Spread Spectrum Communications and Jamming
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Lecture - 49
Rake Receiver

Hello students, after understanding the diversity concept and wireless communication channel characteristics, today we will start understanding the rake receiver, which is the very popular means of combining and getting the diversity gain. It is a basically the main mean of gain receiving the diversity gain or achieving the diversity gain in spread spectrum communication receiver. And we will see how the receiver architecture looks like and also we will learn the performance analysis of this receiver architecture.

So, remember one thing whatever the receiver architecture you have already gone through for direct sequence spread spectrum and frequency-hopping spread spectrum communication system in the first few modules; they are true, but excluding the effect of the diversity gain. If you wish to incorporate the diversity gain into the receiver performance then you need a special kind of the receiver architecture which we today is a discussion point of ours. And today we will hence this architecture is called the rake receiver and today we will discuss on that.

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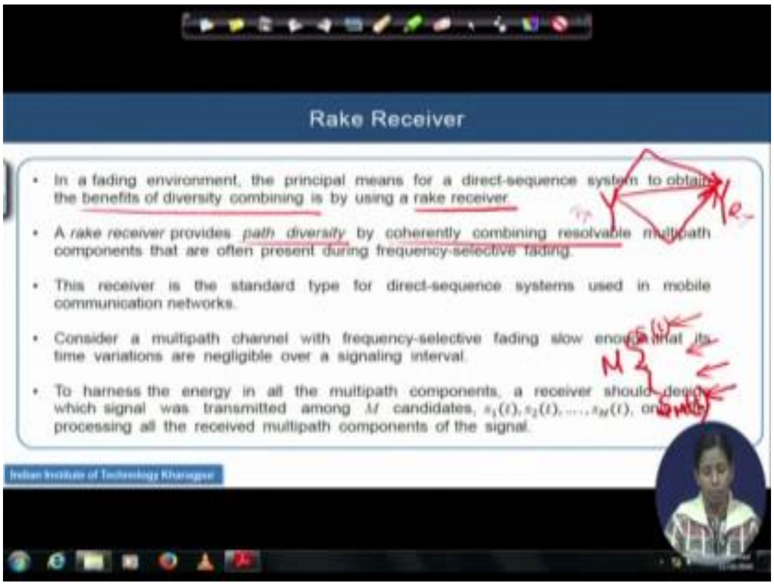

As I have already mentioned rake receiver in a fading environment specially in a wireless fading environment. It is the means of to achieve the benefits of the diversity combining and that combining is possible in the architecture, which we call a rake receiver. And rake receiver is a very standard type of direct sequence systems which are recently which are these days used in the mobile communication network. The way rake receiver works is like this, he provides the path of diversity by coherently combining the resolvable multipath components; and after properly processing them inside the receiver over a frequency selective channel.

So, what he does we understood in the wireless communication architecture, I mean wireless channel when we saw the channel I understood that channel characteristics, we saw that from transmitter to receiver, the rays does not only arrive over the line of side path. It gets scattered it gets reflected or scattered over the multiple scattering environment, and it reaches to the receiver over the multiple paths.

So, rake receiver is supposed to have the capacity to encase this path diversity. If the same signal is coming over the multiple paths and if you can resolve them inside the receiver capture each path segment independently. And hence after proper processing of those paths signals, if you can combine them synchronously, if you can do that then actually you will get the gain of the diversity gain actually we call it we will get the gain huge SNR gain over in the receiver. And this happens over the frequency selective fading, it can give a huge gain. And rake receiver architecture supposed to capable of doing this, we will see how.

And let us consider a situation where the (Refer Time: 04:03) of the frequency selective fading is going on in the channel and but the fading is slow that means, the time variation of the coefficients of the channel is relatively very slow. And relatively it is slow relative to the signalling interval. So, whatever the coefficients frequency selective path coefficients you are estimating, we consider that that is not varying over very fast with respect to the signalling interval.

And to harness the energy to over all paths for each of the for all the multipath component paths, receiver should decide what. It should decide that which signal was transmitted. Suppose, I have a M array signal. And if it is an QPSK signal, you will have at least a two branches of it; if it is an M array signal, you will have M different signals corresponding to the transmitted signal. So, we will have depending upon the Bessel's functions, you were using you will have multiple such signals combinations of such signals constituting your signal. If it is a 16 QUAMP signal, so you will have the numbers of its values. So, consider an array of the signals M array signal, where the number of the signals constituting this array varies from S 1 t to S M t the task of the receiver is to find out that which of these signals have been transmitted, also resolve the corresponding and multiple multipath components corresponding to these transmission. So, that is the task that we are going to discuss.

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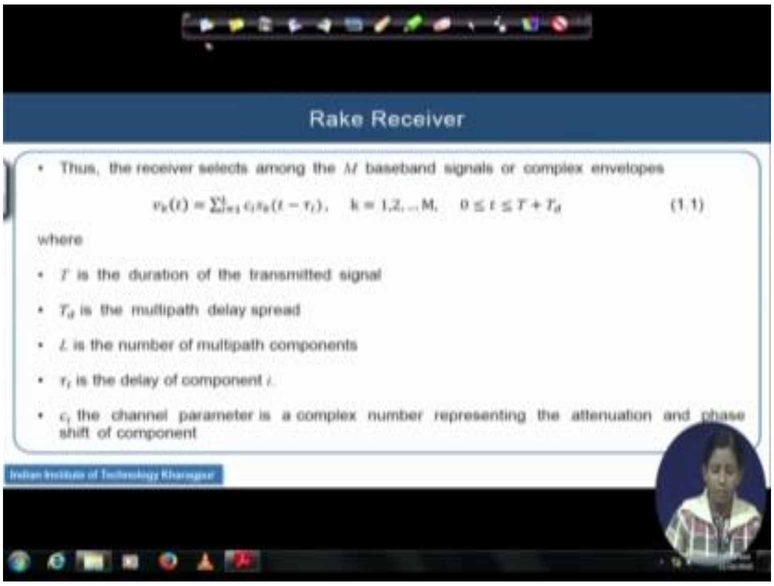

And for that, we have chosen our environment where your frequency selective slow fading environment it is, and hence the estimated channel coefficients and channel phases are not expected to change very frequently over time related to the signal duration. So, the receiver tasks is what, receiver task is to select among all the N number of that is band signals which one is transmitted it selects actually one of those capital M set S 1 to S M. And it actually compute also the complex envelopes of it. But fundamental point is that the correspondent to each and every signal component he actually enables he combines the effect of all the multiple paths in the rake receiver.

So, you see the received signal at the rake receiver is something like that. This is your transmitted signal s k t minus tau i, where k can vary from 1 to capital M, because I understand that this signal is drawn from capital M array signalling. Small t is actually this t will be varying factor, and he will have a variation up T plus T d, where this T is duration of the transmitted signal and T d is a multipath delay spread maximum can happen over the giving channel. And tau i is delay of each and every multipath; corresponding to that multipath delay we are having a channel coefficient c i associated with it is a complex value channel. And to remember that o can vary over i to capital L, we understand that capital L is the number of the multipath components present in a environment.

So, the received signal and the output of the rake receiver will be either combination of all those multipath signals for a typical signal transmitted from the set of the baseband signals. Remember actually which of those in signals we are transmitted in order to decide on that, rake receiver should process similarly by varying k over 1 to all number of the 1 to capital M I mean all available number of the signal set. So, he independently and separately processes all those inset signals and then take the run the decision over all of them and then decide which of the signals have been transmitted. Please remember this equation 1.1; we will refer this equation again and again in the corresponding slides.

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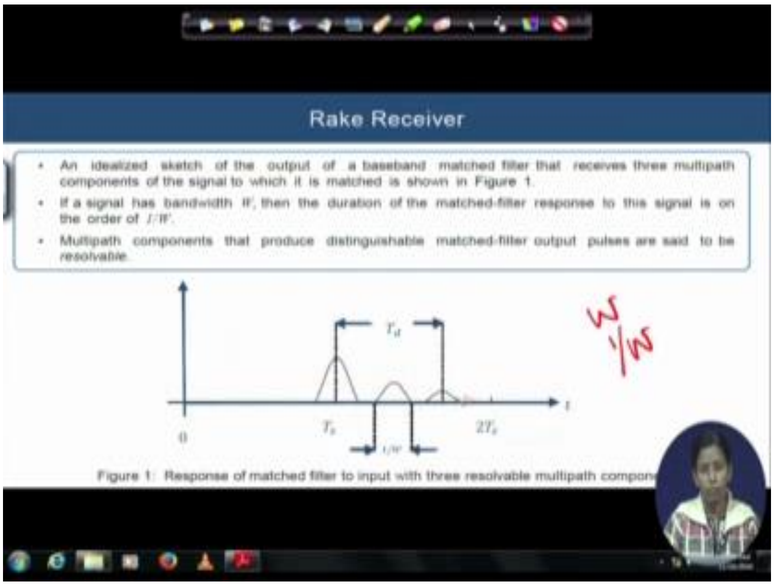

And let us sketch actually and understand what is going on in the environment, and how the receiver is receiving the multipath signals. This is the figure one is an idealised sketch of the whole situation. Remember the output of the baseband matched filter, if the filter is matching with the three components of the multipath, so output of the baseband matched filter for all those three paths will look like this. We have this is the power level receive, so I have reduced the power level considering the fact that more the delay you are tracing the power is getting slowly reduced because it seems that the more delay means your ray has traversed more path. And hence the possibility of absorption of the power over that path will be high. So, you are receiving a very low amount compared to the previous one.

Then if the match filter is matched with the individual three multipath say then the output of these baseband-matched filter will look like this over the delay axis. But remember one thing if my signal is having a bandwidth of capital W then the duration of the matched filter that responses to that signal W, the duration of the matched filter should be 1 by W. And hence the multipath components that produces the distinguishable matched filter output pulses, we call them the resolvable multipath.

So, multipath may be actually also in between, but the resolve based on the zooming capacity of the receiver, I mean the bandwidth of the receiver circuitry, and it is dependent upon that actually the how good resolvable, how resolvability is possible, how much to what extent the resolvability of the receiver works. And hence how many number of the multipath, you can resolve at the output of the baseband matched filter. And then those multiple components are called the resolvable multipath components. So, according to this figure we have three resolvable components. Remember one thing we have considered that s is the sampling the duration of the symbol, and tau d is our delay maximum delay that we they will consider for the analysis.

Another important part to notice here is if the delay this total delay is less than that the symbol duration. So, this be a multipath spreading and this delay spread and hence the dispersion of the signal would not contribute to any inter symbol interference because it would not overlap with the next symbol coming up at the moment of 2 T s. But if this not the situation, if the multipath components are having significant power, and you are continuously watching them over the delay axis's and all are the resolvable multipath, it may happen in such situation.

In certain situation that they are having significant amount of inter symbol interference because of the dispersion time dispersion of the delay dispersion over the delay axis of the signal. It also happens to be creating the inter symbol interference if the data date is very high and hence your time duration between the symbol duration will be less. And if the multipath are having the delay spread of the channel is having very high then in that situation you will ending up with very high amount of inter symbol interference.

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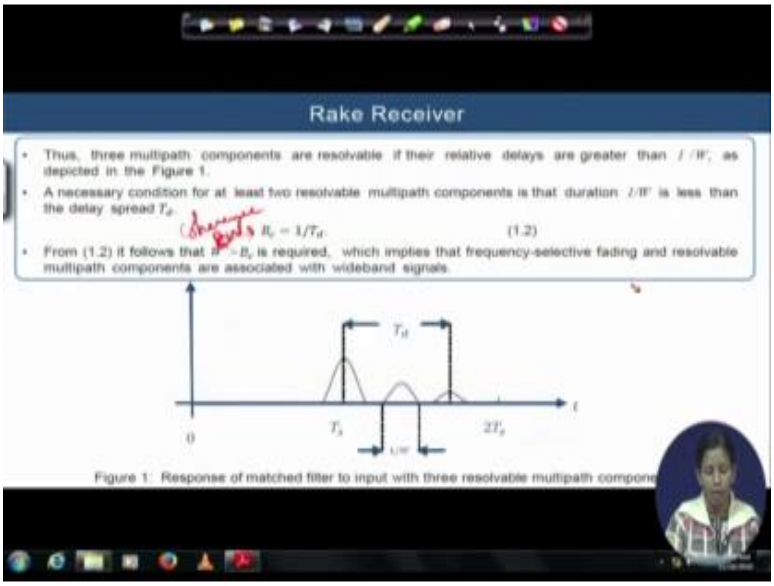

So, they are resolvable the three paths in our consideration will be resolvable. And if their relative delays are greater than 1 by W, in this situation we can resolve them. So, we understand actually the necessary condition that for at least two multipath you can resolve. The condition is such that the duration 1 by W, I mean the delay the duration 1 by W should be less than the delay speed of the tau d. So, in that situation only there is a chance that at least two multipath you will be able to see.

And we have already understood and read that the coherence bandwidth of the channel of the B c it is the coherence bandwidth of the channel. So, the coherence bandwidth of the channel is having some relation with the maximum delay spread. So, this coherence bandwidth can be given by the basically it is the rms having a relation with rms delay spreads, roughly it can be approximated with the T d the maximum delay spread that we are having. So, B c is equal to 1 by T d.

It follows from here that if I now wish to if I have a signal bandwidth which is more than the coherence bandwidth of this and which implies that the frequency selective fading and the resolvable multipath component, so both are actually associated with a very wideband channel, wideband signal sorry wideband signals. So, actually we understand that with the coherence bandwidth in order to have coherence bandwidth actually if it is in our frequency selective situation, when the signal bandwidth will be definitely more than the coherence bandwidth situation, and the number of the resolvable multipath components and all the phenomena is getting associated with this frequency selective fading as well as with the wide bandwidth signals.

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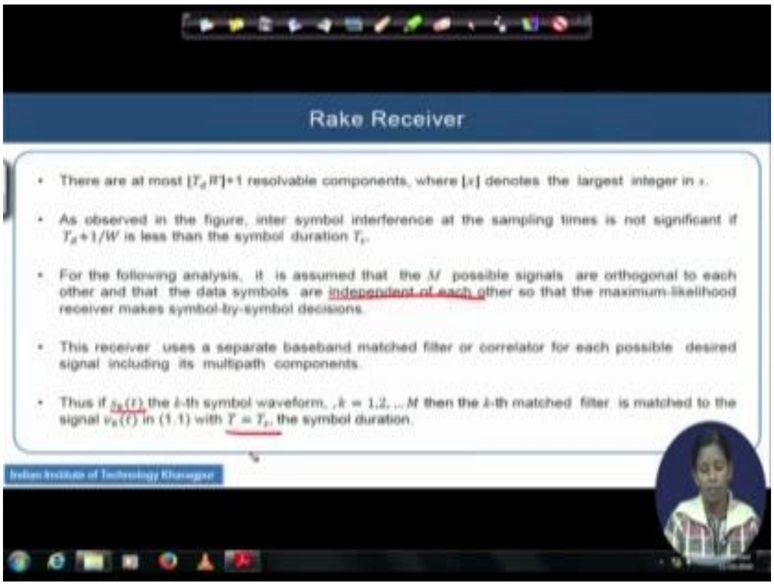

So, what we will expect in such situation is we can expect at most the number of the resolvable multipath or the components should be the T d into 1 by capital T, because W is equal to your 1 by capital; 1 by w is equal to your the time of; T is equal to 1 by W. So, this is the time, this is actually the period by means of which the sampling is going on inside the rake receiver or the bandwidth matched filter bandwidth I should say the matched filter bandwidth is working upon. And hence it is basically for substituting this whole equation is basically your delay divided by the capital T. So, number of the multipath that you are getting is plus 1, because the first one is obviously, will be achieving from that actually your matching should start or the delay competition should start.

So, T d by T, if you take the largest integer part of this expression plus one, this is the number of the resolvable multipath components that you can expect. So, see the number of the resolvable multipath components highly depends on the bandwidth of your matched filter. As we have seen there that the inter symbol interference as I have already informed the inter symbol interference to come into picture either the data rates needs to be high or the T d by 1 by W, if this whole part is less than the symbol duration, you want to get any inter symbol interference. Otherwise unless you will get any one where is changing or if your data rate is increasing.

And for the next part of the analysis, let us assume that this capital M possible signals sets, they are all orthogonal to each other. And also let us think that we are getting we are receiving the data symbols which are all independent to each other and so that in the receiver the maximum likelihood receiver it makes the decision about which symbol has been received that decision is going on symbol by symbol basis. So, this receiver is uses now separate baseband matched filter for each and every symbols that they will be dealing with. And every symbol will have a possible desired number of the multi paths component associated with this processing. So, if we start with the signal called S k T for the kth symbol, and we understand that this is the symbol wave form. And for the kth symbol that k can vary from 1 to capital M, then the kth matched filter output will be that equation 1.1. We started with it is a signal, now it will also hold good for a symbol that is the meaning where actually you have to consider that capital T is not the signal duration now this is the symbol duration T s.

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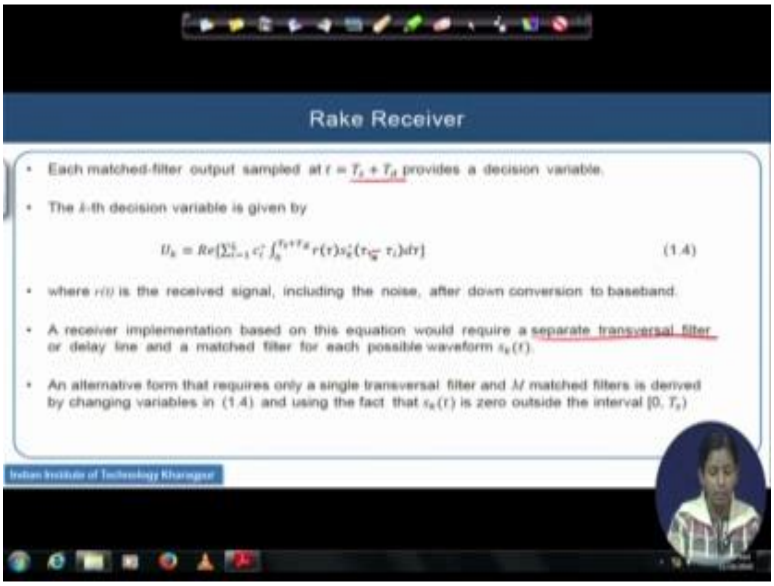

So, now the each matched filter output now you have to sample, the sampling should be done at the gap of the symbol duration plus T d, it is not T plus t d it should be T s plus T d if you are taking the sample. And the decision variable it can be given well established result it is, so it can be given by the taking the real part of this equation 1.4. Remember what we have done here this c i is the summation is going on over the all number of the multipath, you have taken the complex valued coefficients of the channel from each and every path. And this is the signal you have received the matched filter associated with it. And you are integrating it over the duration of T s plus T d. Because that is the symbol that is the sampling duration that we have reached your interrogation is going on that the sample duration over which you are going to get the samples and take the real part of it and take the decisions whether the symbol transmitted is 0 or 1.

And when I am talking about this r t is the received signal, this signal is spread signal with the noise as and it is after down conversion only, you have taken over the down converter signal only it is fade here. So, it is a whole baseband decision variable is running on that your baseband. The receiver implementation now if I try to do based on this, it is seems to be need that it requires separate transversal filter for each and every symbol of the delay line. And then a matched filter is required each and every associated with each and every symbol or alternatively I can do something else. A single transversal filter will be there, and M number of the matched filters I will deploy. And we understand that this s k, it does not exists beyond the duration on this symbol interval 0 to T s.

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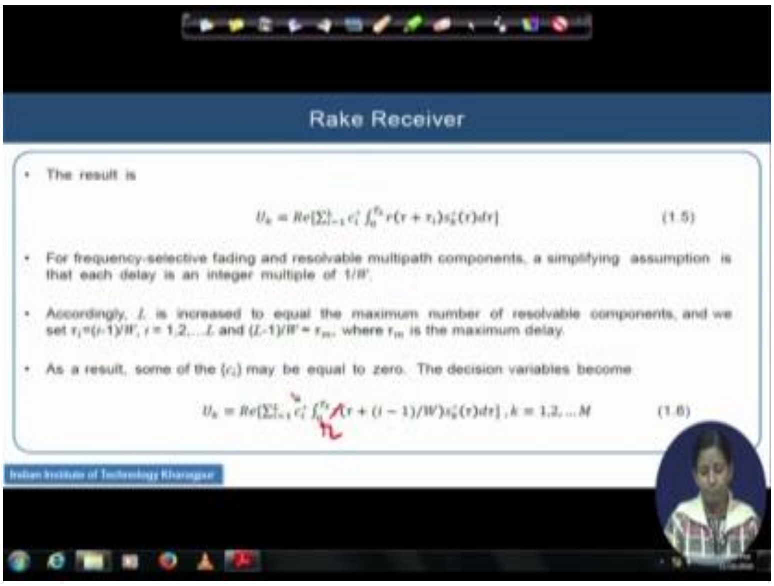

If I try to do that, then the alternative figure would look like something like this. I will come back here, but prior going there, we would like to revisit the decision variable how the change is happening, if I change that is structure like this that I will have a single transversal filter with M number of the matched filter and then how this will change. This will now change like this as if I am having the duration of 0 to T s. And this r tau is having tau plus tau i; tau i will be given by the number of the it is delay associated with each and every multipath, but it has a maximum value which will be defined as the tau n which is the maximum delay. So, tau can vary from tau plus tau n it seems. And then s k star is the matched filter function and then the matched filter will be running over the tau.

And for frequency selective fading and the resolvable multipath components, this simplifying assumption is that each delay is an integer multiple of this 1 by W that is the matched filter duration and filtering duration. So, accordingly this l is increased to equal to the maximum number of the resolvable components only in our discussion. And if I try to go ahead by this equation 1.5 finally, and we may see that some of the components, so for coming from the channel, I mean this c i values may be equal to 0 also. And the decision variable then finally, can be written like this, where this one will be this will be now tau, this will be small r.

So, finally, this guy will be replaced by this r r tau plus i minus 1 by w this i minus 1 is nothing but the expression that we are getting for the expression of the tau m. So, i is varying up to capital L that is why if you substitute L minus 1 by W, 1 by W is the gap of the arrival of the multiple paths as we have understood. So, this is the match to that and then you are ending up with the maximum n minus 1 by w is giving by the maximum delay spread tau m. So, we have just replaced this tau i by this expression where i is varying slowly from 1 to capital L.

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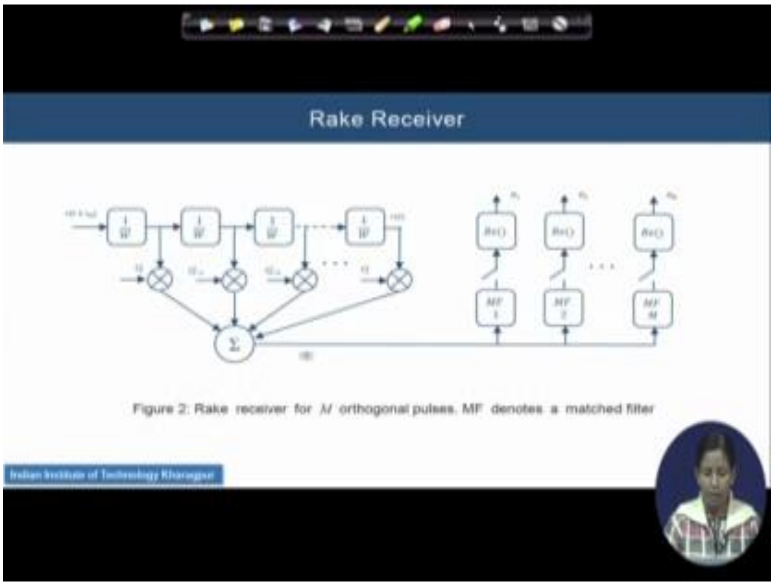

And with this we are now ready to go with the figure. Remember one thing that we have seen in the earlier expression, so let us revisit this expression once more, how this is a final expression we are ending up with. Remember a situation that we are having the received signal, which should have the delayed up to the tau m. So, we are having the final signal r t, this is the first path came and then the delayed paths are actually having a gap of these are the multiple path arriving at a gap of 1 by W. And the last one that we are going to receive is t plus tau n and this is the structure of the transversal filter and at the output of the each tau. So, output of the final top of the filter, you are getting this signal r t and others are giving the delayed pulse multiple pulse also.

This if I draw it like this, so this was the with respect to tau equal to 0, this is my tau axis, so I received the first path say tau is equal to 0 at tau r t; then the second one came, third came, forth came like that the r t plus tau n came at last. So, when I saw the filter structure, I will be here and these are all the before pulses that came earlier. So, all the pulses have been shifted like this, and you are ending at the last one the last multipath is entering here. And all the filtered taps the output of the filtered taps are accordingly adjusted this at the c i’s that you are adjusting. And according to that the traversal filter architecture, we have filter architecture we have added it because this is the addition happening this the filter will be happening multiplying with it and then it is adding up, so that is the addition is done here.

And now the output of this filter, they are fed into the matched filter M number of matched filters we are having. And the real part of all those finally, after having them matched filtered output; this is the multiplied with the matched filter. And then after the matched filtered output there the real part of that you are taking and that operation is going on for each and every k symbol. So, you are having capital M number of the symbols and corresponding to the each and every path you are getting either u 1 to u M.

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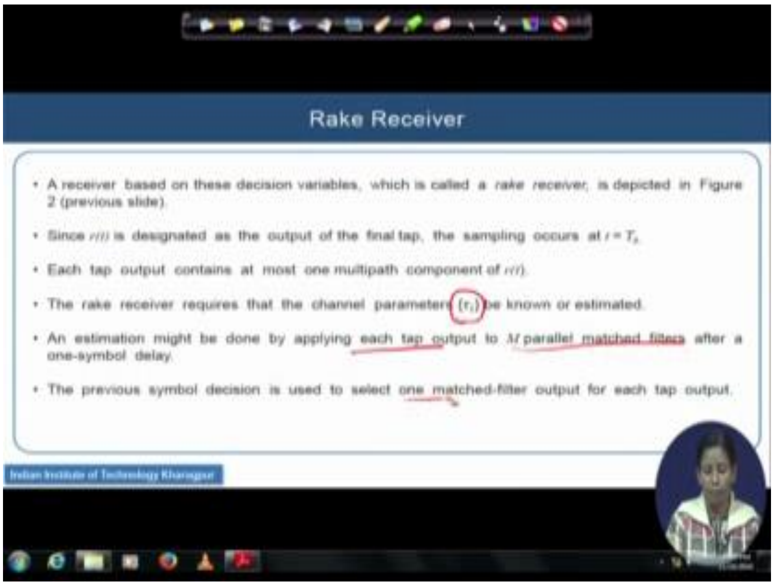

So, if this is the way that we have this is just called the whole architecture is according to the last equation that we have developed and then the decision variable will work saying the taking the actually decision of N symbol by symbol basis. So, remember here one thing that how will you know that the values of the c i, I mean the channel component values how will you compute is a very big question in the whole rake receiver architecture.

So, what I mean is that the channel parameters need to be known with the delays as well as the c i's, otherwise you would not be able to actually design the filter coefficient and the filter can be said that the filter parameters. And this estimation is done by applying the each tap output in the filter to the M parallel matched filters after a one simple delay. And the previous symbol decision is used to select in one matched filter output for each of this tap, for each of this tap output.

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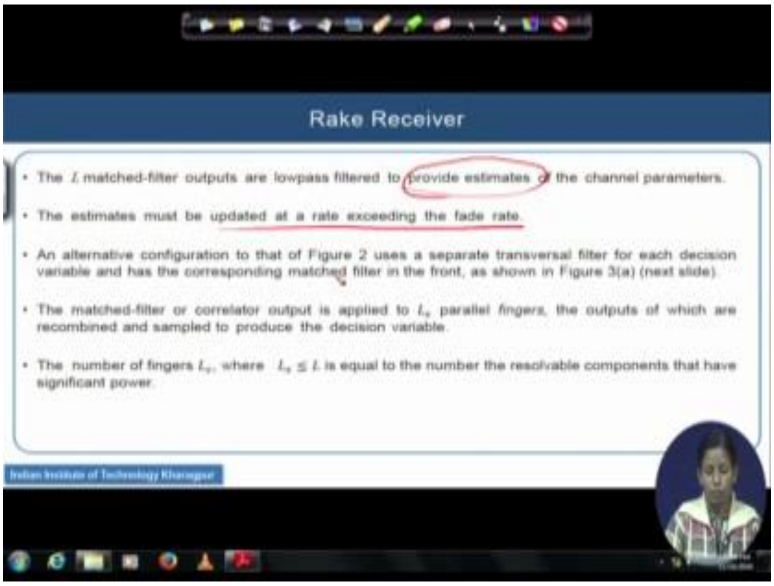

So, now that capital L matched filter outputs are low pass filtered and then it is providing us the estimate of the channel parameters. This estimates we should update at the rate of which should not exceed the fade rate. So, it should be actually at per the rate of the fading and the rate of the slow or fast fading I mean, so that estimation should be according to the rate at which the channel is changing.

So, alternative configuration of this previous one, we can have in the figure two where we separate transversal filters we can also apply for the decision variable and has a corresponding matched filter for each and every symbol in the frontend and then can have the parallel fingers you call the rake fingers and the output of the matched filter. And remember actually for those filters; let us see first the structure, the alternate structure of the previous transversal filter. What we did is we have received the signal and we have associated a matched filter to that signal.

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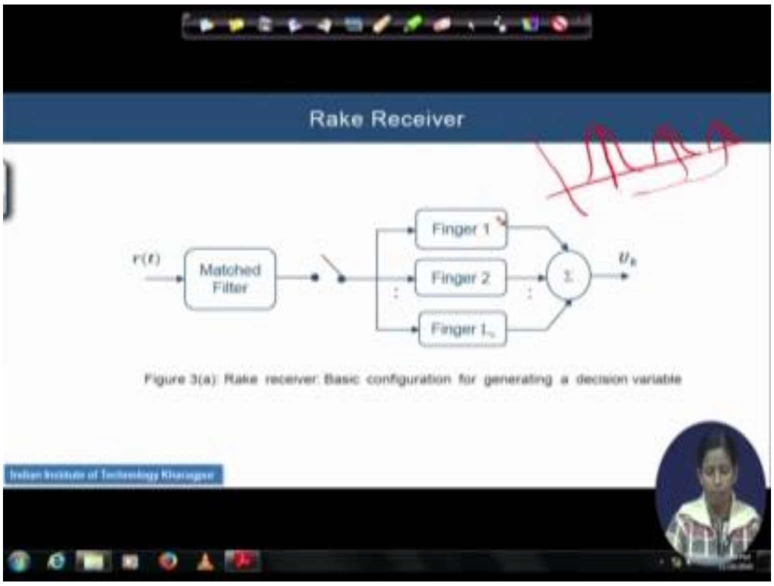

Output of the matched filter is sampled and the sampled output is fed to the fingers. Remember the structure is such that the matched filter output, if it is they are matched to the structure. You remember the first figure the matched filter outputs will provide like this. So, if this fingers are such that the fingers are coinciding picking up actually they are picking up the corresponding multipath they are having that typical delays associated with it; inside the finger you have the correlators like this.

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Inside one finger, we have a correlator. So, if we have basically actually the matched filter output the sampled output is entering into a finger, and it is delayed by a specific amount that delay will be the delay of the corresponding multipaths, and then you have a channel estimator running with it. So, you need to multiply the c i with the delayed path to undo the effect of the channel with this multipath component, and then we take the real part of it, so that is the motivation. And remember how many number of the fingers the number of the fingers are expected to be equal to the number of the multipaths, so resolvable multipaths that we are interested in. But in such a situation, if the channel varies the number of the resolvable multipaths also will be varying. So, how will you keep on changing the finger size, if you deploy the rake receiver finger fixed size of the how come you vary the number of the figures in a practical circuit is a very big question.

What we do is that we try to define here the significant number of the multipaths, multipaths may be a normal seen number that significant multipaths means who are having a significant amount of the power, we actually are interested in all them we shift all that and we actually coherently we process try to process them and adapt. So, if I independently and coherently time shift all that and undo the effect of the channel for each and everybody and then add them up here. So, then finally, I can actually take a decision of the transmitted signal based on the output seen here definitely the real part of the (Refer Time: 29:57) we had interested in.

And remember that in practical circuit, the fingers are not variable. The fingers are always fixed. So, we try to keep the finger fixed, and whatever the multipaths we can capture based on the maximum one. So, l s will be all if the maximum number of the fingers present in a structure the rake receiver architecture is l. So, l will be always greater than or greater or equal to l s that is the concept for the architecture of this fingers. And you know actually definitely process this section or this fingers in the digital domain because the matched filter output you are getting in as a sum. So, it is already sampled and. So, it is the task of the finger to get aligned with each and every multiple path that you are receiving corresponding to one symbol.

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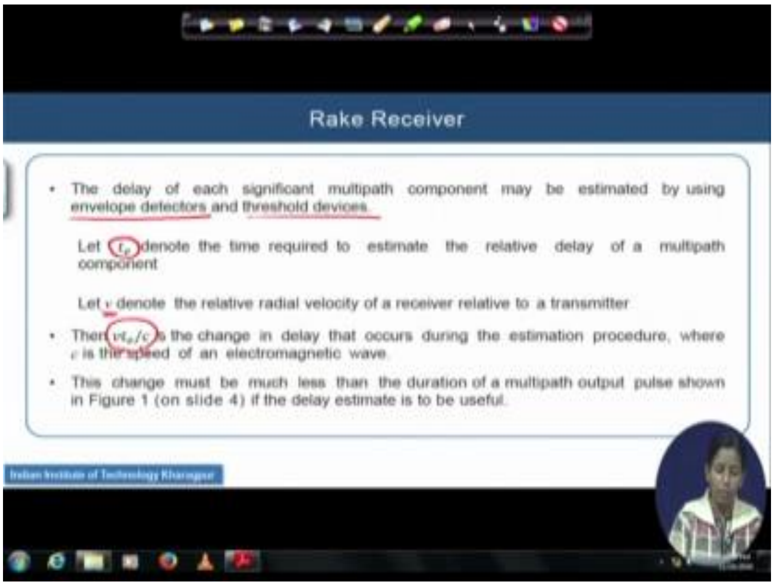

Now, two more important part, the delay of this significant multipath component, they needs to be estimated. So, they can be estimated by using some envelope detectors and some threshold devices also. And suppose the t e denotes the time required to estimate a relative delay of a multipath component. And let v is denoting the relative radial velocity of the receiver relative to the transmitter. So, this receiver velocity and then v into t e by c this is the rate that change of the delay it should be where c is the speed of the light.

So, this is the rate at which the electromagnetic it is the rate at which the delay is expected to change. And basically that of actually if a this change occurs and within that within that your estimation procedure within the estimation procedure this is the change has happened at this rate then your estimation process itself is not going to work finally. So, how actually should be working that is the basic relation between this estimated time t e and the relative velocity and the bandwidth at which at which the selection band bandwidth of the matched filter how does work together, it is like this.

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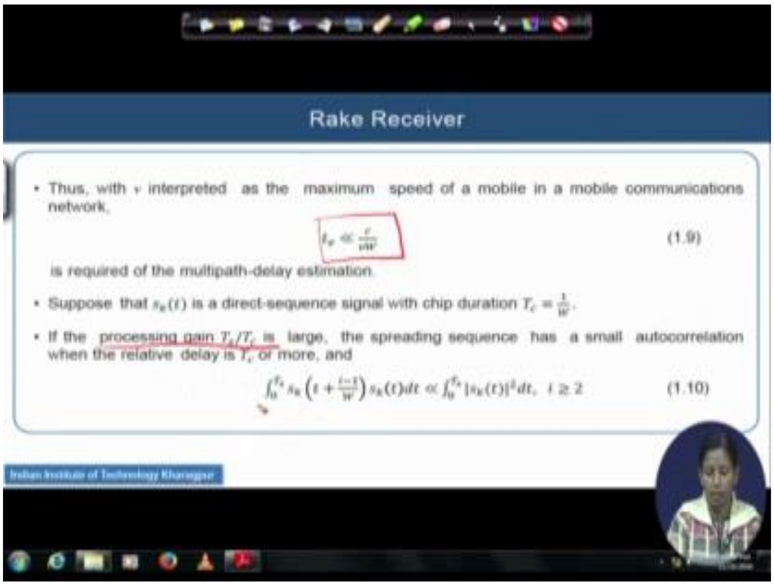

So, if we know the v, it is interpreted as the maximum speed of the mobile in a mobile communication network then the estimated time should be always less than equal to the c by v into W. So, this W is the parameter of your own choice, v may be actually freely moving by it is a velocity of the mobile with which it is moving, but you can actually do the computation with the maximum speed possible in a mobile environment. And you can compute the maximum limit of this estimation t e for the multipath delay estimation inside the receiver, and that is the very golden rule that you have to follow to update the delay estimation inside the rake receiver.

And now the last one is suppose that S k T the symbol that we are transmitting the other sequence signal that has a chip duration T c, which is actually equal to 1 by W. And if then the processing gain is really very large, the spreading sequence has a very small autocorrelation when the relative delay is T c that if the relative delay is itself T c or it is more than the autocorrelation will be actually very small. And hence in this situation, it can be easily written that this expression left side expression of the symbol with every delay if I shift and then I multiply both of them over 0 to T s that is the symbol deviation. Then it should be always less than equal to the actual signal squared it up for i definitely greater than equal to 2.

So, these are the two important part that we will carry forward on the next module when we will do the performance analysis of this rake receiver further in order to find out what is the error probability for a typical modulation scheme coming from the rake receiver. But remember there you have to keep in the mind that whatever the estimation process parallelly running inside the rake receiver, it is following this rule; and also we will consider that the processing range is really very high such that this equation holds good.