Spread Spectrum Communications and Jamming
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Lecture – 59
Multiuser Detection – Part 1

Hello students. We were discussing about the CDMA environment and multi users scenario in a CDMA environment, when a practical system is deployed. Today we will try to understand in a multiuser scenario, how optimum detection is possible in the receiver. This discussion is divided into two parts; part one, we will be discussing in this module and in the next module we will discuss the remaining portions of the multiuser detection section, optimum multiuser detection section and followed by in the third module we will discuss about the error probability calculations on for this optimum multiuser detectors.

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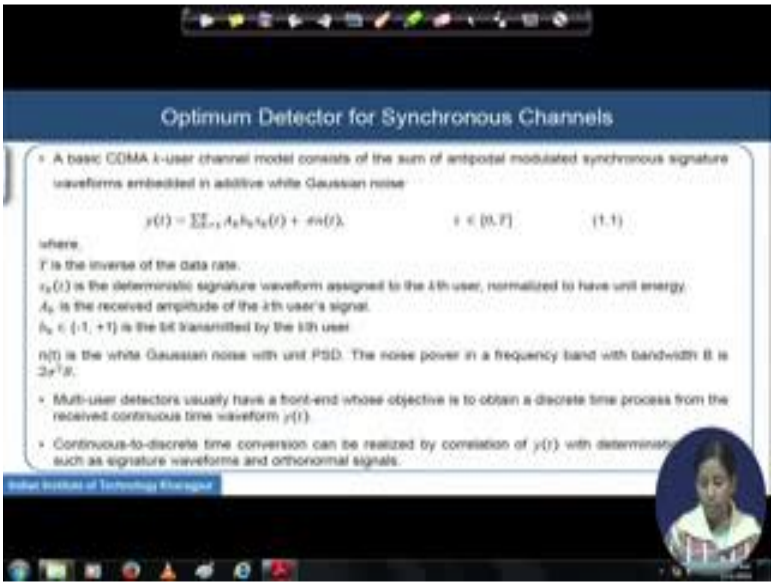

So, let us start the multiuser detection in the problem specifies the fact that we are having in a wireless situation say k users operating simultaneously, trying to access the channel simultaneously and sending the sending the data to a receiver and it is not like that all the k users are targeting to a typical receiver. It is something like that each of these users are targeting their independent intendant to receiver, but as all the receivers are turned on in a wireless environment and going by the fact that the receivers antennas are omnidirectional as well as the transmitting antennas r.

So, in the receiver you are receiving the signals from all the direction and from all the users and. So, in CDMA so the channel that you are visualizing in such an environment is a k user channel model where, k is the number of the users are currently operating or accessing the channel. And the signal in a receiver they received in a receiver front end, it can be viewed as the sum of the antipodal modulated synchronized signatures, signature way forms which are embedded in additive white Gaussian noise. So, the signal received at the front end of the receiver if I denote it as y t, it will be a combination of the signals coming out from the different number of the users. If I have the capital K number of the total users. So, I will get the sum of all the transmitted signals and it is embedded with the additive white Gaussian noise section.

Remember this t of, it is a continuous domain signal received signal and this t can vary between 0 to capital T and this capital T is the inverse of the transmitting data rate. Inside the situation the S k t denotes the deterministic signature waveform, which is basically the spreading code that you have given to independent users, which are from user to user it is varying. It is a signature waveform that is why we are calling and assigned to the k-th user definitely. So, that is why the k is specifying, b k is a transmitted bit which can be either plus 1 or minus 1 for our kind of consideration. So, it is an antipodal modulation going on. Either you are transmitting plus 1 or you are transmitting minus 1 only the two level signaling going on and A k is received amplitude of the k-th user in the receiver terminal

And we understand that n t is a Gaussian noise which is having a power spectral density considered here to be 1 and the noise power in a frequency band which is having a bandwidth of capital B is given by two sigma square B and sigma square is a variance of the noise and remember; that means, our signal transmitted by each and every user is bounded over the bandwidth of capital B.

So, that is our assumption. So, these are all the assumptions associated with this equation 1.1. So, now, you have received this continuous domain signal at the front end of this receiver and in a CDMA receiver. Basically in a multiuser detector, the frontends the objective of the frontend of the receiver is basically to obtain the discrete time information from this continuous time deceived signal at the deceived by the antenna. And this can be easily done where, we have seen all the structures the co dilator based structures, the matched filter architectures which help us actually to change the continuous domain signal, received signal to get the sampled vector or to make it actually change it in the vector form we actually can get the sampled vector, sampled output at the output of the matched filters.

And this can be actually we can go ahead with when we are detecting it where, the matched filter or the co dilator architecture. We simply multiply this continuous domain incoming signal with the corresponding signature waveforms of the different users and then we sample it with the proper rate and hence we get all the orthonomal vectors at the output of the matched filters and the co dilator sample or architecture or the matched filters.

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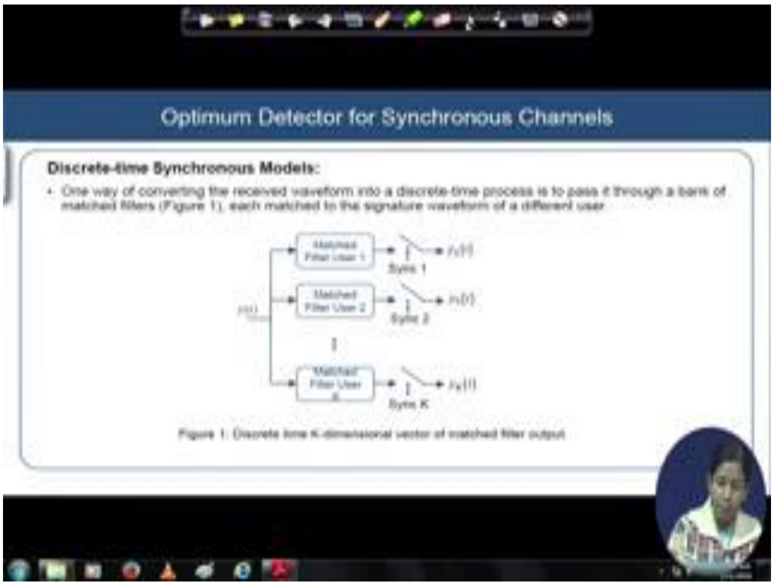

This is a very known architecture that we have discussed several times, when we have talked about the receiver architecture of a spread spectrum communication system as well as we have discussed about the CDMA receivers, co division multiple axis receivers and this is repeated once again. So, y t is a incoming signal of our interest and we have a bank of the co dilators or the bank of the matched filters. Filters are matched, the response of the filters are matched with the user 1 to your user k and hence if it is a synchronous channel; that means, all the user signals are heavily synchronized and with zero synchronization error you are expected that you are getting actually, it is expected that you are getting all the incoming signals aligned to each other and hence as they are using the same bandwidth and most probably the same data rate also then their sampling rate will be also unique and the output of this bank of the matched filters you are going to get the vectors from y 1 to y K.

So, that is all the way actually the initial. This is the set this y 1 to y K is point of our interest based on which the detector starts working and. So, that this is the discrete time K dimensional vector presentation by means of the matched filters, which is a very fast work that a deceiver does in a CDMA receiver does.

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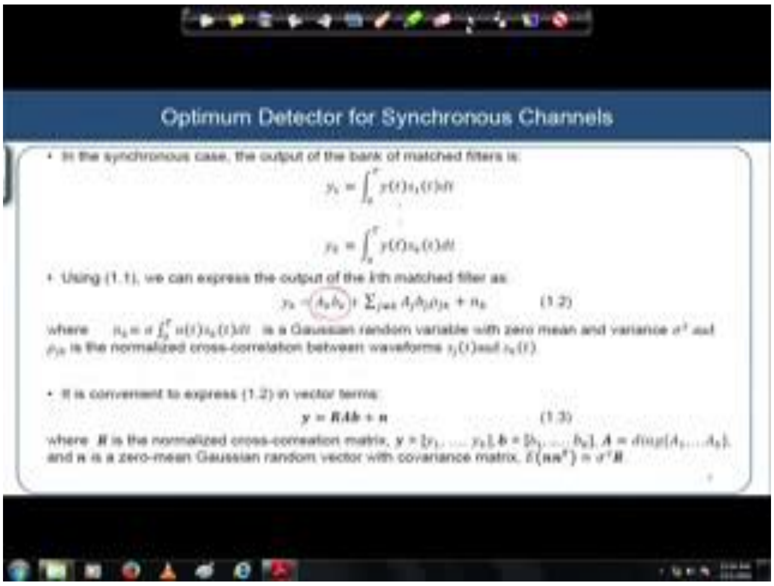

If I express the same theory in the mathematical term; so we will see that inside each and every matched filter we are interrogating the incoming signal, multiplying basically with its own signature waveform, matched to that typical users and then interrogating it over the duration of our interest and like that everywhere the whole bank is working and at the end we are getting y k; we are getting a bank of the discrete random variable. So, y 1 to y K and this y K any one of this sampled output of the matched filter, if you try to see it will basically now this y K can be now read it in as A k into b k plus this summed up terms plus the n k.

This is what the k-th output of the k-th matched filter matched to the k-th user. So, what basically these three terms are saying is. You are getting the intended signal back. You are having a actually this is the b k is a intended transmitted symbol and which is associated with a typical amplitude, which may not be actually equal with which you transmitted in a BSM distorted form of it and then you are getting a interfering parts where, actually the it is the signal (Refer Time: 09:31) signal you are receiving; same signals A j and b j you are receiving where, the j can never be equal to the k-th users because the k-th user term is out.

It is considered in the first term and here for all the j and other remaining users you will get an another term associated with them which is given by rho j k. This rho j k is basically a normalized cross correlation between two waveforms that means S j and S k. So, this is taking care of all the related based on the actually the cross correlation between the two signature waveforms, equivalently two different code set equalized to given actually to other users. This parameter we will contributing or putting the amount of the interference on this received intended deceived signal; n k as usual is a random variable, Gaussian random variable which encounters the noise component and it is a contribution from the noise component associated with the process and it is having a zero mean and the variance sigma square we understand.

Remember this n k can be expressed like this where, in n t with getting multiplied with its independence signature waveforms and integrated over 0 to t and. So, this expression of this 1.2 in the vector form in a easiest way we can also write down as capital R, capital A b plus n where, this each and every body associated with this expression they can be signified like this. Capital R is the normalized cross correlation matrix. This y, this bold y it is actually the combination of this vectors y 1 to y k correspondingly b will be a combination of b 1 to b k. Capital A is the diagonal matrix representing from A 1 to A k.

So, associated to each and every transmitted symbol you are having a amplitude, corresponding amplitude you have received, deceived amplitude variable and that is varying from A 1 to A k with it and the values are actually random and this values are also not deterministic and a diagonal matrix associated with this capital A; n is the zero mean Gaussian random vector with, it is also having some co variance matrix because you see the expression of the n k is observed actually by multiplying the spreading waveform S k t associated with it.

So, as a noise spectrum we understand in the spectral communication that the noise spectrum will be spread in the receiver, once multiplied with the signature waveform. And if we take the hence it will be also associated with the co variance also with the cross correlation matrix and that is why the co variance matrix of this noise which is given by basically the mean of this n into n transpose, it will have actually the relation with this cross correlation matrix capital R. This cross correlation is a contribution of this signature part, signature waveform and multiplied with the variance of the noise.

So, what we are ending up with is once actually at the end of the output series of the matched filters are employed at the output of the matched filters if we see combinely we will be able to see matrix representation of the, which is the combination of the transmitted signal vectors, cross correlation matrix, the noise matrix, the transmitted matrix and the received amplitude corresponding to the transmitted signal matrix and you are ending up with the combination of all these three given by equation 1.3.

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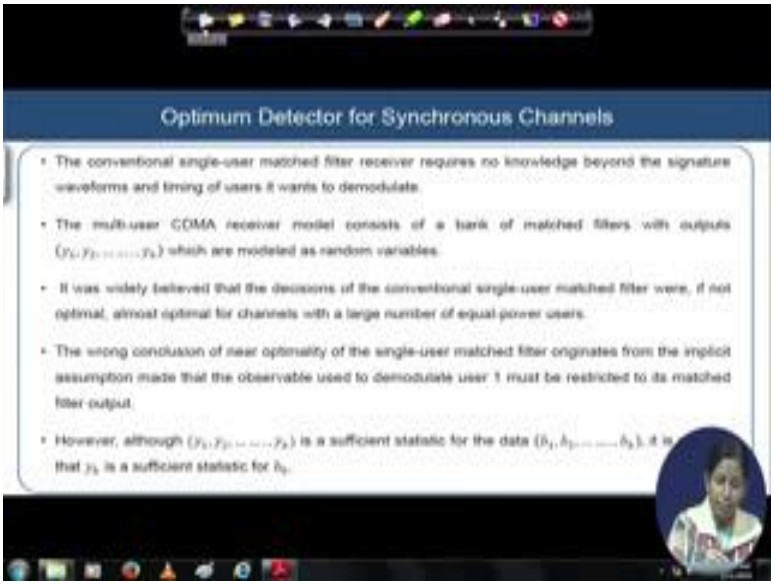

Now, the question is the conventional single user matched filter receiver architecture does it require any knowledge of the other users present in the network? Its answer is no because if it is a single user matched filter case then he only needs to know his own signature waveform and he also needs to know the timing of that user for that is all for his own demodulation. In a multi user CDMA receiver actually this module is extended to the bank of the matched filter. So, you need the bank of the output of the bank of the matched filters so and which are modeled as some random variables.

Remember, in a CDMA situation where the multiple users are own you also need to know the signature waveforms of all the users. For optimum demodulations and detection and then you also need the timing information of all the users. We will show that on the top of it you also need to either know a priori or to estimate the values of the amplitudes for all the users you are receiving, corresponding to all the users the signal you are receiving corresponding to all the users. So, the amplitude of each and every symbol transmitted by all the users and also the variance, noise variance is an important factor that you must be knowing or either you must be knowing or you must be estimating in some ways so that your detected performance is improved.

And remember actually in the CDMA receiver for the long time it was widely believed that the decision of this conventional single user matched filter, if not optimal then it will be also optimal for the channels where the large number of the equiprobable, equal power users are present. So, if they are equal power users and the number is very large we can approximate it with the consideration of the channel with the Gaussian channel and then finally, the single user matched filter performance will be actually sufficient to be close to the optimal performance.

And this conclusion of this near optimality of this single user matched filter came from the fact that it people were believing, that the observables that is used to demodulate suppose, the user one might must be sufficient and it with the restricted actually the demodulation process for a typical user to be associated with its own matched filter output only. So, though actually he was receiving y 1 to y k and the detection process of the matched filter output for a specific user, actually for that only the corresponding matched filter output was getting used. There was the concept of actually utilizing all the vectors available and, but that is why actually this optimality was assumed and this optimality, but in practices of optimality does not hold good.

I mean matched filter based, single user matched filter based optimal solution that people were believing it is not really true. This is due to the fact that though y 1 to y k the matched filter output is a sufficient statistic for the whole data bank b 1 to b k that was transmitted. But remember that y k the only 1 to 1 correspondence I mean y k alone is not sufficient statistics for the b k. As a whole y 1 to y k is sufficient statistics for the data bank of b 1 to b k that is true, but the vice versa is not true saying that the y k will be sufficient statistic for b k.

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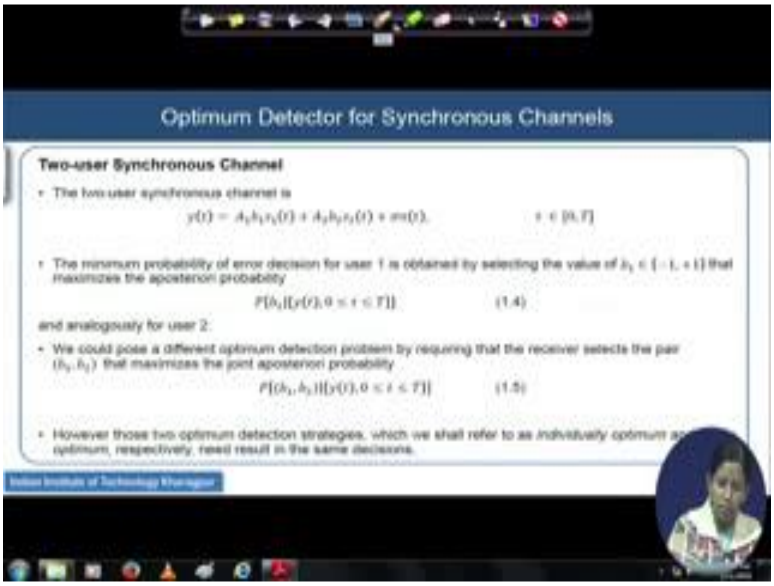

Now, we will slowly enter into to design the optimal detector or understand the optimal detector performance and its architecture. We will take an example of two users synchronous channels. So, in the channel we are having only two users; the transmission is synchronous. So, they are time synchronized I should say and. So, the output of the channel equivalently the input of the receiver should be given like this. So, I have extended the initial expression I showed at the beginning. Now I am breaking it up and you are basically receiving A 1 b 1 s 1 t plus A 2 b 2 s 2 t plus sigma n t.

So, the signal corresponding to the user number 1 plus the signal corresponding to the user number 2 and it is with the AWGN noise. That is all it you are receiving. So, the minimal error probability, the decision that is corresponds to the minimum probability for the user 1 will be obtained by selecting value of this b 1 that belongs to either minus 1 or plus 1, such that this a posteriori probability given by this expression where, you are given the, what is the probability of the b 1 given that this y t is received and the t is varying between 0 to capital T is all about the a posteriori probability. So, the value of the b 1 that maximizes this probability will be the minimum probability of error decision.

So, if this a posteriori probability is maximized the value of b 1 for which this probability will be maximized, that will also minimize the error of received error in the decision process. So, that is the relation, equivalently for the user number 2 also the probability a posteriori probability given for the b 2 given the signal received will be the b 2 value which can actually maximize that probability, will be also confirming that the error probability will be minimum. So, in the similar way for the b 2 also we can device the detection process.

But remember this is actually individually you are trying to, this is the process which we call actually minimum error probability approach, but there is another approach where we do not look into the b 1 only or b 2 only rather we prefer to go to ahead with the joint a posteriori probability is not an individual a posteriori probability given by 1.4 rather he selects a pair of the b 1 and b 2 in such a way that the joint a posteriori probability means probability of b 1 comma b 2 given the signal received over the duration of 0 to capital T, this probability is maximized.

So, now see there are two approaches; one is actually individually you tried to maximize the a posteriori probability. Choose a typical value of the received signal to choose a value of b 1 such that this probability, a posteriori probability is maximized or you try to maximize the joint a posteriori probability by proper selection of the joint pair of the incoming pair of the data. So, these are the two approaches we will discuss about the pros and cons of the both approaches. And remember we say one guy is as a jointly optimum process and another one will be referred as individually optimum. So, the first one is path for individually optimality and this is the path for the joint optimality of the decision of a optimum detector.

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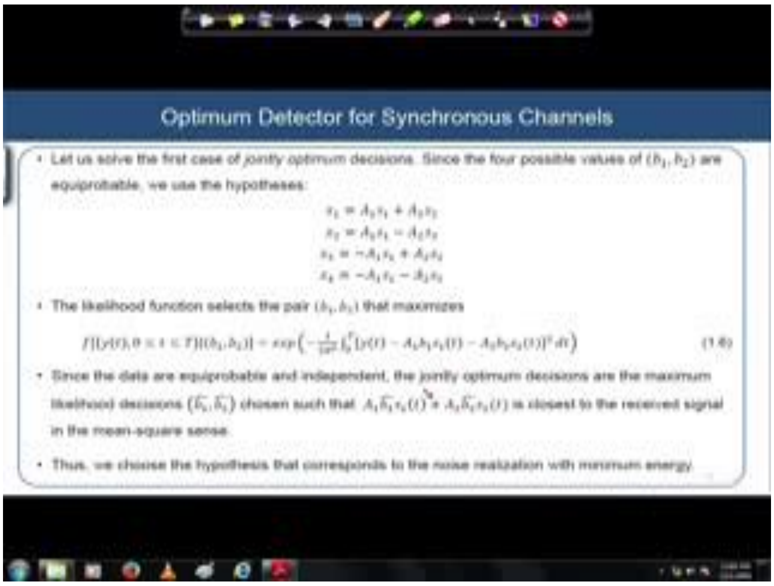

So, we will start with solving the first case, which is jointly optimal decisions and we understand that we have pair of the signals in our hand b 1 and b 2 and both are equiprobable. So, we can have a four different hypothesis mentioned here as; x 1 to x 4 and the hypothesis is such that the first one; x 1 is talking about the A 1 S 1 plus A 2 S 2. It means actually both are having the positive signs and here this is a plus minus situation. This is a minus plus situation or the minus minus situation. So, for each and everyone actually this situation assigned one hypothesis and the likelihood function that selects this pair b 1 b 2 that maximizes. So, likelihood function should select this pair of, I should say this should select a pair of b 1 and b 2, such that those pair should maximize this equation 1.6.

So, basically this maximization process you see actually on the right hand side it is a exponential of this error, it is a noise section basically; y t was your received signal and this is two channel, in a two synchronized channel. So, it is basically the signal that you received A 1 b 1 s 1 t and A 2 b 2 s 2 t the addition of this two we have received. So, when I am subtracting from the received signal this section, I mean the signal portion of it from the received signal you are left with the noise. So, fundamentally you are trying to maximize this function; that means, you are trying to minimize this side and the pair the b 1 b 2 the likelihood function that selects this pair it tries to maximize. It selects such a pair that maximizes this equation 1.6.

And as we understand this b 1 b 2 they are equiprobable, they are independent. The joint optimum decisions are basically the maximum likelihood decision equivalently. It is chosen such a way that this combined effect of this A 1 b 1 estimated with estimated value of this b 1 and b 2 that A 1 b 1 estimate s 1 and plus A 2 b 2 estimate an s 2. They will be very very close with the transmitted signal, equivalently with the deceived signal with the minimum square error sense, minimum square sense. I mean if I my estimation should be such that with the estimated value this expression whatever the value this expression will give and whatever actually I have received, they both of them the error between these two signals if I try to find out the mean square error fundamentally between those two signals which should be very minimal.

So, in that sense actually this with the estimator values of b 1 b 2 these expression value and you whatever you have received both of them should be close to each other. So, that is why if that happens basically then, we see that we call that this b 1 and b 2 the estimated values they are the likelihood decisions, maximum likelihood decisions. So, thus we choose the hypothesis that corresponds to this noise realization with the minimum energy.

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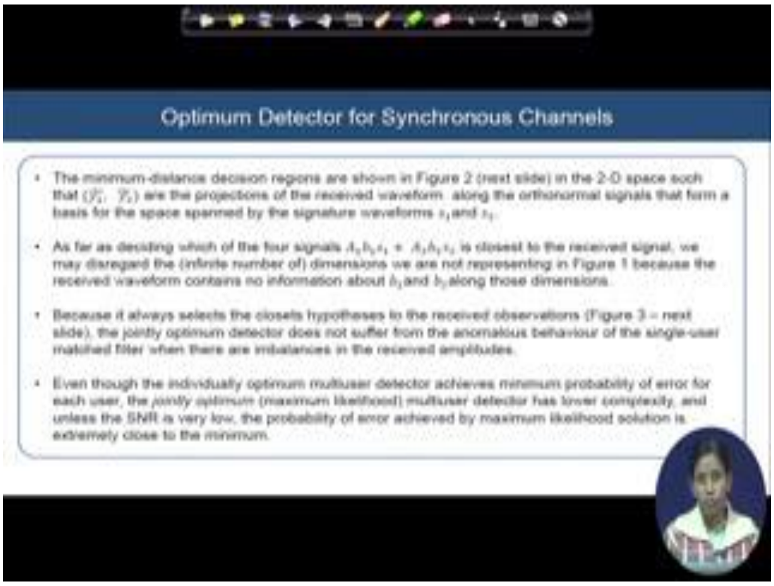

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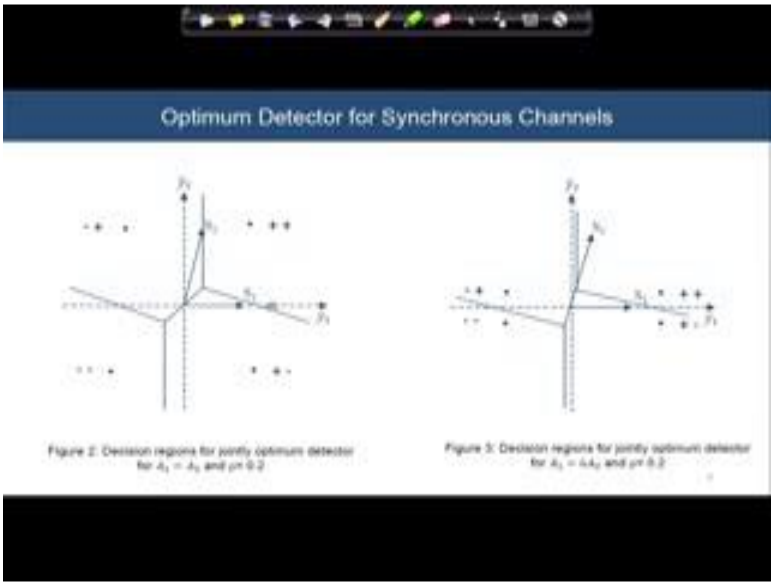

And this minimum distance decision region which we have shown in this next slide. This is a minimum region that we have shown here. Basically actually in the 2D space it is such that these y 1 tilde y 2 tildes which are the projections of the deceived waveform on the orthonormal signal. That forms are a basis, they will form the basis for the space spanned for the signature waveforms s 1 and s 2. So, these decision regions are we have shown for this transmitted signals in signature waveform s 1 and s 2 in that figure.

As far as we understand, we are this trying to decide the four signals and combination will be such that A 1 b 1 s 1 plus A 2 b 2 s 2 is the closest to the received signal. We can disregard all the infinite number of the directions over which actually the contribution of those dimensions on the decision of b 1 and b 2 are almost nil. I mean b 1 b 2 does not have any information to those directions. So, in the 2D space we have not included all those infinite number of the dimensions and only the related dimensions are included in that figure. And remember it always selects the closest hypothesis that is the idea about in the received observations.

So, the jointly optimum detector they never suffer from anomaly they does not suffer from any anomalous behavior of the signal single user matched filter based architecture. So, there is a balance always actually, if you are taking the decision based on the single user matched filter of a single output observation output. So, the anomaly in the behavior that we will suffer with here because of the decision based jointly over the two different users. So, there is a balance incorporated in the whole decision process and that is why actually the anomalous behavior is can be avoided completely.

But remember one thing, that this individually optimum multiuser detector and the jointly optimum multiuser detector if I try to compare between these two; first one the individually optimum multiuser detector it achieves the minimum error probability error for each user and jointly optimum multiuser detector which has a very lower complexity and unless the SNR is really very low, its error probability achieved by this maximum likelihood solution it is extremely close to the very minimum 1.

So, question is actually if I can attain the minimal error for each and every error for each and every user for by the first approach, individually optimum multiuser detection. Why I should go ahead with the jointly optimum case? If the question comes like this, then the answer is the jointly optimum one multiuser detector they are very very low complex compared to this individual optimum multiuser and if the SNR is not only very much low they can actually give low error probability also because maximum likelihood estimation is involved and it can very close to the minimum error probability promised and. So, in we will see actually at one point the fundamentally the individual optimum multiuser and as well as the joint optimum detection process both the way they will try to converge at a point.

So, in this figure we have already discussed about and this was the decision region shown for this kind of the values of the A 1 A 2 and rho; rho is we understand that this is a cross correlation factor and this is actually the another situation where the relation between A 1 and A 2 is something like this and the rho is also having some different values.

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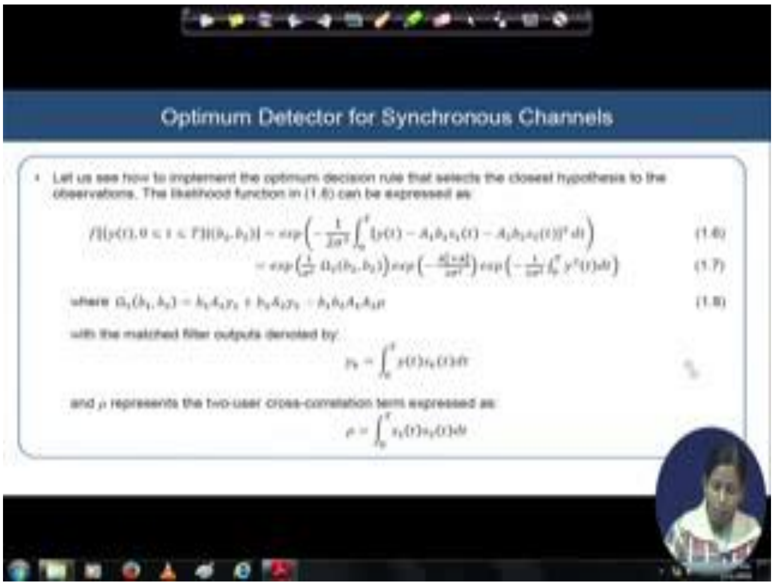

So, let us see how do we implement this optimum decision rule. That selects the closest hypothesis to the observations. So, we understand that this is the likelihood function. We have seen earlier in some few slides back. If this is the maximum likelihood function, the likelihood function sorry it is not the maximum one, the likelihood function based on which the decision is obtained on the b 1 and b 2. I can actually write this expression also like 1.7 where, the factor this big omega 2 can be expressed like this and the matched filter output we understand that it is denoted by this expression y k equals to 0 to t y t s k t and rho represents actually the cross correlation of two signature waveforms.

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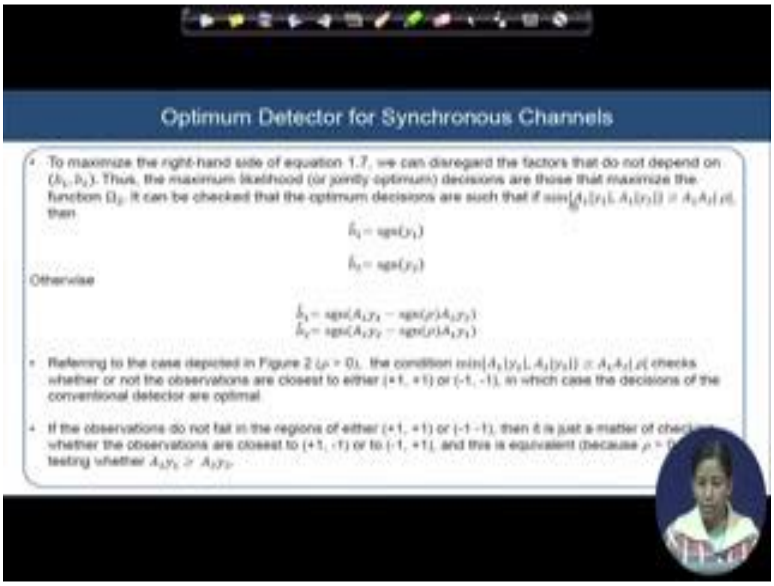

So, these are the identities used or the symbols used inside this expression 1.7 and if these are the expressions, this is the explanation all the symbols with that remember this right side equation of this 1.7 we can disregard the factors that do not depend on the b 1 and b 2. So, basically our target from this expression is to estimate b 1 and b 2. So, the factors which are not de dependent on b 1 and b 2 those factors can be disregarded from the expression itself and we can thus come down and then the maximum jointly likelihood decision, which is jointly optimum decision also are those that maximizes basically the function this b omega 2. So, what are those? It can be checked easily that this optimum decision as such that if the minimum of A 1 mod y 1 and you A 1 mod y 2 whichever is the minimum that should be greater than equal to of this A 1 A 2 into mod rho.

If this satisfies then only we can get the optimum decision. Hence for such optimal decisions the values of the b 1 estimated values of the b 1 and b 2 will be given by the sign of this y 1 and y 2 correspondingly. Otherwise if the situation is not valid then you will get the decisions of b 1 and b 2, the estimates of b 1 and b 2 governed by these two expression. So, think a situation actually if whatever the figure we have shown here referring to this figure number 2 where, actually my rho value is greater than 0 the condition that this minimum value of this A 1 mod y 1 and A 1 mod y 2 greater than equal to this. It checks whether or not the observations are close to either plus 1 plus 1 or minus 1 minus 1.

So, basically it is trying to choose see whether you are in this zone or you are close to this zone. So, and then which case the decisions of the conventional detectors are will be the optimal, if the observation does not fall in this region, regions of either this or minus plus 1 or minus 1 minus 1. So, then actually the basically you are checking whether the observation are close to plus 1 minus 1 or minus 1 plus 1 and this is basically equivalent to testing whether A 1 y 1 is greater than equal to A 2 y 2 or not because rho is greater than 0.

So, these are the two ways the detector is working up on. And we can actually also justify the same results on the other fact where, the rho is having some different values or the relation of the A 1 and A 2 is also having some different values, but this is universally true and for the optimal detector part that this is the fundamental true part. Based on this minimal check points of the optimal decisions can be obtained either this or this and fundamentally when the graphical way also we can understand that finding out that, whether this is greater than this. I mean this guy greater than this finding it means basically you are trying to find whether the observations are close to these two regions or not. Otherwise actually if they does not fall into this region then basically you are trying to find out which one is greater than which one. So, this is the way the optimal detectors or the fundamentals of the optimal detector it is.

In the next module we will see in some details considering the k users situations how the detector performs.