**Power System Analysis**
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**Lecture - 11**
**Resistance and Inductance (Contd.)**

So, before closing the inductance chapter, so, we will take couple of more examples then we will move to the next chapters right. So, come to this example. So, first thing is that determine that you know in between 1 or 2; I brought several examples for you, but time constraint is of course there.

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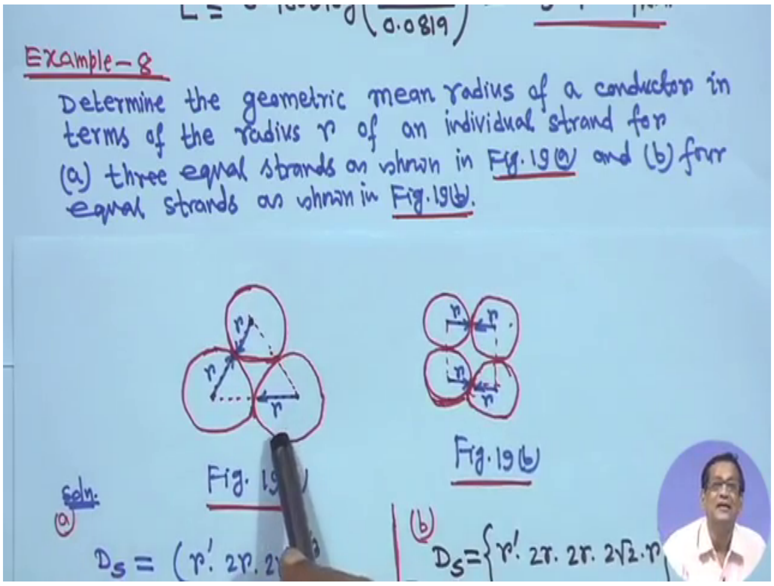


But look at this example figure, I will show you determine the geometric mean radius of a conductor in terms of the radius r of an individual strand for a these 3 equal strands as shown in figure 19 a and b 4 equal strands as shown in figure 19 b right. So, this is your figure 19 a and this is your figure 19 b right.

So, you have to find geometric mean radius of this configuration and this configuration right then for this one very simple it is for this one D s is equal to you will you have to consider all the conductors having the same radius right therefore, you consider for example, this conductors.

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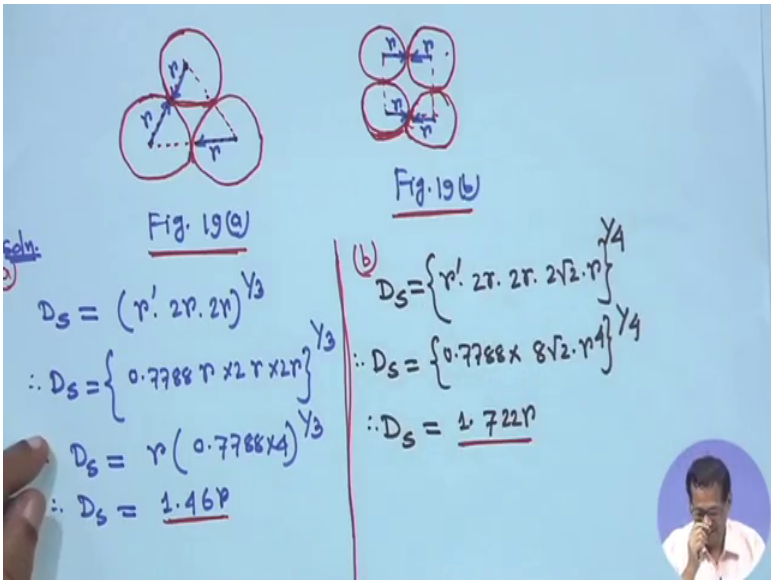


So, D s will be is equal to r dash into that distance from here to here is again 2 r. So, it will be 2 r and distance from here to here is again 2 r into 2 r; look for this conductor will be r dash, but rest it will be 2 r; 2 r please do not make it 2 r dash into 2 r dash then it will be wrong right to the power one third right. So, so D s will be then r dash will be equal to 0.7788 r that you have seen before right and if you simplify this you will get D s is actually 1.46 r the simplest one, but you see this kind of configuration.

Now, 4 second one is that you have 4 such conductors right; so, all the conductors having the same radius r right. So, in this case you have to find out D s right the geometric mean radius consider for example, this conductor. So, it is r dash then distance from here to here this conductor is 2 r then distance from here to here this conductor is also 2 r and distance from here to here; these 2 conductors it will be 2 root 2 r because this is 2 r this is 2 r. So, take the diagonal one. So, it is square root of 2 r square plus 2 r square that will become 2 root 2 into r to the power 1 by 4. So, r dash is equal to 0.778 r. So, 0.778 r, so this one will be r to the power 4. So, 0.7788 into 8 root 2 into r to the power 4 to the power 1 by 4 therefore, D s is equal to 1.722 r right. So, this is that this is your; what you call this is the answer that geometric mean radius if it is 3 configuration and if the configuration like this and if the configuration is like this right.

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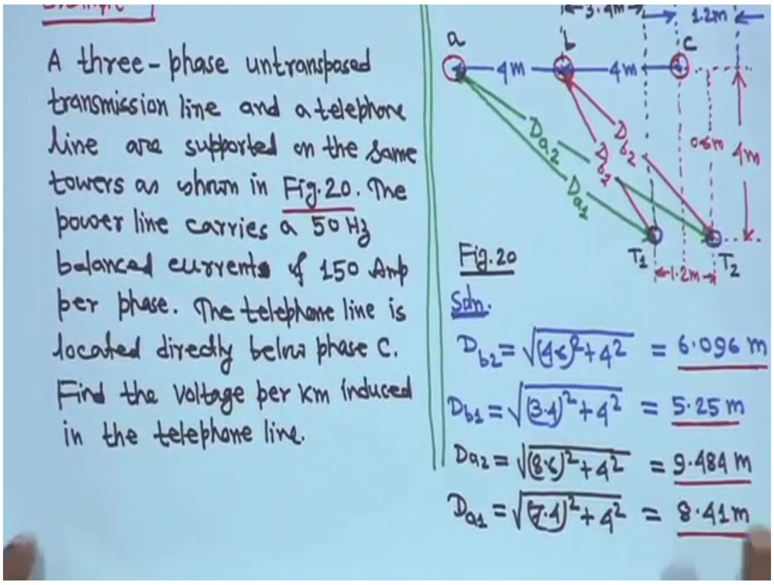


So, next another example we will take and after that we will go to the next chapter right next topic. So, for example, you take this one you take this one first you see the problem a 3 phase un-transposed transmission line and a telephone line I will show you the figure on the right side later line are supported on the same tower as shown in figure 20 right here it is. I will explain later the power line carries a 50 hertz balance current or 150 amperes magnitude 150 ampere per phase the telephone line is located directly below phase c find the voltage per kilometer induced in the telephone line right this is the problem now this is that your 3 phase un-transposed line. So, the horizontal configuration it is a b c these 3 phases distance between phase a and b 4 meter b and c your 4 meter and a and c will be automatically your 8 meter right and the telephone line is there below the; what you call that phase c conductor right. So, it is given here the telephone line is located directly below phase c. So, it is below phase c. So, it is symmetrical.

So, this is T 1 T 2 telephone lines right. So, between these 2 line distance is given one point 2 meter right; that means, from the center point to this one is 1.6; this side is 0.6, right accordingly you calculate the distance because you have to calculate D b 1; D b 2; D a 1; D a 2 all the distances right. So, here to here distance is given your 4 meter right this distance is given. So, therefore, this 0.6 meter actually this 0.6 meter actually this one; this one; this is 0.6 right. So, naturally this distance will be 4 minus 0.6; that means, it is 3.4 meter. So, because you have to calculate D a 1, D a 2, D b 1, D b 2 therefore, D b 2; D b 2 is equal to 4.6 square plus 4 square this is your D b 2 right.

So, if you take from here to here it is 4.6 and from here to here right angle triangle. So, here to here it is 4 and from here to this point I mean this point right this point. So, it is your 4.6 because this is 4 and this is 0.6 therefore, you can calculate the 6.096 meter similarly you calculate D b 1. So, here to here it is 3.4 and here to here it is 4. So, under root 3.4 square plus 4 square, so, is equal to 5.25 meter. Similarly if you calculate D a 2 from here to here, so, from a to c it is 8 and this is 0.6. So, it is 8.6 square and this vertical is 4 square. So, it is 9.484 meter right, similarly D a 1 you can calculate D a 1 as soon as you will calculate up to this it is 4 from here to here it is 3.4. So, total from here to here it is 7.4. So, under root 7.4 square plus vertical is this is given 4. So, it is plus 4 square is equal to 8.41 meter. So, all these distances are calculated.

So, you have to calculate first all the distances right once it is done then from figure 20; this is your figure 20; from figure 20 the flux linkage between conductors T 1 and T 2 due to conductors T 1 and T 2 means telephone lines conductor right.

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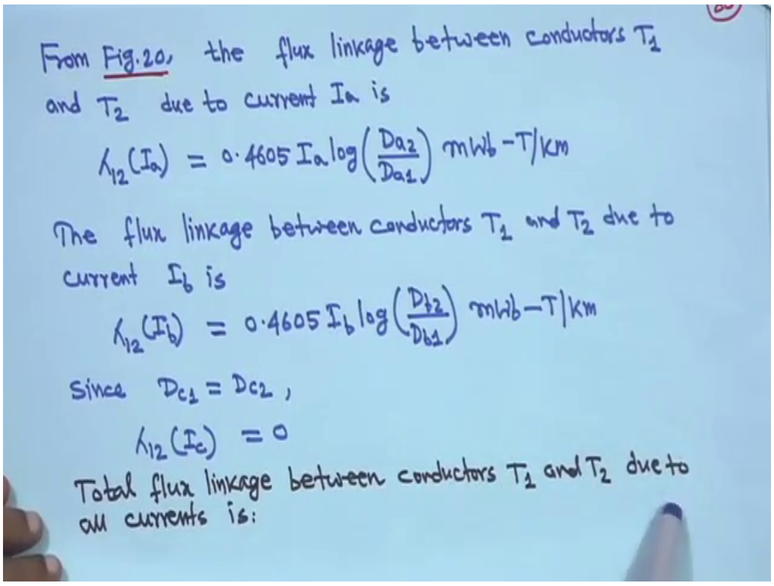


Due to current I a is that same formula we will use lambda 12 the flux linkage right between conductor T 1 and T 2 in bracket I a is writ10 it is due to current I a is equal to the same formula 0.4605 into I a into log D a 2 upon D a 1 Milli Weber tones per kilometer right.

Similarly this is your this is your D a 2 by D a 1 right and due to the current I a right similarly the flux linkage between conductors T 1 and T 2 due to current I b this is that this conductor is in phase b carrying current I b. So, it is also lambda 12; the flux linkage between conductors T 1 and T 2 due to current I b is equal to 0.4605 into I b into log D b 2 by D b 1 right Milli Weber tones per kilometer. Now next one is since D c 1 is equal because this D c 1; that means, this distance and this distance; that means, c to T 1 and c to T 2 if we call c to T 1 D c 1 and c to T 2 D c 2 these 2 distances are same right therefore, that since D c 1 is equal to D c 2 lambda 1 I 2 I c will be 0 because in that case it will be log D c 1; D c 2 upon D c 1, but as D c 1 is equal to D c 2. So, it will be 0. So, directly we are writing that lambda 12 I c is equal to 0 from the symmetry only this because from here to here and here to here the D c; the c to T 1 that is D c 1 and c to T 2 D c 2 these 2 distances are same therefore, it is 0 right.

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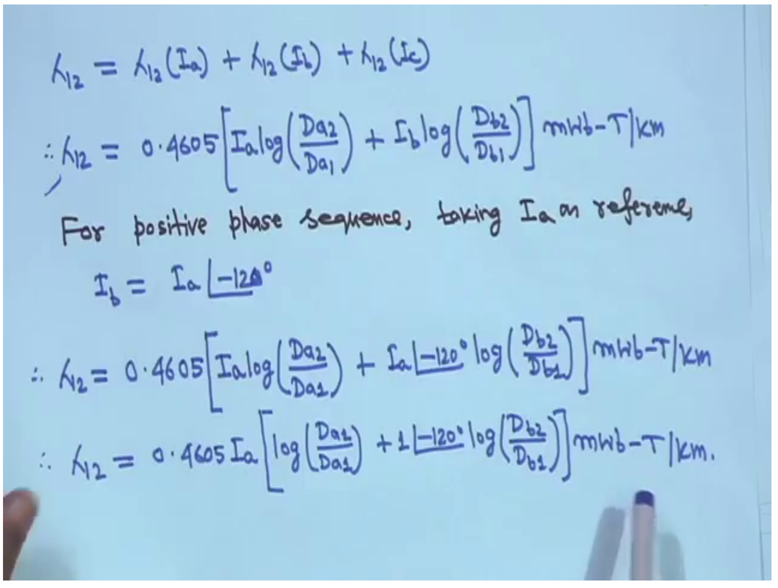


Therefore total flux linkage between conductors T 1 and T 2 due to all currents is right total flux linkage. So, what we will do you sum it up all right therefore, total flux linkage lambda 12 is equal to lambda 12 I a plus lambda 12 I b plus lambda 12 I c lambda 12 I c is anyway it is 0. Therefore, you substitute these 2 expressions that lambda 12 I a is equal to and lambda 12 I b is equal to these 2 expressions you substitute and lambda 12 I c is equal to 0 right therefore, it will be 0.4605 in bracket I a log D 2 upon D 1 plus I b log D b 2 upon D 1 Milli Weber tones per kilometer right now for positive phase sequence take I a as a reference suppose you take I a as a reference therefore, I b will be equal to I a angle minus 120 degrees not showing the Phasor diagram I a I b I c because earlier we had seen. So, it is understandable right. So, I b is equal to I a minus 120 degree for balance system right therefore, here you substitute in this expression I b is equal to I a angle minus 120 degrees then lambda 12 is equal to 0.4605 I a log D 2 upon D 1 and instead of I b we put I a angle minus 120 degree.

So, I a angle minus 120 degree log D b 2 upon D b 1 will Milli Weber tones per kilometer right is equal to that you take I a common. So, lambda 12 is equal to 0.4605 in a in bracket log D a 2 upon D a 1 plus 1 angle minus 120 degree log D b 2 by D b 1 Milli Weber tones per kilometer right. So, this is the expression although you know the distances D a 2 D a 1 D b 2 D b 1 and all are known to you right.

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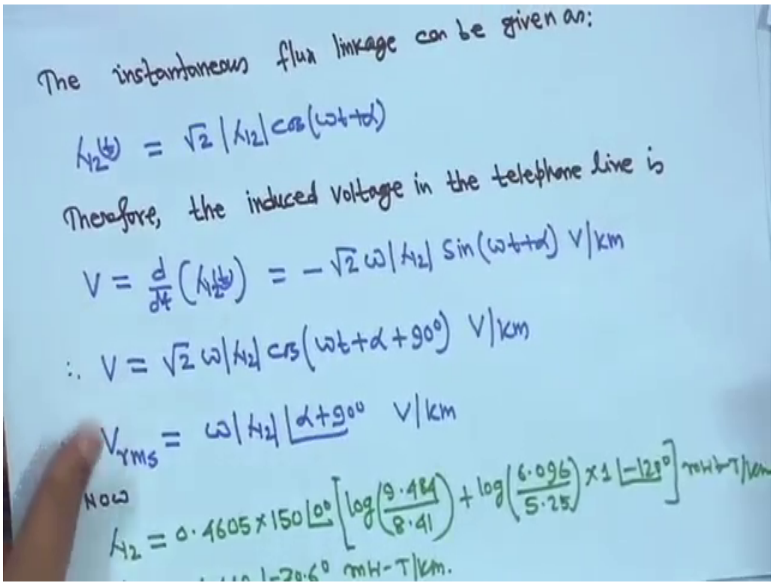


Next is that the instantaneous flux linkage can be given as lambda 12 function of T is equal to root 2 that mod of lambda 12 cosine omega T plus alpha you assume this one that instantaneous flux linkage can be given us right.

So, it is root lambda 12 T is equal to root 2 lambda 12 because we are assuming that this lambda 12 magnitude whatever will come this is RMS value therefore, root 2 multiplies it will be the peak value that is why multiplied by root 2 cosine omega T plus alpha this way you take. Therefore, the induced voltage in the telephone line is that V is equal to d d of d T d d T of lambda 12 T you take the derivative this thing with respect to time for the you take that is the voltage expression is equal to minus root 2 omega mod lambda 1

T sin 1 2 sin omega T plus alpha volt per kilometer right you take the derivative of this equation therefore, V is equal to you can write root 2 omega mod lambda 12 then this minus sign is there.

So, this one you can write instead of sin omega T plus alpha we can write cosine omega T plus alpha plus 90 degree volt per kilometer this you can write therefore, VRMS we can write that root 2 will not be there now right if you write VRMS then omega mod lambda 12 and angle is alpha plus 90 degree. So, angle alpha plus 90 degree volt per kilometer I hope you have understood this right therefore, actually whatever you are getting here this is basically your RMS value; RMS value right and then you therefore, instantaneous flux linkage the way you do for voltage or current RMS to translate into instantaneous current or voltage same thing right.

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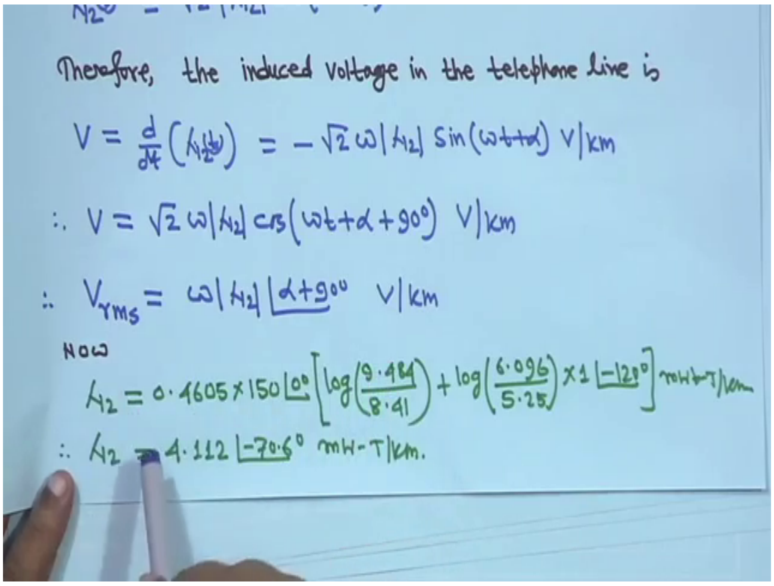


Therefore your VRMS is equal to omega mod lambda into alpha plus 90 degree volt per kilometer right now lambda 12 you substitute in this expression that D a 2 D a 1 and D b 2 D b 1 all you substitute if you do. So, then it is 0.4605 into I a magnitude is the current is given 150 ampere and we have taken I a is the base reference therefore, it is 150 angle 0 degree. This you write into log then all these distances are calculated D a 2 is 9.484 right D a 2 D a 1 8.41 plus your first I am writing this one this one that D b 2 6.096 and D b 1 5.25 into 1 angle minus 120 degree Milli Weber ton per kilometer right.

Then if you simplify this one if you simplify this one; it will be lambda 12 is equal to 4.112 angle minus 70.6 degree Milli Weber ton per kilometer right. So, this is the RMS value this magnitude and this is the angle therefore, so, initially we assume that cos omega T plus alpha lambda 12 this is that initial value we assume no omega T plus alpha.

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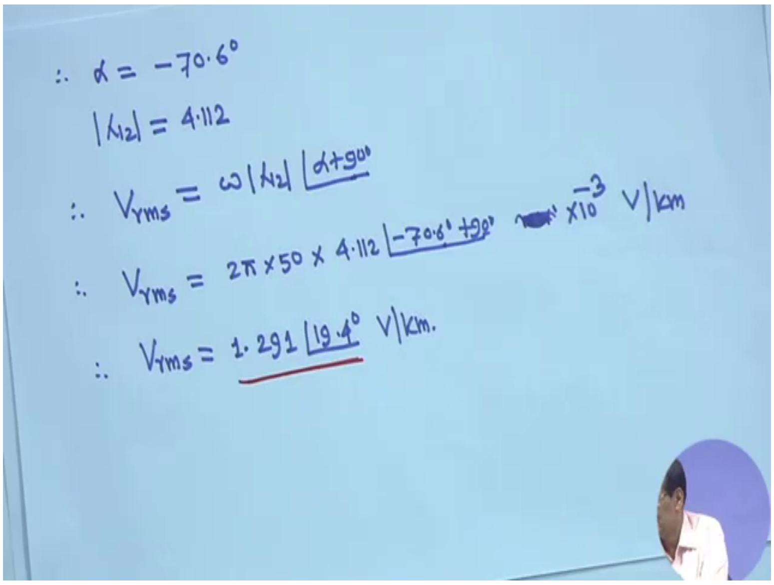


That means your alpha actually is equal to minus 70.6 degree right and mod of lambda 12 magnitude that RMS value 4.112 it is Milli Weber tons per kilometer, but not writing here that understandable right therefore, VRMS will be omega mod lambda 12 angle alpha plus 90 degree because this one your this expression of RMS voltage you got omega mod lambda 2 angle alpha plus 90 degree volt per kilometer right therefore, we it is a 50 hertz system.

So, omega is equal to 2 f 2 pi into 50 mod lambda into 4.112 then alpha is equal to minus seven 70.6 degree plus 90 degree Milli Weber, but this thing into your 10 to the power minus 3 because milli term has been made into 10 to minus 3 this should be volt per kilometer after then VRMS actually is equal to 1.291 angle 19.4 degree volt per kilometer right. So, this is your this is the this is your answer right before going to this right I mean inductance I got few more examples, but time will be limited. So, many examples later we will you know when you will learn this we will give many example to solve and hopefully many interesting problems we will give it to you right.

So, next we will move to that your transmission line capacitance right transmission line capacitance.

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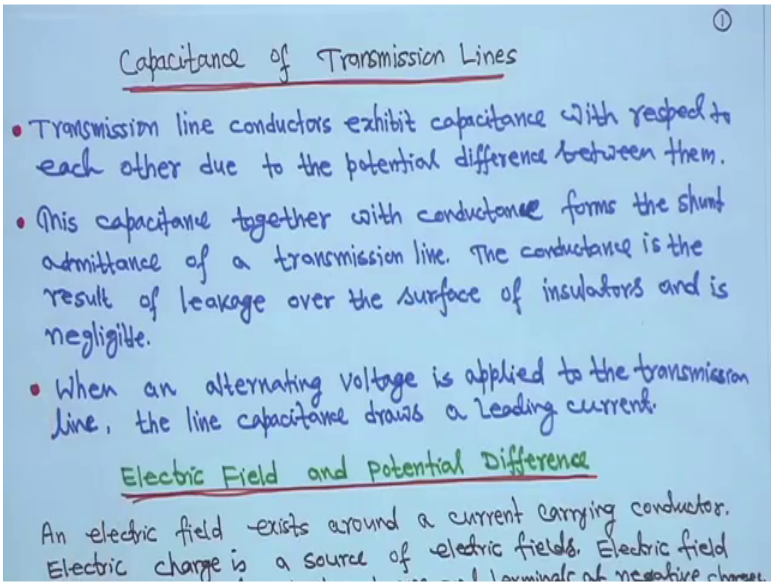


This capacitance of transmission lines as you know transmission line conductors exhibit capacitance right with respect to each other due to the potential difference between them right. So, this capacitance together with conductance your forms the shunt admittance of the transmission line, but as I told you earlier the conductance actually the result of the leakage over the surface of insulators right and is very much negligible. So, conductance we will not consider right and third thing is when an alternating voltage is applied to the transmission line the line capacitance draws a leading current you know that capacitance draws a leading current or in a power system later we will see in a power system that you will find that shunt capacitors are used right. So, basically it injects reactive power into the transmission into your transmission line right. So, later we will see that. So, generally what this thing or what you call our objective.

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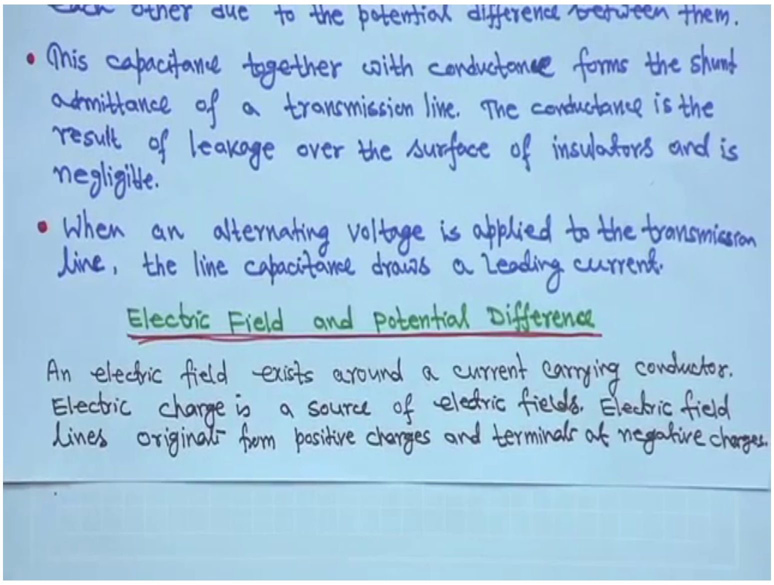


Now, to calculate the capacitance of transmission line for single phase line to line capacitance as well as line to neutral capacitance, so, an electric field actually exists around a current carrying conductor this you know right. So, electric charge actually is a source of your electric field right. So, electric field actually originate from the positive charges and terminate at the negative charges this also you have studied from your higher secondary physics right.

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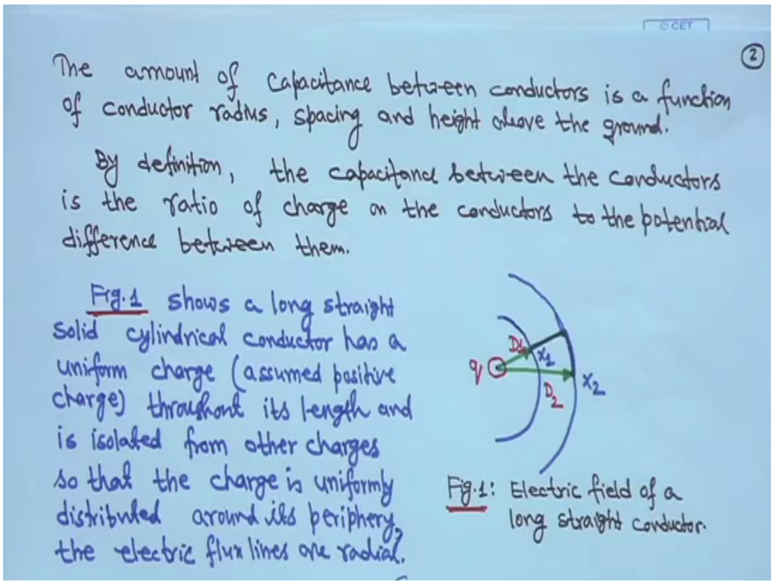

So, the amount of capacitance between conductor is a function of your conductor radius right is a function of conductor radius and your; what you call that space spacing and height above the ground. So, these are these are things, but later we will see that effect of the height on the capacitance right and later we will see.

So, basically 3 things conductor radius spacing and height above the ground; so, by definition the electro that by definition the capacitance between the conductors that you know is the ratio of charge on the conductors to the potential difference between them these are known to you for example, you consider this figure you consider this figure that it shows a long this is the this is your conductor shows a long straight conductor we assume it has uniform charge right. So, assume your positive charge you assume it is a uniform and positive charge right throughout its length and is isolated from all other charges. So, that charge is uniformly distributed; that means, I assume this conductor has positive charge uniformly distributed and near no other no others what you call near other conductors or not affected by any other conductor or any other electric field right.

So, it is uniformly distributed. So, around its periphery so, electric. So, in the electric flux lines are basically radial right. So, so this is the thing. So, we assume this is your what you call this is your conductor carrying current of course, and it has a charge positive charge q and uniformly distributed around say periphery and in this diagram there are 2 points 1 is X 1 another is X 2 from here the distance is D 1 and from here distance is D 2 right since the and it is equi-potential your surface since the what you call equi that is equi orthogonal to the electric flux lines the equi-potential surfaces are concentric cylinders surrounding the conductor. So, basically these are actually concentric cylinders right their radius is say their center is same. So, from the gauss theorem we know that the electric field intensity at distance y right from axis of the conductor any distance here it is not shown, but any distance y you take from the Gaussian theorem you can write that E y is equal to q upon 2 pi epsilon 0 y volt per meter this is equation 1. So, this is a new topic.

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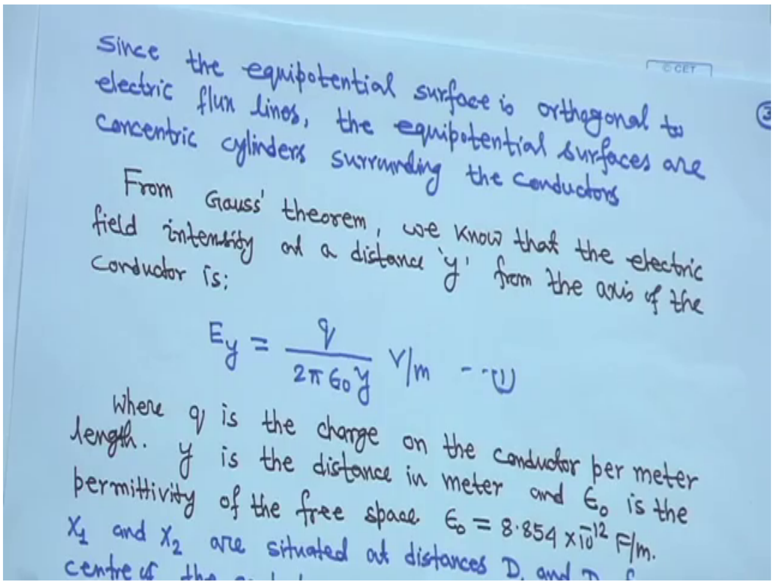


So, again equation number is starting from one right where q is the charge on the conductor per meter length y is the distance in meter.

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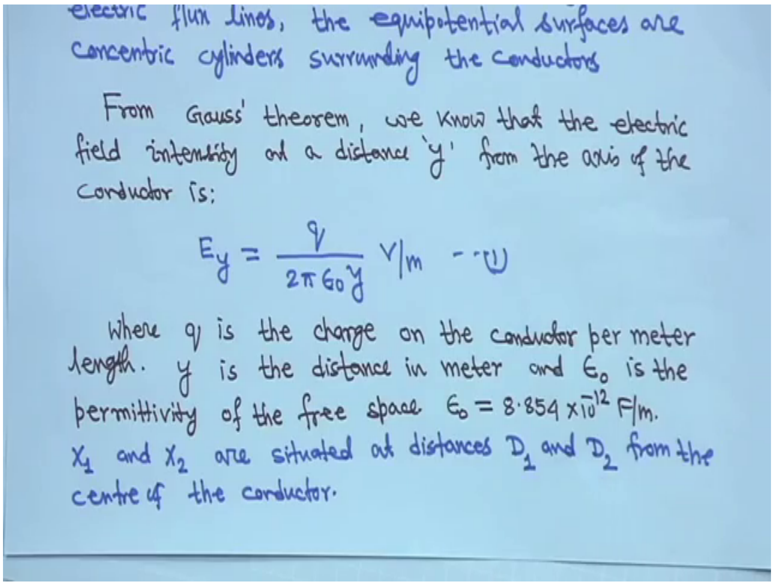


And the epsilon 0 is the permittivity of the free space you know epsilon 0 is equal to 8.854 into 10 to power minus 12 farads per meter and X 1 X 2 are situated at a distances D 1 and D 2 from the center of the conductor this from the center of the conductor X 2 and X 2 this is the distance D 1 and this is the distance D 2 right.

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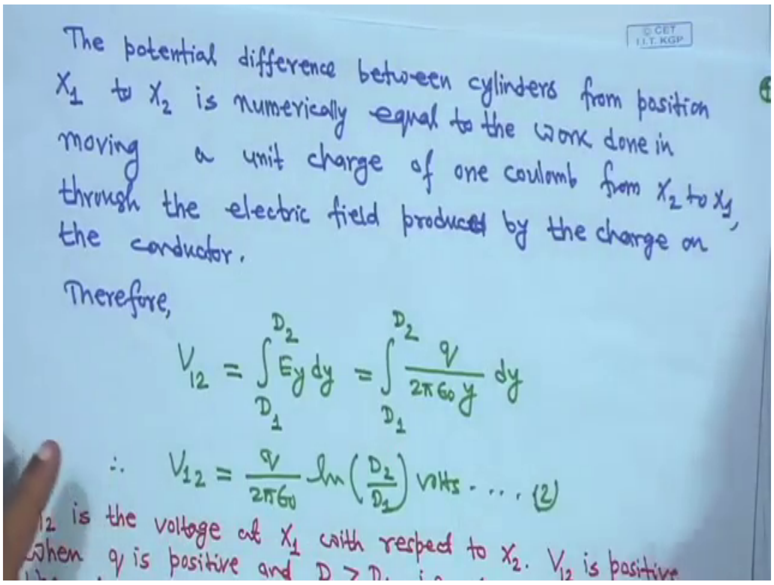


So, this diagram keeping it here right therefore, the potential difference between cylinders from position X 2 to X 2 from position X 2 to X 2 right it goes like this coming like this the potential difference between cylinders from position X 2 and X 2 is numerically equal to the work done in moving a unit charge of one coulomb from X 2 to X 2 right there through the electric field produced by the charge on the conductor; that means, here you have one coulomb of your unit charge of one coulomb.

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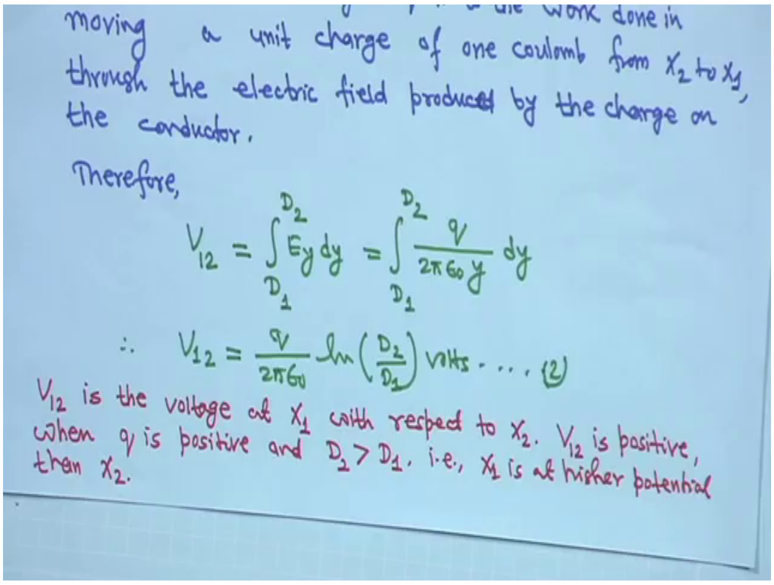

And you are bringing it that is equivalent work done right or numerically equal to work done that you are bringing from X 2 to X 2 right.

So, that is why I have writ10 for you the potential difference between cylinders from the position X 2 to X 2 is numerically equivalent to the work done in moving unit charge of one coulomb from X 2 to X 2 right therefore, this V 1 2 that you are that voltage V 1 2 from integration of it will be D 1 to D 2 because this is from D 1 to D 2 right when we are making 1 to 2 means that it is at that point you are what you are at point X 2 right that is 1 to 2 that V 1 2 is D 1 to D 2 E y d y is equal to D 1 to u y is equal to 2 pi epsilon 0 y into d y; that means, V 1 2 is equal to q upon 2 pi epsilon 0 natural log D 2 upon D 1 volt this is equation 2. So, V 1 2 actually is the voltage at X 2 with respect to X 2 V 1 2 is positive when q is positive right and D 2 greater than D 1 because if D 2 greater than D 1 this will be positive and q has to be positive that is X 2 is at higher potential than X 2 if V 1 2 is positive then your X 2 2 that is your what you call X 2 will be at higher potential than X 2 if it is negative then X 2 will be the higher potential than X 2 right.

So, that is that is your potential or what you call that your voltage because this voltage is necessary because we have to find out the capacitance right.

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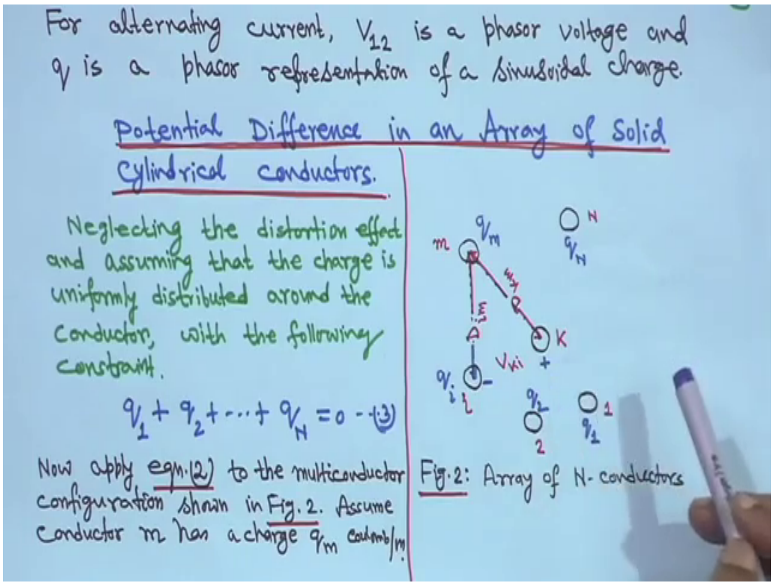


So; that means, for alternating current V 1 2 is a Phasor voltage and q is a Phasor representation of a sinusoidal charge now potential difference in an array of solid cylindrical conductors. So, suppose this is that figure 2 right suppose that neglecting the neglecting the distortion effect and assuming that charge is uniformly distributed around the conductor with the following constants that q 1 to q 2 up to q n is 0 that is equation 3; that means, you have your q 1 q 2 right then q k q I q m q n you many conductors up to n number of conductors right and distance is taken for example, m to k d k m just opposite it is if I move it like this; this distance is actually d k m to k to m and here also I to m distance is D i m right. So, this is the array of m conductors.

Now if you apply equation 2 to the multi-conductor configuration as shown in figure 2 this is figure 2 and this is your this is the your equation 2 right, but now you apply for that is for single conductor, but now you apply this one for multi-conductor right assume conductor m has a charge q m this is the conductor m has charge q m right this is m charge is q m right.

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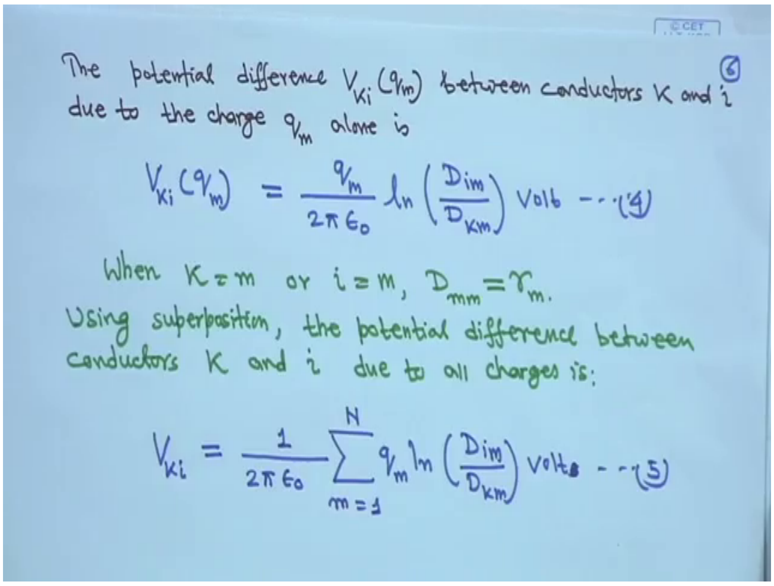


Coulomb per meter therefore, therefore, the potential difference V k I right due to q m that is V k I this is k this is I the potential difference is here I have marked some plus minus right V k I due to the charge your q m right that equation will be that between due to the charge q m alone right.

So, that is why I am writing V k I in bracket q m is equal to q m upon 2 pi epsilon 0 l n D i m upon d k m volt this is equation 4 right. So, this is that this equation we are writing from equation 2 now when k is equal to m if k is equal to m or I is equal to m then d m m will become r m right when either k is equal to m or I is equal to m at that time d m m will be the r m that is radius of the m th conductor right now if you use now there are n number of conductors. So, you have to apply superposition. So, using superposition the potential difference between conductors k and I due to all charges right due to all charges will be V k I is equal to 1 upon 2 pi epsilon 0.

ow all these things putting in sigma that m is equal to 1 to m you have m number of conductors. So, m is equal to 1 to m q m l n D i m upon D k m volt right. So, this is equation 5. So, things are simple this is the most general one right and this one this k to y when you do V k to y q upon 2 pi epsilon 0 right and some plus minus polarity I have shown right it will natural log D i m that is D i m by D k m right this much of volt. So, this formula right is your what you call is generalized here for n number of charges right.

Thank you.