Spread Spectrum Communications and Jamming

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Lecture - 43
FHSS Synchronization Method – II

Hello students, we were discussing the synchronization mechanism, basically the code acquisition mechanism inside the frequency-hopping spread spectrum communication receiver. And in the last module, we have seen the detection probability expression, false alarm probability expression and the expression for the missed detection. And we have also discussed the first method of FHSS code acquisition system that we have deployed matched filter based architecture, we have analyzed it. And we have seen that matched filter based code acquisitions scheme in the FHSS receiver can give us very nice and very fast acquisition. So, time of acquisition is very very less, but the main problem was that the whole architecture is very much hardware intense, and the false alarm probability or the rate of the false alarm may be very high in that system in the match filter base system

The whole architecture gains in terms of the acquisition time because the processing goes on parallelly. So, with respect to the serial search mechanism, the gain is there over the H times whatever be the number of the hopping tones are, so in the order of that approximately we will gain in terms of reduction of the acquisition time with respect to the serial search techniques.

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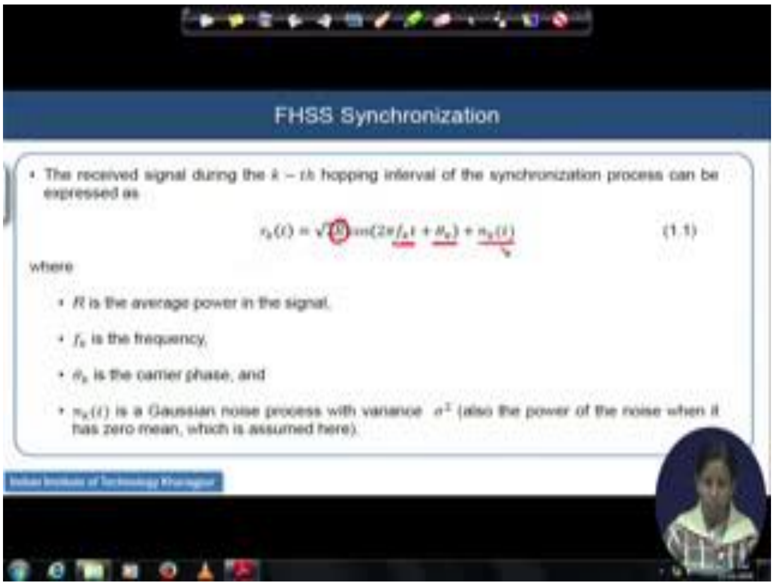

FHSS Synchronization

- The received signal during the $k-th$ hopping interval of the synchronization process can be expressed as

$$r_k(t) = \sqrt{2R}\cos(2\pi \underline{f_k} t + \underline{\theta_k}) + \underline{n_k(t)} \qquad (1.1)$$

where

- R is the average power in the signal,
- f_k is the frequency,
- θ_k is the carrier phase, and
- $n_k(t)$ is a Gaussian noise process with variance σ^2 (also the power of the noise when it has zero mean, which is assumed here).

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We will today discuss the second mechanism of FHSS synchronization. We will quickly revisit the incoming signal model that we took in the last module. Please remember we were discussing based on the push-to-talk network and push-to-talk communication set up where we understand that the communication is going on in the burst mode, where the voice messages will be transmitted at a particular time. Then there would not be any communication over the network over an extended period. Again once the data is enough in one end and the transmitter end, second burst transmission will be going on, so that is the mode of operation.

And we also understood that because of this silence period involved in between two communication within the network, there is a high possibility that the synchronization that was established during the previous communication between the transmitter and the receiver that will be completely lost in the next communication will be happening, next burst communication will be happening. So, in a communication network - burst type of communication network like PTT or push-to-talk network, it is essential that for each and every transmission you align the signals in the transmitter you align the code in the transmitter and the receiver. So, synchronization becomes much more critical in a burst communication or a push-to-talk and that push-to-talk kind of communication network.

And in view of that only we are going to discuss the second method here, because we understood that in the first method the acquisition time got reduced, but false alarm rate has increased and the hardware complexity has increased. We took the model like this that suppose we have capital H number of the hopping tones involved in the synchronization sequence, and hence the received signal over the k-th hopping interval of the synchronization process, we wrote the equation like this.

We also discussed that in this equation r k is the received signal as well as a transmitted signal on the k-th hop interval. Capital R this is the average power inside the signal, if k is the frequency of transmission, theta k was the carrier phase associated with it, this is random and usually uniform a distributed. n k is the additive white Gaussian noise process with the variance sigma square, and we know that as we have considered the mean of this process to be equal to 0 then sigma square is equal to the power of the noise.

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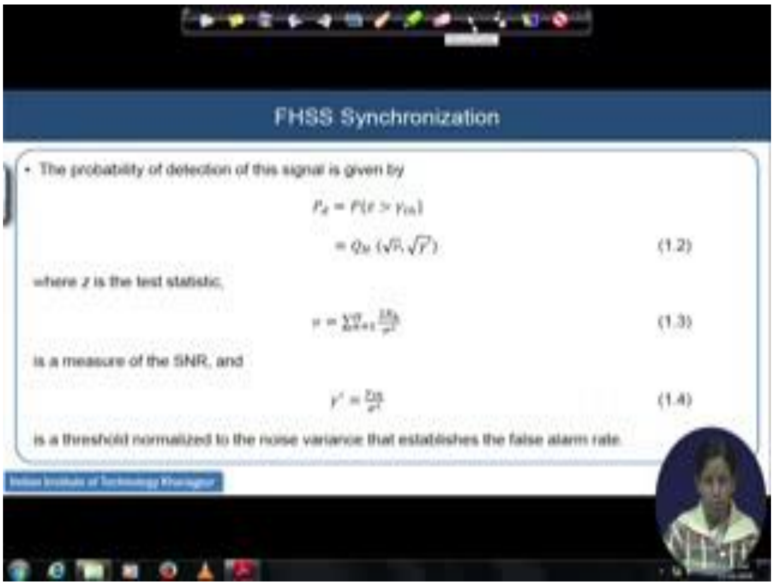

FHSS Synchronization

- The probability of detection of this signal is given by

$$P_d = P(z > \gamma_{th})$$

$$= Q_H(\sqrt{\nu}, \sqrt{\gamma'}) \quad (1.2)$$

where z is the test statistic,

$$\nu = \sum_{k=1}^{H} \frac{2R_k}{\sigma^2} \quad (1.3)$$

is a measure of the SNR, and

$$\gamma' = \frac{2\gamma}{\sigma^2} \quad (1.4)$$

is a threshold normalized to the noise variance that establishes the false alarm rate.

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We also found that by the definition the detection probability will be given by the expression one dot two, where the test statistics there it is a detection probability is the probability that the test statistics z process the predefined threshold value gamma T h inside the correlated detector architecture. And this probability function can be a well approximated by the Q function where Q function, Q is a function parameter is having dependence on two basic parameters; one is the nu, another is the gamma dash where nu is nothing but the sum of all SNRs of each and every hopping interval or every hopping tone. So, we use a sum of overall the hoping tones. And R k is basically the power received over the k-th interval. So, in that sense, you may actually you would like may like to actually change the incoming signal model in the first slide to be not exactly equal to i, you may actually for your easy understanding you may actually write it as R k because it is a power received with the k-th interval associated to it.

And that R k is now getting for each and every path you are taking that R k and normalizing it and dividing it with the signal noise power, and it is equal to gamma square sigma square as we have discussed that we have taken the white Gaussian noise process mean the value to be 0. And similarly the gamma dash to actually we discuss that gamma dash is the normalized value of the gamma T h that threshold selected and the normalization is done with respect to the noise variance or the noise power, because we did this step to minimize the false alarm probability rate. And hence finally, the detection probability becomes the function of this normalized threshold and the total SNR values and it can be well approximated by the Q function, that was understanding about the detection probability.

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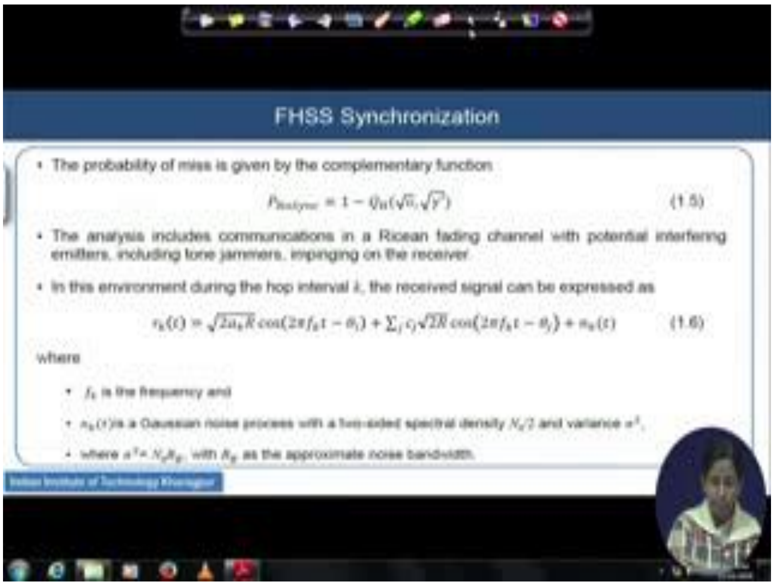

FHSS Synchronization

- The probability of miss is given by the complementary function

$$P_{\text{Nosync}} = 1 - Q_M(\sqrt{\alpha}, \sqrt{\gamma'}) \qquad (1.5)$$

- The analysis includes communications in a Ricean fading channel with potential interfering emitters, including tone jammers, impinging on the receiver
- In this environment during the hop interval k, the received signal can be expressed as

$$r_k(t) = \sqrt{2a_k R}\cos(2\pi f_k t - \theta_i) + \sum_j c_j\sqrt{2R}\cos(2\pi f_k t - \theta_j) + n_k(t) \qquad (1.6)$$

where

  - f_k is the frequency and
  - $n_k(t)$ is a Gaussian noise process with a two-sided spectral density $N_o/2$ and variance σ^2,
  - where $\sigma^2 = N_o B_N$, with B_N as the approximate noise bandwidth.

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And we also saw that the probability of miss hence will be given by 1 minus the detection probability. And this analysis included over a Ricean fading channel and where the interference are present, the jammers are present and all of the receiver side, all other receiver side imperfections, they are also present. So, in this environment of a PTT, where the PTT network operation over a ricean fading channel in presence of the interference jammers and other kind different kind of the impurities and all the impurities and the R f then receivers are also playing a very big role in the detection process itself.

And such an environment during the interval hop interval number k, the received signal we saw that we can write down like this, where there is the first part is the basically the contribution from the direct line of sight path between the transmitter to receiver. This guys says actually the power that you are it is a contribution from the multiple paths in the wireless environment, and this is a Gaussian noise process getting added. We also discussed that there is a bandwidth B IF, which is the noise bandwidth and eta 0 by 2 is that two sided power spectral density of the noise, and sigma square hence will be given by N 0 into B IF.

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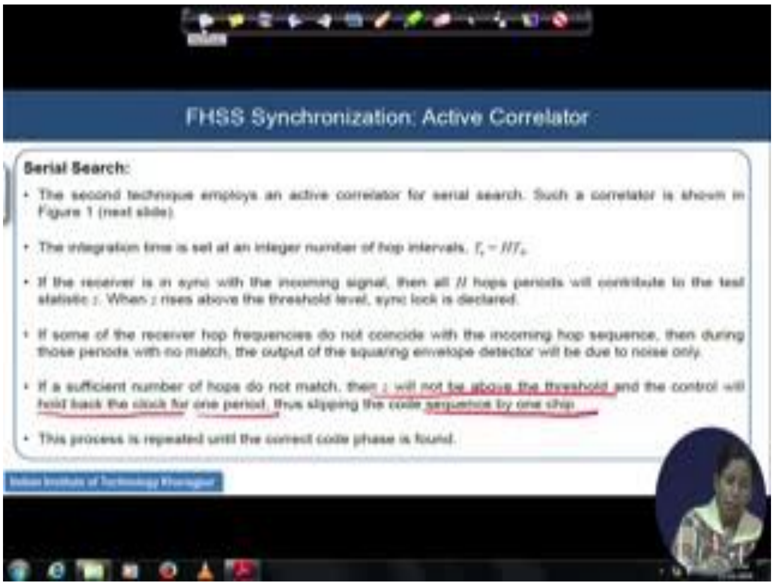

FHSS Synchronization: Active Correlator

Serial Search:

- The second technique employs an active correlator for serial search. Such a correlator is shown in Figure 1 (next slide).
- The integration time is set at an integer number of hop intervals, $T_i = NT_h$.
- If the receiver is in sync with the incoming signal, then all N hops periods will contribute to the test statistic z. When z rises above the threshold level, sync lock is declared.
- If some of the receiver hop frequencies do not coincide with the incoming hop sequence, then during those periods with no match, the output of the squaring envelope detector will be due to noise only.
- If a sufficient number of hops do not match, then z will not be above the threshold and the control will hold back the clock for one period, thus slipping the code sequence by one chip.
- This process is repeated until the correct code phase is found.

So, with this recap, we will enter into the second mechanism of the code acquisition in FHSS which is called the serial search. This serial search employs the active correlators. We understand that matched filter is a passive correlated based architecture. So, now, we will go ahead with an active serial search correlated base search mechanism. And such a correlated can be seen like this can be observed in the next slide.

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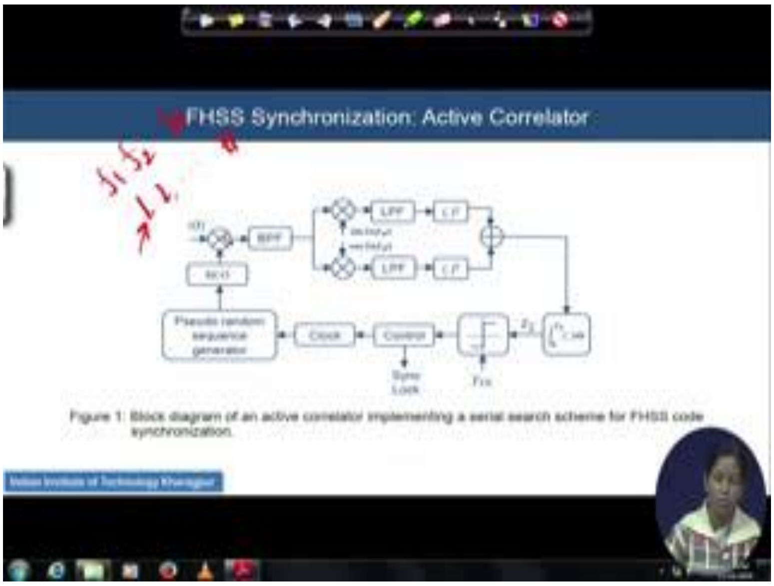

FHSS Synchronization: Active Correlator

Figure 1: Block diagram of an active correlator implementing a serial search scheme for FHSS code synchronization.

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Here is the active correlation architecture. And the integration time inside, integration time with after the low pass filter and the squaring up and all the integration time inside the whole correlation activity that integration will be designed and it will say already set by the hop intervals equal to T s which will be fundamental in the H times of the hop interval. And this is a total interval or the integration time of the observation window where which the decision will be taken, search will be continued.

If the receiver now using sync with the incoming signal, then all these H hops periods will contribute to the test statistic z. And when z rises above this threshold, then you will be declaring that there is sync lock. So, thing is something like that that continuously the incoming signal is getting correlated with the locally generating pseudo random sequence. After the band pass filter and this non-coherent detector architecture the output of this guy the in phase and the Q phase components, they are integrated over the duration of 0 to T h, so they are over duration of the hopping interval.

And each of that hopping interval output is now getting compared with the predetermined threshold, and ones actually it is crossing to the control will declared that there is a sync lock happened. But remember that this stuff that once you are actually you are basically depending upon the statistics of this z, z 2, if this z 2 is whether z 2 will rise above the threshold of predefined threshold of gamma T h 1 or no it which will be below that it is light large depends upon that whether the output of the hops how the output of the hops are contributing to form the value of this z. Suppose, it is such that none of these hops are significantly contributing to the test statistics z, then the output of total output of all of that would be sufficient to cross the predefined threshold value gamma t h. And it may happen that in if some of the disable hop frequencies, they do not coincide with the incoming hop sequence then during those periods there would not be any match. The output of the squaring envelope detector, the output of the squaring envelope detector will give you perfect 0

Basically actually output of this a squaring envelope detector is coming, will give you some value if this guy, incoming signal and it is locally for generated PN sequence for a typical hop perfectly coincides, suppose, I have 1 to capital H number of hops. If I see that when the received signal is coming over hop number one hop number one means frequency hop and the corresponding frequency-hopping sequence is f 1; frequency associated two, it is f 2; and frequency associated with capital H is f H. So, if I see that the locally generated random sequence is exactly in-sync with f 1, when the hop is going on over a 1, then of definitely of the output of the both I and Q detector square load outputs the both the I and Q path will give a significantly high value. And it will contribute actually significant way over the contribution of over the concession of this values is statistic z.

But think a situation that for most of these hoping patterns, so in the hoping frequencies, now locally generated noise sequence generators code is not aligned with it. If they are totally out then square load detector output will be very very low closed to 0. And if it is 0, then for those hoping frequencies the contribution two of the z will be very very low. And hence if I add up all the values of the z with from 1 to capital H for large number of the cases you are totally misaligned to the incoming signal, and most of the cases hence the detector output has produced perfect zero for you.

Then this total value inside the z parameter would not be much; and it will be difficult for the z statistics, this test parameter statistics to cross actually the gamma T h value. And hence the synchronization would not be locked, the control needs to readjust the clock in such a way that most of the favorable situations, most of the hop sequences, you get try to get aligned with it, such that the output of the detector is improved.

Think completely the opposite situation, if you are a perfect sync situation with all the hoping frequency, then for all the cases the detector output will be really high, value of the z will be also very good, so that the gamma T h it will be when comparing the part gamma T h output will be very high. And hence controller will declared the sync this locked. So, it will keep the clock phase exactly the same that you kept in the earlier time span and keep on generating the pseudo random sequences that is going on.

So, question is actually if a sufficient number of the hops are not do not match. So, we understood that z will not be above the threshold. And control will now we are just a clock actually, when there is a no crossing above the threshold, this parameter fails to cross the predefined threshold, then we the controller basically holds back the clock for at least one period. And thus slipping the code sequence by one slip, by one chip and this process is repeated until the correct code phase is found. So, one-by-one chip slip actually is going on till actually the z parameter of the value of the test statistics is z value improves. And you get actually a sufficient high value to cross the threshold; till that point to the control circuit keeps on holding back the clock, and continuously one-by-one chip, chip-by-chip that is the slipping goes on.

So, it is just adjusting of the clock with that. And remember as it is going on in a very serial fashion, so it requires the control circuit may require long time to give declared that there is a sync lock, because it never at one step it never declares or it never actually decreases the clock phase by several chips.

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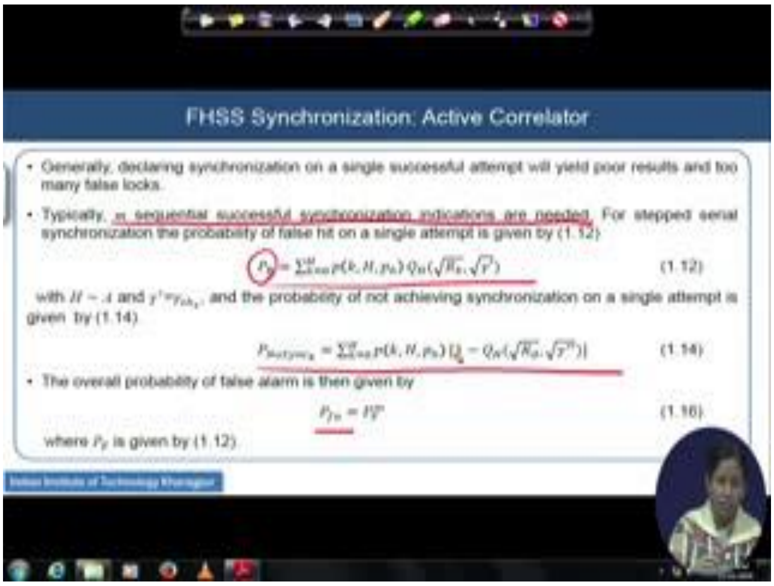

FHSS Synchronization: Active Correlator

- Generally, declaring synchronization on a single successful attempt will yield poor results and too many false locks.
- Typically, m sequential successful synchronization indications are needed. For stepped serial synchronization the probability of false hit on a single attempt is given by (1.12)

$$P_H = \sum_{k=0}^{H} p(k, H, p_h)\, Q_H(\sqrt{H_k}, \sqrt{\gamma'}) \qquad (1.12)$$

with $H = 4$ and $\gamma' = \gamma_{th_1}$, and the probability of not achieving synchronization on a single attempt is given by (1.14)

$$P_{NoSync_k} = \sum_{k=0}^{H} p(k, H, p_h)\, [1 - Q_H(\sqrt{H_k}, \sqrt{\gamma''})] \qquad (1.14)$$

- The overall probability of false alarm is then given by

$$P_{fa} = P_H^m \qquad (1.16)$$

where P_H is given by (1.12)

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Generally by declaring that the synchronization on a single successful attempt is there, if I find even that actually that over a first slot or first loop or first dual time within a first dual time that synchronization is locked. But serial search acquisition mechanism never rely on a single dwell based declaration of the synchronization lock. It is not actually advisable also to lock there if you even if you find that in the first slot event even at the first slot immediately actually after getting the first one, you have got the lock. We try to actually see it over the multiple dwell, so over the combination of the this h tones, we try to check it over the at least two or three times whether the lock is genuine lock or not I mean all the cases whether your test parameter statistics z is crossing the gamma T h value or not.

So, it is the serial search technique not only actually needs lot of time within a dwell within a single search time, if the alignment is not matched. But also actually even after getting locked, it does not declared it to be locked and it takes lot of the others time it takes actually several iterations to finally, declared that there is a sync lock. So, typically here we will see that m sequential successful synchronization indications are basically required. And remember it is a sequential indication, and it is a serial search mechanism. And if you see that for the (Refer Time: 18:38) serial synchronization, the probability of the false hit on a single attempt already we have seen that this expression gives a probability of false alarm. We understood that probability that there is no sync and in the false miss detection is there given by this expression, we established this expression in the last module.

But remember when the false alarm you are declaring, we have to substitute capital H is equal to by capital A and this gamma dash to will be basically the threshold level gamma T h to or gamma t some other threshold value that you are adjusting inside the comparator. And but remember here we are talking about the m sequential successful synchronization indications. So, if I ask what is a total false alarm that will be given by this individual: probability of false alarm raise to the power of small m, obviously.

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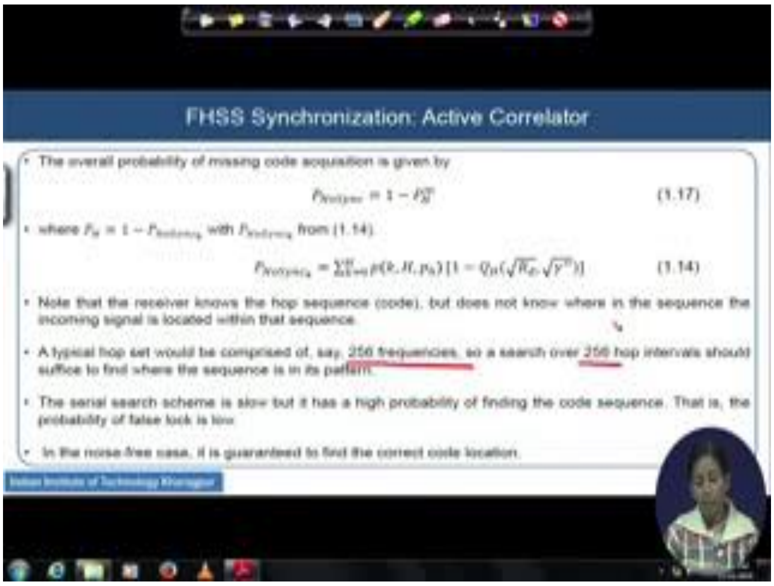

FHSS Synchronization: Active Correlator

- The overall probability of missing code acquisition is given by

$$P_{NoSync} = 1 - P_H^m \quad (1.17)$$

- where $P_H = 1 - P_{NoSync_1}$ with P_{NoSync_1} from (1.14)

$$P_{NoSync_1} = \sum_{k=m}^{H} p(k, H, p_0)\left[1 - Q_M\left(\sqrt{R_c}, \sqrt{\gamma'}\right)\right] \quad (1.14)$$

- Note that the receiver knows the hop sequence (code), but does not know where in the sequence the incoming signal is located within that sequence.
- A typical hop set would be comprised of, say, 256 frequencies, so a search over 256 hop intervals should suffice to find where the sequence is in its pattern.
- The serial search scheme is slow but it has a high probability of finding the code sequence. That is, the probability of false lock is low.
- In the noise-free case, it is guaranteed to find the correct code location.

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Similarly, the overall probability of missing the detection if it is given by the expression of no synchronization it is right. So, it will be in obviously, 1 minus the probability of H whole to the power small m. This probability of H, P of H is basically 1 minus probability of no sync when for the one dual time basis synchronization indication or decision take based one sequence. 1 minus P no sync also is given actually, it gives that there is a probability of overall probability of missing.

And see actually if I am having just a minute, so if I am having this expression given to us, where this P of no sync on in the miss detection will be given by this expression, then over all this detection I will be able to get by 1 minus P of h whole to the power small m. And remember here that receiver may be knowing that what exactly the code is. But receiver does not know that where inside the sequence the incoming signal is located within that sequence.

So, supposed in practice, let me give an example that how much search is required in a practical PTT kind of the networks. Say usually utilize 256 frequency hops. So, search is required over 256 hop intervals, and suffice to find where the sequences is in this pattern in the pattern. And remember if you try to deploy the whole search mechanism by a structure of matched filters we need to 56 parallel matched filter architecture. And very shortly, you will get the acquisition time with a very short acquisition time you will get the result, but remember actually the hardware complexity involved in the 256 parallel matched filter architecture.

Whereas, here 256 serial search may be required and that also we use over a small m number of the consecutive checking of the process, even if you get the synchronization lock to happen within the first dwell time. So, serial search actually in terms of that time consumed serial search over 256 frequency points is really really very high because that is why actually we call the serial search process is basically is very very slow process, but it is a probability of finding the consequences really very high. So, I mean that probability of the false lock will be very low because you are serial searching all the frequencies and that also we are repeating over the multiple dual times and so the false lock is very low. Whereas, in the match filter base architecture at one short, you are trying to see all the frequency points with a typical delay, and you are trying to declare whether the synchronization is achieved or not. So, your false lock probability is very high, acquisition time is very less. In a noise-free actually this serial search can mechanism can give you the guaranteed correct code acquisition or correct code location.

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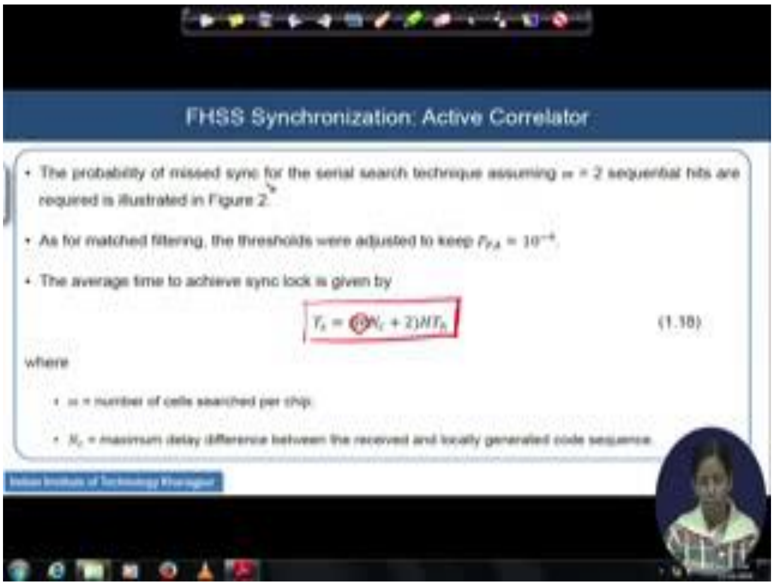

FHSS Synchronization: Active Correlator

- The probability of missed sync for the serial search technique assuming $m = 2$ sequential hits are required is illustrated in Figure 2.
- As for matched filtering, the thresholds were adjusted to keep $P_{FA} = 10^{-6}$.
- The average time to achieve sync lock is given by

$$\boxed{T_s = (mN_c + 2)NT_h} \qquad (1.18)$$

where

  - m = number of cells searched per chip
  - N_c = maximum delay difference between the received and locally generated code sequence

Basically that is why in the practical system deployment, we always be very very critical about choosing the mechanisms, because sometimes the acquisition time becomes very critical based on the kind of application we are doing. Sometimes actually and that moment to the hardware complexity does not play any role, but sometimes no actually the perfection in the code acquisition becomes very, very critical for us. And we do not go ahead and do not care about the acquisition time that you are spreading to give me the correct acquisition. So, in that situation, you go ahead with the serial search. On the top of all that, the number of the frequencies are the hop frequencies you are going to search that plays another important role.

Even if actually my acquisition time is not a very big point or very big point of for our consideration, but remember one thing if the number of the hop tones increases even greater than 256, if it becomes 1024 and all, then in such a situation even if you are giving me the relaxation of giving me a relaxation of may be in (Refer Time: 24:56) restriction putting any restriction on the acquisition time, but then also I will be very much critical of choosing this serial search mechanism. Because I understand that most of the time and hence the bandwidth will be a totally (Refer Time: 24:59), because of this synchronization process, because serial one will not only take the decision over one such over all this 1024 frequency tones. And it will actually repeat this 1024 point search over the multiple times.

So, the acquisition time is actually linear increasing over the tone of the frequencies, hop frequency as well as the hope frequency as well as the number of the times you are choosing actually to repeat it your estimation process and synchronization decision process. So, even if actually in the practice, you are giving the relaxation over the time or the complexity, please try to take a crucial call by looking nicely inside the number of the hopping frequency involved in that synchronization process.

And sometimes, we do actually some mixing up of these architecture, reducing the architecture and I mean that getting the; we try to get the good effect on the plus point of the advantages of both the mechanisms, we will declare actually we will decided in the and discuss it in the next module. And now about getting about the acquisition average acquisition time for the serial mechanical search based active correlated architecture for FHSS, this acquisition time here the expression if you see that it is H into T h. But here N c becomes search becomes m times, and so here the linear increment is involved compared to the matched filter base architecture in the acquisition time.

And N c basically is the maximum delay difference that you are having between the received and the locally generated this in terms of the code chips actually, we can count this delay and let us taken a good example also in the next figure. We will see that when two number of the sequential searches means two times the search was repeated, and if we keep the probability false alarm fix to 10 to the power minus 4, the probability of the missed detection for the serial search technique let us see in the next plot.

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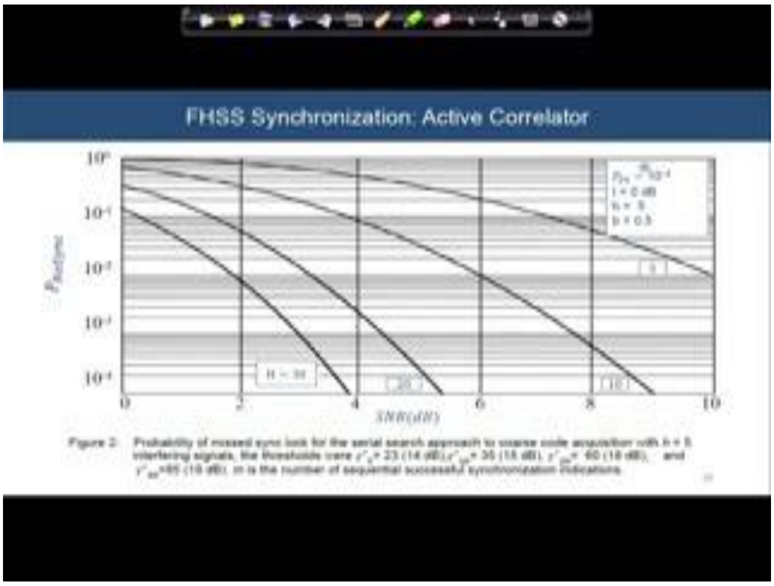

FHSS Synchronization: Active Correlator

Figure 2: Probability of missed sync lock for the serial search approach to coarse code acquisition with h = 5 interfering signals, the thresholds were $\gamma'_{8} = 23$ (14 dB), $\gamma'_{16} = 35$ (15 dB), $\gamma'_{32} = 60$ (18 dB), and $\gamma'_{64} = 85$ (19 dB). m is the number of sequential successful synchronization indications.

See this is a something when you are varying the number of the hop frequencies from 32 very high, that is very low. And you are keeping the probability of false alarm 10 to the power minus 5, because there are parameter set for the experiment. And when the threshold are changing from 14 dB to 19 dB slowly you see that the synchronization is detection probability actually it is reducing, it basically reduced if you increase signal-to-noise ratio. But remember if you are increasing for all the cases we have get to the number of this search values same, but this averaging is going on actually if I am increasing a number of the tones. So, the value of the parameters z that the chances that the z get the accumulator value for the test parameters z over increases the value of the z the chance that the value of the z will cross so the threshold value increases if you have a higher number of the tones also to go ahead with.

So, that is all about the active correlated based architecture and its performance analysis approximately in the super facial way. And to understand what is the basic architecture is about how does it work, and where the problem is in between the serial search technique as well as the match filter architecture. And we realize that serial search technique basically improves the detection probability and reduces the false alarm at the cost of very high acquisition time. So, neither the match filter will be a single solution for all the practical problems not the serial search technique will be actually a unique solution for all the different kind of the application in the practical cases.

So, in the next module, we will learn something new, we will try to combine both the architecture and try to see whether we are getting some advantage out of that combined network or combined architecture or not.