Spread Spectrum Communications and Jamming
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Lecture - 06
Tutorial – I

Hello friends, welcome to this series of tutorial sessions on Spread Spectrum Communication and Jamming. The tutorial sessions will encompass solving certain numerical, which will basically cover several aspects of this particular course. I am Shajahan Kutty, teaching assistant to Dr. Debarati Sen who is basically conducting this course. The problem solved in the tutorial sessions will be in line with the problems that you will be asked to solve as assignments later on.

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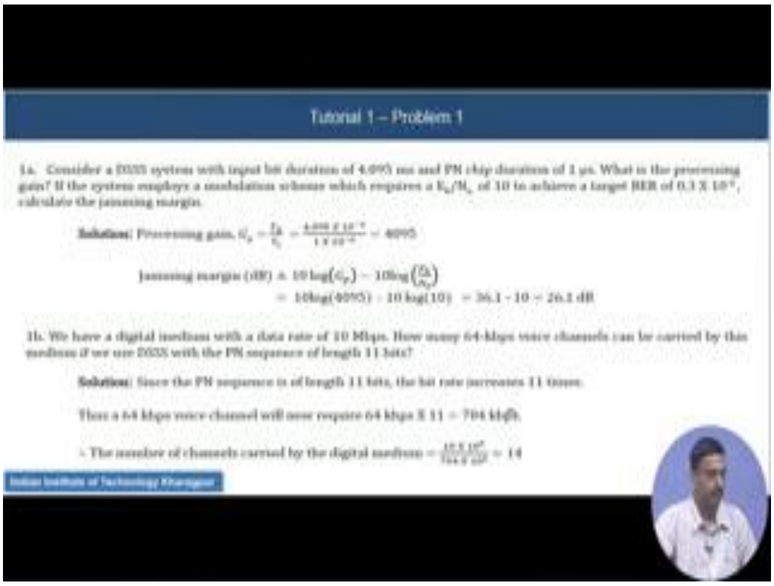

In today’s session, we will discuss five problems related to direct sequence spread spectrum systems. In problem number 1, we see that we have a DSSS system which has a input bit duration of 4.095 milliseconds and a PN chip duration of 1 microseconds. The question is what is the processing gain, and further if this system employs the modulation scheme which requires a E b by N naught ratio that is energy per bit to the noise spectral density ratio of 10 to achieve a target BER of 0.3 into 10 to the power minus 5 then we need to calculate the jamming margin.

As can be seen the processing gain by definition is the ratio of the bit duration to the chip duration, and this is pretty straightforward calculation as you can see. The processing gain for the system turns out to be 4095. Again jamming margin by definition is the ratio of processing gain to the E b by N naught ratio. And in dB terms it is the difference between these two quantities and we since we are given the data corresponding to E b by N naught ratio for a target bit error rate. So, we merely substitute the value over here. And the value of G p as obtained in the previous step, in order to calculate the jamming margin in dB unit. Now, the jamming margin is basically the capacity of the spread spectrum system in order to withstand jamming. And basically what it means is in this case a jammer would require a power equal to or greater than 26.1 dB in order to disrupt this given communication system.

In the next problem, we have a digital medium which is specified to have a data rate of 10 mega bits per second. The problem that we need to solve is to find out how many 6ty four kbps voice channels can be supported if we employ DSSS with the PN sequence of length 11-bits, which is something like a barker sequence. So, we see that since the PN sequence is of length 11-bits, the original input data rate will now be multiplied by this factor 11 which we call as a spreading factor.

So, for every voice channel, we have 64 into 11, which is 704 kbps data rate that now needs to be supported. Therefore, if the medium has the data rate of 10 mbps, and if we have a voice channel which now requires communication rate of 704 kbps then we can see that dividing the two quantities fourteen number of channels can be supported by this particular system.

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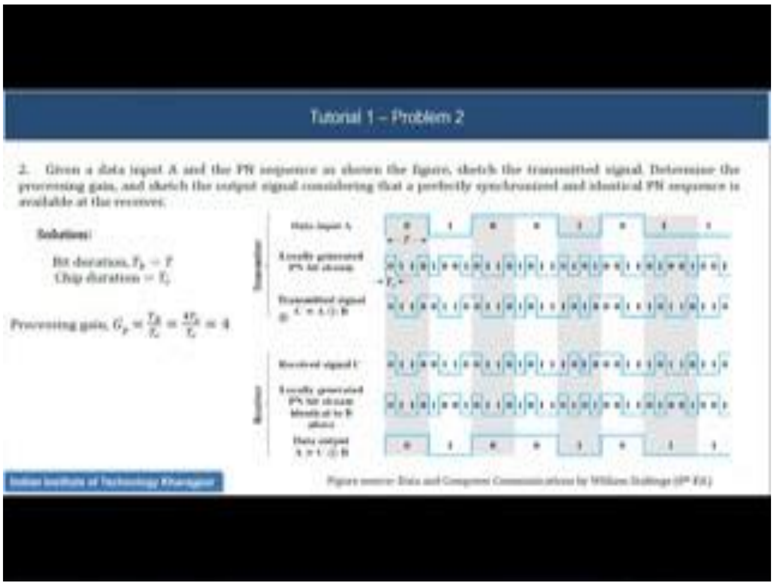

In problem 2, we are given a input data as we see in this figure. And we have a locally generated PN sequence. Now, we need to find out the processing gain in the first place. And we also need to sketch the output signal assuming that the same locally generated PN-bit stream at the transmitter is also available at receiver, and moreover this stream is perfectly synchronized without any delay. So, we first see that for every input bit duration which is T also denoted as T b we have within that duration we have four bits corresponding to the PN sequence and each of those PN sequence bit has a duration of T c. So, basically we have a relationship between T b and T c, T b is basically nothing but 4 times T c as you can see, and therefore the processing gain which is T b by T c is of this system is 4.

Now, when we are given binary data, we basically XOR the input data with the PN sequence data in order to get the output spread signal. And as we can see the input data XORed with this PN sequence gives us the spread transmitted signal. At the receiver end, we take a replica of this signal we assume that the signal is not distorted, and we locally generate the same PN sequence as can be seen which is synchronized with more delay and we see that XORing operation add the receiver enables us to recover the data back in the original form. And since synchronization is perfect, we see that the output data perfectly matches with the input data as observed over here.

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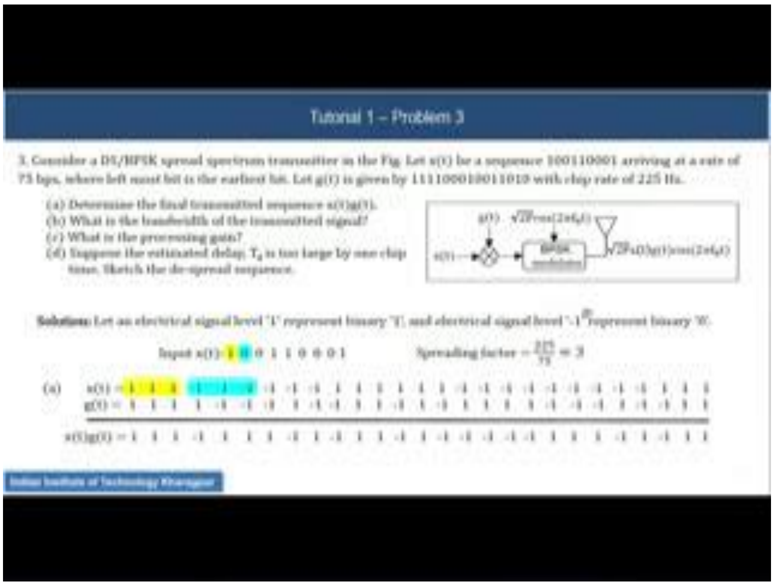

In problem number 3, we see that we have a DS BPSK spread spectrum transmitter, and we observe that the input signal which is a data sequence is multiplied with the pseudo random sequence in order to yield the spread spectrum signal, which is further modulated using the binary phase shift keying scheme for transmission through free space.

So, as we see basically now we try to solve this problem in a slightly different manner as compared to the previous problem, but essentially it means the same. Instead of using binary bits, we represent each binary bit by electrical signal level. As we can see here the binary one is represented by a signal level 1, and a binary 0 is represented by a signal level minus 1 that is a electrical signal level minus 1. So, we also see that according to the data given the chip rate is 275 hertz, whereas the input data rate is 75 bps. Since, we using BPSK, we calculate the spreading factor by merely dividing this chip rate of 225 hertz by 75, and the spreading factor turns out to be 3.

Essentially what it means is that for every 3-bits of the pseudo random sequence, we have the same input data. As you can see the left most bit which is also the earliest bit which is 1 is basically multiplied with the first 3 pseudo random sequence. We are multiplying an not XORing in this case because what we are using is the electrical signal level and not the binary level. So, this multiplication basically gives me an output which is nothing but the spread spectrum signal.

Again for illustrative purposes, the second bit is a 0 and we can see that 0 is represented by minus 1 and for the next 3-bits of the pseudo random data we multiply with the same bit that is 0 because of this spreading factor. And we correspondingly obtain the output. So, in this manner we manage to get the spread signal.

Now, the questions that we need to answer apart from determining the spreads transmitted sequence is as to what is the bandwidth of the transmitted signal, which in this case is pretty easy to calculate, because we already know the spreading factor which is equal to 3. So, if the original bandwidth was 75 hertz, we have now the same bandwidth multiplied that is the same bandwidth multiplied by a factor of 3 in order to achieve our spread signal bandwidth. So, therefore, the bandwidth of the transmitted signal is nothing but 75 into 3 which is 275 hertz.

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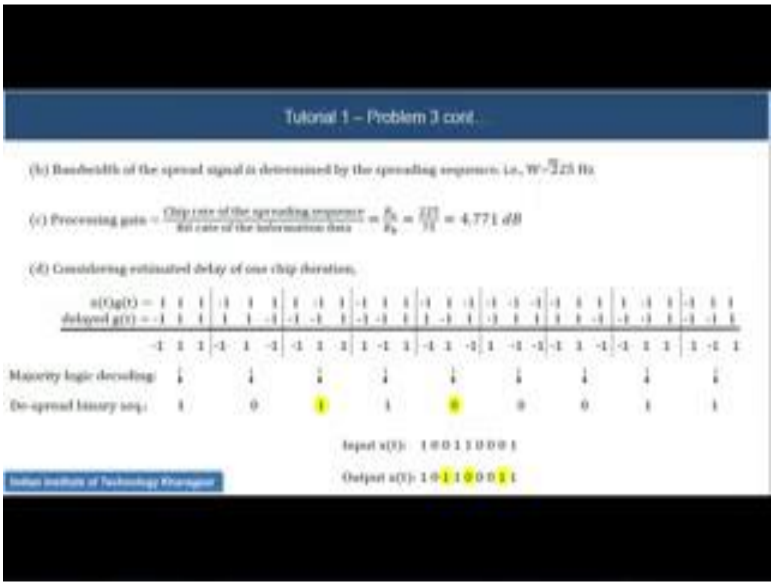

The next question is as to what is processing gain, which is also of fairly simple to calculate, it is nothing but the chip rate which is 275 hertz divided by the input rate that is the input data rate information data rate, which is 75 bps. So, this is equal to 3 which in dB terms that is 10 log of 3 is nothing but 4.771 dB.

Finally if it is mentioned that we have an estimated delay T d which is too large by one chip duration, which now means that at the receiver our PN sequence is not synchronized with the transmitted PN sequence which is now of dygration from the previous problem. If you recall in the previous problem we had the PN sequence at the receiver which was synchronized with the PN sequence of the transmitter, but now it is said that the estimated delay in the PN sequence is more than is too large by exactly one chip time. And therefore, we have a PN sequence which is delayed.

Now as you can see the PN sequence basically starts with 1 1 1 1 and then three minus 1s. So, here we have the PN sequence 1 1 1 and then three minus 1s. So, basically the PN sequence is beginning here where the received signal has already started over here, which basically means we have a dummy bit which we insert which in this case it is a 0 corresponding to electrical level of minus 1. And this basically models the delay in the PN sequence which is too large by one chip duration. So, this PN sequence which is not synchronized. Now is multiplied with the received spread signal sequence, and what we obtain as a result of this multiplication is a new sequence.

What is done now is we use what is called majority logic decoding in order to now convert this set of 3-bits back to our original data. So, the decoding process here is such that we count the number of 1s and 0s in every set of 3-bits. And we determine as to which is greater whether the number of 1s is more or the number of 0s is more; and accordingly we declare the output bit based on which is greater. So, as we see in the first set of 3-bits, we have two 1s and a 0, and therefore we have declared the decoded data as 1, because number of 1s are more. In the second of 3-bits, we see that the number of 0s is more than the number of 1, and therefore this bit is decoded as 0.

Let me take up another random set, we see that we have 3-bits out of which two are 0s and one is a 1. So, we declare it as a binary 0. So, now, what we see is that we have the de-spread binary sequence based on this majority logic decoding as 1 0 1 1 0 0 0 1 1. So, we see that b basically when we compare it to the input, we notice there are three errors input bit positions 3, 5 and 8 I am sorry this bit of one should have also been highlighted because these are the three bits which are in error. And this has happened because we have used PN sequence at the receiver which is not synchronized with the original PN sequence at the transmitter and it because it has a delay of us bit at the beginning. So, we see that synchronization is very important during the process of decoding a spread spectrum signal

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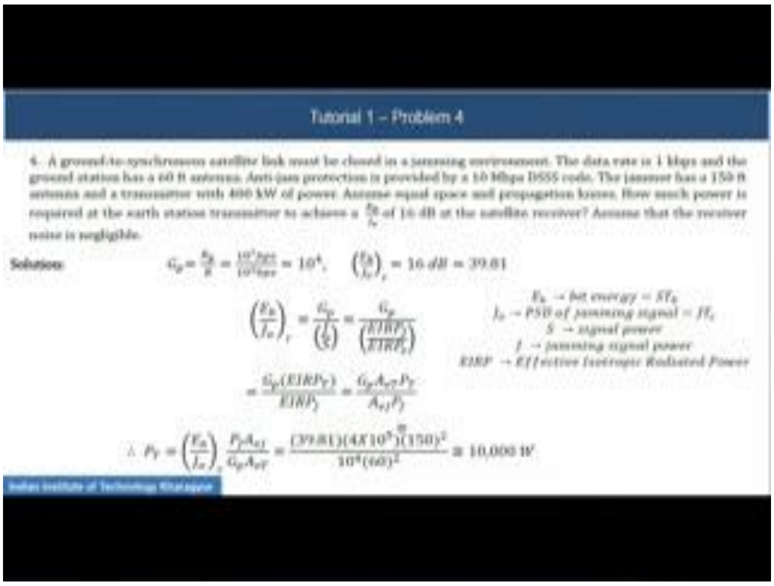

In problem number 4, we have a ground to satellite ground to synchronize satellite link which we need to establish in the presence of a jammer. And what we have is a data rate of 1 kilo bits per second also the antenna dimension of this particular ground station is specified as 60 feet. We provide anti-jam protection using a DSS code with 10 mbps of pseudo random sequence. And the jammer specifications are that it has a antenna height of 150 feet and a transmit power of 400 kilovolts. So, we assume that for both the ground to satellite link transmission as well as the jammer signal the space and propagation loses are same.

The question is to find out the transmit power of the earth station. Assuming that we need E b by j naught ratio, where E b is nothing but the bit energy and J naught is the power spectral density of the jammer signal. So, we need a E b by J naught of 16 dB at the receiver and we moreover assume that the receiver noise is negligible which is a reason why the noise power spectral density N naught is not considered in the denominator of this E b by J naught expression.

So, we begin solving this problem by first finding the processing gain which is nothing but the ratio of this pseudo random sequence rate which is 10 to the power of 7 bits per second to the data rate of the transmitter which is 1 kbps. So, we have a processing gain of 10 to the power of 4. We further convert the specified E b by J naught which is in dB to absolute number. Now, there is this relationship between E b by J naught at the receiver to the processing gain and the jamming signal power as well as the signal power, which is transmitted from the earth station.

So, as you can see processing gain is nothing, but E b by J naught at the receiver multiplied by J by s. In fact, I have just elaborated E b that is the bit energy in terms of signal power and bit duration. So, again J naught which is the PSK of the jamming signal is nothing but the jamming power multiplied by the chip duration. And we see that if we multiply these two quantities what we end up getting is T b by T c or just substitute this values over here, and you will see that G p is nothing but T b by T c which is exactly the definition of the processing gain.

So, basically this formulation make sense. We further express the jamming power and the signal power in terms of the equivalent isotropic radiated power which is also called as the effective isotropic radiated power corresponding to the jammer power and the signal power because it is basically proportional. And EIRP signal power is nothing but EIRP at the transmitter. Once again EIRP as we know is proportional to antenna gain and transmit power. Antenna gain in turn is proportional to the effective aperture of the antenna here A e T is nothing but the effective aperture of the transmitting antenna of at the ground station that is an A e J is the effective aperture of the antenna at the jammer.

Of course, while making this assumption that antenna gain is proportional to the effective aperture we assume that both the transmitting station as well as the jammer have the same wavelength. We also know that EIRP is proportional to the power that is radiated which in this case is P T at the transmitter and P j for the jammer. And we have made use of the fact that the cable losses for the jammer as well as the transmitting station antennas are the same, therefore the ratio cancels out. So, you do not see the cable losses which is essentially inversely proportional to EIRP figuring in this expression.

So, as we can see we are already given E b by J naught which is 39.81. And the rest of the parameters are all known except P T which is basically the transmitted power which we are seeking to calculate. One thing to notice that the effective aperture area of the antennas is written in terms of the squared value of the dimensions specified for the jammer as well as the transmitting station antenna. And the calculation basically yields transmit power of 10,000 watts which is 10 kilo watts. Basically what we notice over here is that the transmit power required is much less compared to the power that the jammer requires in order to disrupt this communication, and that is thanks to this processing gain of a watt 10 to the power of 4, which appears in the denominator of this expression. So, the conclusion basically is that the processing gain which depends on the spreading factor is vital in terms of achieving protection against jamming.

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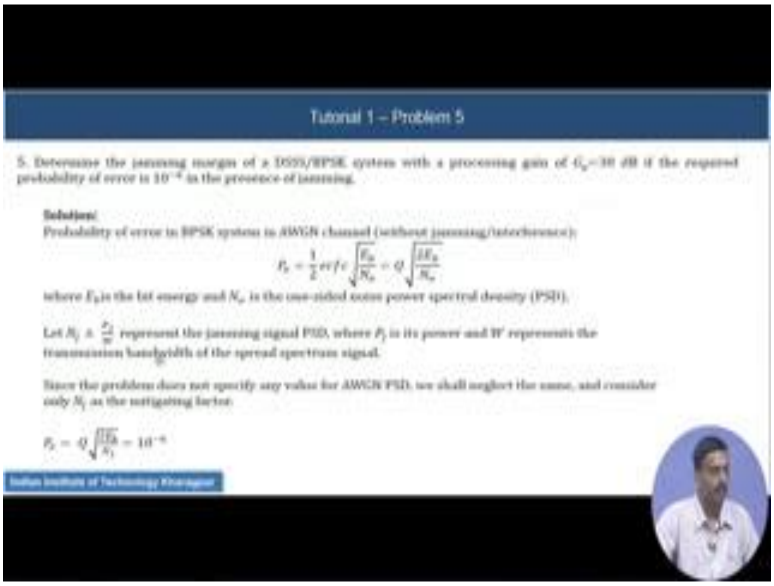

Finally, we have problem number 5, where we need to find the jamming margin of a direct sequence BPSK system. And what we see is we have a processing gain of 30 dB which is specified. And a probability of error of 10 to the power of minus 6 is required in order to make the system communication effective. So, what we notice is that we have a BPSK system, and therefore we begin the solution by invoking the definition of probability of error for a BPSK system in AWGN channel. And this is specifically without considering any other interference. And from the standard relationship which is derived in well known textbooks, we have probability of error given in terms of the error function over equivalently in terms of the cube function.

Now, what we need to know here is of course, that there are some simplistic assumptions made while calculating this P e 1 of them being that the probability of 0s and 1s in this particular communication system is the same. So, what we do is we calculate the E b; I am sorry what we do is we find out what is this ratio E b by N naught which basically requires calculation based on the specified probability of error, since N naught which is the noise power spectral density in this AWGN channel is not specified. We will neglect it and we consider instead N j has the mitigating factor, which is nothing but the power spectral density of the jammer.

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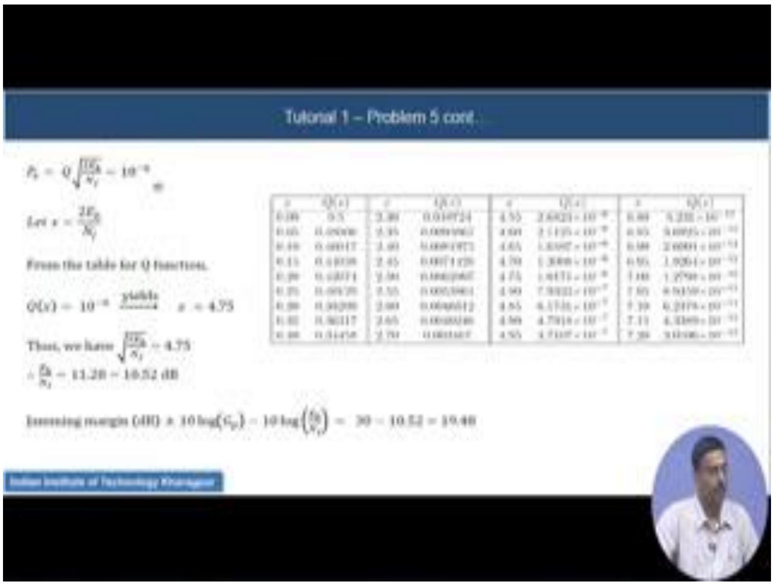

So, merely replace N naught by N j because, N naught is not specified, whereas we know that we are in a position to calculate this N j. Once we can map this probability of error to the Q function table. So, what we see is that the probability of error which is equal to twice Q root twice E b by N j is already specified in the problem. We merely look up the table in order to find out what is the value of this factor corresponding to 10 to the power of minus 6. So, what we see is there is a close match at 4.75 which is equal to this factor.

I think there is a small mistake over here; this x should be equal to root of twice E b by N j. So, basically in this case, we get root of twice E b by N j equal to 4.75, which is what is obtained from this table. So, we basically find out from this what is E b by N j which is equal to 11.28, and in turn in dB terms it is 10.52. And the jamming margin is nothing but by definition in dB terms the difference between the processing gain and dB and the ratio of E b by N naught or N j as the case is that in dB. And we now subtract the quantities in order to get the final result. Please pardon me for this error. Once again please note that this x should be root of 2 E b by N naught and not 2 E b by N j as specified here.

So, this concludes the session one for tutorials on direct sequence spread spectrum system. In session two, we shall deal with problems on frequency-hopped spread spectrum signal.

Thank you.