Spread Spectrum Communications and Jamming
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Lecture – 48
Diversity for Fading Channels

Hello students. Today's discussion will be a diversity concept over the fading channels. In the last module, we have learnt about how wireless communication is about what is the meaning of fading in wireless channel, what are the different kinds of the fading that we deal with in a particle system design, and we have seen based on the RMS delay spread of the channel and the coherence time, coherence bandwidth. We can have the four different combination of the channel situation namely a frequency selective flat frequency selective slow, frequency selective fast fading, flat slow fading or flat fast fading channels.

And also we understood that was the signal is released from the transmitting antenna, it never reaches to the receiver via single path rather it reaches over the multiple paths after getting reflected and scattered by the difference scattering objects and elements of the wireless environment and that complete phenomena is called the multipath propagation of a signal.

Today, we will try to learn what is the meaning of diversity over this multipath channel propagation. So, it is a situation where we can is there any gain of this multiple path reception of a same signal in a receiver frontend, can we get a gain out of it or it is totally a loss always. Already we have seen in the last module that because of this multiple path propagation signal gets dispersed - the bandwidth gets dispersed, and the over the time axis it is getting dispatched, over the frequency it is getting dispersed, because of the dispersion may be lot of the signal processing we need to do in the receiver that is easily understandable. But can we get any gain out of this multipath propagation today we will try to learn that.

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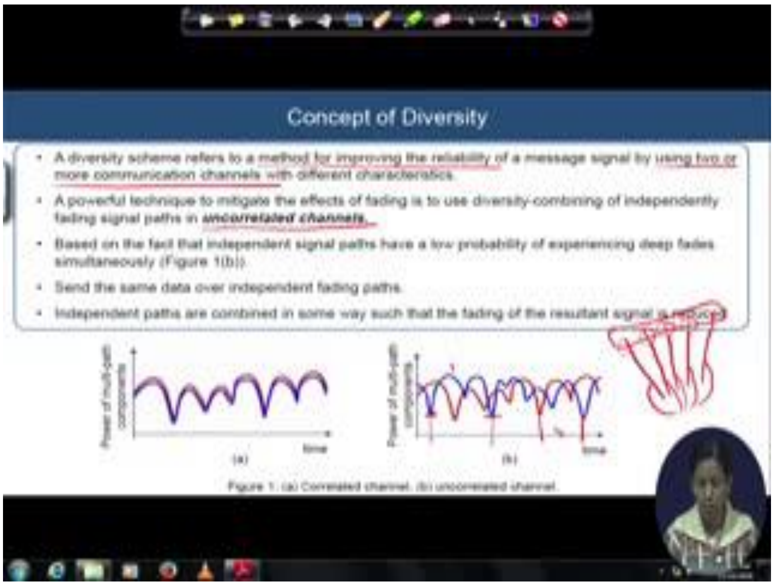

So, what is the diversity? See, diversity schemes they refer to a method, a method for improving the reliability of the message that we are receiving; and if we can improve the reliability of this message by using two or more communication channels, and remember all those communication channels needs to be different with in characteristics, they need to be completely independent with each other. It is something like that if I am giving you the multiple channels or say multiple pipes, and if you are about to send the same message signal, and if you replicate the same signal over the multiple paths, and here all the paths are actually independent to each other and having no correlation with each other.

Hence, there is no interference actually, and if you receive there is a replica of the same signal independently in a receiver, so by combining them in some way you can get a huge gain over the noise. A signal power can be boosted up several times over the noise as well as over the in presence of the interference also. So, this is a fundamental concept of the diversity. So, you used that multiple (Refer Time: 04:10) replica of the same signal over the multiple channel. So, by some means of the way you would multiply it and receive it over the different way, and combine it in such a way that reliability of the message can be improved if I can improve the received signal-to-noise ratio.

So, diversity actually here can be done by several means we will discuss little bit later. But if it is doing it for through the channel we need to have the uncorrelated paths or uncorrelated channel that I have explained here. This uncorrelated channels I have drawn here into at least I have considered two different channels here where the channel properties are actually different over the different portion over the different if a moves along the time axis.

And this channel condition is such that the channel number one it is a blue line he is having some deep fade; that means, the amplitude is getting heavily down and it is totally it will be down actual for this time periods, time axis, this time axis over the time instant this over time instance this and over the time instant this for the sorry whether time instant this. So, these are the conditions say actually for the blue one the time instance one the time instance two and the time instance three, the amplitude will be heavily down of the transmitted signal.

For the channel number, if it is a channel number one - for the red channel, the amplitude will be heavily down for the channel instance say 1 dashed and then the channel instant 2 dash. So, remember one thing the wireless channel if they are multipath channels I mean to say if they are uncorrelated with highly impossible that both the channels will give you this exactly same kind of the profile over the same time domain. So, at a particular instant, if you check the condition if you check the profile of the two different paths one path you may see actually heavy fade channel heavy deep condition, but another channel they are wound be are that much deep situation.

So, if I combine the received signal of these two instances at a particular instant from these two different channels, so by combining the effect properly way I can receive my signal properly. Whereas, if I use only the blue channel at this instant one, my signal received signal power will be heavily down and I may not be able to detect the signal also. But if I take the combination of the channel one of the blue and as well as the red channel there is a high chance that I can actually discard one reading and I can take the other reading or I can take the average of these two reading. And actually I will get a very pretty good amount of the received signal power over the noise and which will help me to properly detect the message. So, that is the way actually we there is a meaning of the diversity that we are trying to bring here.

So, remember one thing, on the left side, I have drawn a channel that two certain multipaths over is certain instant of the time if such that they are heavily correlated, so their profiles are actually correlated in the means the change of their amplitude and the phase profile of both the channels are going hand in hand almost. There is not much difference. So, in such cases whatever the effect the signal will experience in the blue channel, he will experience the same kind of the fading path in the red channel. And hence there is not much diversity gain possible if I combine if you choose these two channels for the transmission for diversity.

So, channel to get the diversity effect or diversity gain, the fundamental point is that you have to have some differences between the two paths, and they should be uncorrelated that is why we say they are uncorrelated; here correlation between the two channels are very high, and here the correlation is almost zero kind of; so, more the correlation is less, more you will get the gain for the diversity combining.

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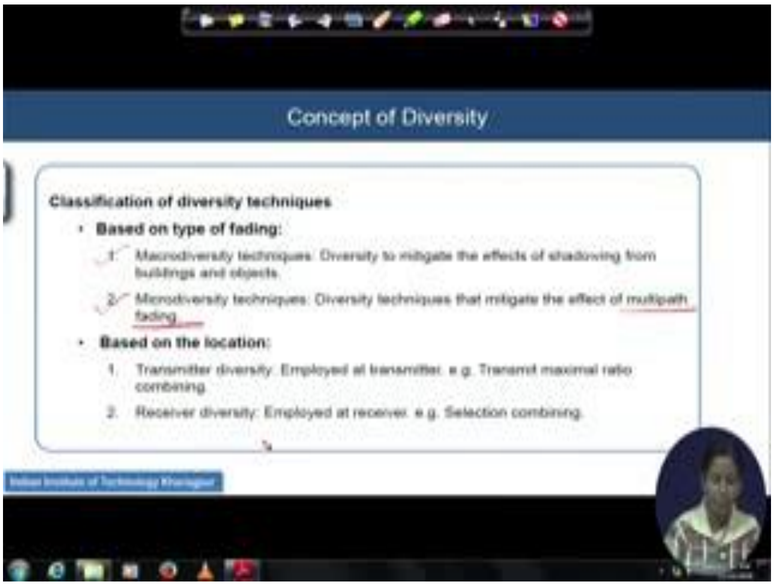

So, all this independent paths will be combined in such way in the receiver. Remember there are several ways of combining the signal received over the multiple path that we will discuss later on. So, the types of the diversity techniques let us quickly revisit. Based on the type of the fading we have two different kind of the diversity one is the macro diversity as well as and second one is the micro diversity. The macro diversity technique is to mitigate the shadowing effects that which are very common situation happens when outdoor communication goes on and it is effect coming from the buildings and different objects. So, to avoid the fading that is happening because of the shadowing from the buildings, and the objects in a wireless environment, we go ahead with the macro diversity techniques.

There is another thing which is called micro diversity which is essential to avoid how to mitigate the effect of the multipath fading. And most of our schemes that I will discuss here is about the micro diversity. And based on the location also, there may be two different kind of the diversity; one is the transmitter diversity another is a receiver diversity. So, diversity is employed at the transmitter I mean the transmit maximal ratio combining schemes and all they are going ahead or you are doing the diversity combination in the receiver, where you are providing the diversity effect, where you are actually employing the gain.

The gain will be wherever you are where actually the diversity effects is employed based on that within the transmitter side or in the receiver side then we call it transmit diversity or receive diversity. The example of a good example of the receiver diversity is the maximal ratio combining or selection combining or equal power gain combination all that.

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Based on the resources diversity classification will be like this. Number one is time diversity. What you are doing, in time diversity in time diversity the same signal you have certain time to for transmission, and you can divide that time over the smaller epoch. And this over thus small epoch, you are repeating that signal over the each and every time in a path. So, the signal that you are transmitted here, here, they are in basically same, but you are repeating the same signal over the multiple time intervals. How you do you feel that it will give you gain it will give you gain because we understood already that the channel condition never remain same for a wireless environment.

So, it is a expected that even if the channel here is in the first time duration normally first three time duration channel condition is extremely bad, bad in the sense actual means the effort to the fading effect very deep fade are happening. So, I am not going to expect actually very good signal in the receiver frontend, but after sometime there is a chance that the channel condition or the fading situation will change in the time duration fourth and fifth and then you will get a very good signal power, so receive signal power will improve. So, this is a called time diversity where the same signal is transmitted over the multiple time intervals.

Frequency diversity and finally, the decision over the time diversity channel will be going on by taking an average over all the time approximately or based out of all that or may be a combination of the base two or base three maybe any three logic you can apply in the receiver. Frequency diversity case, what we do we have a given bandwidth for transmission, we will actually divide that bandwidth over the small chunk of the frequencies, and we will keep on resending the signal over the same signal over the multiple frequencies. So, it is something like that your signal is repeated over the multiple frequency bands.

So, this is a frequency over the frequency axis wise, you are having the diversity and by combining the signal over the multiple frequency bands in the receiver, you can have a narrowband filters and you can filter out and you can decode detect the signal over every frequency bands. And finally, at the output of the detectors, you can combine them by the best possible way you think is possible.

So, in this situation also again the consideration is that the condition may be very bad over any part of the frequency. So, some part of the frequency bandwidth channel is behaving heavily badly, but fade is very high, but for the other frequency bands actually there would not be a condition. This situation is very good actually to avoid the interference also because interference of the narrowband interference. Sometimes if present, they present in any one of the narrowband. And even if actually your signal is getting heavily affected because of the presence of interference maybe one of the frequency bands, you will be get rid of them if you combine thus portions of the other frequency bands. So, frequency diversity can give you an inherent resilience against the interference also.

The third one is the space diversity. We use the multiple antennas, which are different, which are spaced apart. And we transmit the same signal from the multiple antennas in the transmitter side and the way it traverse via the multiple paths, and it is reached to the receiver frontend, and we combine them by different logic. So, it is a space diversity that you are getting, this is a time, this is a frequency and this is a space.

We can also have the angle diversity. So, we can restrict the receiving antenna beamwidth to a certain way that this is actually heavily focused maybe towards this antenna sometime, then it may be focused on this and this may be focused on this. So, it you are restricting that you are zooming you are zooming basically at certain portion of the part and you are receiving actually the energy over the small selected zone of the angle in the receiver section. So, we call it the angle diversity as the angles are different from in this three beam patterns, so you can actually get rid of the interference also if the interference is coming from this angle or from this angle. So, it is the angular basis filtration you are doing, and you can go ahead with the combination of this filtered output in a best possible way you feel actually good for the signal detection.

The fifth one is the polarization diversity. Using the antennas with the different polarization as derive the vertically polarized or the horizontal polarized or circularly, symmetric one, you can go ahead with different diversity over the polarization axis. But remember, the more a time frequency are the easiest one to go ahead with, and space polarization angle diversity they demand the complicated transmitted receiver architecture.

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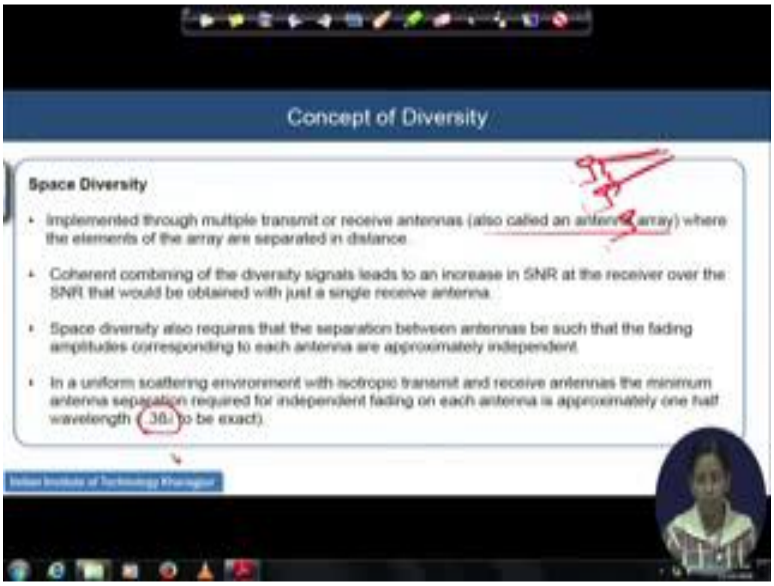

In the space diversity, we implement the multiple transmit or the receiving antennas, we called them the antenna array, I draw it in the last slide, where this elements and the arrays are will be separated by a distance. The coherent combination of all these array elements I mean this signal received from these array elements, we will be actually in help us to increase the signal-to-noise ratio compared to a reception via single received antenna.

So, this is basically this diversity or the transmitter the receiving side, you prefer you have both way, if you wish to get the maximum receiving a space diversity and the enhancement on the signal-to-noise ratio is largely dependent upon the number of the antenna array elements so involved inside the process we at a cost of large hardware complexity. And this space diversity also requires that the separation between these two antennas must be such that the fading amplitudes corresponding to these paths are approximately independence. So, we cannot bring both of them too close such that the paths, the correlation the paths to paths associated to these antenna elements are becoming correlated.

Once it becoming correlated then the fundamental actually gain of the diversity will be heavily damaged. Because the total gain of the diversity relies on the fact that you have to keep the un-correlation of the different paths, we have to ensure that and to into a ensure that we have to maintain certain distance between these antenna elements in the antenna array. Usually we have seen that of a half of the wavelength one half of their wavelength or 0.38 lambda exactly to be specifically to be said it is actually a good difference between the antenna elements to keep to go ahead with the independent fading over the multiple channels when the space diversity is deployed.

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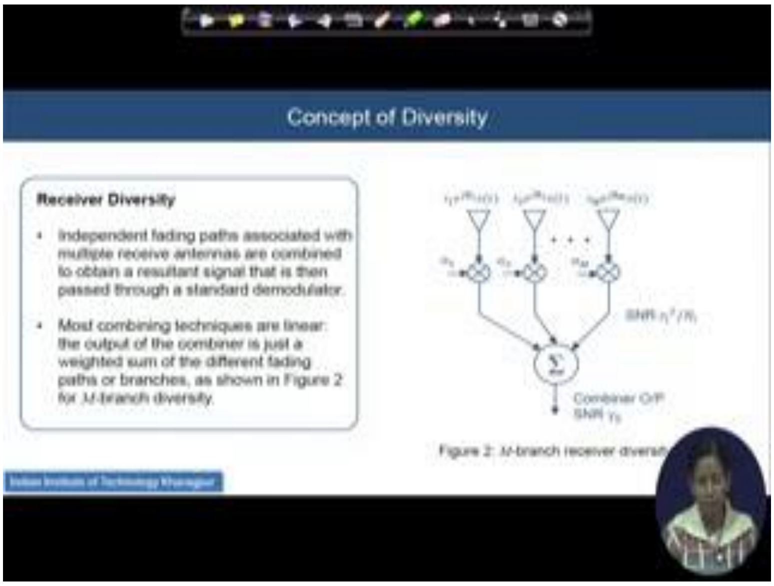

And this slide is talking about an example of a receiver diversity. What is going on here is your receiver front end is now having an array of the antenna elements. And now you are thinking that each and every antenna, the distances between the elements are such that they are capable to capture the signals who are uncorrelated to each other. The channels that are associated to reach of the antenna elements are uncorrelated, hence we expect that the signals received by the array elements, they are uncorrelated to each other. In the receiver diversity, to achieve the diversity, we actually the output of each and every antenna element, we adjust them by some gain factors.

And finally, we add them of and combine them of, so the SNR actually it is expected to see that this addition will be co phased addition if we can align the phase of each of them, if you consider that they are well phase aligned all the elements are. So, then actually the output will give you the maximum SNR. And you can fit this output to a detector to detect the message signal.

Remember actually we call it if I am having capital M number of the antenna elements, so varying from 1 to capital M here; we call it a M branch diversity in the receiver. And once we are receiving this signals SNR of each and every path if they are a co phased, well phased aligned I mean to say then the SNR of each and every path will be giving by given by the r i square by N i. i is the number of the path, r is a received signal power of that path, where r i square will give you the signal power received in that path. And if i consider that the noise bandwidth also over the all of them are exactly same. And the power spectral density of the noise, they are also same. And they are the additive white Gaussian noise I am considering for this analysis. Then finally, the noise components this SNR values for each and every path that is associated here will be same and will be governed by the r i square by N i.

So, most of these combination techniques if you look into this combiner they are basically the linear, and this output of this combine arrays just will be the weighted sum of all these fading paths are the branches and this M ordered diversity will be very easy to get at the output.

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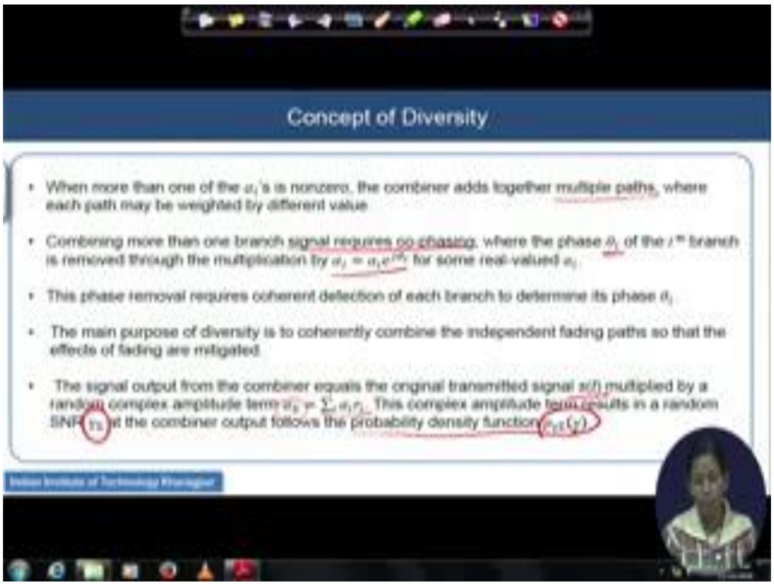

So, with this example, so let us quickly look into the facts when one or when more than one of this weight vectors, they are nonzero. The combiner adds together the multiple paths. If all are zeroes and you are having a single path value of alpha equal to 1 then this is a single path communication with may be a line of sight path only we are receiving. Otherwise, actually if the alpha is a nonzero then you are going to combine the weighted sum of each of the paths multiple paths.

So, once we consider that the signal are co-phased and where the phase theta i of the i th branch is called is theta i is the phase of the i th branch associated to it. So, then the phase theta i of the i th branch can be removed through the multiplication by this. And hence we remember hence with say that alpha i is equal to the a i of e to the power j theta i for some real value of this a i. Once actually this phase removal is done, and these phase removal definitely demands the coherent detection of the each branches phase properly, and let us assume that it is properly detected and it is removed accordingly. So, the coherent combination of these independent fading paths is possible to mitigate the effect of the fading over the paths.

And what we will be ending up with in a co-phased such kind of the co-phased situation and the removal of the phase information from the receive signal over the multiple path is that if I am transmitting the signal original signal s t and I f it is multiplied by the random complex amplitude term, so the overall the path that total complex amplitude term will be given by a i into r i. This summation is running over all the total number of the multipaths, multipaths i is denoting the number of the multipath. This complex amplitude term results in a random signal-to-noise ratio definitely and at the combiner output which would should have some probability density function, here we are writing that probability density function of the SNR the summed up SNR as P gamma sigma gamma.

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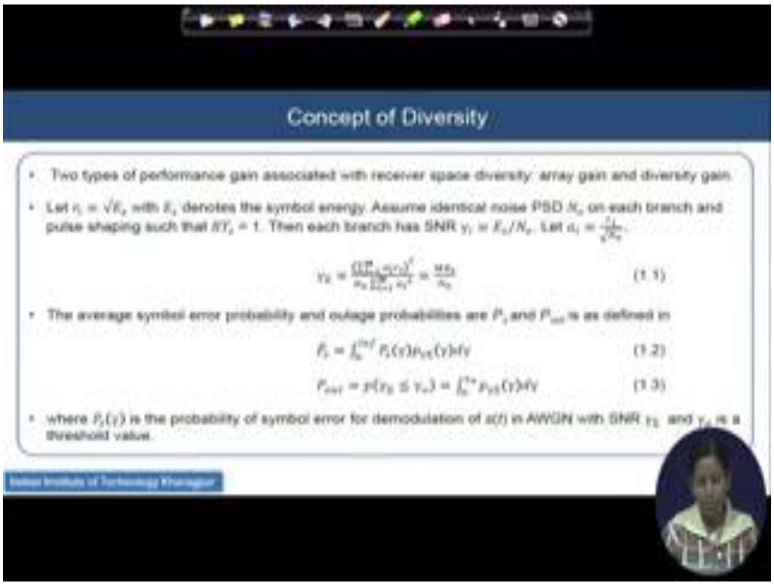

Next we understood that there are two different types of the performance gain associated with this concept of the diversity. One is receiver space diversity mainly, one is the array gain, and another is the diversity gain. So, let us first look into the gain that we are getting from the diversity. And let we are having r i -the received signal is equal to this square root of this E s, E s is the symbol energy. Let us also assume that there are the identical noise power spectral density over the each and every branch, and we have use the pulse shaping in such a way that B into T s is equal to 1 the bandwidth - time bandwidth product of that design is equal to 1. And such that the every SNR each and every branch of every branch can be now talked about by E s by N naught. And let the a i is nothing but the r i divide by square root of this power spectral density N 0.

Then this summation of this SNR is basically will be given by the power of the received signal a i into r i that summed over all the paths squared up. And here also this is the divide which is the normalized factor by a i square, and then this is the noise power. And then finally, if I am doing some calculation we will be ending up with M into E s by N 0. And remember here that if the if the signals are coherently added, if they are co-phased and they are coherently added, then in only in the presence of the noise in absence of the interference, we can have the received signal-to-noise ratio. I mean, additive signal-to-noise ratio is in times higher than the single then the case of the conventional receiver where the diversity is not deployed. Remember this capital M is given by the number of the antenna array elements and deployed for these are involved in this combination process, diversity combining process.

And if you go further for this situation we can compute the outage probability and the symbol error of it is symbol error probability by these two equations. Where this P s gamma is the probability of the symbol error for demodulation of the signal over a AWGN channel. And this one is we understand we described in the last slide; it is the probability of getting this additive sigma SNR, additive SNR. And outage probability is given by this expression where there gamma 0 is the selected threshold value.

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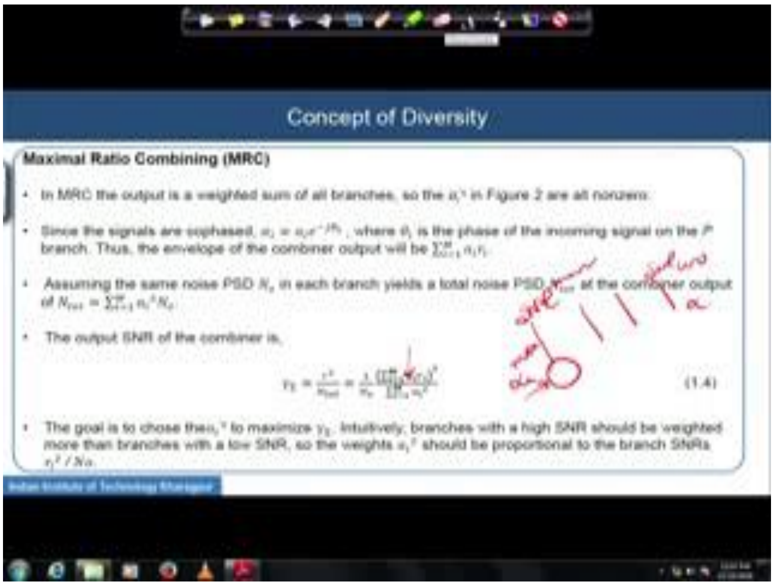

We will describe here one at least one of the diversity combination scheme, which is the maximal ratio combining, the mostly used technique in the practice. In MRC the output is weighted sum of all the branches. So, in this figure-2 that we have seen here the MRC will be the weighted sum of all their branches at the output, so the way we will write it is like this. So, since the signals are co-phased I understand that this is possible and then the output of the combiner will be given by a into r i, where i is moving from i to M. And we assume that the power spectral density in each and every branch is same, and hence the total noise power combined output for the noise power will be given by a i square into N 0 because a i is getting activated over the signal as well as the noise. And now the signal-to-noise ratio at the output of the combiner will be nothing but the r square by the total and here we are ending up with this whole factor.

So, now what is the goal of this maximum ratio combining is that you will try to choose the a i is in such a fashion that it maximizes the signal-to-noise ratio, this gamma some of the gamma. So, intuitively the branches who are having the high SNR values, we would try to give the weightage higher will try to choose the higher weightage factor there compared to the branches where low SNR values are there. So, it is something like that we will compute the SNR values of each and every branch of the received paths and you choose the alpha i is in such a way that wherever the SNR is high, wherever the SNR is maximum, you put give the value of alpha also maximum there. And the value is where the SNR is a lower one, you choose the alpha where you also minimum.

So, we do not put the weight or adjusting the weights, in a such a way that giving the maximum weighted to the hard SNR paths our total sum of this summation will be very, very high. So, target is to maximum this by choosing this value of the a i in such a way that it is performed and this ones we are doing this SNR where maximizing the SNR, finally the gain will be over the detection process.

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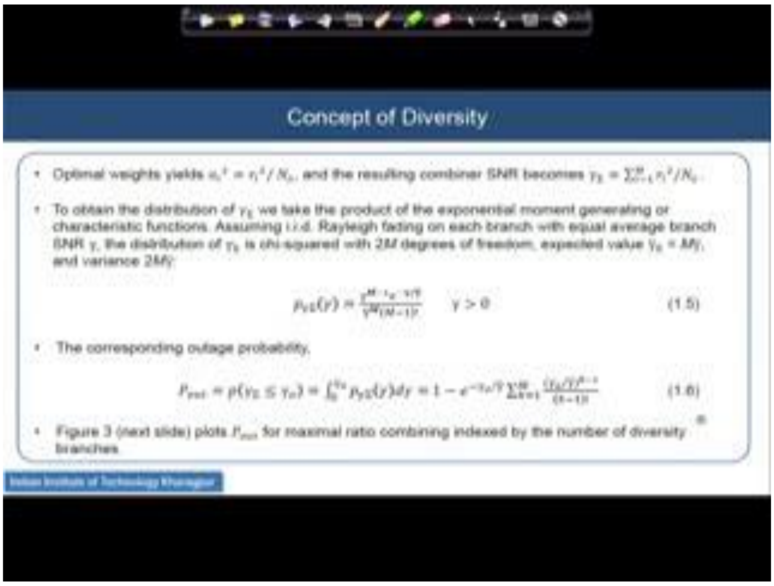

So, optimal weights now that we have selected is given by this I understand. Resulting combiner has come like this. So, if I try to see what is the distribution of this summed up weighted SNR values summed up proportion is coming up. We take the product of the exponential moment generating or the characteristic functions. If you consider that the fading is the Rayleigh fading only each branch and the equal average branches SNR is equal to gamma.

And then the distribution of this additive function this sum of this is SNR values will be chi squared distributed with 2 M degrees of freedom and its mean value will be given my M of gamma bar, and this variance will be given by this. So, this chi square distribution is invoked here. The corresponding outage probability which is nothing but the integration of this distribution function over an interval of 0 to the selected choice of this gamma value, this threshold value will be due to this exponential distributions.

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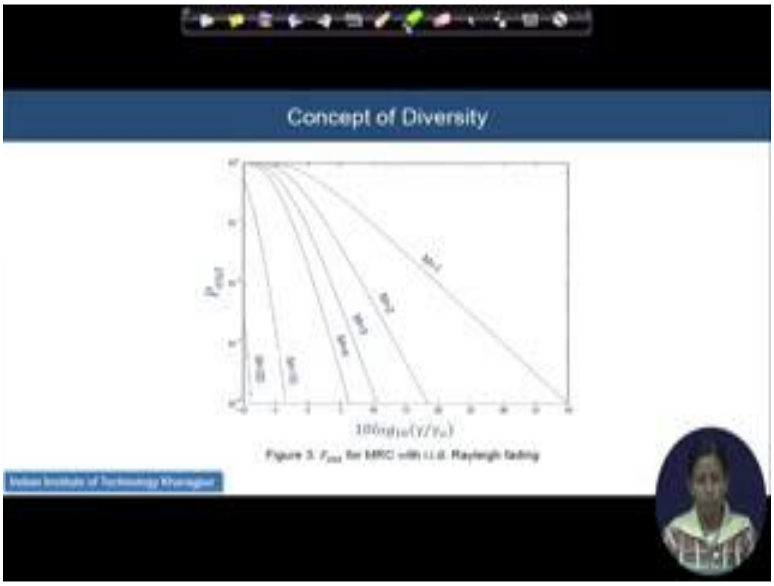

And if I try to plot this value of those outage, it will be able to see that if I am increasing the number of the diversity the outage probability is improved, and we are getting actually over the normalized value of this gamma with respect to the threshold if I try to plot on x-axis. So, with the increase of this diversity, outage actually a improves.

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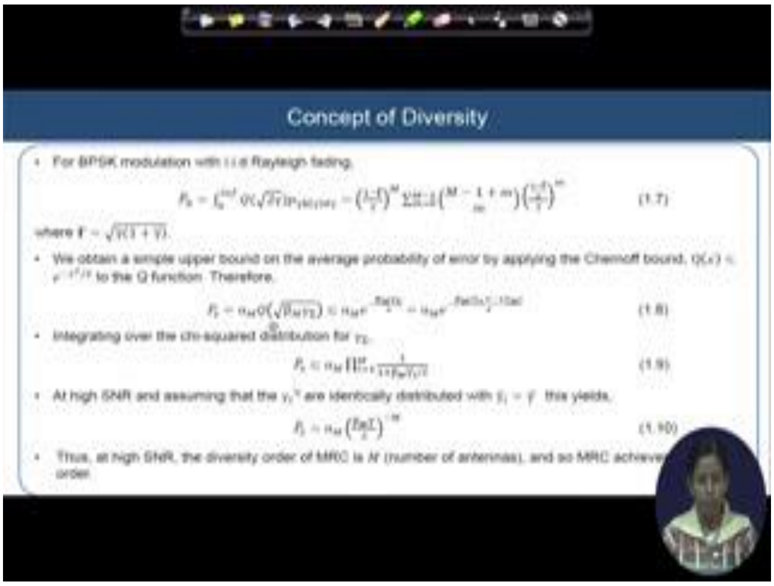

If I now check what will be the error probability calculation, if taking an example of the BPSK over the independent and identically distributed Rayleigh fading channel. So, it will be average error probability will be given by the Q function we understand and Q function is the actually a it is variable whether parameter gamma on it. And it is now getting affected will be by the distribution of this sum SNR. And finally, it will be resolved it will be given by this expression where this capitol gamma is nothing but given by the average value of the SNR.

Now, in order to solve this expression, we take a simple upper bound on this average probability by taking a applying this Chernoff bound on this Q function given by this Q x is less than equal e to the power minus x square by 2. And therefore, the symbol probability with this structure of this capital M number of the branches as we have shown in this figure will come down to the formula here. And if I actually integrate this sum integrate this part over the chi squared distribution over all the branches of capital M then we will be ending up with this. So, this final equation of 1.10 simply says that at higher values of the SNR, the diversity order of this MRC, the higher values of the SNR, at high value of this gamma bar, a diversity of order of this capital M is the diversity MRC is capital M, so MRC can achieves the full diversity order itself.

So, symbol error rate will be heavily improved if I am keeping on increasing the diversity order. So, you need not to increase the both, so but if the SNR value is high only you can have the high order of the diversity effect and the gain from the diversity is easily visible for your MRC. And this is the very simple example the way we do the diversity combining in the receiver.

There are several other processes for example, equal gain combination, and if you are interested you can refer all those different diversity techniques from the standard Wireless Communication book by (Refer Time: 32:57) Wireless Communication by (Refer Time: 33:00). And we will actually apply any mostly by MRC combination in the further related to our kind of the discussion in the spread spectrum communication. So I have explain, the MRC mainly in this module.