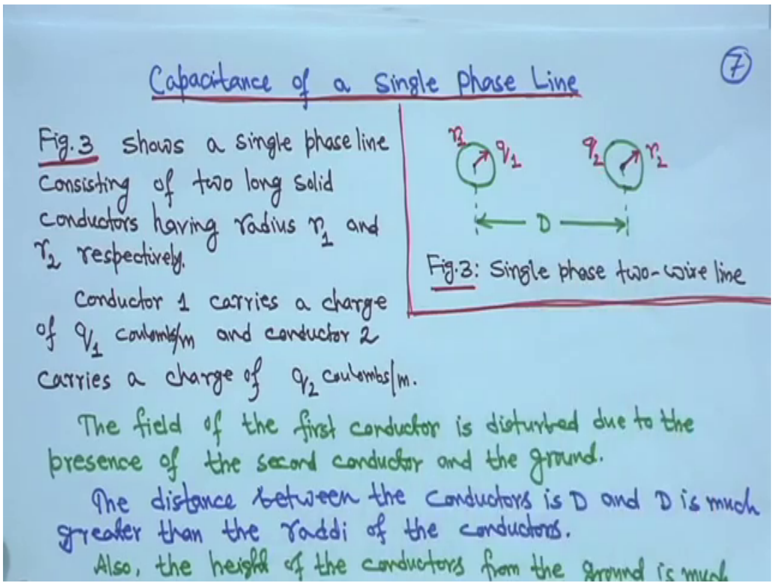
Power System Analysis
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Lecture - 12
Capacitance of Transmission Lines

(Refer Slide Time: 00:19)

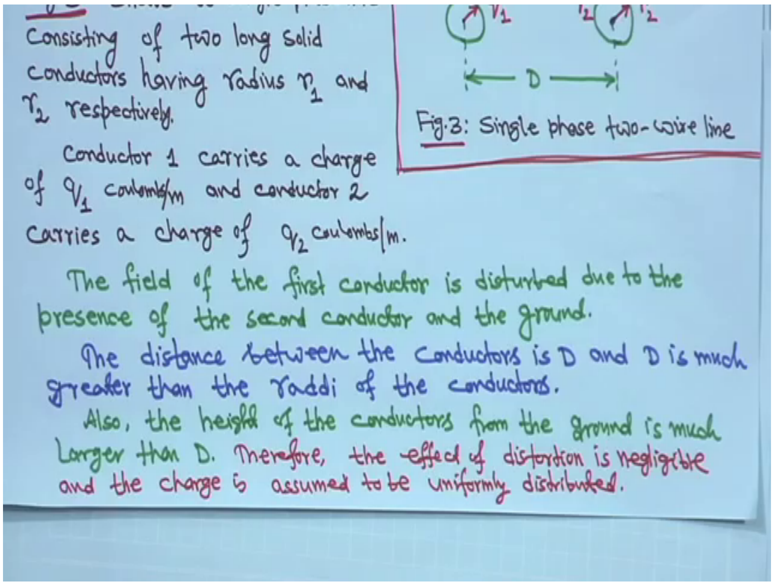


So, next is the capacitance of a single phase line right now look at this figure you have 2 conductors listen one thing for the capacitance case there is no question of r dash right like inductance. So, this should be in your mind. So, now, you consider a single phase line this is there are 2 conductors this is q 1 and q 2 the charge is q 1 q 2 and if q 1 is equal to q is a positive charge then q 2 will be your minus q right. So, later we will see.

Now, the radius of this conductor is r 1 right the radius of this conductor is r 2 we have initially we have taken that what you call radius of 2 conductors are different and center to center distance is D right center to center and of course, D is much much larger than r 1 and r 2 right. So, this is single phase 2 wire line right. So, conductor one carries a charge of q 1 coulomb I told you per meter and conductor 2 carries a charge of q 2 coulomb per meter right the field of the first conductor is disturbed due to presence of the second conductor and the ground right.

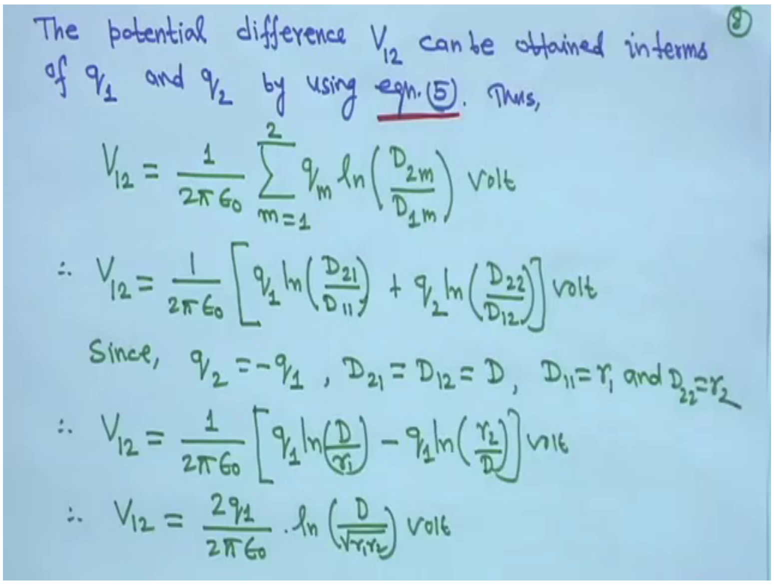
So, basically if you take now you are considering single phase dual line naturally field of this one will be disturbed by this one as well as by the ground; we will see later right. So, the distance between the conductor is D and D is much much greater than the radius of the 2 conductors r 1 and r 2 right also the height of the conductor from the ground is much larger than D because conductors are always above the ground right. So, that height h is much much greater than D, but when you will consider h that we will see later.

(Refer Slide Time: 02:14)



So, therefore, the effect of distortion is negligible and charge is assumed to be uniformly distributed. So, we are assuming that the effect of distortion is negligible and the charge is assumed to be uniformly distributed right.

(Refer Slide Time: 02:32)

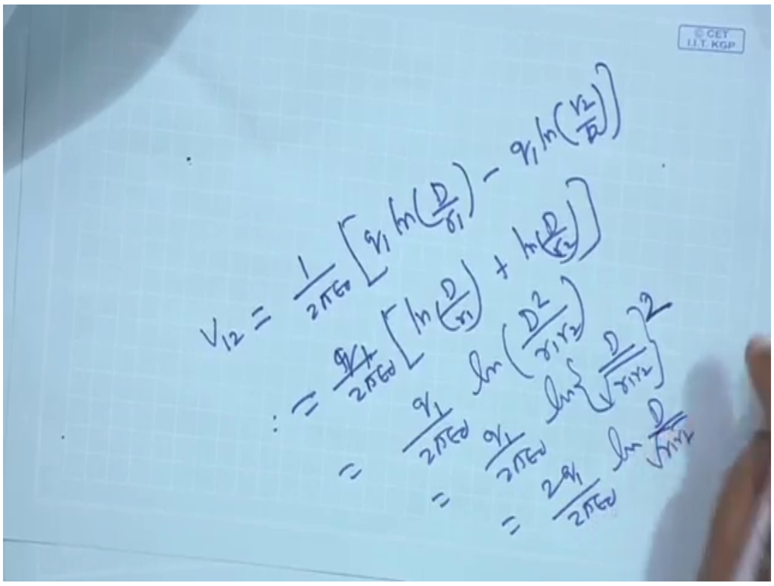


The potential difference V 12 can be obtained in terms of q 1 and q 2 using equation 5 that general equation we have given m is equal to 1 to n. So, the same equation for 1 to 2 you are using 1 upon 2 pi epsilon 0 2 conductors are there m is equal to 1 to 2 this is q m l n D 2 m upon D 1 m right volt right.

So, this expression this expression just now we given no this expression D I m upon D k m right. So, I is equal to 2 and your k is equal to 1 right. So, therefore, the m is equal to 1 to 2. Now V 12 is equal to 1 upon 2 pi epsilon 0 then q 1 l n upon D 21 upon D 11 plus q 2 l n D 22 upon D 12 volt now since q 2 is equal to minus q 1 and D 21 equal to D 12 actually 2 conductors are there conductor 1 and conductor 2. So, that is why D 21 and D 12; that means, this is here to here 1 to 2 1 basically this distance is D, but this is a generalized formula after that we are replacing this right and D 11 I told you that if I k is equal to m or I is equal to m then D m m is equal to r.

So, when D 11 is equal to r 1 and D 22 is equal to r 2 right therefore, V 1 is equal to 1 upon 2 pi epsilon 0 q 1 l n D upon r 1 minus q 1 l n r 2 upon D because q 2 is equal to minus q 1 that is why minus q 1 l n r 2 upon D volt right. Therefore, V 12 is equal to 2 q 1 upon 2 pi epsilon 0 l n D your square root of r 1 r 2 because if you if this is k if you take q 1 common and it will be basically plus l n D upon r 2 because it is minus l n r 2 upon D it will be D upon r 2. So, that your after that you simplify this it will be D I 2 q 1 upon 2 pi epsilon 0 l n D upon under root r 1 r 2 I mean these things are simple just for your clarification just writing one line.

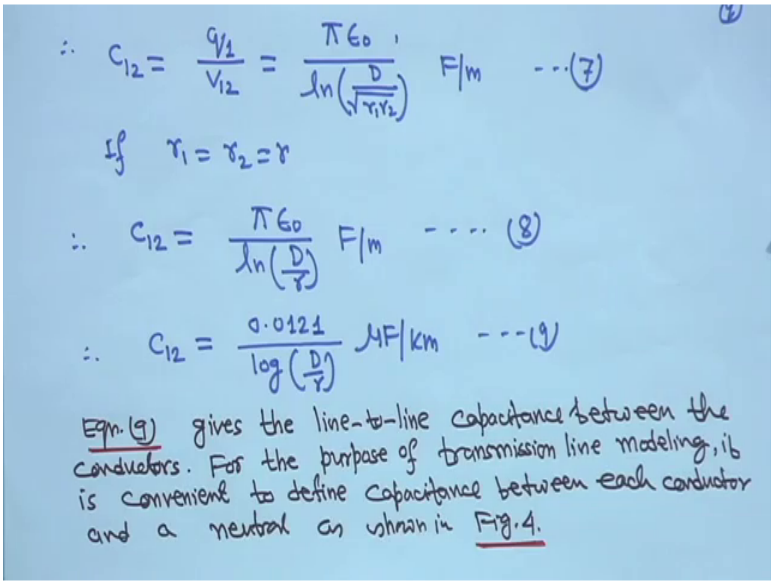
(Refer Slide Time: 04:58)



So, V 12 is equal to your 1 upon 2 pi epsilon 0 then q 1 l n D upon r 1 right minus it is written q 1 l n r 2 upon D right therefore, this is equal to 1 upon 2 pi epsilon 0 you sorry you take q 1 common right this 1 is l n D upon r 1 plus l n then D upon r 2 right that is equal to q upon 2 pi epsilon 0 then l n D square upon r 1 r 2 right this one. You can write no q upon 2 pi epsilon 0 l n right then you can write D root over r 1 r 2 right this square this is square is equal to 2 q 1 it is q 1 actually it is no you are putting your this thing it is q 1. So, it is q 1 then 2 q 1 upon 2 pi epsilon 0 l n D upon root over r 1 r 2 right. So, that is the simplification.

So, that is why we were writing here this one is your what you call 2 q 1 upon 2 pi epsilon l n D upon root over r 1 r 2 volt right; that means, therefore, our objective is that you have to find out capacitance between the 2 lines C 12.

(Refer Slide Time: 06:38)

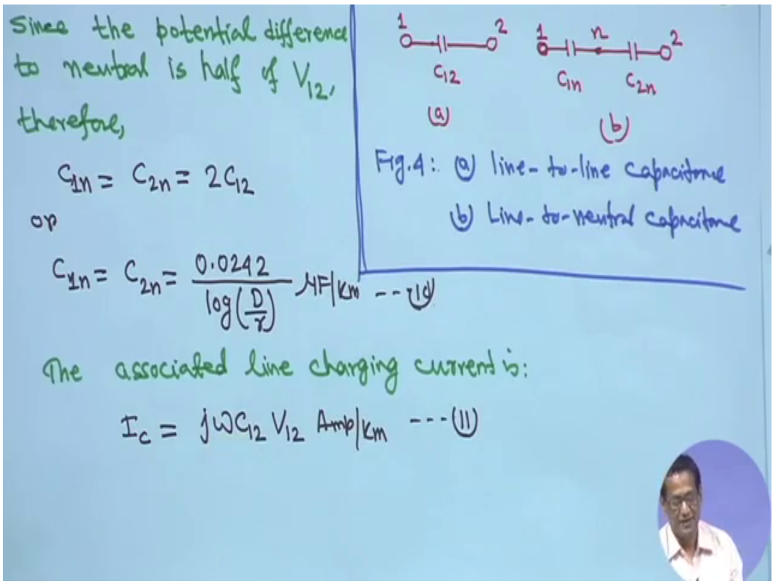


Therefore this C 12 is equal to it is q 1 upon you know q is equal to C V in general therefore, C is equal to q by V therefore, C 12 is equal to q 1 upon V 12 is equal to pi epsilon 0 upon l n because this; this 2 2 will be cancelled right 2 2 will be cancelled I did not make it here, but this 2 will be cancel right. So, therefore, C 12 is equal to q 1 upon V 12 is equal to pi epsilon 0 upon l n D under root r 1 r 2 farad per meter this is equation seven right therefore, if both the conductors' radius same then r 1 is equal to r 2 is equal to r therefore, C 12 is equal to pi epsilon 0 l n divided by l n D farad per meter this is equation eight right.

So, epsilon 0 you know 8.854 into 10 to the power minus 12 farad per meter this is known right and this is this is your natural log, but you convert you convert it to this log right. So, this conversion is up to you right. So, this simple thing you can do it. So, generally for all the numerical this that we will use log of this thing right not the natural log right because many things easy to remember therefore, C 12 is equal to if you simplify it will be 0.0121 divided by log D upon r microfarad per kilometer right. So, this is equation 9 right. So, this is the capacitance. So, this equation 9 actually it gives the line to line capacitance between the conductors right, but for the purpose of transmission line modeling it is convenient define the capacitance between each conductor and the neutral as shown in figure 4.

So, this is line to line your line to line capacitance, but in between line 1 and 2 if you have a neutral then you have to find out that capacitance from each conductor to neutral that is C 1 n and C 2 n.

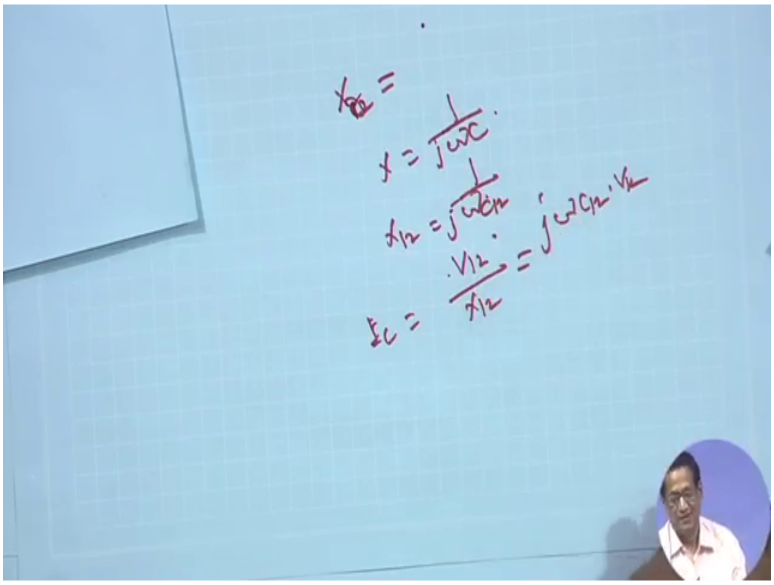
(Refer Slide Time: 08:55)



So; that means, if you come to this come to this part when 1 to 2 this is C 12 that is whatever you got whatever you have got C 12 is this one right equation 9; now if you have a neutral in between this is a neutral n right. So, this side 1 to n this is the C 1 n and this is C 2 n right and these 2 are in series right this is conductor 1 this is conductor 2 right. Therefore, C 1 n is equal to C 2 n is equal to 2 C 12 because they are in series. So, C 1 n is equal to C 2 n is equal to 2 C 12; that means, C 1 n is equal to C 2 n is equal to this will be multiplied by 2 right; that means, it will be 0.0242 divided by log D by r microfarad per kilometer this is equation 10 right.

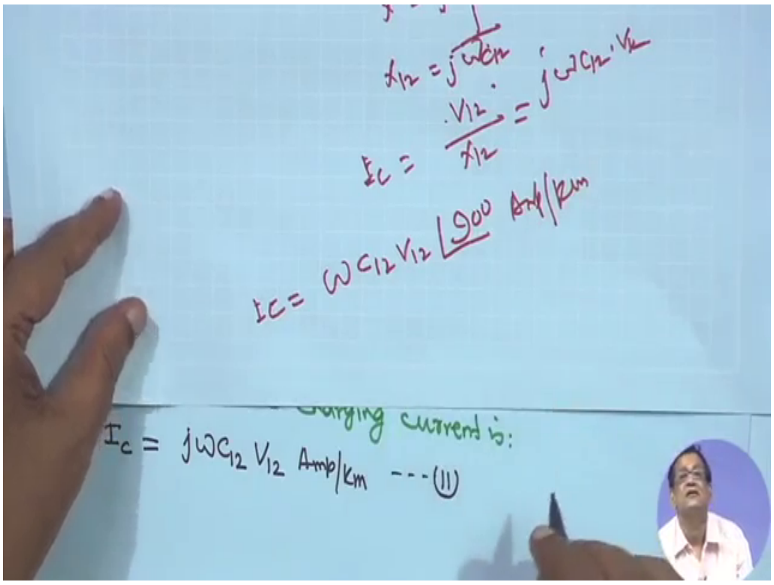
Now, the associate line charging current will be I C is equal to j omega C 12 V 12 ampere per kilometer right. So, it will be if you think like this something like this that if you try to find out the reactance say I put X 12 say X 12 your what you this thing your generally you know that X is equal to 1 upon omega C right.

(Refer Slide Time: 10:02)



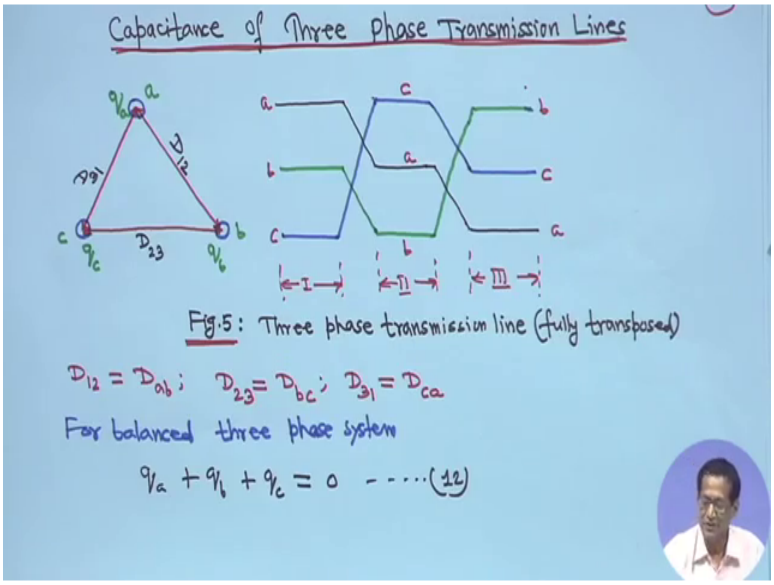
Therefore if I make it X 12 is omega by C 12 right. So, this is that reactance therefore, when you try to find out the charging current say I C say I C right is equal to in general I am putting. So, voltage is V 12 divided by your X 12 is equal to your omega C 12 into V 12, but as it is a as it is a capacitance right. So, this; what you call then you instead of magnitude here you put j here you put j therefore, it will be j therefore, right. So, that is why that is why this line charging current will be I C will be equal to j omega C 12 into your V 12 ampere per kilometer, but you take the magnitude then j should not be there if the magnitude is taken right.

(Refer Slide Time: 11:25)



So, it is a leading current it is a leading current right so; that means, this one, this one; one more line I am writing for you; that means, this one will be I charging current will be omega C 12 V 12 angle your ninety degrees because j is there. So, this ampere per kilometer right therefore, this is this is your general thing.

(Refer Slide Time: 11:48).

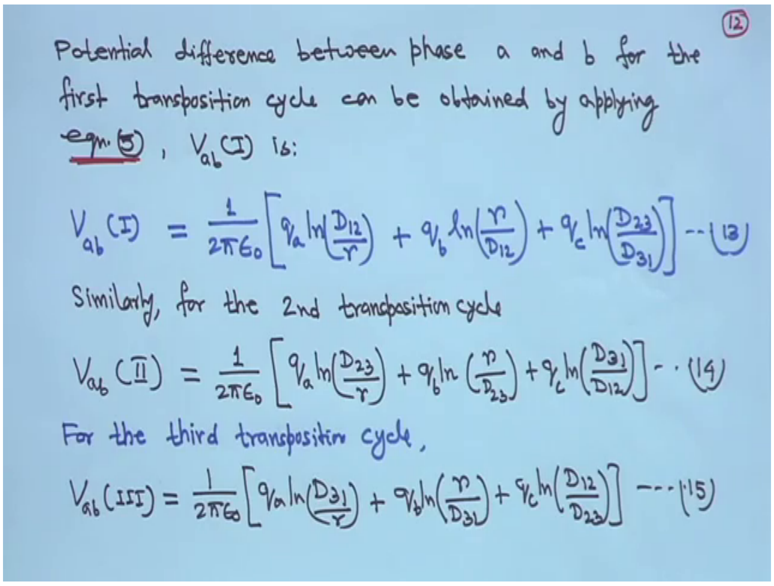


Next is capacitance of a 3 phase transmission line right. So, for transposition no need to explain here now because already for inductance case we have explained that transposition of transmission line right. So, you assume your 3 phase transmission line right and this capacitance of 3 phase transmission line and lines are transposed. So, this is phase a phase sorry phase a phase b and phase C right distance between a to your a to your C actually D 3 1; that means, D a c actually D 31 I have not written it, but I am telling from my mouth. So, you too know it right.

Similarly, D a b actually is equal to D 12 right and similarly D b c actually D 23 capital right not making it here, but at the time of derivation we will see if. So, you want you can make it, but D a b is literally D 12 D a c is equal to a b b c c a. So, D b c will be D 23 and D c a will be D 31 right. So, this is the line transpose. So, first section; section 1, it is a b c, next section it will be c a b that is section 2 and third section it will be b c a in the third section. So, this already we have seen for the inductor right. So, 3 phase transmission line this is figure 5 fully transpose, but here I have written here D 12 D a V D 23 D b c and D c a is equal to D 31 now for balance 3 phase system q a plus q b plus q

C is equal to 0 this is equation 12 right like your current I a plus I b plus I c is equal to 0 right assuming no neutral right. So, next is we will apply the same formula right.

(Refer Slide Time: 13:50)



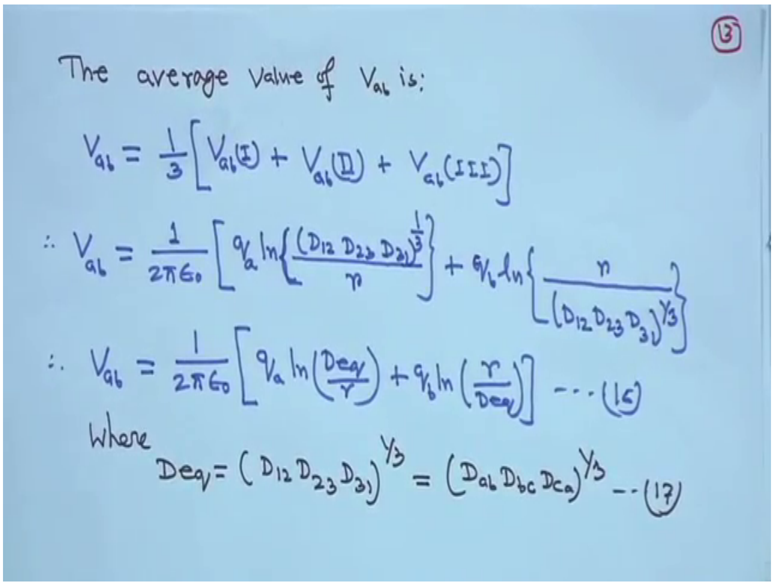
So, potential difference between phase a and b for the first transposition cycle I mean for the first one for the first transposition that is section 1 section 2 section 3. First we will see the first one here right so; that means, can we obtain by applying equation 5 the same equation again we will use the generalized equation therefore, V a b in bracket I means it is not current it is section 1 right V a b bracket; that means, for the this section 1 this section 1 right is equal to 1 upon 2 pi epsilon 0 q a l n D 12 by r plus q b l n r D 12 r by D 12 plus q c l n D 23 upon D 31 this is equation thirteen, but same equation 5; the generalized equation we are applying for for inductance also we for cross linkage we applied 1 generalized equation for capacitance for voltages also the same equation we are putting in this form right.

That means q a l n D 12 upon r plus q b l n r by D 12 plus q c l n; that means, q c l n; that means, q c l n D 23 by D 31 that is q c l n D 23 by D 23 by D 21 right because V a b your a b right due to further section 1. So, this is that configuration of section 1 this triangular configuration for this one and this one you can draw right for inductance case we have shown it. So, no need right therefore, your this one will be 1 upon 2 pi epsilon q a D 12 upon r D 12 is nothing, but D a b upon r right then q b this q b l n r upon D 12 right plus q c this is the q c l n right D 23 divided by D 31 right b that because you want to find out the potential that is your V a to b right if you recall voltage 1 to 2 D 2 upon D 1 that very first diagram that C 12 calculation V 12 calculation. So, if you think that way; that means, it is your from here to here it is D 23 and then D 31 right I hope you are understanding this right.

So, similarly for the second transposition cycle I mean this one this one you make this configuration and this configuration of your own because inductance we have seen it right therefore, for the second transposition cycle it will be V a b this is for this 2 means this is for second transposition cycle this is the first transposition cycle section 1 is equal to 1 upon 2 pi epsilon 0 right then q a l n D 23 upon r plus q b l n r upon D 23 plus q c l n D 31 upon D 12 this is equation 14 right.

Then for the third transposition cycle that is V a b this is third transposition cycle 3 is equal to 1 upon 2 pi epsilon 0 q a l n D 31 upon r plus q b l n r upon D 31 plus q c l n D 12 upon D 23 this is equation 15 from your imagination also you can write this equation without looking into that diagram also if you understood then no need to look into the diagram from your intuition you can write it right.

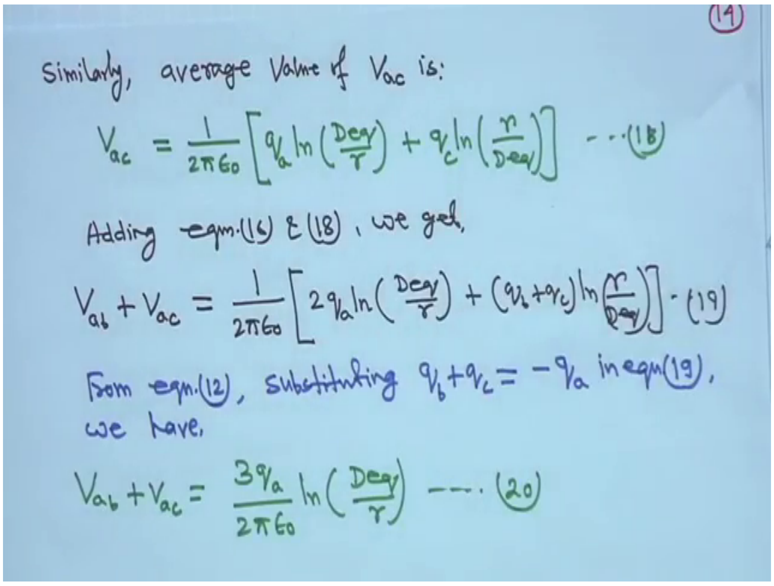
(Refer Slide Time: 17:35)



Therefore 3 3 different transposition cycles we got V a b for section 1 V a b for section 2 and V a b for section 3 now you take the average value of V a b right.

So, the average value of V a b will be equal to 1 third V a b for first transposition cycle plus V a b the second transposition plus V c b or V a b for the third transposition cycle right you substitute all and then simplify you will get the step you please do it I have made this one final one that 1 upon 2 pi epsilon 0 q a l n it will be D 12, D 23, D 31 to the what you call to the power one third divided by r plus q b l n r upon D 12 D 23 D 3 to the power your 1 by 3 right cube root. Therefore, V a b is equal to 1 upon 2 pi epsilon 0 q a l n D e q upon r plus q b l n r upon D e q where D e q is equal to D 12 D 23 D 31 to the power one third cube root is equal to I am writing here capital D a b into D b c into D c a; one third this is equation seventeen right. So, this is average value of V a b right similarly this is an exercise for you because I have done for you V a b right.

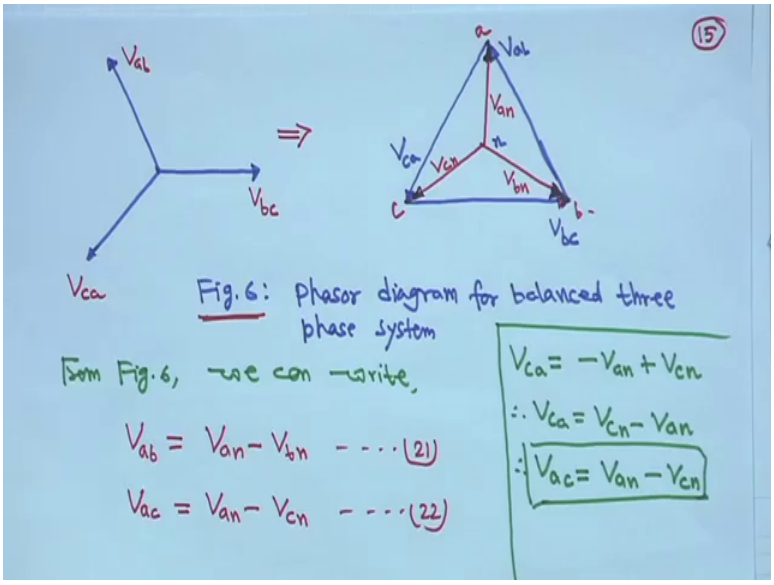
(Refer Slide Time: 19:00)



Similarly, average value of V a c from the inspection you can write V a c will be 1 upon 2 pi epsilon 0 look here it is V a b q a and q b right. So, q c will be eliminated automatically when you will substitute no q c will automatically be eliminated and V a b will be in terms of q a and q b. So, here also from your intuition you can write V a c will be 1 upon 2 pi epsilon 0 it will be q a l n D eq upon r plus q c l n r upon D e q this is eighteen. So, straightforward you can write after this you add equation sixteen and eighteen; that means, V a b plus V a c these 2 add if you add it will be 1 upon 2 pi epsilon 0 please add of this thing it will be 2 q a l n D q upon r plus q b plus q c into l n upon r D e q which is equation nineteen, but we know that q a plus q b plus q c is equal to 0.

Therefore this from your equation 12 therefore, q b plus q c is equal to minus q a equation 12 it was q a plus q b plus q c equal to 0 therefore, q b plus q c is equal to minus q a substitute here in the equation nineteen substitute here if you do simplify just put it directly you can write them it will become V a b plus V a c is equal to 3 q a upon 2 pi epsilon 0 l n in bracket D e q upon r this is equation twenty that is V a b plus V a c is equal to your 3 q a upon 2 pi epsilon 0 l n D e q upon r this is equation 20 I hope you have understood no things are simple right.

(Refer Slide Time: 20:52)



Therefore after that you have to make it some mathematical you know Phasor representation because you have to calculate the capacitance right. So, make it this is V b cs reference this is V c a V b c V a b and V c a right.

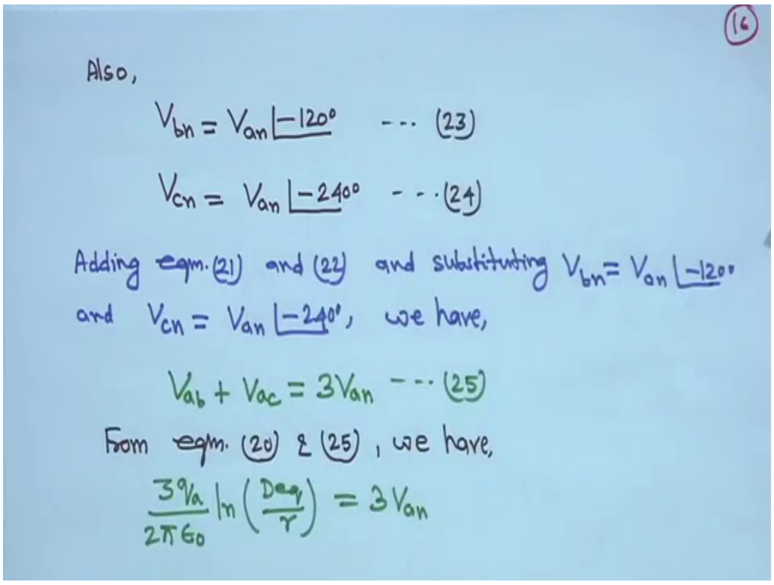
Now, this one you make it like this if you make it like this; this is your V b c, this phase is your V b c right and this is V a b. So, from here I am drawing the triangle that this is V a b right and this is V c a. So, drawing the triangle arrow is downwards V c a; that means, this Phasor you cannot triangle this is balanced system not unbalanced balanced system right neutral will be here right. So, if it is a neutral. So, this tip actually this is V a b this tip will be a and this is V b c. So, this tip will be b and this is V c a. So, this tip will be c; that means, for your understanding this will be actually a right and this is V b c. So, this is b and this is V c a; this is C right therefore, and this is the neutral.

So, this voltage keep it here this voltage is V a n this voltage is V b n this voltage is V c n and this angle actually thirty degree this angle thirty degree and this angle 120 degree right, but not writing here then figure will become clumsy, but this is understandable right. So, what I am doing that what we are doing is this V b c is this reference. So, this is V a b. So, this is V a b. So, this is V a b and this is V c a. So, make this V c a the arrow direction will be same as it is just triangular format and this is neutral side right therefore, Phasor diagram for balance 3 phase system now from figure six; that means, from figure six you can write V a b is equal to V a n minus.

Now, you do vector the Phasor also V a b is equal to V a n minus V b n that is your this V b b this look this V a b now you do the vector Phasor V a b is equal to V a b is equal to this tip is here opposite. So, minus V b n plus V n I am writing this one first and then this one, but if you take for your understanding I am telling this V a b equal to V minus V b n because arrow tip is here opposite. So, minus V b n this tip is here plus V a n. So, writing V a n first minus V b n this is equation 21 I hope you have understood similarly we want a b b C c a first you see that V a c right. So, in this case V a b plus V a c. So, V a c will be equal to V a n minus V c n first see the V c a; V c a is equal to it will be minus V a n because V c a is equal to the way you do the vector Phasor also same thing that Phasor is a rotating vector right this is the difference between Phasor and the vector right.

So, your V c a is equal to minus V a n writing fast plus V c n right therefore, V c a is equal to V c n minus V a n, but if we write V a c it will be just opposite V a c will be then you just interchange that c and a it will be V a n minus V c n right therefore, V a c is equal to V a n minus V c n this is the equation 22 I hope this simple thing, but sometimes, but sometimes confusion arises. So, hope there should not be any confusion I hope you have understood right.

(Refer Slide Time: 24:44)



Therefore then from this Phasor diagram your V b your what you call from this Phasor dia here also V b a is equal to V a n minus angle 120 degree right. So, this is your V a n this is your V a n. So, V b n actually lagging from lagging by 120 degree because this angle is 120 degree therefore, V b n is equal to V a n angle minus 120 degree we are writing twenty 3 understandable.

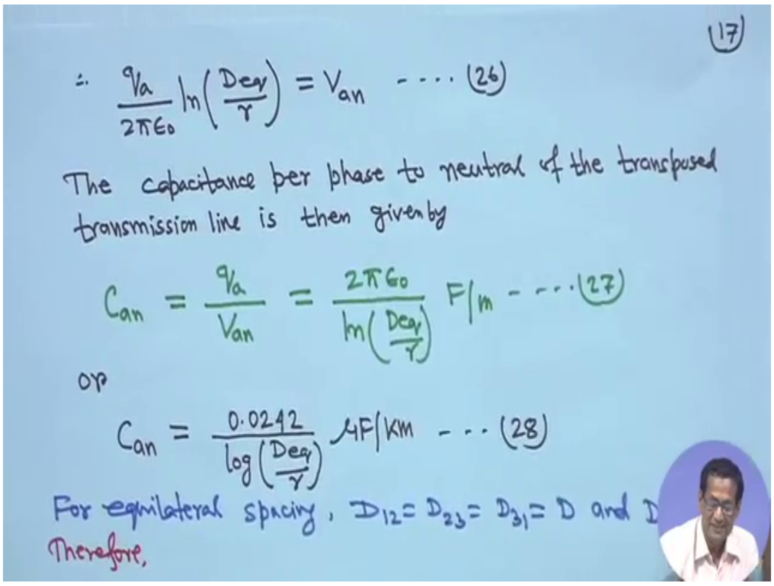
Then V c n V C n is equal to V a n from here to here from here to here it is 240 degree, but lagging. So, V c n is equal to V a n angle minus 240 degree this is equation 24 hope this is no no confusion right straight forward this; this V b n is reference then V b n is equal to V a n angle minus 120 degree and V c n move this way V a n angle minus 240 degree this is equation 24 right.

Therefore adding equation 21 and 22 and these 2 equation you add V a b plus V b c these 2 equation you add right and substituting V b n is equal to V a n angle minus 120 degree and V c n is equal to V a n angle minus 240 degree you substitute and you add substitute and simplify this is your job right. All these things we have discussed and explained right there should not be any confusion anywhere right therefore, if you do. So, V a b plus V a c actually is equal to 3 V a n this is equation 25 this relationship you have to establish that V a b plus V a c is equal to 3 V a n this is 25.

Now, from equation 20 and 25; that means, this is 25 this is equation 20 this is V a b that is V a b plus V a c this is actually equal to 3 V a n therefore, 3 q a upon 2 pi epsilon 0 l n

D e q upon r is equal to 3 V a n. So, 3 3 will be cancelled both sides right cancel both sides. So, you can write q a upon 2 pi epsilon 0 l n right.

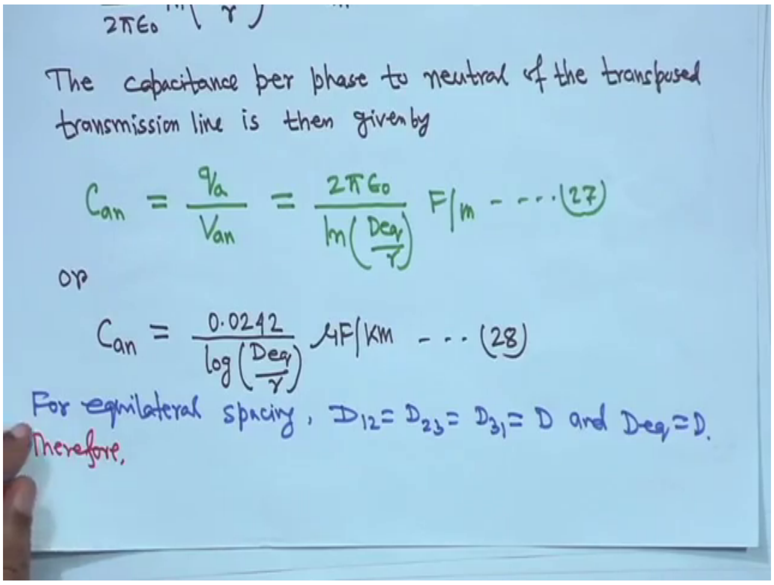
(Refer Slide Time: 27:12)



D q upon r is equal to V a n this is equation twenty six right therefore, capacitance now directly you can use that the capacitance per phase to neutral of the transpose transmission line is then given by C a n is equal to q a upon V n plus q equal to C V you know in general. So, C a n is equal to q a upon V a n is equal to 2 pi epsilon 0 l n D e q upon r farad; farad per meter. So, natural log you convert to this log right or you can get it C a n is equal to 0.0242 log D e q upon r micro farad per kilometer this is equation 28 right.

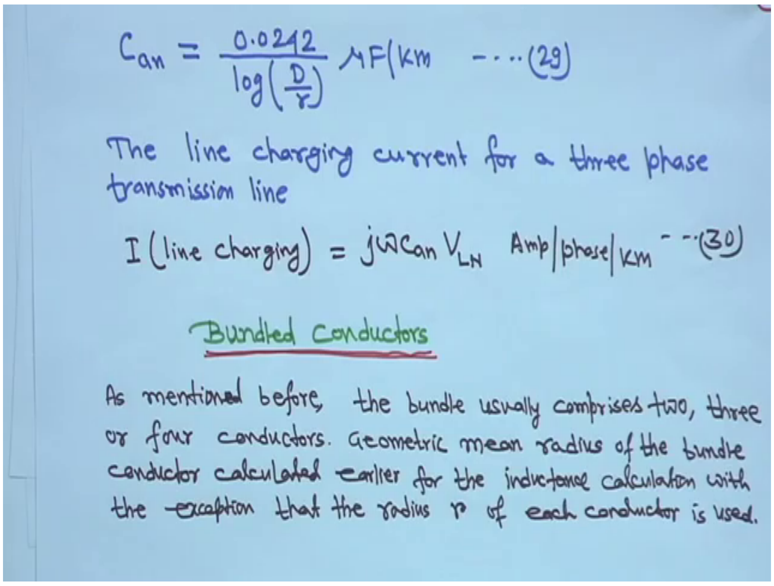
So, up to this things are explained right therefore, for equilateral spacing that if we assume that equilateral spacing that D 12 is equal to D 23 is equal to D 31 is equal to D.

(Refer Slide Time: 28:03)



Therefore D e q will also be equal to D because D into D into D to the power one third is equal to D therefore, therefore, C a n is equal to 0.0242 by log D upon r microfarad per kilometer this is equation 29 right.

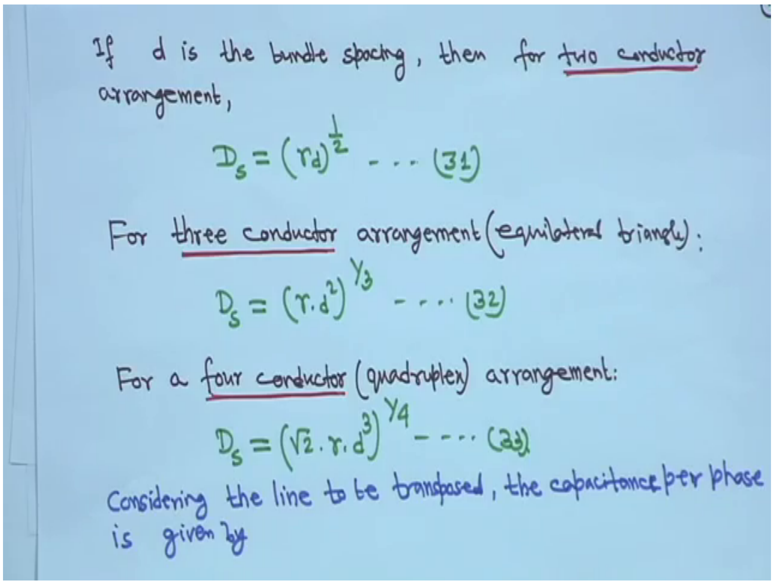
(Refer Slide Time: 28:25)



Therefore the line charging current sorry the line charging current for a 3 phase transmission line I in bracket I am writing line charging same as before j omega C a n V l n l n V l n means line to neutral voltage that is if you have a phase a b c; that means, it is either V a n or V b n or C a n in this case we are using C a n. So, in this case l will be equal to V l n line to neutral, but not writing right because your C a n; C b n; C c n for balance thing all are same, but anyway this is ampere per phase per kilometer this is equation thirty right.

Next one is bundled conductor. So, up to this charging current is required later you will see right particularly for the for your characteristic of transmission line then your this at that time we will see all the details right, but this is this is necessary now bundled conductor bundled conductors we have seen for 2 conductors 3 conductors and four conductors right. So, it is 3 2 already we have seen for induction cases. So, here I am not giving you the diagram the same diagram. So, the geometric mean radius of the bundled conductor can be you have calculated earlier for the inductance calculation with the exception that the radius r with conductor is used right. So, in that case for bundled conductor case for inductance we have used r dash right that is for calculation of internal inductance here no internal your capacitance. So, nothing is there no internal capacitance is there. So, that is why we use r only, but not r dash right.

(Refer Slide Time: 30:31)

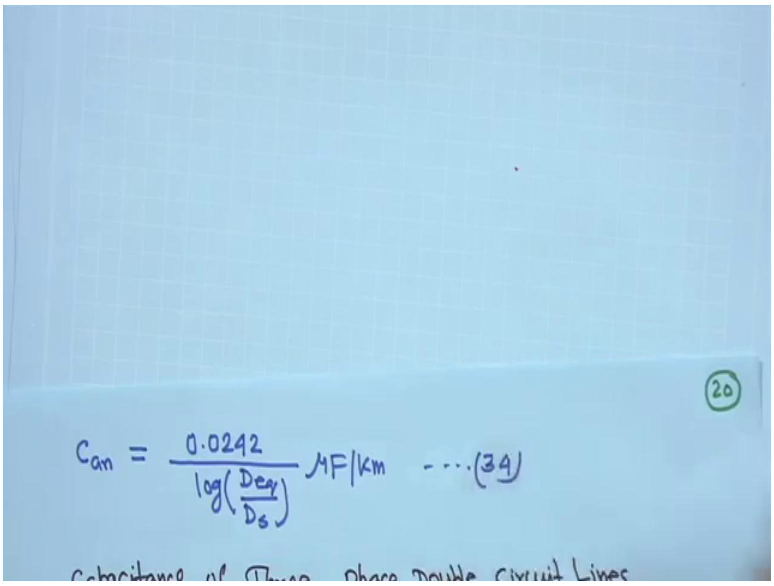


Therefore if D is the bundle spacing then for 2 conductors arrangement just imagine 2 conductors 3 conductors four conductors we have taken no. So, D s is equal to r D to the power half right the geometric mean radius for 2 conductor case in the case of inductance it was actually r dash, but in the case of capacitance it is r go back to the diagram.

Similarly, for the 3 conductor arrangement that is equilateral triangle this diagram also shown for inductance cases, but in this case D s will be r D square to the power one third. So, here no question of r dash again only r this is your thirty 2 and for a four conductor that is quadruplex arrangement D s will be root 2 r into D e q, but in the case of inductance it was r dash. So, here it is no r dash to the power 1 by 4 this is equation thirty 3 right. So, considering the line to be transposed the capacitance per phase is given by if we assume the line is a transpose line right.

So, these are the conductor arrangement is given if you have this kind of arrangement instead of a b C right if you have this; this equation basically this equation that log of D upon r instead of this equation we can write if we assume that considering the line to be transpose the capacitance per phase is given by in general right.

(Refer Slide Time: 31:57)



This one that C a n is equal to 0.0242 log D e q upon D s micro farad per kilometer this is equation 34 we are writing we are writing D upon r that is for equilateral spacing, but if we have that kind of configuration. So, generally we write log D e q upon D s microfarad per kilometer this is equation 34.

Thank you.