Spread Spectrum Communications and Jamming
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Lecture - 64
Tutorial – IX

Hello friends. Today we will take up tutorial- 9, which is the last tutorial in the series of all tutorials in this particular online course. We shall deal with problems and multi user CDMA.

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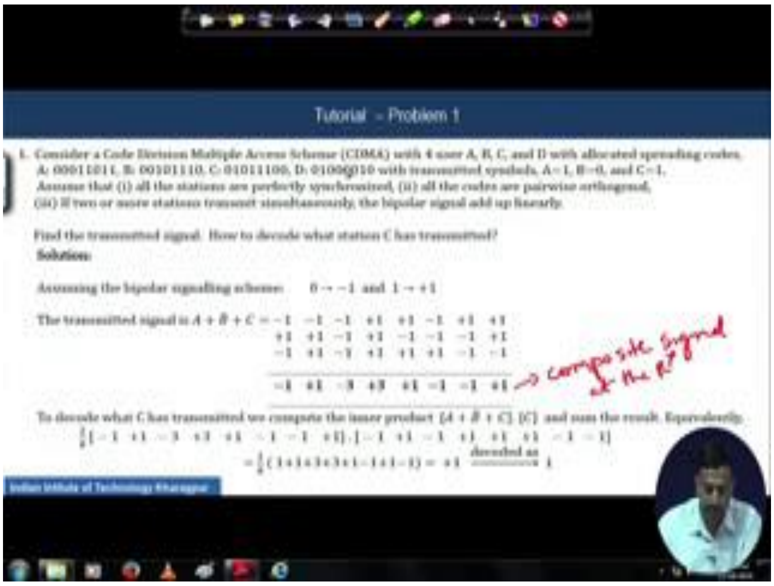

As the first problem, let us consider a code division multiple access scheme. And therefore, uses A to D with allocated spreading codes. So, the spreading codes are preallocated. As you can see they are distinct codes. I am not making any comments on the nature of these codes. These are just randomly picked up just in order to illustrate this problem. But, these are not PN sequences or any such code, which is been specially designed. These are just in order to highlight that we, you, shall take some data bit and we shall process the data bit with this spreading code, in order to realise our spreading signal.

The important thing here is that we have four users. So, it is a multi user system. And using that, we have transmitted symbols which correspond to bits 1 and 0 binary. For A, B and C, these transmitted bits are A equal to 1, B equal to 0 and C equal to 1. So, essentially what the problem states is that we have, if we have a bit 1 transmitted by user A, it is going to be spread with the corresponding code of user A, which is 8-bit sequence, as you can see here. B transmits a bit 0 with another 8-bit sequence and C transmits a bit 1 with the corresponding 8-bit sequence shown over here.

Some assumptions to start with using that all the stations are perfectly synchronised which is debatable assumption, because usually CDMA transmission is synchronous in nature. However, for the sake of simplicity we shall assume that all stations are transmit at the same time. And at the receiver, the received signals from these stations are also synchronised. Secondly, all code, we assume that all codes are pairwise orthogonal. We also make another assumption that if these stations transmit simultaneously which is what is going to happen in our problem. Then, the signals that are binary bits are going to be mapped using a bipolar signalling scheme. And, these bipolar signals add up linearly to form a composite signal which is received at the receiver. And then, we use the despreading procedure, in order to decode the bit at the receiver from a given user.

So, as I said the bipolar scheme signalling scheme that we assume is that binary 0 is mapped to a minus 1 and binary 1 mapped to a plus 1 signal level. The transmitted signal which is the linear combination or addition of the three signals are given as shown. Now, what is to be noted is how we obtain this is that this first row here, corresponds to the bit 1 which was transmitted by A, which is mapped to plus 1. And, this plus 1 is multiplied to the sequence, code sequence corresponding to A. So, I will just quickly show you how the, we obtained this. So, this is obtained by taking this A, signal by A, which is plus 1. And, we have merely multiplied it with the sequence corresponding to this A, which is nothing but minus 1 minus 1 minus 1 here, plus 1 plus 1 minus 1 plus 1 and plus 1.

So, this multiplication is this first row. So, in a similar manner we can obtain the signal, spread signal due to B, which is now 0, multiplied with the code sequence of B. So in effect since 0 get mapped to minus 1. So, what we are going to do is this 0 is a minus 1. And, this is going to be mapped, rather multiplied to the code sequence; that is, a bipolar code sequence of B. So, since we are multiplying with it minus 1, effectively the resultant sequence is nothing but the inversion of this B because this is B is being multiplied by minus 1. So, that is why we have denoted it by B complement. Here, the compliment sign basically means that we are inverting every bit and then mapping it to the bipolar equivalent. C is 1. So, 1 multiplied by the code gives you the code itself. It is like A. Once this is done, these signals as I said are synchronised and add up linearly. So, the linear addition of signal levels is given by this row. So, this is nothing but a composite signal at the receiver.

So, now we assume that the signal has been received as it is. And, the noise has not corrupted the signal. So, normally what happens is we usually assume that there is going to be some kind of additive white Gaussian noise. But, in this case we neglect white Gaussian noise, additive white Gaussian noise or for that matter, any interference. And, we assume that a composite signal is received as it is. So, the way to decode what station C has transmitted is to take the inner product by representing this in a vector form as you can see. And, merely take the inner product of this received composite vector with the code vector corresponding to C.

So, we scale it by 1 by 8 because that is the length of the code. So, the inner product is nothing but element wise multiplication of the two vectors or component wise multiplication. And then, summing it; summing the result. 1 by 8 as I said is a scaling factor. And as you can see, keep multiplying element by element and note down the result. Sum up all the resultant inner product terms, scale it by 8. And, we get plus 1, which can be mapped back to binary 1, which was the bit that was precisely transmitted by user of station C.

So, you can repeat this example by choosing to find out over D code what station B has transmitted, for example. So, all you need to do is once again take the composite signal, and these times multiply by the code of B. So, just remember that although B was 0, and in order to represent the spread signal, we have used the inversion or compliment sign. While multiplying the composite signal, we have to multiply it not with the compliment, but only with the original code B because that is what is available at the receiver; because receiver does not know what the transmitter had sent. So, what it has is, are the codes A, B, C and D. And, so multiply the signal with B. And, you will get back minus 1. You can verify that separately.

D, user D, of course in this problem has been used as a dummy. It basically has no role to play, in the sense that we do not assume that it is transmitting. So, this is kind of a silent station. So, I hope you got an idea of what, basic idea of what multiuser CDMA transmission is.

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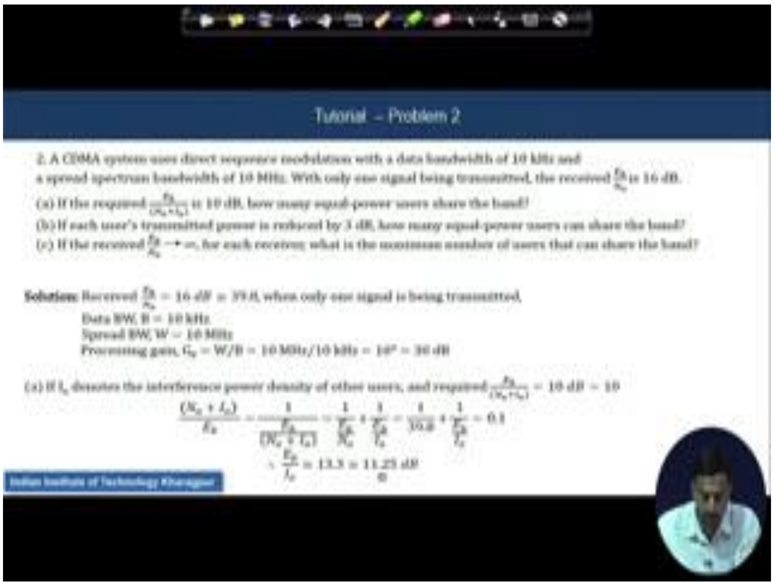

Now, let us proceed to the second problem. So, in the second problem we have a c d m a system which uses direct sequence modulation. We have a data bandwidth of 10 kilohertz and a spread spectrum band width of 10 megahertz. Now, if only one signal were to be transmitted, then we have E b by N naught matrix at the receiver. And that is specified to be 16 dB. However, since ours is a multi-user system, we now proceed to find out what happens if we have a requirement of multiple users and interference there in. And, so now we are keeping in mind that E b by N naught for a single user is 16 dB. What will, how many users can the system support if we have E b by N naught plus I naught, that is, interference PST also specified and this ratio now required is 10 dB.

Secondly, if the user transmitter is transmitter power, that is, every user’s transmitter power is reduced by 3 dB. Now, how many equal power users can share the band? And finally, if E b by N naught tends to infinity, that is almost negligible noise case, and then in that case how many, what is the maximum number of equal power users that can share the band. So, this is essentially a multi-user CDMA problem where we deal with E b, that is, signal to noise plus interference power ratios. And, we try and figure out what is the capacity of the system, in terms of the number of users supported.

So, now we first take up the data which is given. And, E b by N naught with only one user without interference, convert it to a linear value. We have the bandwidth of the data specified as well as the spread spectrum bandwidth. So, as we know the; by definition, the processing gain which has been seen in a previous class is it has been derived as well is the ratio of the spread bandwidth to the data bandwidth. So, that turns out to be 10 megahertz by 10 kilohertz which is 30 dB.

So, we tackle the first part of the problem. And, we see that the E b by N naught was found out. For what we do if we take the reciprocal of this term, this is in order to facilitate or for ease of the evaluation basically. So, as you see the required E b by N naught plus N naught is given as 10dB, which is nothing but 10 in the linear scale. Reciprocal of this is 0.1. So, we rewrite these terms. And we, this kind of taking this reciprocal helps us splits this term into E b by N naught and E b by I naught. So we already know; what is E b by N naught, in linear term. So, we substitute it and the rest of it is just simplification. And using simple mathematics, we calculate the E b by I naught ratio. So, it turns out that E b, that is, the bit energy; energy per bit divided by interference of PST is given by (Refer Time: 14:27) found out to be 11.25 dB.

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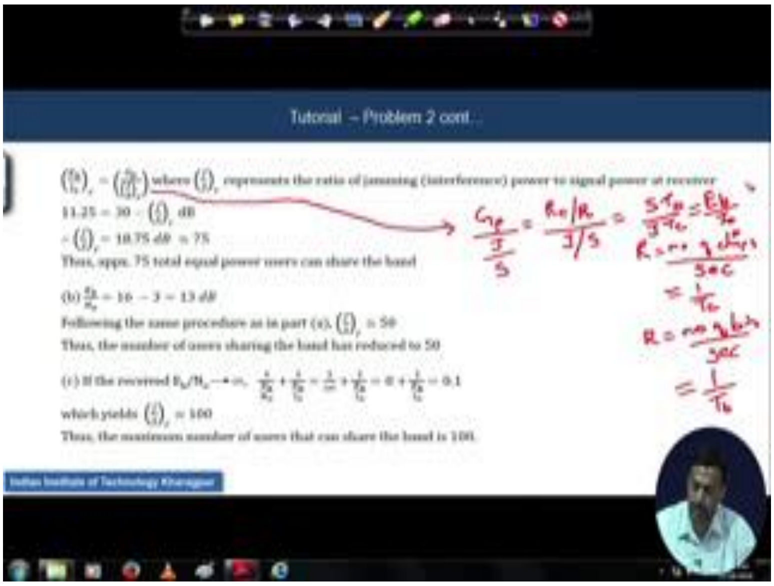

Now, how to use it in order to find out the total number of users supported. So, what we do is we have already calculated this. And, this expression again is quite common where in E b by N naught is written in terms of the processing gain and the ratio of the jammer power to the signal power. So, let me just quickly elaborate this. G p by J by s. G p, as we already know is nothing but the chip rate divided by the original bit rate. And J by s, as I said is a jammer to signal power ratio. So, what this basically means is that we have this; s can go in the numerator. And, R c is nothing but chip rate; that is the rate, chip, number of chips per second.

So, this is nothing but; R c is nothing but number of chips per unit time. So, essentially the denominator over here what we get? And this, we know is also equal to 1 by T c; so in the denominator. In the numerator what you get over here is signal power. This is jammer power into T c, and R which is nothing but number of bits. It is of the original signal per second. So, this can be per bit. It is 1 by T b. So, this becomes s into T b. This as we know, has been equated to E b by I naught, which is true because s into T b gives you E b and J into T c because this is the interferer power multiplied by unit time. This gives you the interference PST. So, basically we see that we express E b by N naught in terms of the processing gain, which in turn is expressed in terms of R c N R. And, we are in a position to equate it to the jamming, jammer power to signal power ratio.

It is easier to evaluate it in terms of dB. So, we write this E b by N naught, which we had obtained in the previous slide. So, that was 11. 25 dB and the processing gain is already found out to be 30 dB. So, that is mentioned here. And, now we get the jammer power to signal power ratio to be equal to 18.75 from this expression. And, this is equal to 75, which is precisely the number of users that can share the band provided that we limit our E b by N naught plus I naught to 10 dB. This is because the numerator, we have the total jammer power and the denominator is a individual signal power. So, this much was there. This is the margin. This many number of users can be supported based the; you will be see basically found out using this ratio; that is, the total jammer power, the total interferer power divided by the desired signal power.

In the next problem, we had been asked; we have been told at the trans each users transmit power is reduced by 3 dB. Since n, noise PST remains the same. Reduction in users transmits power results in the users reduction in users energy per bit. And so therefore the original value of 16 dB, which was specified is now subtracted, and by an amount of 3 dB. So, the new E b by N naught ratio is 13 dB. So, you can proceed with a similar analysis in order to prove that the jammer to signal ratio now reduces. And, so the number of users that can be supported also reduces. This is because of the reduction in the transmit power of the signal.

Finally, we have been asked, we have been told that noise power is almost negligible. So, E b by N naught tends to infinity. And, we use this expression which we had used previously for this part a of the solution, where in this first time turns out to be 0. And, now we straight away get E b by N naught as 0.1, which once again following a similar analysis, gives us a jammer to signal power ratio of 100. So, the number of users supported has now increased because the E b by N naught is; we have taken a limit as E b by N naught tends to infinity, which is a good thing because what we are saying is no signal power increases manifold compared to noise power with, in which case the number of users supported is more than what is, has been found out in part a.

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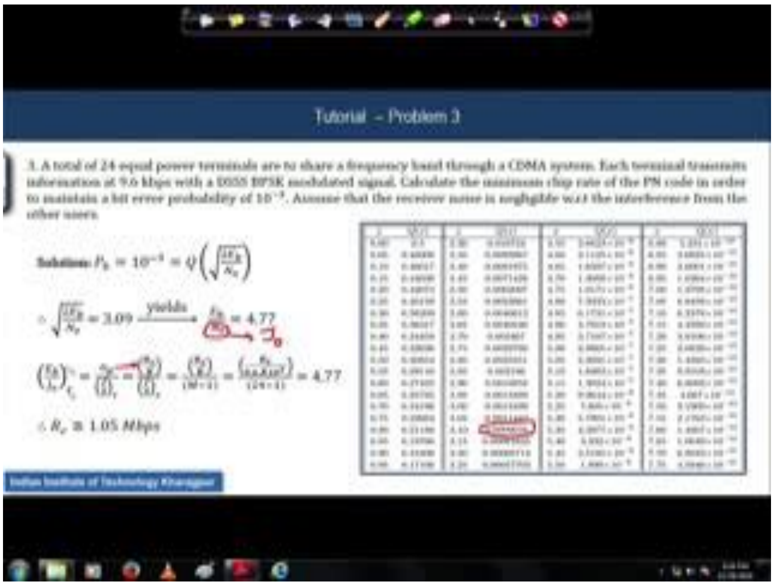

So, we move on to the third problem. So, in this problem we have, we have been told that we have 24 equal power terminals of which have to share a common frequency band using a CDMA system. They have been given the rate of data transmission, which is 9.6. And, we have to calculate the rate of, the chip rate of the PN code, that is, R c. And, the specification is that the minimum bit error probability should be 10 to the power of minus 3. Importantly, the system is a direct sequence spread spectrum; BPSK system. So, this information is crucial because we need to map 10 to the power of minus 3 probability to this subsequently, and important assumption that the receiver noise is negligible with respect to interference from other users.

So, proceeding with the problem what we see is that we invoke the bit error rate probability expression for BPSK, which is in terms of the Q function. And, the argument is root 2 E b by N naught. We have been given the desired value of probability of error which is 10 to the power minus 3. Also, we see that 10 to the power of minus 3 corresponds to an argument value, which is somewhere close to 3.10, somewhere in this range. This is point, almost 0.0001 or rather 0.001, I am sorry. So, presumably the value is just before this 3.10; an argument of 3.10. So, the precise value of x over a root of 2 E b by N naught, in this case is 3.09. And, this can give you the value of E b by N naught, which is 4.77.

Now, actually noise, receiver noise is an negligible. So, we can, since that is negligible, the only mitigating factor over here is the interference due to other users, which we shall call it or denote it as J naught, where a J in a sense is for jamming. But, this is more of unintentional jamming, just like in the previous problem. So, it is nothing but multiuser interference term. So, we do not have N naught, but what we have is a requirement of E b by this J naught should be equal to 4.77. So, we use that in the expression which we used in the previous problem. And, we are in a position; as I said G p can be expressed as R c by R b or R b sometimes also denoted because the R bit rate is also denoted as R.

And that is already specified to be 9.6 k b p s. The J by s ratio, we know is the number of users in other; that total number of users minus their desired users. So, it is M minus 1. So, approximately what we have is R c of 1.05 MBPS. M, of course has been specified to be equal to 24. So, this problem is pretty simple. And it, what it does is it gives you an idea of what is the chip rate required, provided the probability of error, number of users and the information rate is specified.

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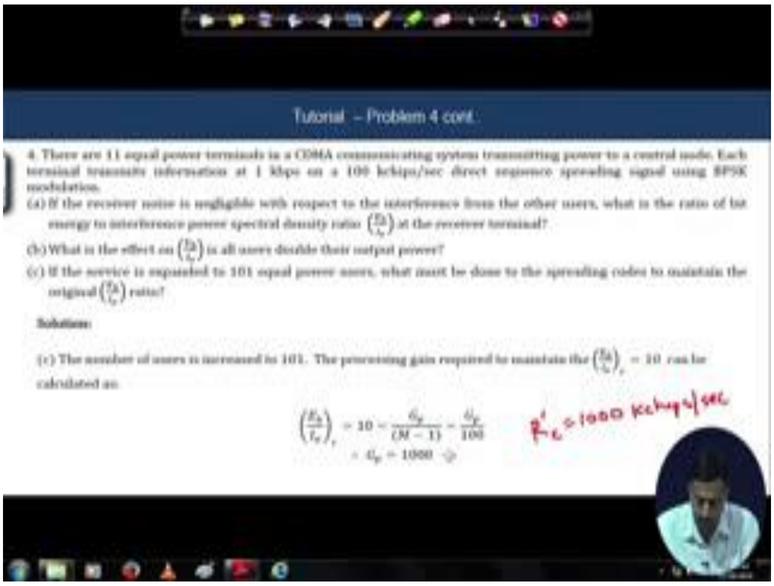

So, we move on to the last problem. What we see here is we have 11 equal power terminals in a CDMA system. And, all these 11 terminals are transmitting to a central node. So, there is going to be interference from one user to all other users. That is, each user to all other users. So, the information rate is 1 k b p s. And, we have chip rate of 100 kilo chips per second. And, we have been asked as to what is the ratio of bit energy, that is, E b to the interference power spectral density ratio, which is I naught and receiver terminal in this case. Further what happens if all the users double their output power? And, moreover if the service is to be expanded to a specific number of users; in this case 101 users. Then, what must be done to the spreading code in order to maintain the E b by I naught ratio.

So, we see that the processing gain is the chip rate divided by the bit rate, which is 100. We have been told for problem for part a, there are 11 equal power users. So, M minus 1 is 10. And, we once again use that same equation E b by I naught is equal to processing gain divided by J by s, which is M minus 1 approximately. So, we have 100 divided by 10. And therefore, the first solution for part a is the ratio is 10, which is also 10 dB.

Now, in part b what happens if all the users double their power? If all users double their power, the corresponding energy also doubles. However, since all users, as each user has doubled the power, the interference power also doubles because for a given user signal from all other users is like interference. So, if since all users have doubled their power, the interference power also naturally doubles. And therefore, we see that E b by N naught, sorry, E b by I naught ratio remains the same.

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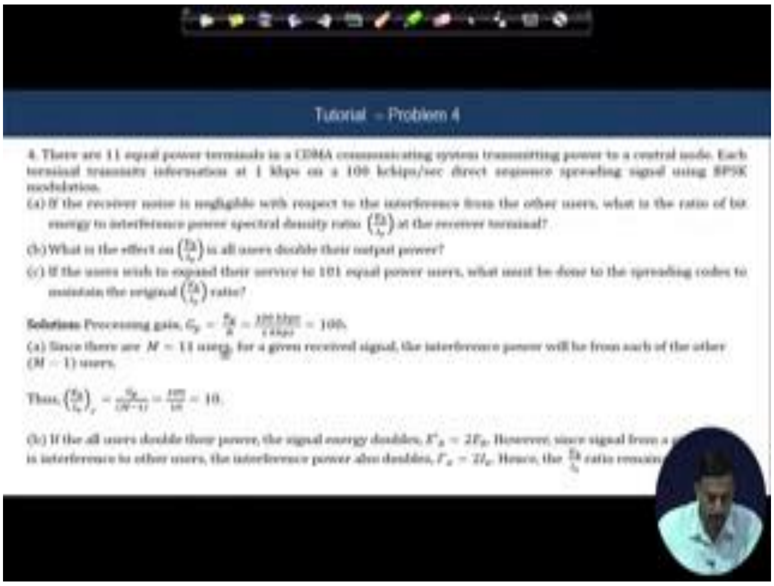

Finally, just a small change in the original problem that is what if now the number of users is given, but the E b by N naught, sorry, E b by I naught ratio has to be maintained. So, we maintain this at 10 and then we increase the number of users to 101. So, M becomes a 101. So, the denominator is 100. And, the processing gain required now is 1000. So, now what has to be done because 1 k b p s is we have fixed the transmit rate at 1 k b p s. We originally had a sequence transmitting at 100 kilo chips per second. But, now we therefore need the code rate to increase, so that we get a ratio of processing gain ratio of 1000. So, we need R c also to be increased by an order; that is to 1000 kilo chips per second. So, this is a new R c. Now, we could call it R c dash so that the processing gain turns out to be 1000.

So friends, this concludes the tutorial series. I hope you enjoyed the problems. And, I am sure you will be able to now solve problems, a variety of problems similar to the ones that we have sold in these tutorials. Of course, we had three tutorials dealing with numericals. In the remaining tutorials, we try to introduce Matlab implementation of simple basic spread spectrum systems. I hope you appreciate those codes as well and the results and the interpretation. I would like to thank you for your patience and your attention.

Thank you.