2. So, this is 0.0478 angle 22.1 degree divided V 2 P conjugated plus because it was minus 0.3846. So, it is plus 0.3846 V 1. And here also it was minus 0.6153, so it is plus 0.6153 V 3 P. So, these are this is the equation three this is for V 2 equation.

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$$\frac{P_3 - jQ_3}{Y_{33}} = \frac{(-1.386 + j0.452)}{67.23 \angle -67.2^\circ} = 0.0217 \angle 229.2^\circ$$

$$\frac{Y_{31}}{Y_{33}} = \frac{31.62 \angle 108.4^\circ}{67.23 \angle -67.2^\circ} = 0.47 \angle 175.6^\circ$$

$$\frac{Y_{32}}{Y_{33}} = \frac{35.77 \angle 116.6^\circ}{67.23 \angle -67.2^\circ} = 0.532 \angle 183.8^\circ$$

Therefore eqn(ii) can be written as:

$$\Rightarrow V_3^{(p+1)} = \left[ \frac{0.0217 \angle 229.2^\circ}{(V_3^{(p)})^*} - 0.47 \angle 175.6^\circ V_2 - 0.532 \angle 183.8^\circ V_2^{(p+1)} \right] \quad \dots (iv)$$

Similarly, for  $V_3$  you again compute  $P_3$  minus  $j Q_3$  upon capital  $Y_3$ . So,  $P_3 Q_3$  all your computed  $Y_3$  capital  $Y_3$  capacitated you will get 0.0217 angle 229.2 degree. Now, the ratio you have to compute capital  $Y_3$  1 by capital  $Y_3$  3; so  $Y_3$  1 known to you,  $Y_3$  3 known to you, so that is actually coming 0.47 angle 175.6 degree. Similarly, another one that  $Y_3$  2 upon  $Y_3$  3 capital one, capital  $Y$  put this values 35.77 angle 116.6 degree and 67.23 angle minus 67.2 degree, you will get 0.532 angle 183.8 degree.

Therefore, that equation 3 that means, that these equation that means, these equation it can be written as  $V_3 P_1$  is equal to 0.0217 angle 229.2 degree by  $V_3 P$  conjugate minus 0.47 angle 175.6 degree into  $V_1$ , then minus 0.532 angle 183.8 degree into  $V_2 P$  plus 1 where your putting  $V_2$ . This is equation 4. Now, this equation you put it you put it equation three and this one you make it as equation four this you have to solve iteratively. So, all the constant parameter first you compute because those parameters will not change throughout your iterative process.

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Now solve eqns (iii) and (iv) iteratively.  
First Iteration:  
 Set  $p=0$

$$\therefore V_2^{(1)} = \frac{0.0478 \angle 229.1^\circ}{(V_2^{(0)})^*} + 0.3846 V_1 + 0.6153 V_3^{(0)}$$

$$\therefore V_2^{(1)} = \frac{0.0478 \angle 229.1^\circ}{(1+j0)^*} + 0.3846(1.05+j0.0) + 0.6153(1.0+j0.0)$$

$$\Rightarrow \therefore V_2^{(1)} = 0.98305 \angle -1.8^\circ$$

$$V_3^{(1)} = \frac{0.0217 \angle 229.2^\circ}{(V_3^{(0)})^*} - 0.47 \angle 175.6^\circ V_1 - 0.532 \angle 183.8^\circ V_2^{(1)}$$

Then you solve equation three and four iteratively first iteration you put  $P$  is equal to zero therefore, when you put  $P$  is equal to zero it will  $V_2$  1 and this will be 0.0478 angle 22.1 degree  $V_2$  0s conjugate plus 0.3846  $v_1$  plus 0.6153  $V_3$  0. So,  $V_1$  - 1.05 you know that 1.05 plus  $j$  0.0 you know that.  $V_3$  0 also 1 plus  $j$  0 you know that. And your this thing your  $V_2$  initial value also 1 plus  $j$  0. So, here also it will be 1 plus  $j$  0 conjugate means it is 1. So, substitute this value  $V_2$  1 is equal to this one divided by 1 plus  $j$  0

conjugate that means it one actually one minus j 0 means one only. Then 0.3846 v 1 is 1.05 plus j 0 and your other term 0.6153 initial value of V 3 0 1 plus j 0. Then you compute all this. If you do so V 2 1 will come first iteration 0.98305 angle minus 1.8 degree.

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Handwritten mathematical derivation on a blue background:

$$\Rightarrow \therefore V_2^{(1)} = 0.98305 \angle -1.8^\circ$$

$$V_3^{(1)} = \frac{0.0217 \angle 229.2^\circ}{(V_3^{(0)})^*} - 0.47 \angle 175.6^\circ V_1 - 0.532 \angle 183.8^\circ \cdot V_2^{(1)}$$

$$\therefore V_3^{(1)} = \frac{0.0217 \angle 229.2^\circ}{(1+j0)^*} - 0.47 \angle 175.6^\circ (1.05 + j0.0) - 0.532 \angle 183.8^\circ \times 0.98305 \angle -1.8^\circ$$

$$\Rightarrow V_3^{(1)} = 1.0011 \angle -2.06^\circ$$

Similarly, for V 3 also you put again that P is equal to 0. So, V 3 1 will be this one already you have got it divided by V 3 0 conjugate minus 0.47 angle 175.6 degree V 1 then minus 0.532 angle 183.8 degree that is your V 2 that is 1 in first it has sent. I mean these values do not put here V 2 zero immediate value should come in the next equation. So, here V 2 1 is these one. So, you make V 2 1 and all then you substitute on V 2 V 3 is 1 plus j 0 conjugate and minus 0.47 angle 175.6 degree 1.0 into 1.05 plus j 0.

And this is minus 0.532 angle 183.8 degree into 0.98305 angle minus 1.8 degree. So, V 3 1 will come is come after calculating all you will get 1.0011 angle minus 2.06 degree.

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After first iteration

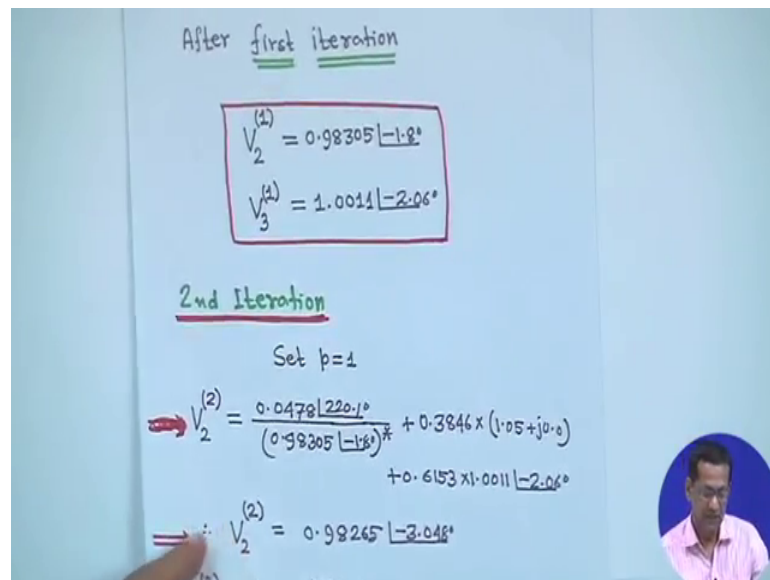
$$V_2^{(1)} = 0.98305 \angle -1.8^\circ$$

$$V_3^{(1)} = 1.0011 \angle -2.06^\circ$$

2nd Iteration

Set  $p=1$

$$\Rightarrow V_2^{(2)} = \frac{0.0478 \angle 220.1^\circ}{(0.98305 \angle -1.8^\circ)^*} + 0.3846 \times (1.05 + j0.0) + 0.6153 \times 1.0011 \angle -2.06^\circ$$

$$\Rightarrow V_2^{(2)} = 0.98265 \angle -3.046^\circ$$


That means that after first iteration this is the result  $V_2^{(1)}$  is equal to this much and  $V_3^{(1)}$  is equal to this much. Now, second iteration when you will go for second iteration these values remain constant all constant parameters are computed before. So,  $V_2^{(2)}$ , when  $P$  is equal to 1, it will become  $0.479 \angle 220.1$  that is fixed divided by the first iteration value whatever you have got for  $V_2$  that is  $0.98305 \angle -1.8$  degree.

So,  $0.98305 \angle -1.8$  degree conjugate plus  $0.3846$  into  $1.05$  plus your  $j 0.0$  this term actually remains constant because  $b_1$  is also constant because your slack bus. This term also will not say plus  $0.6153$  into  $1.011 \angle -2.06$  degree that your this  $-2.06$  degree this value you got for  $V_3^{(1)}$ .

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2nd Iteration

Set  $p=1$

$$\Rightarrow V_2^{(2)} = \frac{0.0478 \angle 220.1^\circ}{(0.98265 \angle -3.048^\circ)^*} + 0.3846 \times (1.05 + j0.0) + 0.6153 \times 1.0011 \angle -2.06^\circ$$

$$\Rightarrow \therefore V_2^{(2)} = 0.98265 \angle -3.048^\circ$$

$$V_3^{(2)} = \frac{0.0217 \angle 229.2^\circ}{(1.0011 \angle -2.06^\circ)^*} - 0.47 \angle 175.6^\circ \times (1.05 + j0.0) - 0.532 \angle 183.8^\circ \times 0.98265 \angle -3.048^\circ$$

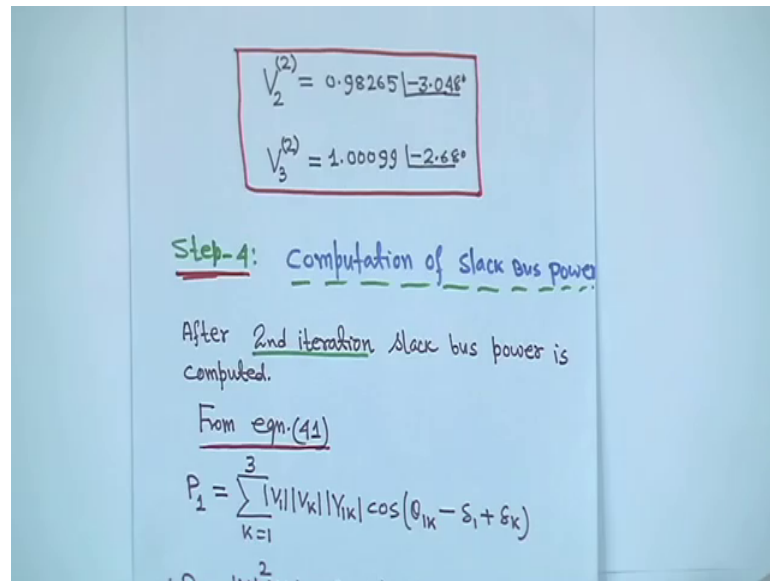
$$\Rightarrow \therefore V_3^{(2)} = 1.00099 \angle -2.68^\circ$$

So, if you make it so  $V_2^{(2)}$  in the second iteration 0.98265 angle minus 3.048 your degree. Similarly,  $V_3^{(2)}$  also you calculate here also say this term you have got earlier and this is 1.0011 you got in first iteration. So, angle minus 2.06 degree conjugate, it is conjugate minus 0.47 one angle 175.6 degree into 1.05 j 0. This term actually remain constant because  $V_1$  also not change in slack bus voltage then minus 0.532 angle 183.8 degree into that this  $V_2^{(2)}$  voltage you have got it here 0.98265 angle minus 3.048 degree these values directly substitute here that that one you do it.

So, if you do so so a put it there. So, if you do so  $V_3^{(2)}$  will come 1.00099 angle minus 2.6 degree. So, this is up to this is only two iterations. We are not (Refer Time: 23:58) converge or not. So, if you take if you do by are calculator another two, three iterations then you may be knowing. So, this is the steps to you show that how one can compute using Gauss-Seidel method, how one can solve the problem; so only two iterations are been made so that is sufficient to understand this.

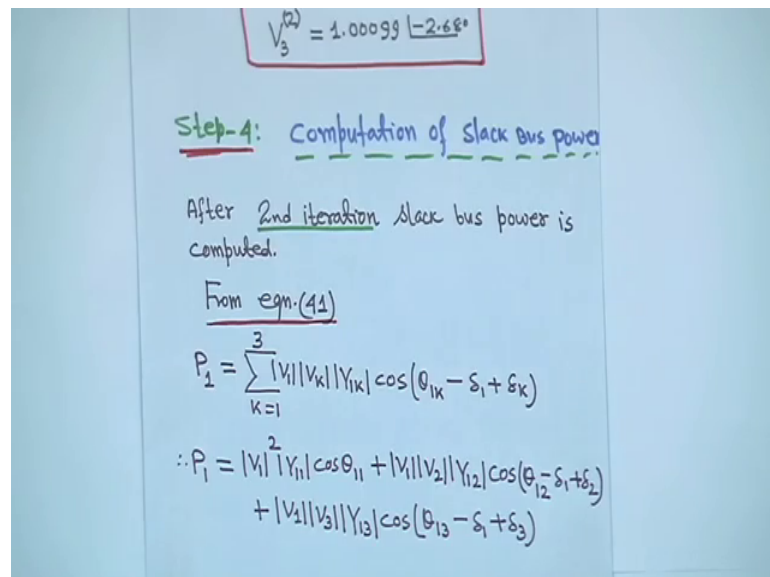


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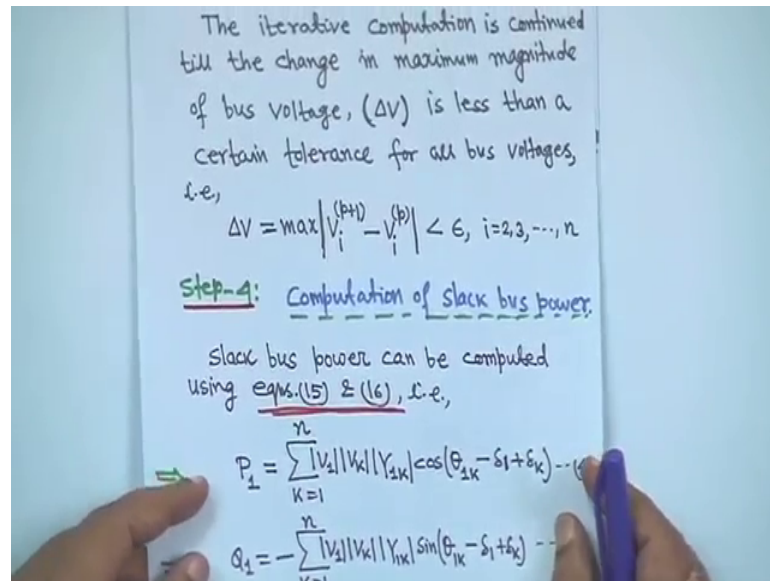
So, next is that after second iteration your this is your what you call this is your V 2 0.98265 angle minus 3.048 degree and V 3 2 1.00099 angle minus 2.68 degree. So, this is your V 2 V 3 after second iteration only you have been as to compute the slack bus power.

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So, after second iteration, slack bus power from equation 41 that means, this is equation 41 we have when we are explain algorithm for the gauss algorithm for the Gauss-Seidel method.

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In step 4 computation of slack bus power. So, when for slack bus power we have taken  $i$  equal to 1. So,  $P_1$  is equal to  $k$  is equal to 1 to  $n$  then  $V_1 V_k Y_{1k}$  all are magnitude cosine  $\theta_{1k} - \delta_1 + \delta_k$ , this was  $P_1$  similarly for  $Q_1$  this expression. So, after second iteration and your  $n$  is equal to for our case  $n$  is equal to 3, because we are considering your three bus problem. Therefore, equation 41 for  $P_1$  consider has  $k$  is equal to 1 to 3  $V_1 V_k Y_{1k}$  all are magnitude then cosine then cosine  $\theta_{1k} - \delta_1 + \delta_k$ .

So, when will do such computation, you have to a lot of patience; otherwise that is every possibility to write this equation the term you can means there it is possibility. Second possibility is that calculation error that calculator error that means, if you try to compute very fast then you have to very careful that your calculating correctly; otherwise there is a possibility of mistake. So, if you expand this it will become magnitude  $v_1^2$  then magnitude  $Y_{11}$  then  $\cos \theta_{11} +$  magnitude  $V_1$  magnitude  $V_2$  magnitude  $Y_{12}$  cosine  $\theta_{12} - \delta_1 + \delta_2$  plus magnitude  $V_1 V_3$  magnitude  $Y_{13}$  cosine  $\theta_{13} - \delta_1 + \delta_3$ .

Now, after second iteration you have got this voltage after second iteration. And all the angles are known  $\theta_{11}$  that is that angle of  $Y_{11}$  that angles are known angle are  $Y_{12}, Y_{13}$  all are known so and the  $\delta_1$  is what you call  $\delta_1$  is 0,

because slack bus voltage 1.05 plus your j 0. So, delta 1 is 0, and delta 2 after second iteration is minus 3.048 degree and delta 3 after second iteration minus 2.68 degree.

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(46)

$$\begin{aligned} \therefore |V_1| &= 1.05, \quad \delta_1 = 0^\circ \\ |V_2| &= 0.98265, \quad \delta_2 = -3.048^\circ \\ |V_3| &= 1.00099, \quad \delta_3 = -2.68^\circ \\ |Y_{11}| &= 53.85, \quad \theta_{11} = -68.2^\circ \\ |Y_{12}| &= 22.36, \quad \theta_{12} = 116.56^\circ \\ |Y_{13}| &= 31.62, \quad \theta_{13} = 108.4^\circ \end{aligned}$$

$$\begin{aligned} \therefore P_1 &= (1.05)^2 \times 53.85 \cos(-68.2^\circ) \\ &+ 1.05 \times 0.98265 \times 22.36 \cos(116.56^\circ - 0 - 3.048^\circ) \\ &+ 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.68^\circ) \end{aligned}$$

So, here you have to substitute all this data. So, in this case magnitude V 1 is 1.05 and delta 1 is 0 that is slack bus angle. Then magnitude V 2 is this much we have computed after second iteration just now I showed delta 2 is minus 3.048 degree. Magnitude V 3 1.00099, and this delta 3 is minus 2.68 degree. Now, capital Y1 1 53.85 and is angle was minus 68.2 degree all you are computed. And capital Y1 2 is to 22.36 and theta 1 2 is 116.56 degree all computed capital Y1 3 is equal to 31.62 and theta 1 3 108.4 degree.

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$$|Y_{12}| = 22.36, \quad \theta_{12} = 116.56^\circ$$

$$|Y_{13}| = 31.62, \quad \theta_{13} = 108.4^\circ$$

$$\therefore P_1 = (1.05)^2 \times 53.85 \cos(-68.2^\circ)$$

$$+ 1.05 \times 0.98265 \times 22.36 \cos(116.56^\circ - 0 - 2.048^\circ)$$

$$+ 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.66^\circ)$$

$$\therefore P_1 = 3.84 \text{ pu MW} = 3.84 \times 100 = 384 \text{ MW}$$

From eqn. (42),

$$Q_1 = - \sum_{k=1}^3 |V_1| |V_k| |Y_{1k}| \sin(\theta_{1k} - \delta_1 + \delta_k)$$

So, with this you substitute, you substitute all these things for P 1, this P 1. So, if you do so then you will get P one is equal to 3.84 per unit megawatt base MVA is 100 if you multiply, 384 megawatt. Similarly, for equation 42 again in step for of the algorithm that mean these equation again these equation, you put n is equal to 3 for this bus and find out what is the your reactive power of the slack bus Q 1. So, in this case you have put in from equation 42 that means, this equation this is your equation 42, this equation 42.

Therefore, Q 1 is equal to minus k sigma k is equal to 1 to 3 magnitude V 1 magnitude V k magnitude Y 1 k sin theta 1 k minus delta 1 plus delta k expand this expand this when you expand this will be like this.

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$$Q_1 = -|V_1|^2 |Y_{11}| \sin \theta_{11} - |V_1| |V_2| |Y_{12}| \sin(\theta_{12} - \delta_1 + \delta_2) \\ - |V_1| |V_3| |Y_{13}| \sin(\theta_{13} - \delta_1 + \delta_3)$$
$$\therefore Q_1 = -(1.05)^2 \times 53.85 \sin(-68.2^\circ) \\ - 1.05 \times 0.98265 \times 22.36 \sin(116.56^\circ - 3.046^\circ) \\ - 1.05 \times 1.00099 \times 31.62 \sin(108.4^\circ - 2.68^\circ)$$
$$\therefore Q_1 = 1.9786 \text{ pu MW} = 1.9786 \times 100 = 197.86 \text{ MW}$$

Step-5: Calculation of Line Flows and Line Losses.

From Eqn. (34)

It will be like this and then you substitute all the parameters whatever you have given here you have whatever you have given here all the parameter here, all the parameter here. So, you substitute and after sub after substitution you compute it is becoming 1.9786 per unit we call sometime we call per unit megawatt because that is actually per unit megawatt, megawatt not megawatt this is Q that is multiplied by 100. Some it is per unit is sufficient per unit megawatt you want mean that is really unit is megawatt that is you call per unit megawatt, but otherwise you just put per unit no problem. So, it is equal to 1.9786 into 100, so 197.86 megawatt this is after second iteration.

Now, again from equation 34 that whatever your what you call that last formula we sorry not last formula that your line power flows formula line power flows formula that P ik expression you have got in equation 34. Same expression you are writing here just put just put i is equal to 1, k is equal to 2. So, this is your line flows formula just put k i is equal to 1, k is equal to 2 and put all the parameters.

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$\therefore P_1 = 1.9786 \text{ pu MW} = 1.9786 \times 100 = 197.86 \text{ MW}$

Step 5: Calculation of Line Flows and Line Losses.

From Eqn. (34)

$$P_{ik} = -|V_i|^2 |Y_{ik}| \cos \theta_{ik} + |V_i||V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k)$$

$$\therefore P_{12} = -|V_1|^2 |Y_{12}| \cos \theta_{12} + |V_1||V_2| |Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2)$$

And put all the parameters. So, in this case all that when you put all the data you will get P 1 2 is equal to 1.8189 per unit megawatt you call. So, that is 181.89 megawatt multiplied this per in value by 100.

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$$\therefore P_{12} = -(1.05)^2 \times 22.36 \cos(116.56^\circ) + 1.05 \times 0.98265 \times 22.36 \cos(116.56^\circ - 0 - 3.048^\circ)$$

$$\therefore P_{12} = 1.8189 \text{ pu MW} = 181.89 \text{ MW}$$

$$P_{13} = -|V_1|^2 |Y_{13}| \cos(\theta_{13}) + |V_1||V_3| |Y_{13}| \cos(\theta_{13} - \delta_1 + \delta_3)$$

$$\therefore P_{13} = -(1.05)^2 \times 31.62 \cos(108.4^\circ) + 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.68^\circ)$$

$$P_{13} = 2.0 \text{ pu MW} = 200 \text{ MW}$$

Similarly, when your i is equal to 1, k is equal to 3, so P 1 3 easily you can get it i mean in this expression that equation 34 you can find out i is equal to 1 then k is equal to 2, i is equal to 1, k is equal to 3, i is equal to 2, k is equal to 3, P 2 4 all will you get you. So, similarly P 1 3 also you this is the thing and then you will get after substitution all are the

data put all the data here you will get P 1 3 is equal to 2.0 per unit megawatt that is 200 megawatt multiplied by 100.

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$$\begin{aligned} \therefore P_{13} &= -(1.05)^2 \times 31.62 \cos(108.4^\circ) \\ &\quad + 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.68^\circ) \\ \therefore P_{13} &= 2.0 \text{ pu MW} = 200 \text{ MW} \end{aligned}$$

$$\begin{aligned} P_{23} &= -|V_2|^2 |Y_{23}| \cos \theta_{23} + |V_2| |V_3| |Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \\ \therefore P_{23} &= -(0.98265)^2 \times 35.77 \cos(116.6^\circ) \\ &\quad + 0.98265 \times 1.00099 \times 35.77 \cos(116.6^\circ + 3.048^\circ - 2.68^\circ) \\ \therefore P_{23} &= -0.4903 \text{ pu MW} = -49.03 \text{ MW} \end{aligned}$$

Similarly, P 2 three also and your when your i is equal to 2, and k is equal to 3, so this is the expression put it you will get it. P 2 3 is equal to minus 49.03 megawatt the minus sign indicates actually power is flowing from 3 to 2 actually as 2 to 3 is showing minus 49.03 means power actually flowing from 3 to 2.

Thank you.