Spread Spectrum Communications and Jamming
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Lecture – 50
Performance Analysis of Rake Receiver

Hello students! We, in the last module we were discussing about the rake receiver architecture. We have discussed several match filter based rake receiver architectures and we ended up with symbol basis, symbol by symbol basis rake receiver architecture design, where the decision in the receiver is going on the symbol by symbol basis basically. And, we have done the analysis over a slow frequency selective fading channel for direct sequence spread spectrum communication. In continuation of that, today we will learn and discuss a little bit more about the performance of this rake receiver.

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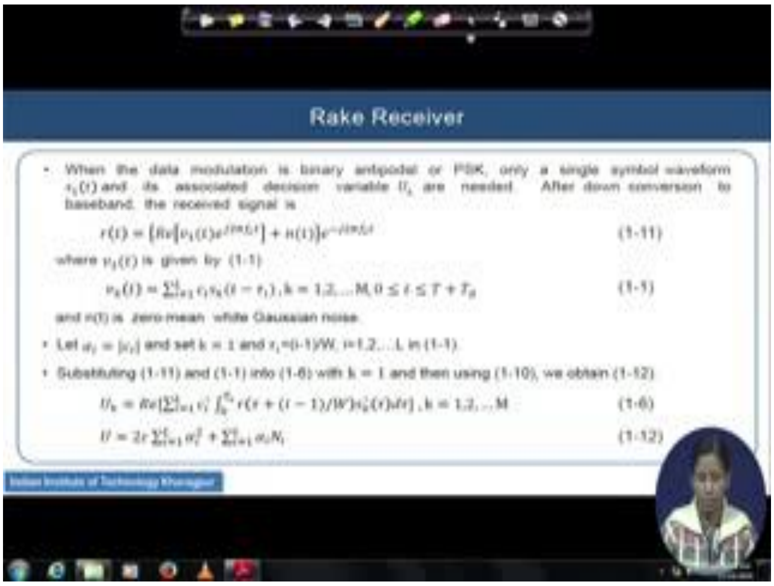

Let us consider that we are using BPSK symbol. It is the data modulation running with binary antipodal or PSK signal. So, then only a single symbol waveform of s 1 t will be associated with us. We do not have anything else. So, do you remember we considered the analysis in the last module as if that we have an M array signals. And, the signals inside that array varies from hence s 1 to e 2 to s m t. If it is a BPSK signal, we have only a single bit going over one basis function. And, hence this signal is consisting of only s1 t.

Hence, the decision variable will be also single. And, only u1t will be correspondingly. And that is your decision variable will be associated with the L number of the multipath components that is received with respect to single symbol transmission. After down conversion of this s 1 t, the BPSK modulated signal, the down converted signal will be look like the equation 1- 11. It is nothing but we understand that v 1 t was; remember we started with a signal v k t that was received. That expression for v k t is there in the last equation, in the last module; v k t was looked like this. Remember, where my t was the symbol duration, and for which was the symbol duration. And, the t can vary from capital T plus T d we wrote. T d is the delay and, so T plus T d; up to that this signal can vary. And, we have capital L number of the multipaths.

So, the signal at the output of the receiver will be the combination of all the yield number of the multipaths we receiving. Signal over all L number of multipaths. Where this c i for each and every path i, c i is the coefficient of the channel. Complex coefficient of the channel getting introduced; t tau i is the delay, multipath delay associated with the i th multiple path, multipath. So, this is the multipath delay associated with the i th path; this is the channel coefficient, complex channel coefficient associated with the i th multipath. And like that you are having capital L number of the multipaths. Capital L number of the c i values and t i values, you are having for each and every s. In our situation, s k is having only single signal. So, we are having s 1 only. So, correspondingly v t will be v 1 only. Hence, I have written in the equation 1 hyphen 11 that the received signal will be the real part of this v 1 t e to the power j 2 pi f c t plus some noise associated with it.

And, now actually after you are receiving it, you are coming down here. The n t is 0 mean white Gaussian noise process that you have received. And, this was a received signal, and then again you have to multiply it with e to the power minus j 2 pi f c t to bring it down for the down conversion, this section came. And, now let us consider two things. Number one is let us substitute a value mod c i as alpha i. I told that c i is a complex quantity.

So, we will take a mod value of it. And then, we are defining a new symbol as alpha i. And, we understand that k is equal to 1, so we will put all these values slowly here. And, we also saw that the maxim, the i, the delay will be given by i minus 1 by w. And, so for the first one the delay will be 0 and the next one onwards the delay will becoming as a part of the, as a part of this. Following this equation, where 1 by w is the time duration of the match filtering operation and w is the bandwidth of the signal as well as actually bandwidth of the matched filter.

And, if I substitute now all these expressions in this one point eleven, and finally, actually we try to put the value of all these in the expression of this one point six that we developed in the last module, then finally we will be ending up with this decision variable as 1-12; where alpha i will be given by mode of c i. N i is associated with; capital N i is associated with the noise samples for each and every, associated with each and every path. And, so we are getting actually the two halves. And, this is the total decision variable based on which actually at the output of the receiver will take the decision about which signal was transmitted; 0 or 1.

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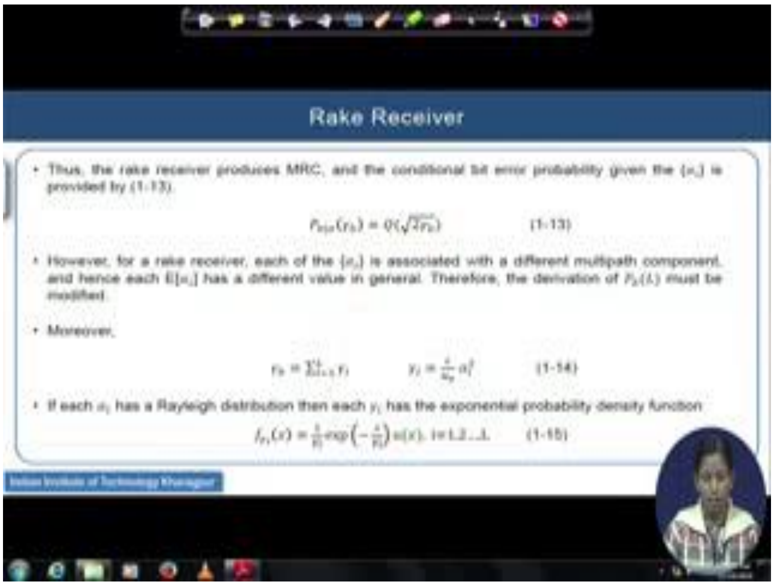

And. So, if I see the output we will be seeing; when seeing the last equation that the rake receiver produces the maximum ratio combining. And, the conditional bit error probability for the maximum ratio combining, hence which is the probability given by the alpha i will be given by this expression; where this gamma b is basically the (Refer Time: 06:47) SNR of each and every path. And then, you have taken the summation of all the overall the paths. And gamma i, the SNR, instantaneous SNR of each and every path is given by a signal power by the noise multiplied by alpha i square. So, sigma into (Refer Time: 07:06) into alpha i square by n 0; where EPSA is a deduction in the total alpha i. Epsa is the signal power multiplied by with the amplitude or the power of the, it is a power of the channel coefficient that we are getting. So, basically it is the deduction of the signal power that you are taking in the top and there is a noise power in the bottom.

And, see in the rake receiver, we understand that this alpha i associated with the the computation of the signal power, final received signal power. It is actually will be different for the different multipaths. That is why it is having a variation over i. So, we need to find out the mean value of i. And, therefore, actually this is the parameters who will largely deviate the computation of this bit error probability in the receiver that we understand.

If in a practical scenario, most the cases we understand that this channel coefficients are Rayleigh distributed. And if it is so, then this power, this SNR value, the gamma i, the SNR of each and every path will be exponentially distributed. And, it will have the exponential probability density function given by the expression 1 hyphen 15.

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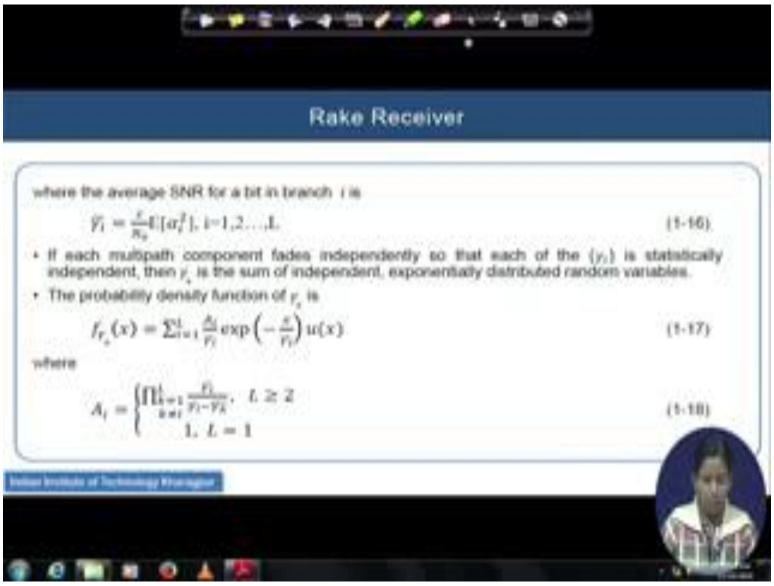

Remember, in the exponential probability density function, we have got a factor called gamma i bar. This is the average s i n L value, that SNR value, that we will discuss next year. This average SNR for a bit in a branch i that will be given by this, where actually the power is calculated see, why this is called a average SNR because we understood that the alpha i values, largely values over if the multipath number changes, so it is the alpha i square. And, we are taking the mean value of it. And then, you are calculating over the mean say channel power, you are multiplying with the actual signal power to get the equivalent signal power or received signal power at the end of that path, in the frontend of the receiver. And, remember each of these multipaths also fades independently, so that this gamma i is statistically independent. And if this is true that the gamma i is statistically independent from each other in each and every path, then gamma b can be expressed as a sum of all this independent and exponentially distributed random variables.

So, then in the such situation, the probability density function of gamma b will be given by this, where actually this factor A i will be given by the another expression written in 1.18.

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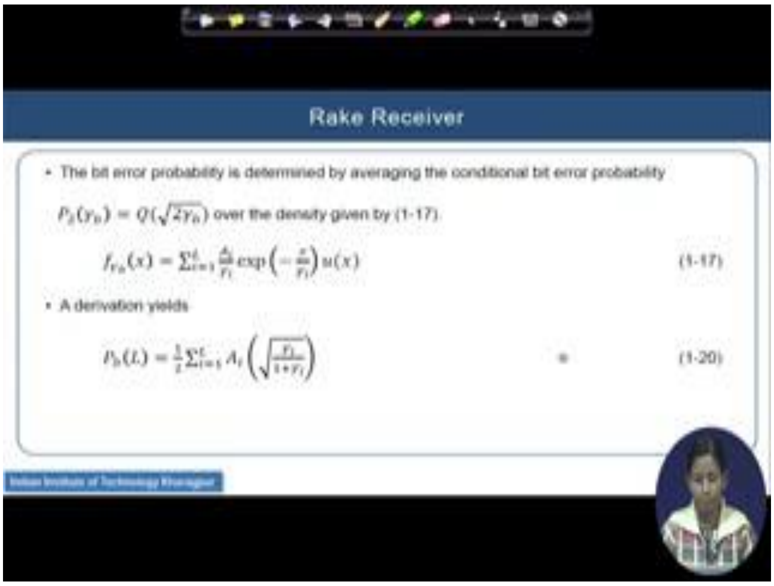

The bit error probability; it is determined by averaging the conditional bit error probability, which is this, and, over the density given A by; this expression. This expression we have seen in the last slide. So, if I go on deriving it and then over the whole L number of the multipaths finally, so a derivation yields that the error probability will be finally the summation. If all the paths SNRs are independent to each other, then it can be given by the summation. And finally, the derivation will lead us to the equation 1.20 for computation of the bit error probability for a BPSK kind of situation.

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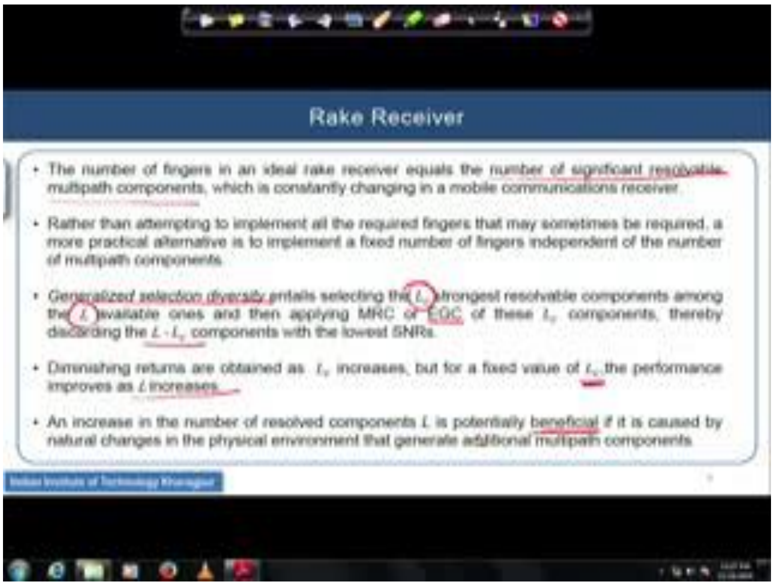

Now, some good points to remember. The number one is that the number of the fingers in an ideal rake receiver usually is equal to the number of the significant resolvable multipath components. But, in practice we understand that it is continuously changing in a mobile communication receiver. And then, instead of attending the variable or dynamically choosing the number of the fingers on the fly, in the practical communication system, what we do is actually we fix it to a high number. And, out of those high number of the fingers, who for, currently how many number of the fingers are giving the significant result, we try to combine them only in the combiner. And, there is a generalized selection diversity that entitles of selecting this L c number of the strongest resolvable multipaths among the all capital L available number of the multipaths.

And then, we can actually combine the maximum ratio, combining in the receiver or equal gain combining of these L c number of the strong multipath components to get the output of the rake receiver. And, L minus L c components are actually rejected because they are having the lowest SNRs. In order to combine, we need to know in order to choose how many number, what will be the typical value of L c. We can put also on the threshold on the SNR values, so that it will be helpful to choose those number of the multipaths having the SNR higher than the selected or the predefined threshold.

Now, if returns, if diminishing returns are obtained as L c increases, but for fixed value of L c, the performance improves as the L increases; because if the number of the multipaths are more and the resolvable capacity is also there in the rake receiver in the receiver architecture. So, the more number of the multipaths you can actually accommodate the more channel energy basically you can capture. And, it will help a lot in the channel estimation process as well as in the detection. So, actually if a number of the capital L increases, definitely it is the probability that number of L c will also increase. And, hence it is preferable to go ahead with the combination, with the higher number of the multiple components.

But, remember it involves huge complexity in the receiver and, so an increase of this resolvable number of the components is potentially beneficial also to ask, as I have already explained because it is the cause by some natural changes in the physical environment. And, this additional multipaths are coming (Refer Time: 13:20) that will help us actually in improving the signal to noise ratio in to the rake receiver, into the receiver architecture.

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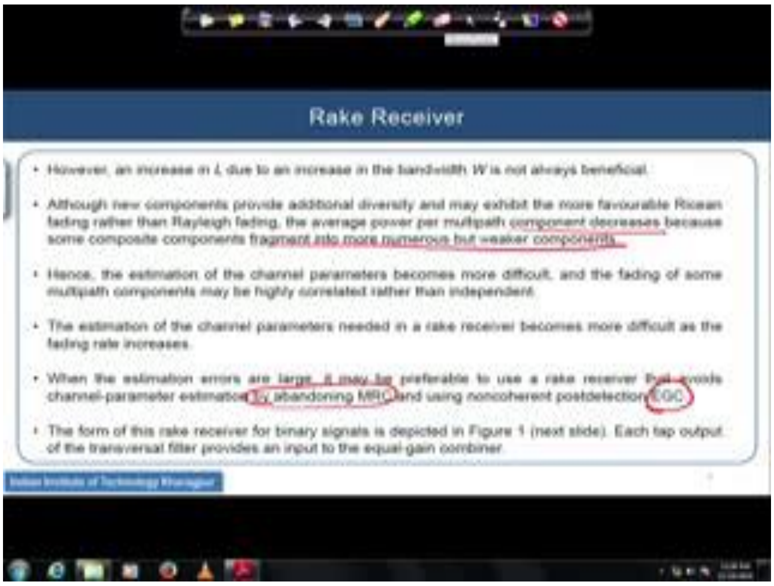

But, remember one thing. As L is increasing due to the increase, due to an increase in the bandwidth W, it is not beneficial means L can increase in two ways. One is actually there. Some changes has happened in the scattering environment, that is why. And, another point is because that W has changed. So, if it is, change is because of the W only, then it is not a good situation for the receiver. And, if it is from the multiple paths actually happening from the environment, then it is beneficial because some of the paths may give a very good return after reflection.

So, some new components, so, provide some additional diversity in such cases. But, they can exhibit some favorable Ricean fading also because rather than the Rayleigh fading. And, hence that average power per the multipath component, in that cases if it goes from the Rayleigh to the Ricean distribution, the component bit will be decreasing because some composite components fragment into more numerous, but the weaker components kind of. And, so if it happens like this, the estimation process in the channel parameters they become much more difficult. And, the fading of this multipath components maybe highly correlated also.

We remember the whole computation of the error probability in the last few slides that totally depends on the fact that all the multipath components are receiving or getting received, they are all independent to each other. That is why we can combine them independently, they can be processed independently. And, they are, there is no correlation to each other. But, if it happens know that the Rayleigh fading has changed to the Ricean because the new components have got added from the environment. And that edition is such that they are a highly (Refer Time: 15:28) correlated to each other. And then, the fundamental assumption will be lost and then they all the components would not be independent anymore.

The estimation of the channel parameters needed here in a rake receiver it becomes more difficult as the fading rate increases so, when the estimation error is really very large. So, if you are happening to, if you are facing such kind of the environment where actually the fragmentation is occurring because of the weaker components. And, your channel estimation is becoming, is it is degrading factor, such that the errors are very large, in that case the use of the rake receiver that avoids the channel parameter estimation that can should be, that should be avoided.

So, MRC always receive, always requires the estimation of all those channel parameters coherently. And, means when estimation error is very high, in such cases MRC combination is not a good option to go ahead with. Preferably, you please proceed with the non-coherent postdetection, your equal gain combining procedural for the combiner circuit. And, to form of this kind of the rake receiver for binary signal, we can actually then replace the earlier architecture that we have seen in the last module by this architecture.

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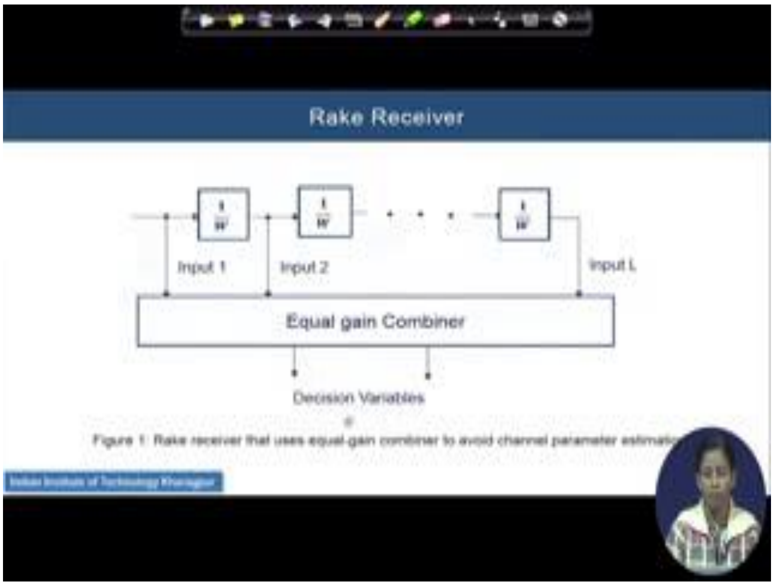

So, all the tabs are coming out. And finally, they are having the equal gain inside the, getting added up in the equal gain combiner. And, here is the decision variable which is taking the decision about the transmitted symbol.

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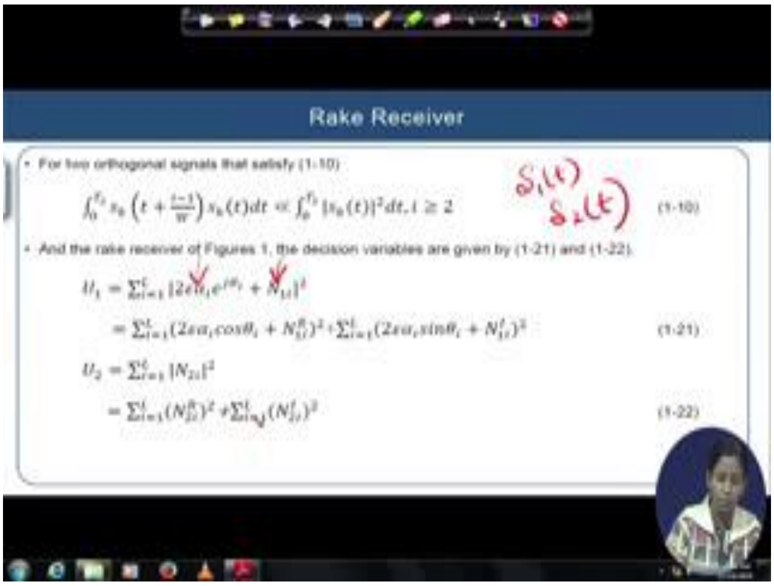

We can extend actually the error propagation and the error probability calculation by taking two orthogonal signals. We did it first with a BPSK, where the orthogonal, there is a single signal is 1 t only. Now, we are having s 1 t and s 2 t, both for consideration.

And so, such that these two orthogonal signals is satisfied this equation. So, this was the last equation that we have talked about in the last module. And, I told that this in order to happen this, we have a certain condition. The conditions are such that the processing gain is very high, autocorrelation function is relatively very small, coherent compared to the t c and over the duration of the t c, and such that this happens.

So, in the rake receiver definitely you are corresponding to two signals. You will get two decision variables. And, remember the equations are coming such that this is the contribution from the noise component. And, this is the computation; I mean, this is the contribution from the signal part. And, remember inside that whole, we can have the real part of the noise. And, if I consider the real part of the noise and imaginary part of the noise, hence u 1 will be divided into two halves. Decision variable for two also is having two parts. One is the real part of the noise and also is the second part is a, from the noise also.

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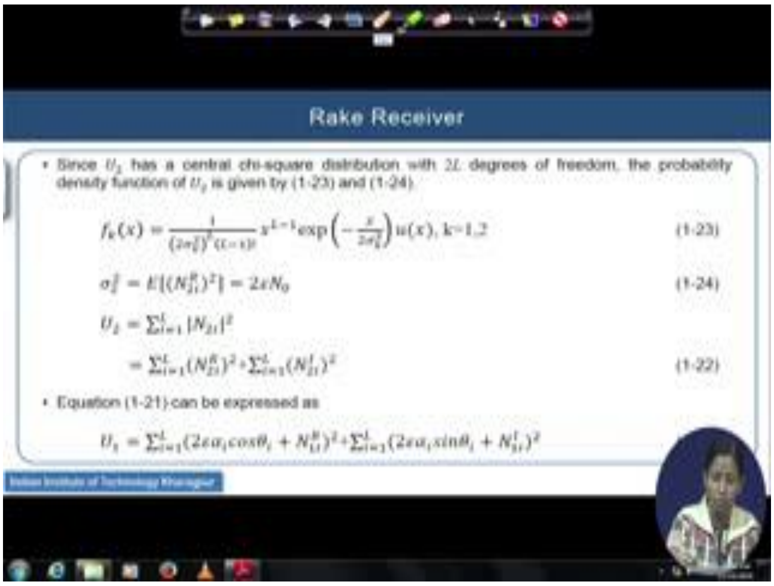

Remember, this second decision variable that we have shown it has a central chi squared distribution with 12 degree of, 12 degree of freedom. L is the length of the multipath channels. And hence, the probability distribution of that guy can be represented by this two. These are the chi squared distribution, probability density function for chi square distribution. And, where the variance is denoted by this and, so we, here we have reproduced the two parts. Once again U 2 is given by this term and U 1 is the decision variable, which is given by this term. And, we will be interested in proceeding with the portion of U 1 because that is considering the signal section measure.

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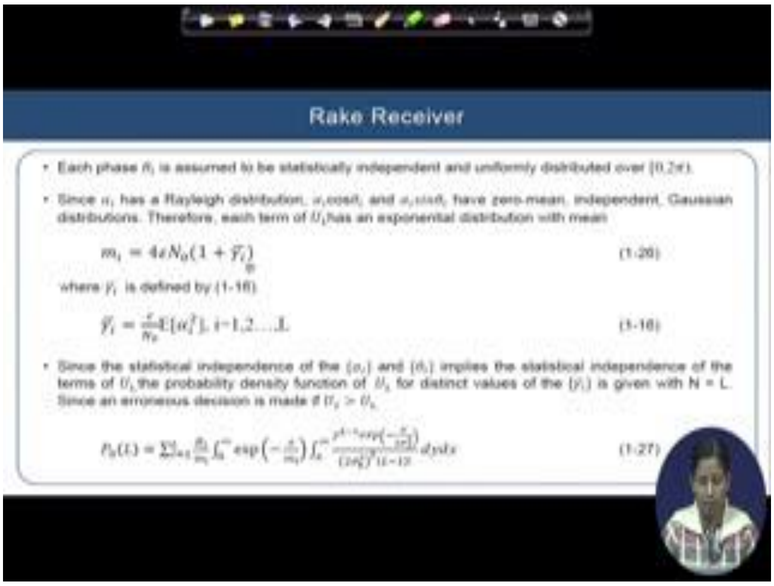

And, remember that in the previous slide, when we are considering the phase theta i. Theta i is a unknown phase associated with the signal part. And, this theta i is uniformly distributed over the duration of 0 to 2 pi. And, also remember that we have a Rayleigh fading channel. So, alpha i is all the amplitude of the channel coefficients. They are distributed by the Rayleigh distribution only. And, so if it is the Rayleigh distributors, so alpha i cos theta i alpha i sin theta i. They are having some 0 mean and independent Gaussian distributions associated here in the equation 1.25.

So, this is the real part of it and imaginary part of it, and both of them are having a 0 mean independent Gaussian distribution. So, you will get, if it is a Rayleigh distribution, you will get a mean of it. So, U 1 has an exponential distribution finally with the mean of this where this is the average SNR for the branch number i. We understand.

And, we saw in the last slides that this gamma i bar is basically taking the mean of over the power of the, it is a mean power taken from the channel that is actually now controlling the received signal power. That is why the mean is about, and, the statistical independency now this, over this alpha i and theta i that should be there and, if they are statistically independent implies also that the terms inside the U i, the probability density function of the U i, all the U1s, sorry not U i, U 1s. Are they are having some distinct values for each and every this i? I mean for each and every branch? And then, actually this gamma i bar can be given with your capital N is equal to L.

So, as an erroneous decision can be made, if your U2 is coming greater than U1; that means, the whole noise components that basically are very high with respect to the U 1. If this is the situation happening, then actually the error probability will be given by this expression. What are the independent components inside it? So, for example, B i and all, we will be discussing it in the later slide.

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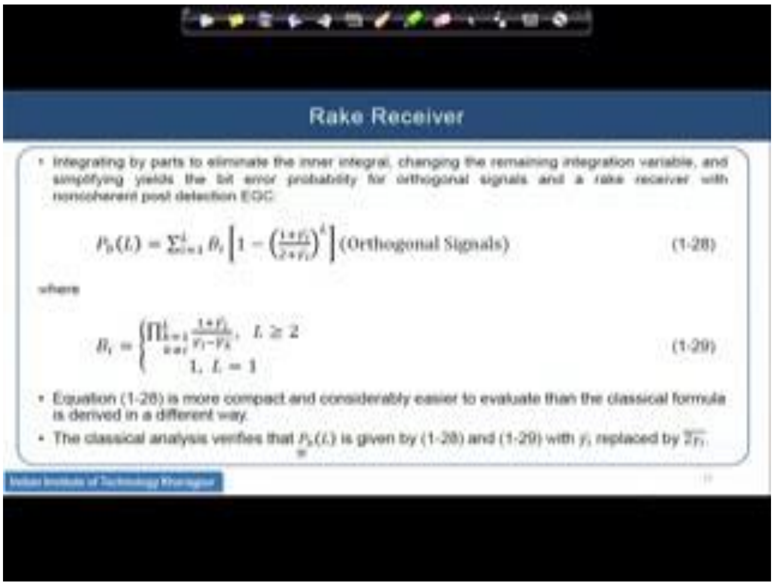

We will do the integration, integrating by parts for computing and eliminating the inner integral. And, which do some changing in the remaining integration variables. And, some simplification will yield the equation 1 hyphen 28.

Remember that we consider that that the output, the post detection, equal gain combining, combination is going on. It is not the MRC that is going on in the rake receiver. And, we will be ending up with the detection probability given by the equation 1 hyphen 28. But, this B i will be given by another expression 1.29. Remember, actually this is the more compact and that is considerably easier to evaluate, then the classical formula in a different way. And, the classical analysis of this, classical analysis of computing this probability of error over the L path, number of the paths actually that main difference of this classical one. And, this new one is basically if you substitute the value of this gamma i bar with 2 i 2 gamma i bar, you will be ending up with the nu 1.

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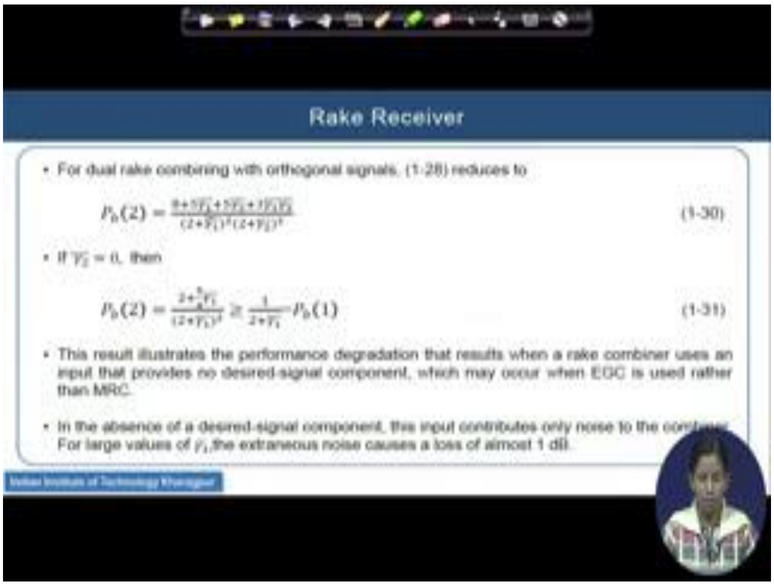

So, we had, we started with only the two different orthogonal signals we understand. So, if it is two orthogonal signals having in our hand, those two orthogonal signals, for two orthogonal signals that probability, the L will be now substituted with a value of 2. And then, this P b 2 will be simplified; form will be like this. Remember this gamma 1 bar is the average SNR over the path 1 and for corresponding to the signal 1. And, gamma 2 bar is actually the path, instantaneous SNR path, actually corresponding to the value path number 2. And so the path, total having the two paths and actually combination of these two will be easier to understand over the whole path. And, actually if my gamma 2 bar now finally if it is 0, hence I am not combining the signal receiving from the second one, then I will be ending up with the expression like this, which is it basically equal to the expression of the path 1 probability of getting the bit error rate over a single path basically.

So, this result shows what that the result illustrates that the performance of heavily degrades. If we are not combining the information over the multiple paths, the error probability will be high over a single path arrival and decision taken based on the single path and error probability will be definitely less. If you are combining the signal to noise ratio or if you are combining the information basically over the multiple paths, so the gain that is coming from the diversity, if you can combine it properly either by the EGC or by MRC, you will be finally ending up with the fact that the error probability will be reduced.

And, now remember one thing that when a rake receiver, there will be a severe degradation actually because one of these paths are giving the combiner output equal to your 0. And, it is not giving the desired signal component. And, this, not happening the desired signal component. Even if sometimes it happens, even if the path is there, it, no desired signal component may occur if you are combining sometimes by the EGC by equal gain combiner and then, rather than MRC because MRC, each and every paths is estimated. The parameters are estimated and the combination goes on.

So, when the error probability is high and if you understand that there is the existence of the multiple path in even after that this was happening, so we may actually need to take a call whether we should go ahead with the choice of EGC combination or we will switch towards the MRC. And, in absence of this desired-signal component, this input contributes only the noise to the combiner. And for the large values of gamma 1 bar, this extra noise causes the extreme loss of around one dB kind.

So, what we did actually in this rake receiver that we try to establish in this module, we try to establish the fact that if we are having, if we are having BPSK signal and only single waveform is there, the where we are ending up with actually the target was to calculate and see the error probability with this rake receiver architecture, then we try to extract the signal. We try to understand actually which kind of the combiner will be based for what kind of the scenarios. If the number of the multipaths are increasing, where the MRC will be the good way to opt or EGC will be the good way to opt. And, how actually this increment of the L is, whether it is a beneficial to us in the signal combination and gain detection or it is not, in that cases we have suggested to use EGC if the capital Ls is increasing heavily.

And then, we have considered that instead of one orthogonal signal, if you are having two orthogonal signals and how this decision variables will keep on changing us and how the decision variable will be changing and how finally the error probability will be effected. And, if actually the signal component, I mean the effective signal component is not detected at the output of the combiner, what we have already signified here, that what is the effect of the noise that will lead us to the extreme changes. Or actually, it will cause how much amount of the changes can happen or the losses can happen on the detection process and we found that approximately 1 dB loss is expected to happen in this situation.