

Design Principles of RF and Microwave Filters and Amplifiers
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Module No # 4
Lecture No # 16
Amplifier Design for Specified Noise Performance

Welcome to this sixteenth lecture of this lecture series today we have already seen the RF amplifier design for maximum gain and for specified gains. Now today we will see one amplifier where gain is not so important but noise performance is important you know that amplifier is called low noise amplifier it is a integral part of any RF receiver. You know in any RF receiver we have very small signal because in when we transmits the signal and receive the signal in wireless path due to large path losses.

We get very weak signal so there you know any amplifier in its input there are noises also the amplifier or any electronic device it produces its own noise. So the signal and noise both gets amplified by the any amplifier now if the its own noise is very high of any amplifier because of active devices there is chance of quite good amount of noise the amplifier itself adding up now.

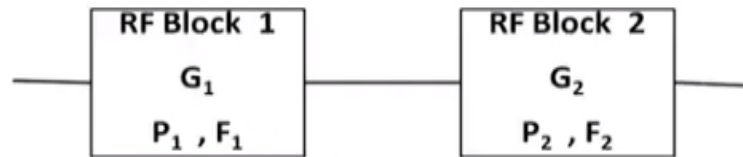
Now if that is not content then you may have the whole signal gets buried in noise. So it is very important to design for low noise amplifiers and this amplifier is generally the first block after the antenna, antenna receives the signal antenna as we already seen that antenna is basically a narrow band filter. So it gets one RF frequency to which it is supposed to be tuned because it is a narrow band device.

So that RF signal is first fit to an amplifier because you need to amplify the signal for further processing and giving to the other subsystems of (02:20) subsequent blocks of receiver. Now their the first one that if it is adding noise then the whole strategy of deduction may get disturbed that is why we take high precaution for this and this LNA's is are not supposed to have maximum gain but they are supposed to minimize the noise.

(Refer Slide Time: 03:01)



Low Noise Amplifier Design



$$F_{overall} = F_1 + \frac{F_2 - 1}{G_1}$$

- Low Noise crucial for LNA
- Moderate Gain required to suppress higher of subsequent blocks.

That is why the design is separately seen so let us see that in today's lecture, so you see suppose I have an RF block 1 this is an LNA after that there will be mixers, after that there will be amplifiers etc., then will be detectors. So there are various blocks so if there are various RF blocks let us say that we have the first RF block 1 with the gain either gain or attenuation both we are calling gain. So if it is attenuation it will be a gain.

So if it is attenuation it will be a less than 1 absolute value of dB scale it will be minus and let us say that it has the power is given by P_1 and the noise figure that is coming to be F_1 . All of you know noise figure it is basically the output noise power divided by the input noise power mapped to the output and we all know that if we do simple those mathematics which all of you done in B.Tech that it is basically the SNR at the input divided by SNR at the output.

The input is kept at a fixed temperature generally that is to 90 degree kelvin and there is no mismatch in between the source or the input and input side. So under that it is the SNR ratios so let us have that noise figure which is also a characterization of noise. Instead of noise temperature we can have noise figure anyone is will do but when we have amplifier blocks it is more convenient to work with noise figure that is why are saying that this block as a noise figure of F_1 .

Now it is connected to a another block suppose RF block 2 with the corresponding gain 2 the signal power P_2 and F_2 and we know this you all know from your basic B. Tech classes that over all this whole 2 blocks together the overall noise figure that is given by $F_1 + F_2 - 1$ by G_1 . You see that this F_2 etc that means the second blocks noise figure even if that is quite high that means you have good amount of noise.

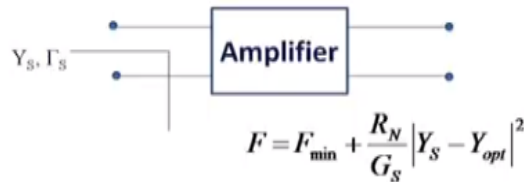
Still it is being divided by G_1 the gain of this amplifier or gain of this block so that becomes low but this F_1 it is not divided by anything. So whatever the noise figure of first block that will be always will be dominant factor in the overall noise figure whole visible system. So it is very important that this F_1 is quite kept low because this F_1 is high is no chance of making this overall noise figure low that is why the design for this if it is become an amplifier LNA then it is important because I will have to keep this F_1 low.

Low noise as I said crucial for LNA moderate gain required to suppress higher noise obviously if I want to have low noise I cannot get moderate gain because you see that. When we do that for conjugate matching etc the maximum gain that time we make the amplifiers operate with the full activeness and time we will have full noises. When we try to make low noise amplifier the gain may not be very high but moderate gain is required because some gain is required for any amplifier type of thing.

(Refer Slide Time: 07:21)



How Noise Figure is Related To Source Admittance ?



$Y_S = G_S + j B_S \rightarrow$ source admittance seen by RF Amplifier

$Y_{opt} \rightarrow$ Optimum source admittance that results in Minimum Noise Figure F_{\min}

$R_N \rightarrow$ equivalent noise resistance of Amplifier



And as of subsequent blocks so this gain is because it will suppress the subsequent blocks noise so some gain is required it may not be very high gain of this block it should be substantial . So this is the challenge so noise figure is related to source admittance you see that this has been shown by people that this as been shown by people that this noise figure of any amplifier that is related to his source admittance we are calling Y_S and you know with any impedances we have the corresponding reflection coefficient.

Similarly, with this source admittance means obviously we have a source impedance. So we have corresponding reflection coefficient of source and people have shown that this relationship that any F of this amplifier that is given by this $F_{\min} + R_N / G_S \cdot |Y_S - Y_{opt}|^2$. So Y_S we have already seen it is a source admittance.

So what are this F_{\min} R_N G_S and Y_{opt} so Y_S this Y_S is nothing but g source admittance seen by the RF amplifier Source admittance the real part is called the source conductance and this part is called source susceptance B_S that you all know. Y_{opt} is an optimum source admittance that results in the minimum noise figure for the amplifier F_{\min} . It is a specification given by the amplifier manufacturer that optimum source admittance if I make that then the noise will be minimum.

(Refer Slide Time: 09:18)



Noise Parameters of Active Devices

For transistors $\rightarrow F_{\min}$
 Γ_{opt} specified by
 R_N Manufacturer

- Sometimes measured
- Define Noise figure Parameter

$$N = \frac{F - F_{\min}}{4R_N} \left| 1 + \Gamma_{\text{opt}} \right|^2$$

- For specified F & noise parameters, N f



And R_N is the equivalent noise resistance of the amplifier as we are saying that this amplifier has a its own noise it adds that equivalent noise resistance. Any electronic circuit has a noise resistance that we are calling R_N . That also specified by manufacturer and so you see that R_N G_S F_{\min} and Γ_{opt} there are this thing G_S obviously the source admittance part so this is excitation part but this three parameters the F_{\min} then R_N and Γ_{opt} they are characteristic of the amplifier.

This are called the noise characteristic of the amplifier so for transistors F_{\min} Γ_{opt} + R_N they are specified by manufacturer and also you can sometimes measure it measurement procedure for measuring this So let us say that this is known so then we define a noise figure parameter out of this things and also the characteristic impedance. So let the noise figure parameter is called N , N is given by this formula $F - F_{\min}$ by $4R_N$ by Z_0 $1 + \Gamma_{\text{opt}}$ Square sorry there are two vertical buds actually there should be one only.

So also this is not T this is actually Γ_{opt} or Γ_{opt} of capital Γ for specified F and noise parameters N is fixed. You see that suppose we are asked to design LNA for a specified noise figure then once that is fixed and F_{\min} R_N and Γ_{opt} are the parameters of noise parameters of transistors and Z_0 it is the characteristic impedance of the character. So for the specified F and this three noise parameters we can see that N is fixed for particular characteristic impedance.

(Refer Slide Time: 11:15)

Relate Y to Γ

$$F = F_{\min} + \frac{R_N}{G_S} |Y_S - Y_{opt}|^2$$

$$Y_S = \frac{1}{Z_0} \frac{1 - \Gamma_S}{1 + \Gamma_S} \quad Y_{opt} = \frac{1}{Z_0} \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}$$

$$G_S = \text{Re}[Y_S] = \frac{1}{2Z_0} \left[\frac{1 - \Gamma_S}{1 + \Gamma_S} + \frac{1 - \Gamma_S^*}{1 + \Gamma_S^*} \right] = \frac{1}{2Z_0} \left[\frac{1 - |\Gamma_S|^2}{|1 + \Gamma_S|^2} \right]$$

$$F = F_{\min} + \frac{4R_N}{Z_0} \frac{|\Gamma_S - \Gamma_{opt}|^2}{(1 - |\Gamma_S|^2) |1 + \Gamma_{opt}|^2}$$

$$N = \frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2}$$

Now you see this formula that in terms of admittance we know how to convert the reflection coefficient we know Y_S given by this formula Y_{opt} is given by this formula and G_S is the real part of Y_S . So in terms of this from this Y_S I can write this so this is finally this. So with this we will have to change this Y_S and Y all this Y things to gamma things so that can be done.

So that conversion if you just do the manipulation this conversion is this you see this formula here I do not have any source admittance part but I have this in terms of reflection coefficient we have written and we know that so already we have found that value of N what is the value of N this is the value of N . So if we put that then we get that this equation N is this. Now for a specified noise figure N is fixed so now you see also gamma opt. Gamma option optimum that is also a parameter for transistor.

So you see this is a one equation in one unknown that is gamma S so we can then solve for Gamma S from this equation consideration N and gamma opt as given constant. So that will be doing next and that is the the manipulation always we do. So from that equation we can write like this then we can this is the next step of this you see and that means I have already got Gamma S gamma S star.

(Refer Slide Time: 12:58)

Manipulation for Γ_s

$$(\Gamma_s - \Gamma_{opt})(\Gamma_s^* - \Gamma_{opt}^*) = N(1 - |\Gamma_s|^2)$$

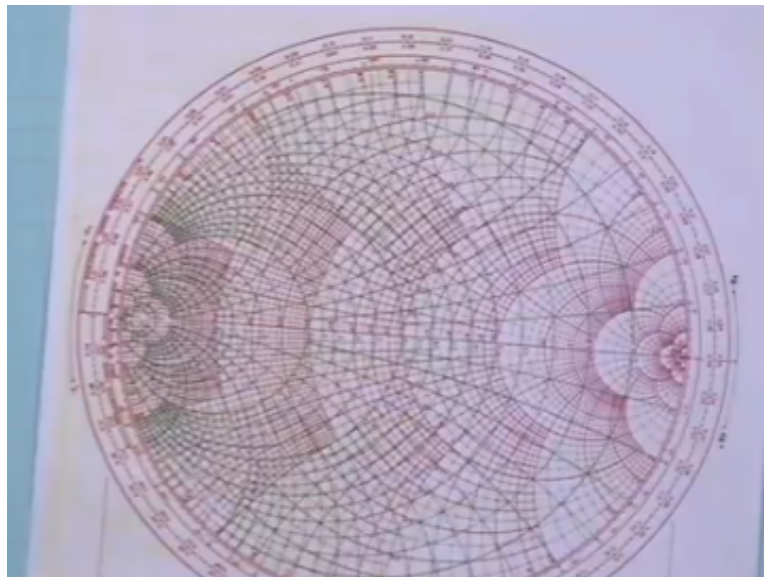
$$\text{or } \Gamma_s \Gamma_s^* - (\Gamma_s \Gamma_{opt}^* + \Gamma_s^* \Gamma_{opt}) = N - |\Gamma_{opt}|^2 - N|\Gamma_s|^2$$

$$\text{or } \Gamma_s \Gamma_s^* - \left(\frac{\Gamma_s \Gamma_{opt}^* + \Gamma_s^* \Gamma_{opt}}{N+1} \right) + \frac{|\Gamma_{opt}|^2}{(N+1)^2} = \frac{N - |\Gamma_{opt}|^2}{(N+1)} + \frac{|\Gamma_{opt}|^2}{(N+1)^2}$$



So that if I just I am making the whole square thing as we do always so finally the solution is this Γ_s – some complex quantity because this Γ_{opt} is a complex quantity N is a real number. So and this side is fully a real number so by now you know that this is locus of the circle. This is the in the smith chart in this smith chart if I now locate Γ_s for a given N that means given noise parameters then I know that this will represent a circle.

(Refer Slide Time: 13:50)



What is the center of the circle we are calling C with the subscript F to show that for a given noise figure this value is this center is this and radius is this. So obviously here if $F = F_{min}$ you can check that If $F = F_{min}$ let us see the N value so where is N so if F is F_{min} N becomes 0. So

if N becomes 0 you see the radius becomes 0 and the center is at Γ_{opt} so basically the F is F_{min} that means specified noise figure is same as minimum possible noise figure from the amplifier then that represents a point.

(Refer Slide Time: 14:04)

Choice of source Impedance

$$\left| \Gamma_s - \frac{\Gamma_{opt}}{N+1} \right| = \frac{\sqrt{N(N+1 - |\Gamma_{opt}|^2)}}{(N+1)}$$

Locus is a circle \rightarrow

$$C_F = \frac{\Gamma_{opt}}{N+1}$$

$$R_F = \frac{\sqrt{N(N+1 - |\Gamma_{opt}|^2)}}{(N+1)}$$

When $F = F_{min}$
 \rightarrow circle degenerates to point Γ_{opt} .

Which is equal to then gamma source resistance just equals the gamma opt obviously that is also the definition of gamma opt so circle degenerates to point that part is easy. That means if we always design for F_{min} we get it but sometimes we do not because if so much noise stringent noise figure is not required then we try to design for a bit higher noise figure which will serve our purpose but that will give us some more gain because we know gain is also important.

(Refer Slide Time: 15:55)

Example


Noise performance of GaAs FET is superior to BJT above 4 GHz.

The scattering parameters of a transistor measured at a bias point for low noise operation ($V_{CE}=10V$, $I_C=4mA$) at 4 GHz are,

$$S_{11} = 0.552 \angle 169^\circ \quad F_{min} = 2.5 \text{ dB}$$

$$S_{12} = 0.049 \angle 23^\circ \quad \Gamma_{opt} = 0.475 \angle 166^\circ$$

$$S_{21} = 1.681 \angle 26^\circ \quad R_N = 3.5 \text{ dB}$$

$$S_{22} = 0.839 \angle -67^\circ$$


Though we are not maximizing it for LNA but some gain if you get then subsequent stages they are noise figure will be improved. So that we do so let us take an example of this whole thing suppose I want to find noise performance of Gallium Arsenide FET. You know gas FET, their FET is always superior to BJT because in BJT there are actually current is controlled. Current means flow of charges so that will add noise whether in any FET it is basically field operated so not much so much charge carrier they cause that is why their noise is less .

So noise performance of GAS FET is generally superior to BJT that is why if LNA is generally designed with FET's not with BJT's. Now let us say for that particular transistor FET we have the parameters given that means this are all S parameters. You see S parameters S_{11} is magnitude of S_{11} is less than 1. Magnitude of S_{22} is also less than 1 So you know that we can we have unconditional stability extra also the minimum noise figure for this particular transistor is given as 2.5 db it typical value gamma optimum.

That means optimum source resistance for which I can achieve this is given by this $\angle 166^\circ$. So you know that optimum source resistance will be somewhere here because here it is 166° and point something. So this whole thing is one so half of that roughly and R_N the real part of the source or real part of source impedance that is 3.5 db ohm So design a LNA for minimum noise figure here so that is R_N here they are saying minimum noise figure means we will try to design for 2.5 db itself.

So we know first will any amplifier we need to do the amplifier analysis you know Rollett criteria so check for amplifier stability to find out this values K Rollett stability factor that is you see 1 greater than 1 so no problem also you check that delta its magnitude is .419. Delta is a complex quantity we should have written the delta magnitude of delta that is .419 so that is also less than 1 ok. So we have passed the K delta test or Rollett test.

(Refer Slide Time: 18:04)



Stability Analysis

- Check for amplifier stability

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.0179$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.419 \angle 111^\circ$$

↳

- Device is unconditionally stable



So we can say that device is unconditionally stable so you need not worry we can proceed for LNA design. So LNA design you see for minimum noise figure already Γ_{opt} is given so that gives us the source reflection coefficient. Now minimum noise figure of 2 db 2.5 db now we plot constant noise figure circles from 2.5 db to 3 db actually if you want to have minimum noise no problem. But for any general design we are showing that ok we are starting from 2.5 db.

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LNA Design (contd.)

For minimum noise figure $\Gamma_s = \Gamma_{opt} = 0.475 \angle 166^\circ$

Minimum noise figure 2.5 dB.

↳

We plot constant noise figure circles from 2.5 dB to 3 dB.



So at whatever step you want you find out those circles that can be done let us say we have taken that instead of minimum noise will take 2.8 db noise figure so then we can calculate the value of NI that will turn out to be remember this is that absolute thing so I need to convert this db's to the actual figure and that is like here.

(Refer Slide Time: 19:32)



Design (contd.)

For, 2.8 dB F_i ,

$$N_i = \frac{F_i - F_{\min}}{4 R_N} |1 + \Gamma_{opt}|^2$$

$$= \frac{1.905 - 1.778}{4(3.5/50)} |1 + 0.475 \angle 166^\circ|^2 = 0.1378$$

Centre of the constant noise figure circle,

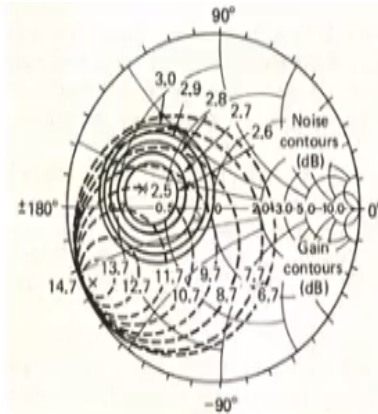
$$C_{F_i} = \frac{\Gamma_{opt}}{1 + N_i} = \frac{0.475 \angle 166^\circ}{1 + 0.1378} = 0.417 \angle 166^\circ$$

Radius of the circle $R_{F_i} = \frac{1}{1 + N_i} \sqrt{N_i^2 + N_i (1 + |\Gamma_{opt}|^2)} = 0.312$

So you see the N_i which is the noise parameter for the 2.8 db noise figure that subscript using i that means we are specifying i noise figure. So noise parameter that will be become this point .378 center of the constant noise figure circle we can easily plot because that C_f expression we have so by putting i here you get this is the values you get radius of the circle that will be here that you can calculate that is .312.

So that means center is as I said somewhere here and it is having a circle of radius not much so that we have done you see this is 2.5 db, 2.6 db, 2.7 db, 2.8 db 2.9 db. We see that you see the circles are closely spaced so we can say for this transistor the noise figure is not sensitive to small variation of Γ_{opt} and S_{11} . So no it is upto us to choose where from this circles on the circle somewhere you choose.

(Refer Slide Time: 20:47)



- For this transistor F_i is not very sensitive to small variations of Γ_s around Γ_{opt}
- Choose Γ_s

So that will determine your noise figure if you decide that I will choose from 2.7 ok you can choose it and advise is to try to make it as near to center as possible. So if you are on this you come down here as much as possible so that you get less mismatch etc. It is no unique it is upto you now whatever noise figure you choose that will determine your noise performance as well as gain.

(Refer Slide Time: 22:12)

$$\text{Find } \Gamma_L = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right)$$

$$= 0.844 \angle 70.4^\circ \text{ (for } \Gamma_s = \Gamma_{opt} \text{)}$$

- If other Γ_s chosen, Γ_L will change accordingly

Determine Gain $G_T = 11$ dB

$G_{ob} = 12.7$ dB



You see though it was said you can do for 2.5 db we are lets taking it 2.8 db and doing it . Now the question of you will have to now match design the any amplifier finally the input matching network gain and output matching network gain that you will have to do. So you see it is better

to find out that since you have already chosen γ_S . You can find out γ_L from this formula last lecture we have extensively used this.

So that already makes that you are makes that load reflection coefficient is this for $\gamma_S = \gamma_{star}$ db. If other γ_S is chosen that means we chosen a particular γ_S but if you choose something else then you remember that you will have to choose the γ_L also accordingly. Then determine gain in our case once this is there you know how to find the gain. Because once you have this γ_S γ_L value we know that what is the maximum gain or what is the specified gain.

Let us say the determine our gain is 11 db though we could have got 12.7 db upto to that it goes we will decide 11 db will be sufficient . So design means you see there is no harden first thing in practice will see that what value will choose etc ., It is a guideline you should not go beyond this range. You see that we are going from 2.5 db noise figure to 3 db if that is the whole application can sustain that we will go there and we will get this gain.

Similarly we could have got by proper input and output matching this 12.7 db but we say 11 db sufficient to a choosing that . Now another example let us take that transistor is this another parameter thing F_{min} is this here it is saying ok design for a noise figure of 2 db. You know noise figure the unit is kelvin unit less it is wrongly here written.

(Refer Slide Time: 24:10)



Example 2

Let a transistor has following parameters

$$S_{11} = 0.6 \angle -60^\circ$$

$$S_{12} = 0.05 \angle 26^\circ$$

$$S_{21} = 1.9 \angle 81^\circ$$

$$S_{22} = 0.5 \angle -60^\circ$$

$$F_{\min} = 1.6 \text{ dB}$$

$$\Gamma_{opt} = 0.62 \angle 100^\circ$$

$$R_N = 20 \Omega$$

Design for a noise figure of 2 dB K.

We should design for maximum gain at this fixed noise figure because we since noise figure is specified we keep upto that what we should design for maximum gain. So first we find out the noise figure parameter N that turns out to the points this so immediately we got constant noise figure circle for that 2 db noise figure that is here RF is here. So now input gain on the smith chart we plot input section constant gain circles.

(Refer Slide Time: 24:29)



Find Constant Noise circle

We should design for maximum gain at this fixed noise figure

$$N = \frac{1.58 - 1.445}{4(20/50)} |1 + 0.62 \angle 100^\circ|^2 = 0.0986$$

$$C_F = \frac{0.62 \angle 100^\circ}{1 + 0.0986} = 0.56 \angle 100^\circ$$

$$R_F = 0.24$$



For input say we are 1 db to 1.7 things. Once then I know will be find out value of gain have already

section gain let us trying to make from db these are the I have GS like this that normalize GS these. Then we can what is the specified what is this that we seen. So CS and RS

input matching circles maximum gain circle 1.7 db out of which intersects 2 db constant noise figure circle at this point.

(Refer Slide Time: 24:59)



Input Constant gain circles

On the Smith Chart, we plot input section constant gain circles for input section gain 1 dB to 1.7 dB

$G_s(\text{dB})$	g_s	C_s	R_s
1.0	0.805	$0.52\angle 60^\circ$	0.300
1.5	0.904	$0.56\angle 60^\circ$	0.205
1.7	0.946	$0.58\angle 60^\circ$	0.150

- Maximum gain circle 1.7 dB intersects 2 dB constant noise figure circle at

$$\Gamma_s = 0.53\angle 75^\circ$$

So we will take gamma S to be that point also we assume unilateral device and we make conjugate match because we are trying to maximize whatever we have in that we are trying to maximize the gain. So here the you this all we saw so it determines the load impedance also S22 is given value from that we can always find gamma L it is this will be your gamma L.

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Output Section

- We assume Unilateral device and conjugate match

$$\Gamma_L = S_{22}^* = 0.5\angle 60^\circ$$

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}$$



Then the gain from the load side that you can get as 1.25 db so conjugate matching already the transistor. Its own gain is 5.85 we have seen that input we are taking 1.7 just now we have decided that output side we will take 1.25 db. So you see that we are getting 8.53 db so by conjugate matching this LNA which is designed for 2 db noise figure.

(Refer Slide Time: 26:18)



Conjugate Matching

$$G_0 = |S_{21}|^2 = 5.58 \text{ dB}$$

$$G_{TU} = 1.7 + 5.58 + 1.25 = 8.53 \text{ dB}$$

So, by conjugate matching this LNA will give 8.53 dB gain

It will give you the gain 8.53 db 53 that is ok for LNA this type of gain is possible if not LNA in other cases generally it should be a double digit factor. But here since we are enforcing that ok we will be keeping our noise figure low. We are satisfied with this 8.5 db gain now also when since we are made unilateral assumption we need to give the error or upper bound or lower bound of our answer. So yesterday in the last lecture we have discussed that.

(Refer Slide Time: 27:11)



Error Calculation for Unilateral Assumption

$$U = \frac{|S_{12} S_{21} S_{11} S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = 0.059$$

$$\frac{1}{(1+U)^2} \leq \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

$$\text{or } 0.891 < \frac{G_T}{G_{TU}} < 1.130$$

$$\text{or } -0.5 \text{ dB} < \frac{G_T}{G_{TU}} < 0.53 \text{ dB}$$

- So, ± 0.5 dB error in gain calculation



How we can have find this parameter that unilateral assumption parameter U that is this so note this is the formula and you put this value of you. You know that GT by GTU that this is the error in unilateral assumption that various from this in db terms various from -0.5 db to (0.5 db) 0.53 . SO we know that it is the assumption can be made. And we have made that so our matching section that becomes simpler than a classical conjugate match thing so $.5 + - .5$ db error will be there in gain calculation.

So we know we have calculated 8.5 we may 9 db may be 8 db its not more than that. That is important with this we have seen an low noise amplifier design now this is basically extension of the earlier that constant gain circle part but here we have given special attention to noise figure. So we have introduced noise figure parameter and with respect to that basically that specify we design and then we match to get some gain out of it that follows the whole procedure. Thank you.