Power System Analysis
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Lecture - 10
Resistance & Inductance (Contd.)

Now, come to example 3.

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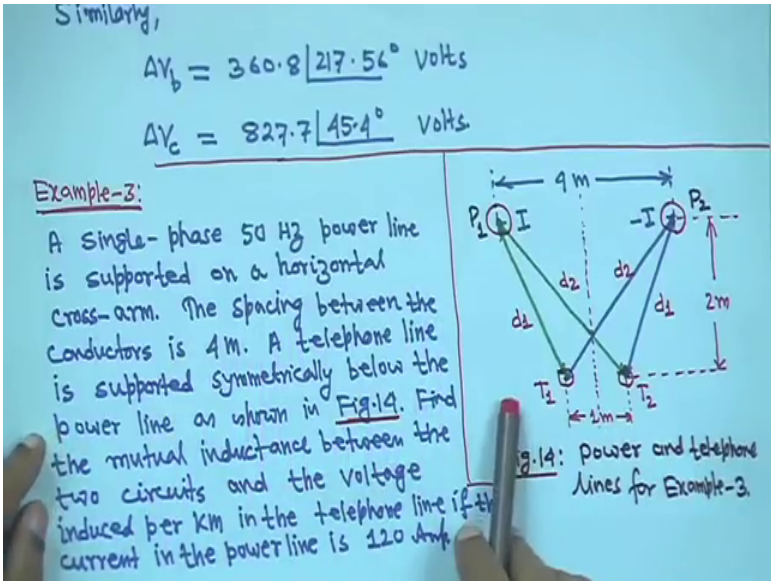

So, one thing that, you have seen just 2 examples and from that you assume that how much time it takes to explain that each and everything. But try to understand hopefully when I will go on giving the numericals 1 or 2 things I will leave up to that explanation, right. When we will this thing you only 1 or 2 cases I will put a question to you that why it is coming, because if everything I tell then you have to then you will not use in many thing.

So, 1 or 2 things I will skip; for example: this example one particular thing I will just leave it to you that you find out that what is the reason.

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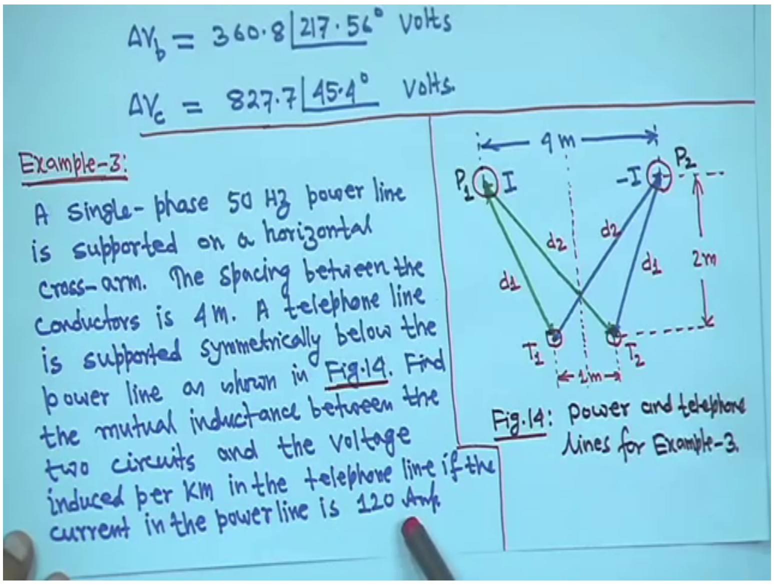

I give everything in details and I will give you some hint, but you find out what will the reason. For example, take this example that first time this thing then I will explain this right. So, a single phase 50 hertz power line is supported on a horizontal cross-arm the spacing between the conductor is 4 metre. A telephone line is supported symmetrically below the power line as shown in figure 14, this is figure 14. I will come to that, find the mutual inductance between the 2 circuits and the voltage induced per kilometre in the telephone line, if the current in the power line is 120 ampere.

Now it is basically as assume it is a single phase line. So, this is 2 conductor, this is incoming this is what you call outgoing. That is why current here is I here it is minus I right; that means, one is going into the page, another is leaving the page this is the convention.

So, between these 2 conductors P 1 and P 2 power line that distance is 4 meter, right. And this 2 are nearby telephone lines right. So, from this conductor the distance is d 2 this is d 1 from this is symmetrical this is symmetrical. So, here also this is d 2 this is d 2 they are same, similarly this d 1 this d 1 they are same. And from here to here vertical distance is 2 metre. But now we have to find out d 1 and d 2 first we find out, so power and telephone lines for the example. And you have to find out voltage induced in the telephone lines.

So, first thing is that d 1, you have to find out that what is your distance d 1 right. So, this is 2 meter your what you call this is, this is 2 this is 2 meter, right. And this one, your it has gone this is this is your, this height is this vertical line you will drawn this is 2 meter, right. At 2 meter and from and this one you are from I mean, I just hold on hold on I have to I have to make it for you.

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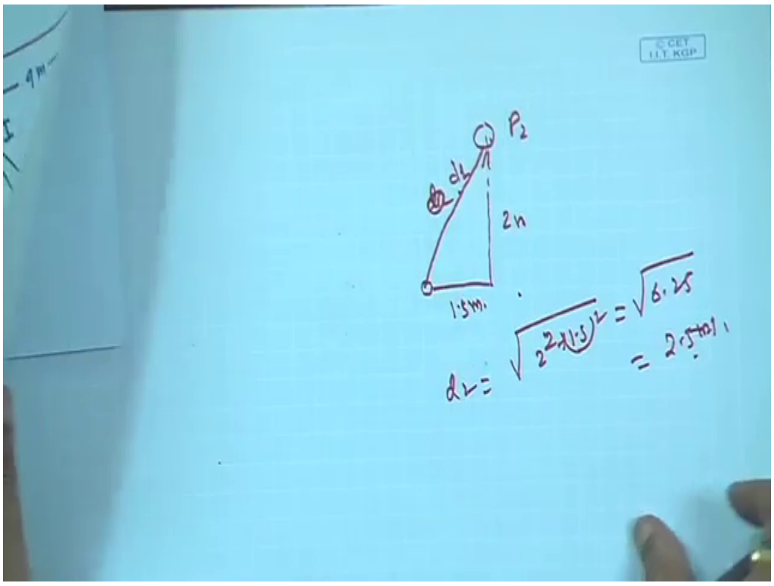

So, this is the conductor P 2 say this is conductor P 2, right. Draw a vertical line this distance is 2 meter, this distance is 2 meter and from here it telephone line is here, telephone line is here right.

So, this is the thing. So, in this case this is your this distance if I make it here only, here only, right; if I make it here only. So, this is this between this 2 telephone line it is one meter from the symmetry from here to here it is 1.5 meter, right. Sorry 0.5 meter, right. And here to here it is 2 meter; that means, these distance is 1.5 meter right; that means, this distance from here to here to here it is 1.5 meter, right. And this is 2 meter, and this is your d 2.

Therefore d 2 is equal to root over 2 square plus 1.5 square is equal to root over your 6.25 is equal to your 2.5 meter, right. That is not d 2, d 1 this is d 1, not d 2. This is d 1. This distance is actually d 1, not d 2 d 1 right. So, this d 1 is equal to 2.5 meter similarly, when you calculate d 2, d 2, right. In this case this is your, this is your what you call, this is your 2 meter 2 meter as usual, right. And this from here to here you have to find out that it is 1.5 and this is your one 2.5.

So, this portion is this your here to here if you take this to here it is 2 and from here to here it is 0.5. So, from this distance is 2.5 I mean if you take like this hold on right.

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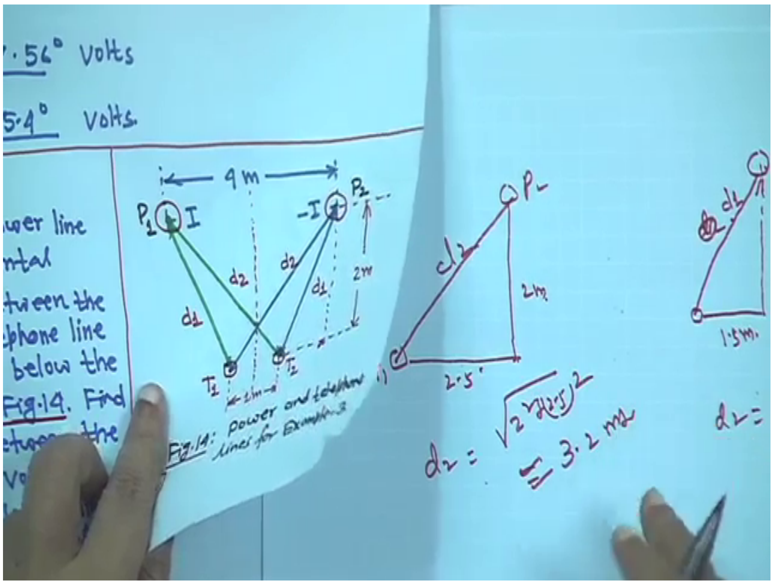

This is your P 2 and this is your 2 meter and this is your telephone line one this is T 1, right. This is T 1. So, this distance is 2.5 your and this is 2 and this is your d 2. Why it is why this 2.5 from here? From here to here it is 4. So, here to here it is 2 and between this telephone line is 1. So, from here to here is 0.5; so 2.5.

So, d 2 is equal to actually 2 square plus 2.5 square that is approximately 3.2 meter right. So, first you have to calculate these distances, one these distances are computed these distances are computed.

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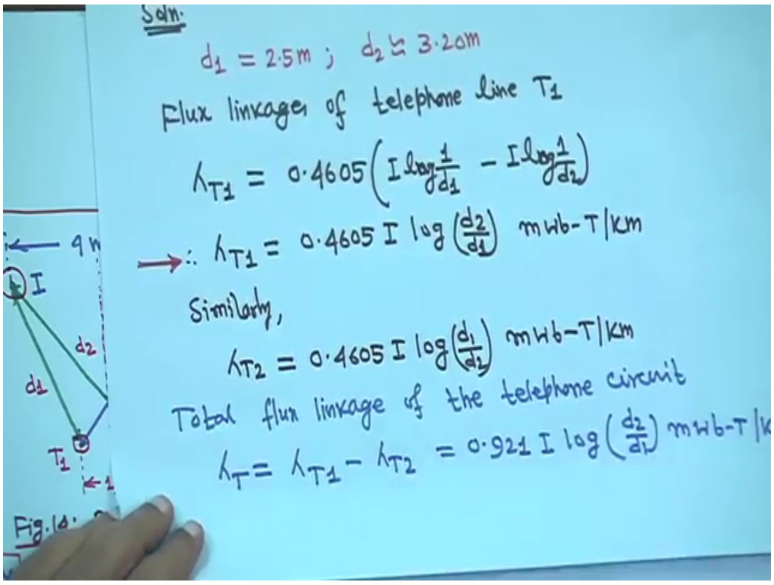

Then you have to find out that flux linkage you are of a telephone line T 1 right. So, next is that your flux linkage of telephone T 1.

So, lambda T 1 I write 0.4605 I log 1 upon d 1 minus I log 1 upon d 2, right. This is the flux linkage of telephone line T 1. So, both you have to consider this current here it is I here it is minus I. So, lambda 2 one will be coming this much of milli waver tons per kilometre. Similarly, for lambda 2 you find out it will become 0.4605 I log D 1 upon d 2 milli waver tons per kilometre.

So, this is I am writing know you have understood one current is I another is minus I here also. Now total flux linkages of the telephone circuit when you are doing it we are writing lambda T equal to lambda T 1 minus lambda 2, this is leaving up to you that why minus you think this 2 equation lambda 2 and lambda 2, but you think total one in other way, right. Then we will get the answer.

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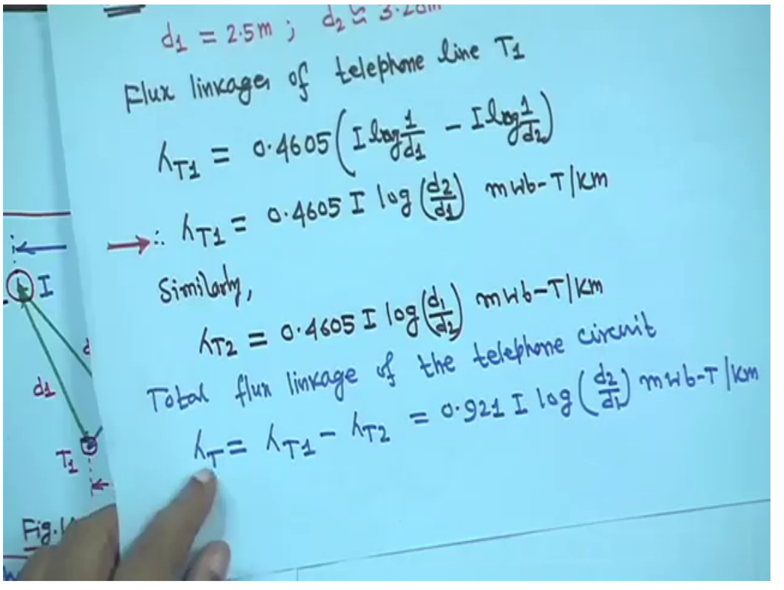

Now, lambda T is equal to lambda T 1 minus lambda 2 2 is equal to it will come 0.921 I log D 2 upon d 1 milli waver tons per kilometre. This is up to you, right. See little bit I will you leave it up to your imagination that why. That means, flux linkages we got now mutual inductance then mutual inductance m is equal to lambda T upon I, right.

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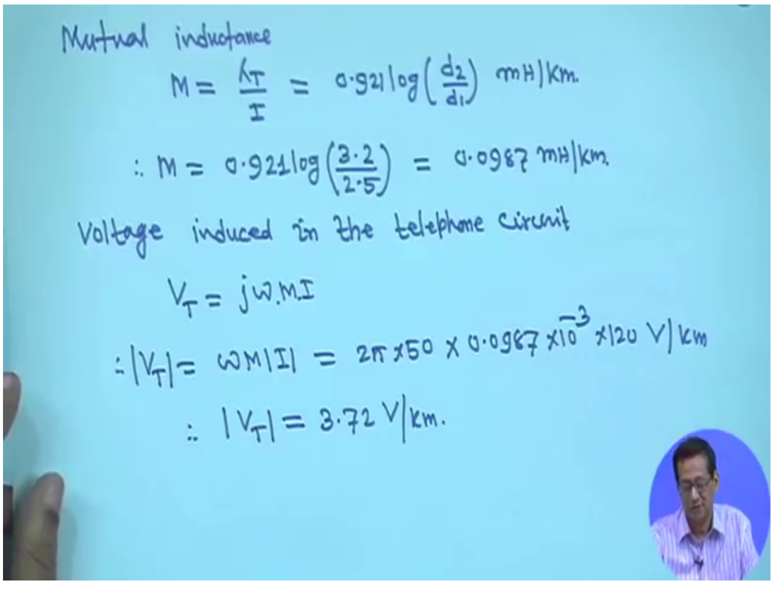

Is equal to 0.921 log D 2 upon d 1 milli Henry per kilometre.

Now, d 2 we got 3.2, d 1 2.5 substitute here you will get 0.0987 milli Henry per kilometre. So, voltage induced in the telephone circuit j omega m into I omega m L omega is the reactance m omega is that mutual your what you call, we define mutual 1. So, this is actually mutual reactance. So, V T is equal to j omega m into I, right. I is given I think 120 amperes I is given right. Therefore, magnitude we are putting only j remove magnitude omega m into I.

So, 2 pi 50 hertz system 50 into 0.0987 into it is milli is there. So, that is why into 10 to the power minus 3 into 120, because current magnitude is 120 ampere volt per kilometre. So, that voltage induced telephone line 3.72 volt per kilometre. This is the answer I hope this problem is you have understood.

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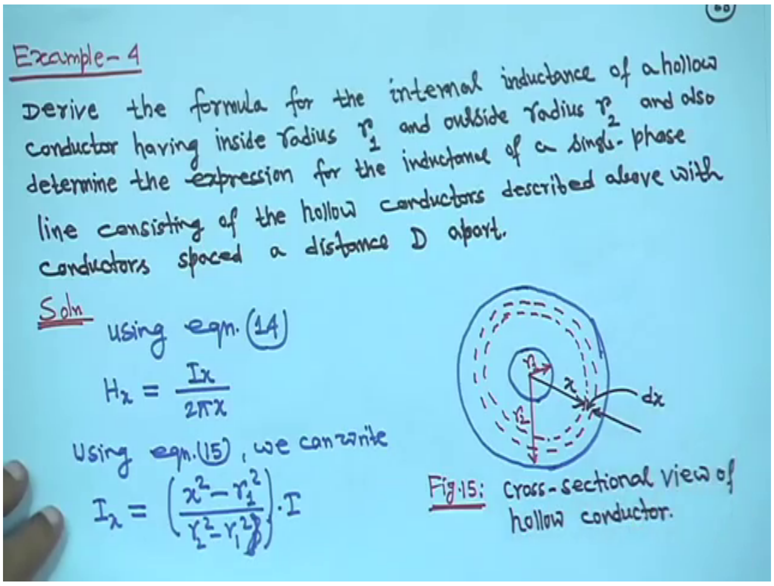

Next another one, this is also from your basic concept. This is something like it is numerical, but it is something like theory right. So, example 4: so all derivations we have seen, and all you have gone through also now look at this example, right. Derive the formula for the internal inductance of a hollow conductor having inside radius r 1 and outside radius r 2. And also determine the expression for the inductance of a single phase line consisting of the hollow conductors described above with conductor’s phased distance D apart.

So, first is the internal inductance of a hollow conductor. We have seen internal inductance of a this thing what you call that solid conductor half into 10 to the power minus 7 Henry per meter, but we have to find out the internal inductance of a hollow conductor right. So, this is the cross section view of hollow conductor. Inner radius is r 1 this is that inner radius and outer radius is r 2, at same as be polite distance x, right. We have taken a small reason d x and length is 1 meter. So, I told you that area will be d x into one square meter right.

Now, using equation 14, that coming from that amperes law H x at a distance such that your H x is equal to I x upon 2 pi x, right; that magnitude field intensity.

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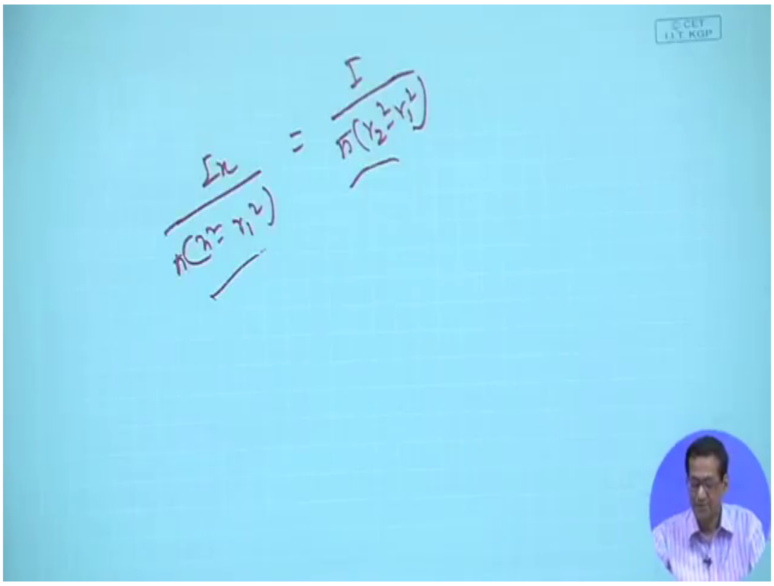

Now using equation 15, we have we are assuming the current density, current density is same; that means, at a distance x that your I x divided by x pi x square minus your r 1 square is equal to I divided by pi r 2 square minus r 1 square.

So, at a distance x area of this one is pi x square minus pi into x square minus r 1 square pi into x square minus r 1 square is equal to I that, solid portions area that I upon pi r 2 square minus r 1 square current density same. From that only that is equation 15, equation 15 I x we write x square minus r 1 square divided by r 2 square minus r 1 square into I.

Therefore this I x, this I x you substitute here, you substitute here, right. If you substitute then H x will be x square minus r 1 square divided by r 2 square minus r 1 square into 1 upon 2 pi x into I.

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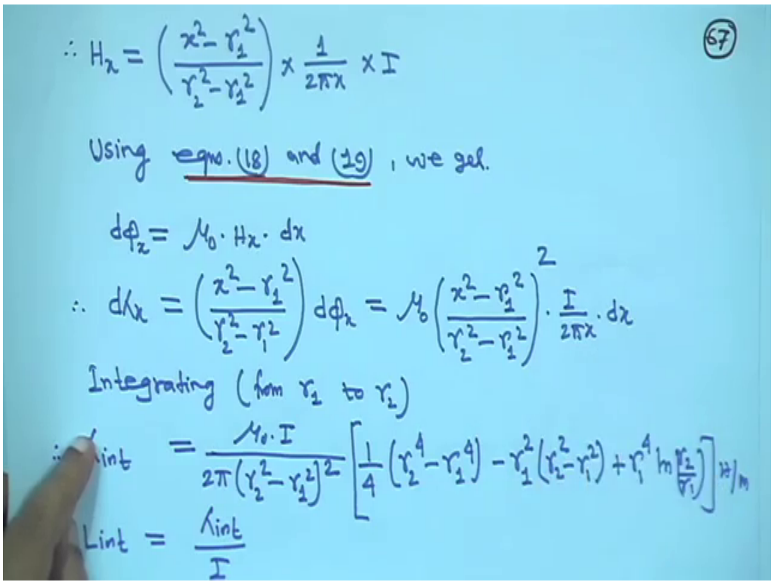

Now, not going back to the equation I have already you here for using equation 18 and 19 you will get that differential flux linkages that d phi x is equal to mu 0 into H x into d x; that means, here only; that means, in this reason only the differential flux your d phi x, right. Is equal to mu 0 into H x into your d x.

Actually it is I should have written d x into one that length is one we have taken, but all this things we have gone through, so no need; so H x into d x Multiplied by one not doing it. Therefore, d lambda x that d d lambda x is equal to that fractional your what you call that fractional tons ratio we have said no. So, d lambda x is equal to x square minus r 1 square divided by r 2 square minus r 1 square into d phi x.

So, earlier we have given the term that fractional tons ratio. So, here also flux linkages because it is internal. So, it will be this is the fractional term, right. This is the fractional term. Earlier we made x square your what you call, x square upon r square, but that is a hollow conductor. So, this term is coming into d phi x, right. Is equal to your, this is a understandable because for solid conductor we have made it at that time it was x square upon r square there was no r 1 r 2 only one radius r.

But in this case it is hollow conductor. So, this is the fractional term into d phi x. So, this is the flux linkages now this d phi x you this d phi x mu 0 H x d x you substitute here. H x you substitute, you substitute H x here, and d lambda x already d phi x is equal to substitute H x and finally, you put everything here you will get u 0 into x square minus r 1 square by r 2 square minus r 1 square whole square, into I upon 2 phi x into d x right.

So, integrate because it is hollow conductor. So, integrate from r 1 to r 2. If you integrate it is integration is your job, right. I am giving you the final expression. So, this integration you can do it a simply integration, right. You just break it in the square term and just divide this by x then integrate right. So, in this case you will get mu 0 into I upon 2 pi r 2 square minus r 1 square whole square, in bracket 1 upon 4 r 2 to the power 4 minus r 1 to the power 4 minus r 1 square into r 2 square minus r 1 square plus r 1 to the power 4 natural log L n r 2 upon r 1, bracket Henry per meter this is L internal.

Therefore sorry this is lambda internal right. So, this is actually not Henry per meter this is flux linkage waver tons right. So, in this case L internal is equal to lambda internal upon I right. So, divided by I lambda internal by I. So, this I should not be there. So, this is the expression of upon some simplification, right. The expression will be something like this L internal half into 10 to the power minus 7 into 1 upon r 2 square minus r 1 square whole square and bracket all this term; so Henry per meter.

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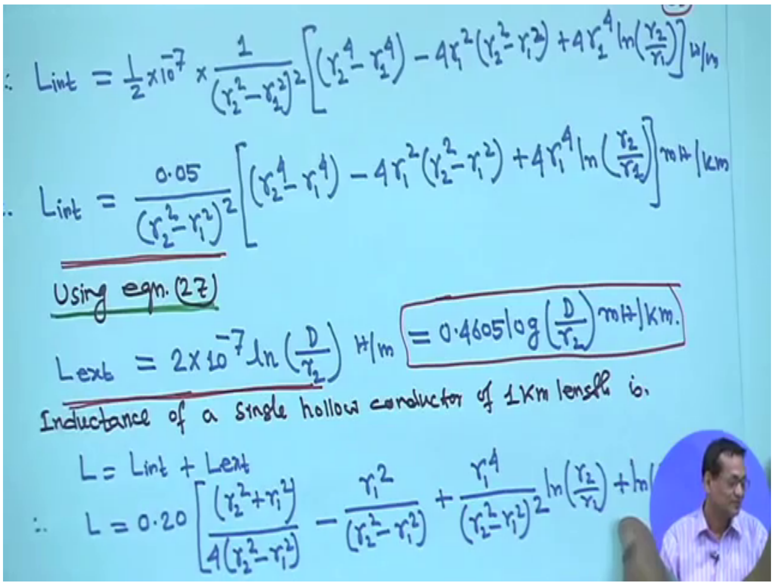

This can be further simplified right. So, it is 0.05 divided by r 2 square minus r 1 square whole square, and bracket all this terms will come in milli Henry per kilometre right.

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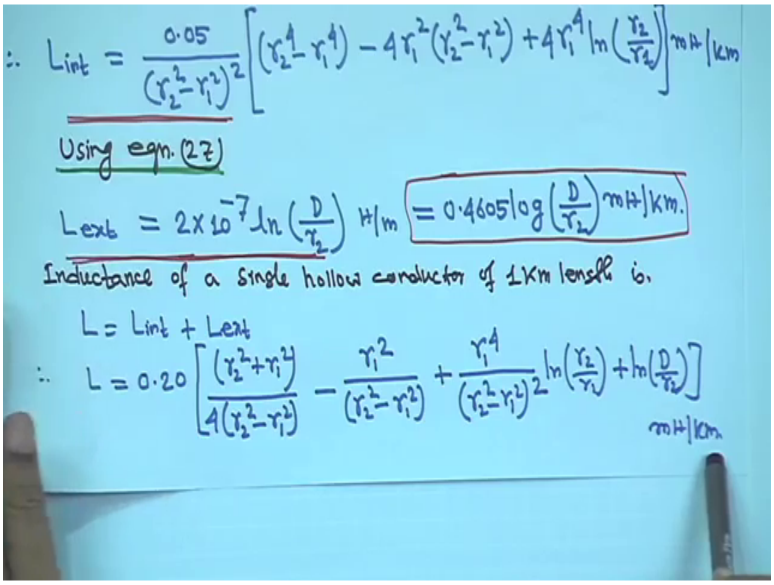

So, using equation 27, we know that L external that already we have derived that equation 27, if you go back 2 into 10 to the power minus 7 natural log L n D upon r 2 Henry per meter. And this one, this one although I have written is equal to 0.4605 log D upon r 2, as in this expression natural log is coming we did not converted to the log.

So, this portion, this portion will not use here, right. Only this natural log will use. So, this 2 if you add inductance of a single hollow conductor of 1 kilometre length- if you add L internal plus L external. So, you add broth and simplify you will get L is equal to point 2 and in bracket all this terms. R 2 square plus r 1 square by 4 into r 2 square minus r 1 square minus r 1 square upon r 2 square minus r 1 square plus r 1 to the power 4 r 2 square minus r 1 square whole square, L n natural log r 2 upon r 1 plus L n D upon r 2 milli Henry per kilometre.

So, this is the expression for internal inductance and this is the expression for the total right. So, this is for what you call internal total inductance of the hollow conductor. So, when conductor is hollow: so naturally the expressions also totally different.

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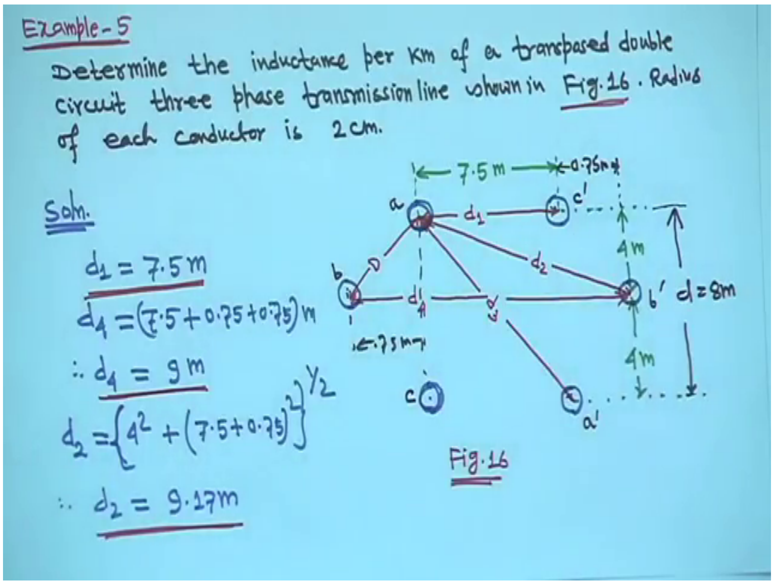

Next, this is another example right. So, here I have to give. So, I have to take many example for you otherwise problem will arise, right. Or I mean varieties of problem for that whatever possibility problems have come to my mind I have made it right. So, determine the inductance per kilometre of a transposed double circuit's 3 phase transmission line shown in figure 16, right. Radius of conductor is 2 centimetre.

So, this is actually a transpose this is the configuration given a, b, c then you are a dash b dash and c dash, right. Between a and c dash and a dash c distance is given 7.5 meter, right. And this vertical height is given 4, meter here also it is 4 meter and total is 4 plus 4. So, 8 meter. So, all other distances like your d 1, d 2, d 3, d 4 or you have to compute right. So, d 1 is equal to 7.5 already given, right. Then d 4 d 4 is equal to it will be 9 meter, just I am telling you from here this is 7.5 this side is your what you call this side is this portion is 0.75 and this side is also 0.75.

So, d 4 is equal to 7.5 plus 0.75 plus 0.75 is equal to 9 meter, right. Similarly d 2, d 2 is this one. This is 7.5 and plus 0.75 and this is 4. So, d 2 is equal to that square root this half square root that is 4 square; that means, this 4 square plus this is 7.5 plus 0.75 whole square to the power half. So, d 2 is 9.17 meter. Because first you have to find out all the distances, right. Then you have to then you have to find out another distance the D this symmetrical. So, a b or c dash b or b c or a dash b dash this all distances are same symmetrical. So, you have to find out D. So, d d also this is your, this is your if you make it like this height is 4 this is 4 means this is 4.

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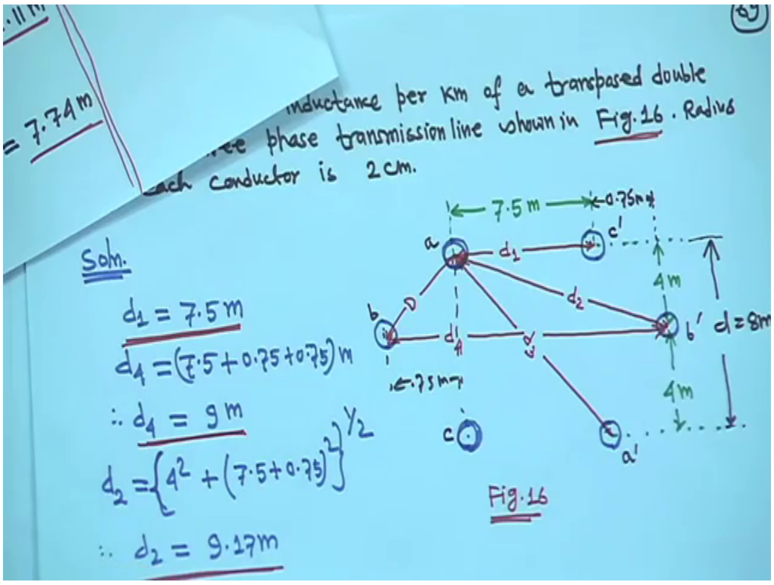

And this one this is your this height is 4 this side is 0.75, that is this is also 0.75 meter.

So that means, D will be, D will be d will be is equal to your 4 square plus 0.75 square. That is why D is equal to 4 square plus 0.75 square to the power half that is square root is equal to 4.07 meter, right. Similarly d 3, d 3 means this one, d 3 means this one.

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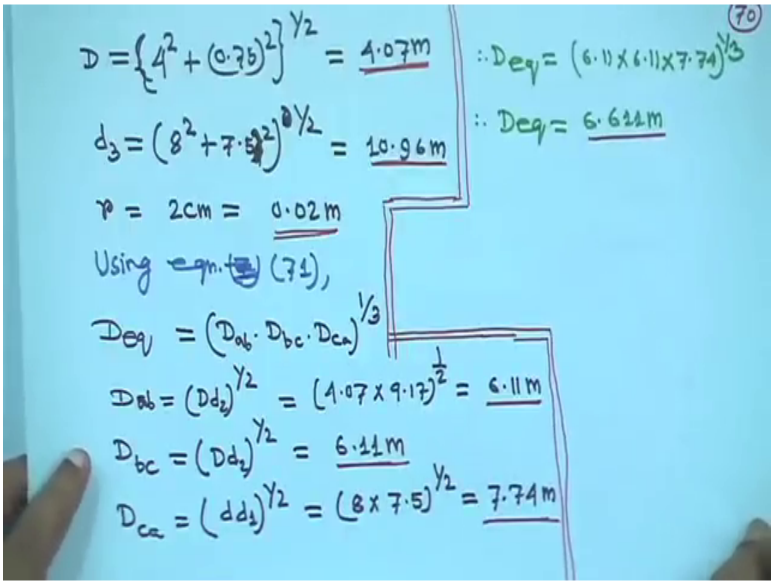

So, here it is 7.5 and from here to here it is 8 right; that means, d 3 is equal to 8 square plus 7.5 square to the power half. So, 10.96 meter.

R is given 2 centimetre that is 0.02 meter. So, first you calculate all the distances, after that equation 71 first D q; so D a b, D b c, D c a to the power one third, this symmetric symmetrical thing. So, D a b; that means, between phase a and b just previously also we have seen expression of D a b, D c c a same way. So, it is between D a b if you take D a b and your this thing what you call, that that what will that your D a b part.

So, it will be d into D d 2 to the power 4, 4 things will come D d 2 D d 2 into D d 2 to the power 1 by 4, but ultimately it will be D into D 2 1 upon 2, right. Because it is b 2 in the phase a and b all 4 possibilities will be there a 2 b, then, what you call your a 2 dash, right. A to b a to b dash similarly your what you call a a dash b dash.

So, D a b and a dash b dash they are same, right. And similarly d b dash and a dash b dash they are same 4 times it will come to the power one by 4 identical. So, that is why D d 2 to the power half 4.07 into 9.17 to the power half 6.11 meter similarly from the symmetry a b similarly b to c they are also same from symmetry; so d b c also D d 2 to the power 1 upon half 6.11 meter.

Now, D c a only again it will come D d 1 into D d 1 to the power 1 by 4, but ultimately it will be D d 1 d into D 1 to the power half, because D will be repeated twice D 1 also will be repeated twice, that is why directly we can D d 1 to the power half. You can see yourself you can see yourself. So, many things we have seen right. So, it will be 8.8 into 7.5 to the power half. So, 7.74 meter.

So, D a b, b c c a you got now Deq is equal to D a b into D b c, D c a to the power one third. So, substitute all this values a b b c c a. You will get Deq is equal to 6.611 meter, right. That is Deq, next is your equation.

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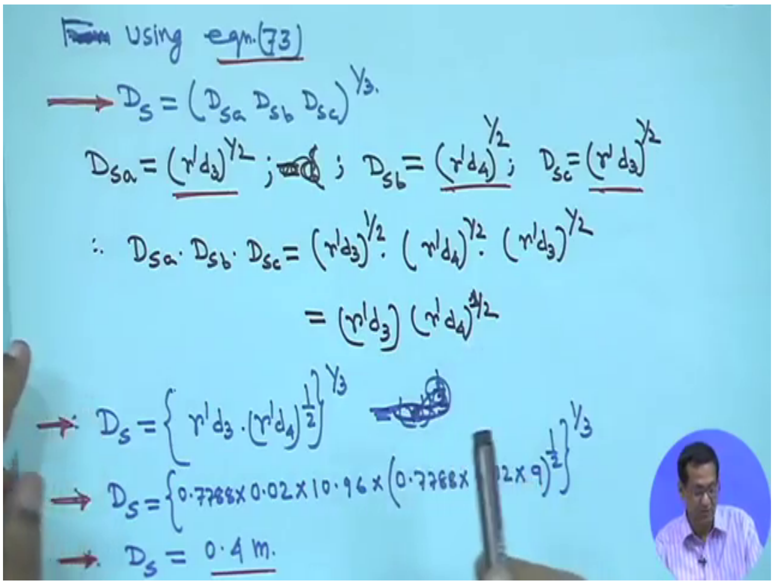

Using equation 73, right; the D s is equal to D s a, D s b, D s c. Now find out that yourself, gmd for phase a phase b phase c. So, D s a now easily you can calculate r dash d 3 it is it is phase a. So, a a dash no, so it is d 3 it is there right. So, r dash d 3 to the power half.

So, similarly for D s b also that phase b right. So, it will be your r dash d 4 because b b dash it is b 2 b dash distance is d 4. So, it is r dash d 4 similarly D s c will be r dash d 3 to the power half; so c r dash c 2 c dash right. So, same distance a to a dash c to c dash symmetry same distance d 3 means c to c dash here also d 3. So, it is r dash d 3 to the power half right.

So, D so, multiply this D s a into D s b into D c D s c. If you multiply all it is coming r dash d 3 into r dash d 4 to the power half after multiply and simplify you will get this one.

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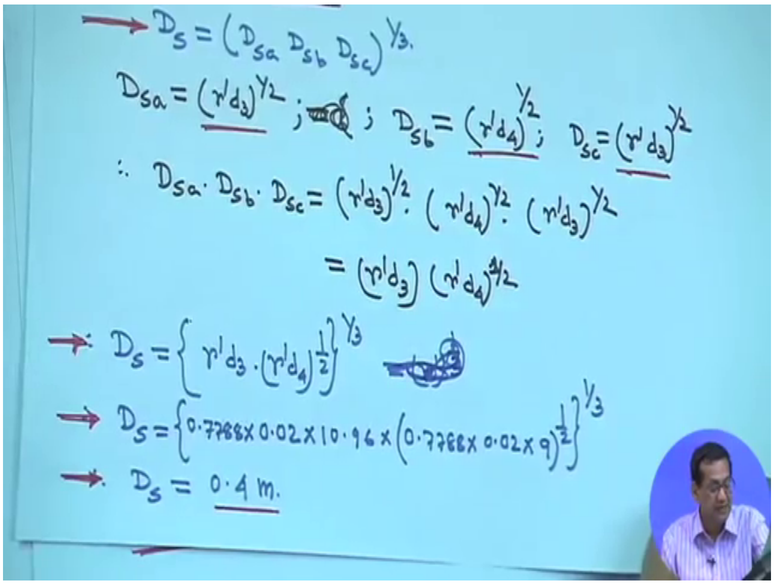

Therefore d s is equal to the D s a, D s b, D s c to the power one third that is this one is this one. So, r dash d 3 into r dash d 4 to the power half whole to the power one third.

Substitute all this values r dash is 0.7788 r r is 0.02 similarly, here also and simplify you will get, D s is equal to 0.4 meter right. So, Deq obtain D s also you have obtained. Next is that formula directly we use the inductance for phase L is equal to 0.4605 log Deq upon D s milli Henry per kilometre Deq 6.611 and D s point 4.

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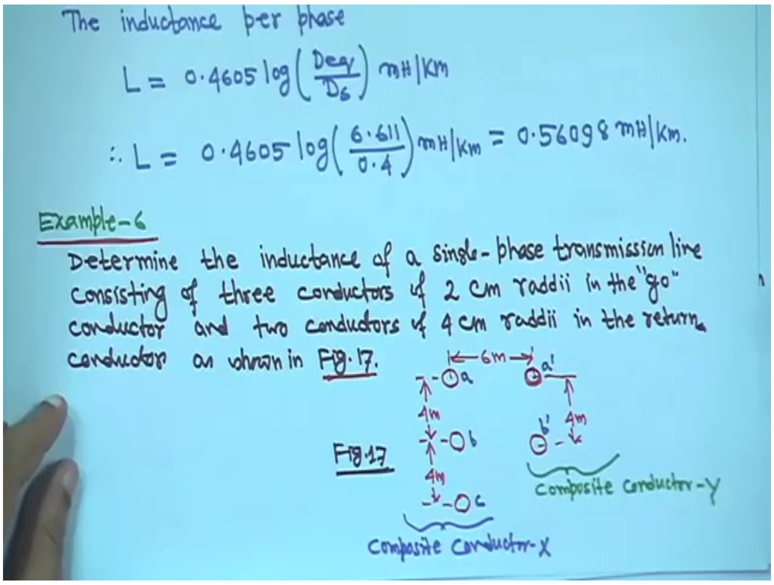

So, 0.6098 milli Henry per kilometre: so this is the answer. Next example we will take for composite conductors we have seen know inductance of derivations we have seen right, but here also we have to obtain this. So, I have taken little bit smaller small example, if I take bigger one then it will take more computation. So, determine the inductance of a single phase transmission line, consisting of 3 conductors of 2 centimetre radii in the go conductor go means, you assume current is going inside into the page, right. And 2 conductors of 4 centimetre radii in the return conductor as shown in figure 7.

So, earlier for the mathematical derivations, we have taken group of conductors x and group of conductor y, we have taken in group n number of conductor general formula and group y we took m number of conductors, right. And total mutual possibilities will be n into m and for each group it will be for the group x n number of conductors. So, n into n that is n square possibilities and for the group y m number of conductors within that yourself one m into m square possibilities, but mutual will be n into m right.

So, this is composite conductor x, right. Here a b c, 3 conductors are there. So, here n is equal to 3, right. And here 2 conductors are there. So, m is equal to 2 distances are given a a dash 6 meter, 6 meter a to b 4 meter, b to c 4 meter and you have to find out that your what you call inductance right. So, before going to that here I have written group x n is equal to 3 group y m is equal to 2 and mutual possibilities m into n totally 6 right.

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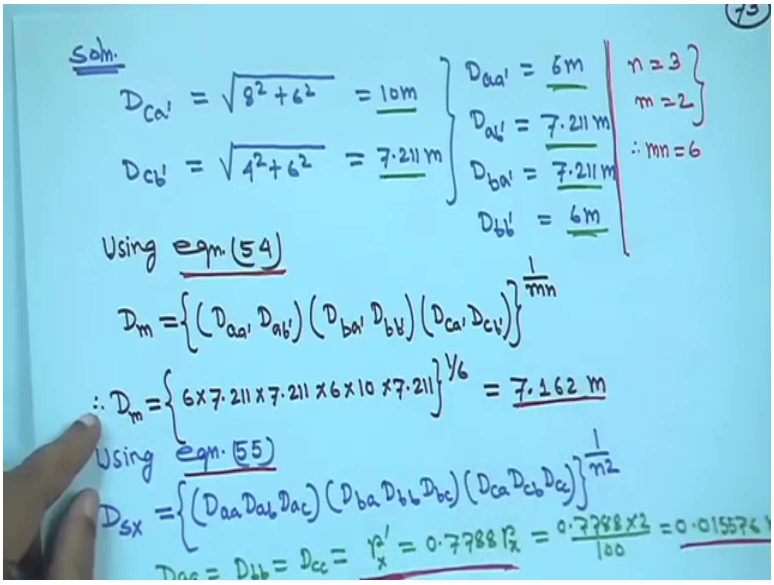

So, you calculate all the distances. D c a dash under root 8 square plus 6 square is equal to 10 meter. Then D c b dash under root 4 square plus 6 square is equal to 7.21 one meter, right. And all D a a dash D a b dash D b a dash D b b dash all are given, right. All are given, right and this is the, now using equation 54, first you find m nth root, right. Using so, in that case what will happen that you the D m the mutual one, right. Mutual one all possibilities D a dash D a a dash D a b dash.

So, D a D a a dash D a b dash next you will take D b a dash d b dash D b a dash D b b dash. Next we will take c D c b dash and D c b dash and D c a dash. So, all for mutual possibilities you take to the power 1 upon m n, right. Here m is equal to 2 n is equal to 3 right. So, here it is written m n is equal to 6, but directly I have not written here 6 I have just written m n, right. After that I am putting here m n is equal to 6.

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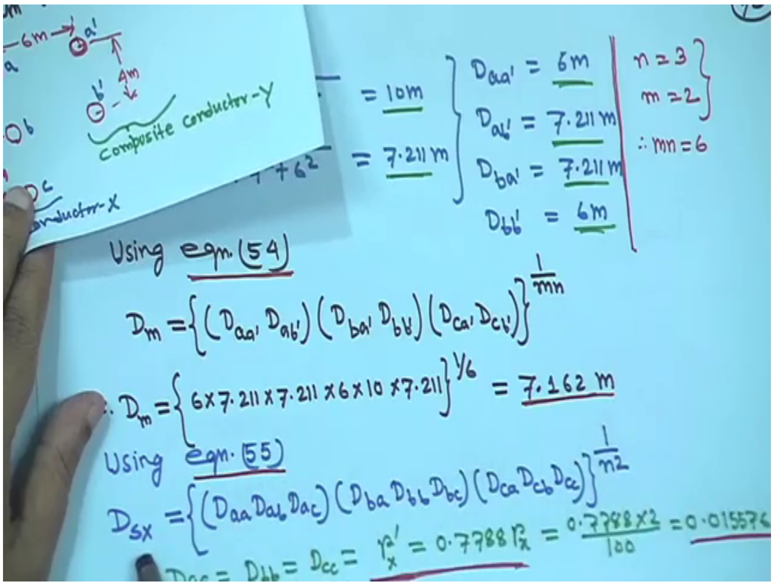

So, now getting this example how to compute D m, now will be clear to your, right. Now put all this values you will get 7.162 meter. Similarly using D s x group self gmd of group x we have to compute. So, using equation 55, look at that you will get D a d a that is basically it will be r dash later we will see D a a dash D a then D a b then D a c D a a D a b D a c. Then you take b D b a, D b b, D b a, D b b then d b c then you take c then D c a then D c b and then D c c. So, all these n square all 9 product, there are 9 product and n is 3. So, it will be 9.

So, D a a is equal to D b b is equal to D c c I am making r dash x we assuming that group x conductor radius is r x. So, r dash x is equal to 0.778, r x that r x is given 2 centimetre. So, it is basically 0.015576 meter; that r dash x, right. Once you made it all this all this calculations then, D s x you put you put all this distances you put all this distances all this multiply, you will get d s x is equal to 0.011576 cube 4 to the power 4 into 8 square to the power 1 upon 9 ninth root, right. If you see the pair the power you will get 2 plus 4, 6 plus 3, 9.

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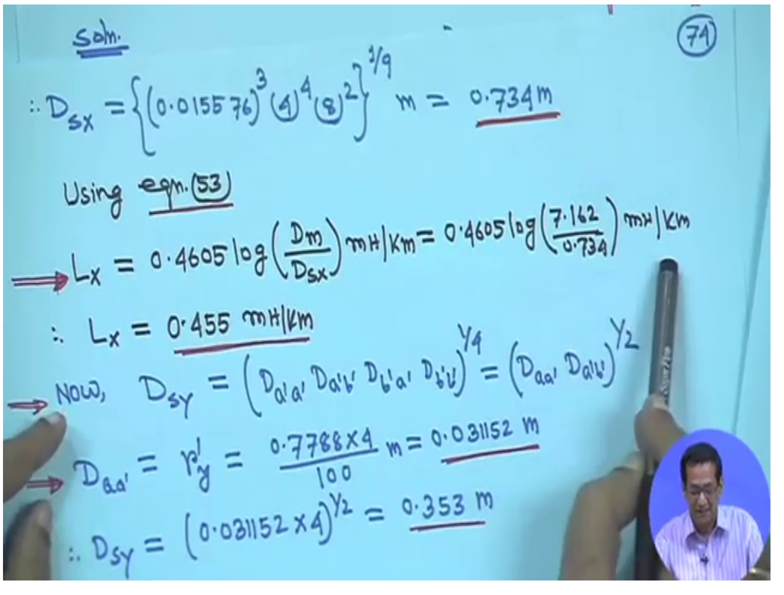

You have to see that it is matching with this, then dimensional it is correct, then the things are correct if you have not made the wrong calculation here, right. I mean distance calculation, but you have to check this right. So, it is 0.734 meter. So, using 53 equation 53 this is the expression from group x conductor L x is equal to 0.4605 log D m upon d s x milli Henry per kilometre. You substitute D m and d s x right. So, you will get L x is equal to 0.45 milli Henry per kilometre.

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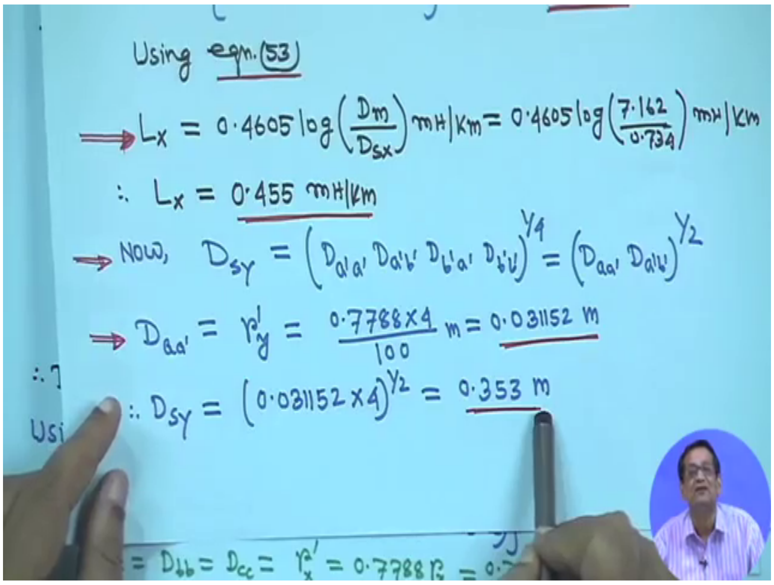

Now, for mutual case that D m will remain same when you will calculate L y. We mutual distance will D m will unchange only D s y will different here also, here you have 4 conductors sorry, 2 conductors in your this thing where it as gone just here, here you have here you have 2 conductors in composite conductor y 2 conductors right. So, in this case here it will come D s y D a a dash then D a dash b dash then D b dash a dash then D b dash b dash. So, D a dash a dash D a dash b dash D b dash a dash.

So, D a d a dash and D b b D a dash a dash and d b dash b dash, right. That is D a a dash and this is r y dash right. So, again radius of the group y conductor 4 centimetre. So, it is r dash. So, 0.7788 r divided by hundred you have to convert to meter. So 1.52 meter. So, D s y is equal to put all this values it will be because repeated values same value that is why 0.03152 into 4 to the power half because 4 will come twice this one also become twice that is why it is half.

So, 0.353 meter right. So, similarly; that means, next is your use this alloys sane formula D m will remain same.

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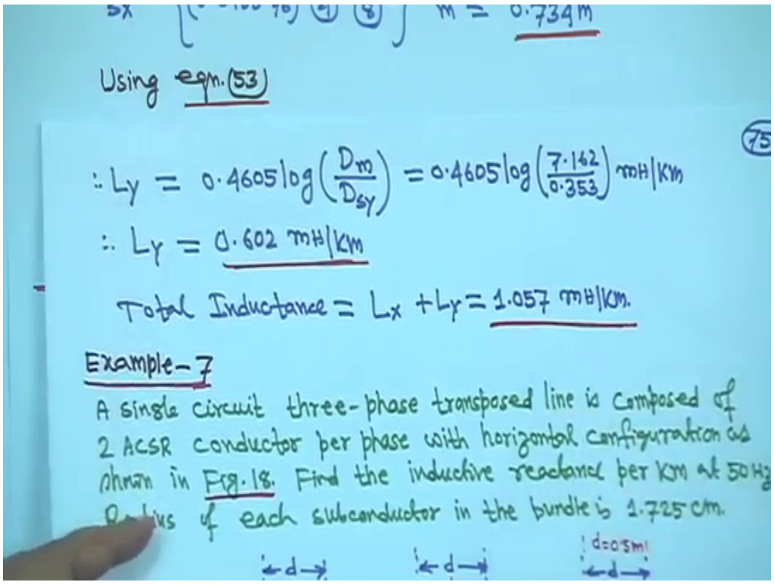

So, 0.4605 log D m upon D s y 0.4605 log 7.162 upon 0.353 milli Henry per kilometre, right. If you solve L y is equal to 0.602 milli Henry per kilometre. So, total inductance of this thing L x plus L y you had both L x and L y 1.057 milli Henry per kilometre from this one thing you have noticed, that for when number of you are what you call number of conductors are more in a group that inductance is less 0.455, because here 3 conductors are there and group y 2 conductors are there. So, if you put more conductors. So, inductance will become less hence the reactance.