**Spread Spectrum Communications and Jamming**
**Prof. Debarati Sen**
**G S Sanyal School of Telecommunications**
**Indian Institute of Technology, Kharagpur**

**Lecture - 04**
**Concept of Jamming Margin**

Hello students, we were discussing the direct sequence spread spectrum communications. And in this module, we will continue that discussion. And in this module, we are mainly focusing on the fact that we understand direct sequence spread spectrum system, any spread spectrum system in that sense was designed to give resilient against the jamming environment. We have also discussed in the last module about the processing gain, and we saw that it is a measurement which can give you how much resilient you are, how much resilient by what factor, you can have the resilience against the jamming environment. But apart from the processing gain, there is another term used in the system design which is called the jamming margin.

Today, we will also try to concentrate about this jamming margin, we will try to get the concept of jamming margin and we will try to see, what is the typical relation between this jamming margin and the processing gain.

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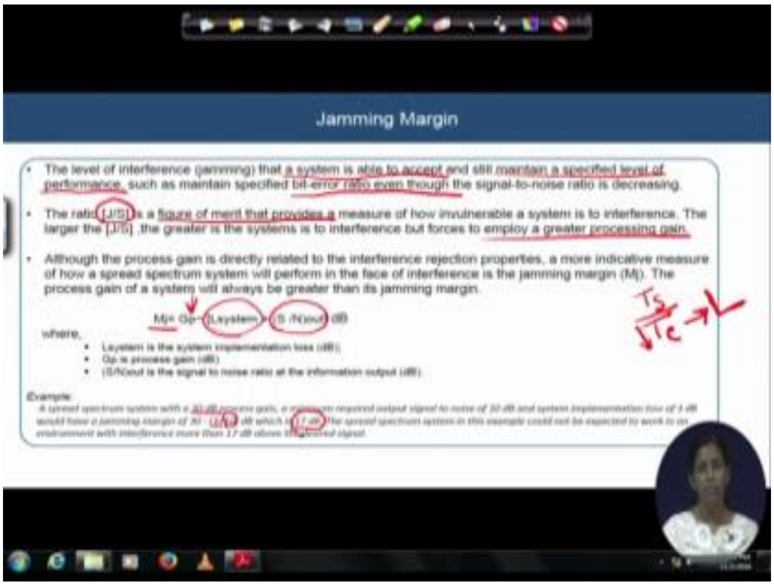


See, this jamming margin it is a level of the interference that a system is able to accept. So, if I tell you the jamming margin is again it is expressed in dB it is the this much dB;

that means, from the system design perspective, it is the amount of the dB level where actually any amount of the interference exactly straightforward it says the amount of the interference that the receiver can withstand. And withstand it means withstand with specified level of the performance without actually diminishing the quality of the error performance or diminishing the value of the bit error rate, this is the extra amount of the interference it can accommodate. Such as to maintain its bit error rate ratio exactly what you were getting; so without compromising in any error increment or decrement in the signal to noise ratio, the extra amount of the interference power that it can accommodate.

The jamming margin usually is considered as the jamming power to the signal power, it is a figure of merit and that provides a measure that how vulnerable your system is to interference. And obviously, we would like to have a larger value of the J by S. And because it is system is greater is the system is to the interference part resistance to the interference, but it forces to employ a greater processing gain.

Remember if you wish to get higher resilience, this factor needs to be higher which finally, boils down to the fact that you have to have a higher processing gain. If you wish to have a higher processing gain then the T s by T s that ratio it needs to be high; in order to make this high, you have to decrease the value of that T s keeping this thing in consideration that the symbol duration we are not changing. So, you have to decrease the value of that T s once I am decreasing the value of that T s hence the length of the sequence that I am dealing with that is increasing. So, you are boiling down to the fact that you are increasing very long length sequence for the spreading which has lot of its own kind of disadvantage as well as advantage in the practice, but the relation is something like this.

Though the processing gain is having some direct relation with the interference rejection properties as we have discussed in the last module, but remember that there is another there are some few more factors which relates the processing gain with the jamming margin. If I tell that jamming margin is given by my M j then and G p is my processing gain then processing gain subtracted by the minimum required E b by N naught at the output. And it depends the requirement depends on the kind of the modulation, you are utilizing definitely if you go for higher modulation scheme S by N requirement at the output will keep on increasing. And L system talks about the amount of the losses that can happen once you go to implement the system. So, considering these these two dB values the subtracted value of this the summation from the processing gain will tell you what is the jamming margin you are left with.

For example, I have a spread spectrum system with its processing gain is equal to 30 dB, where 3 dB is the loss in the system, and 10 dB is the required SNR output for a typical kind of the modulation system. Then you will be ending up with the 17 dB of the jamming margin that means, the system can work nicely up to the interference power level go as high as 17 dB above the desired signal. But beyond that the environment where the jamming power goes 17 dB, beyond 17 dB above the signal power level, the design system cannot work, this is the meaning. So, today we will also have the mathematical analysis, we will do the mathematical analysis to understand in detail the relationship between this processing gain, jamming margin, how does it they look like.

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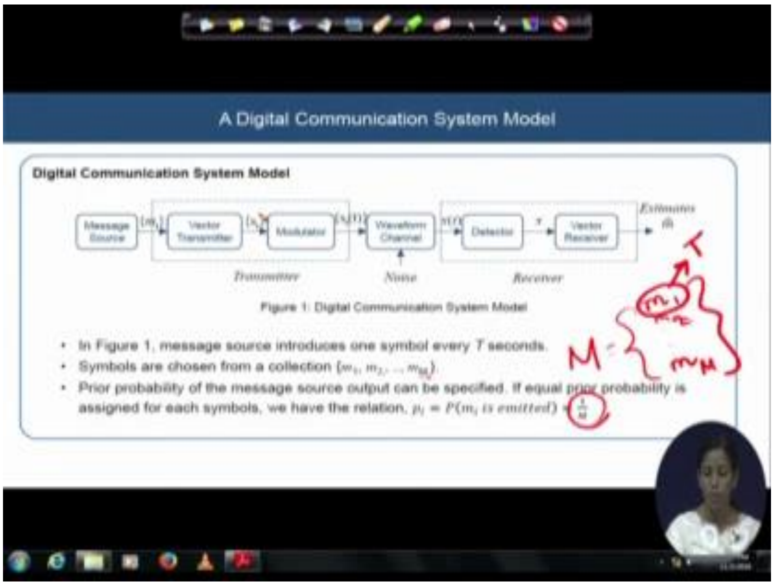


Before going in detail in that we would like to revisit the fundamentals of the digital communications system, because lot of understanding we need to revisit before entering into the spread spectrum communication system analysis. Figure one, we have shown digital communication system model. See, a digital communications system starts with a message source. Let us assume that this message source is generating the messages m i's where I can go from 1 to capital M. So, as if I have a set capital M, who is having the number of the message symbols who is running from m 1 to capital M. And this is an we can call an alphabet also, and this message source each of these messages of this alphabet is called a symbol; and each of this symbol is having a duration of t seconds.

And moreover, we are thinking that this message source is drawing randomly one of these symbols from this set and he is transmitting and sending to the vector transmitter. And we also assume that apriori we know the probability associated of assigning each symbol to the vector transmitter; and that probability, the all the probability of getting any one of this symbol values from this set are equal. And hence the probability of p i associated with the i th symbol of choosing that typical symbol is equal to 1 by capital M.

The selected message symbol is sent to the vector transmitter. The task of this vector transmitter is he will actually for every real-valued signal, he will develop some vector symbol. And he at the output of this vector transmitter hence for every m i we will be able to see a vector representation of this m i, which is given by s i. How s i looks like let us see in the next ppt - in the next slide.

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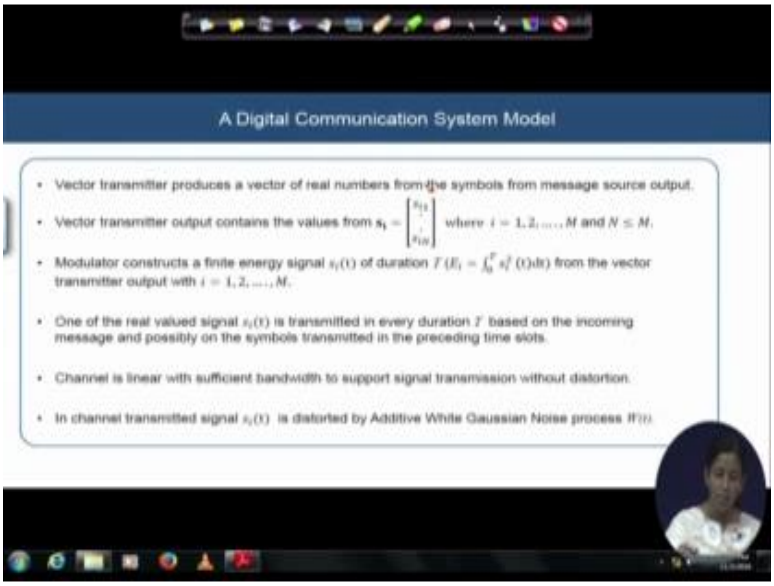


S i is a vector. So, it is written in the bold; and it combines the actually for every ith sample it has capital N numbers of the vectors. So, for each and every symbol, it has a linear combination of capital N number of the vectors it generates actually, the and you can actually have a with your signal s i t that is generated at the output of the vector messenger or the vector transmitter. Where actually this s i t is the combination of the the multiple number of this vectors which is drawn from this capital, it is drawn from this s i. And remember the constitution of this s i t is such that and over the duration of this capital T, the total energy of this s i will be normalized to 1 and s i can vary for over the symbol. So, hence it can have I can go from 1 to capital M, and it will you will get the signals s 1 to s m.

So, one of the real-valued such signals once this signal is constituted by the combination linear combination of these vectors, and in one of those real-valued signal now will be transmitted over the duration of capital T over air. And which one will be transmitted it will be based on the incoming messages and also it depends on the message transmitted in the last time duration. The transmission that will be done it is on the air and hence we need to look little bit about what kind of the channel wireless channel we are looking about.

See, the output of this s o is now getting modulated by the modulator and you have constructed s i t, and now it is released to the channel. So, for our understanding, we will consider a channel, who will have at least the bandwidth equal to the signal bandwidth or more than that. And we are also considering that the noise the that is getting added from the channel will be additive white Gaussian noise process and that noise process we will be now mentioning as W t and hence the transmitted signal s i t that the modulating block is giving you that will now be distorted by this additive white Gaussian noise process.

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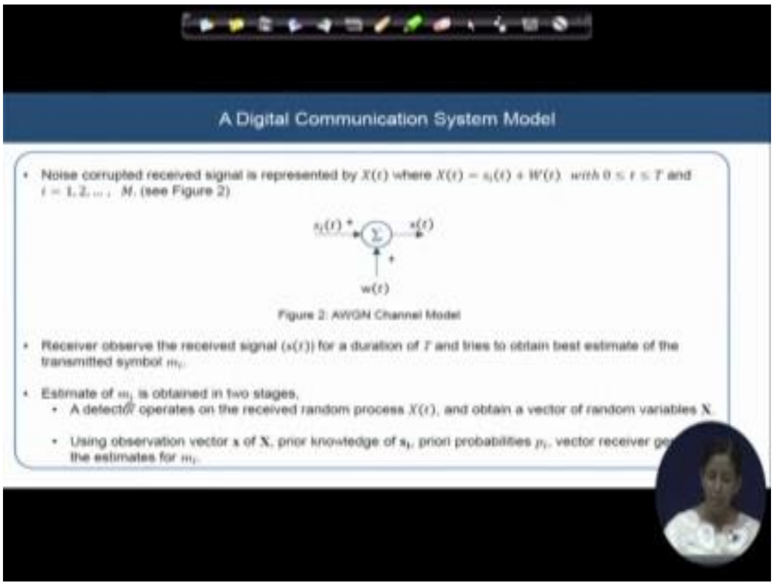


If I try to depict it in a figure, see the figure number two is taking the signal from the output of this modulator, and the channel is adding the noise remember and generating the signal. This x t is the received signal. Remember actually w t and x t they are the sampled value of this random process capital X t and the capital W t. And from now onwards this x t will be considered as the received signal for us. And receiver will now have the information about x t. Once observing the value of x t over the duration of capital T of course, the task of the receiver is to estimate the value of this m i. So, he is having a received signal contaminated by the noise; and based on that received observation, his task is to estimate that what was the transmitted signal m i.

Receiver does this job into two half. In the first process, there is a detector who works or operates on that random process x t received random process x t, and he generates a vector of a random variables which is capital X. And now by observing the small x which is a random sample value of this capital X with the prior knowledge of this s i, I mean the vector transformation in done in the transmitter with the probability of occurrence of all those samples probabilities p i. And this vector receiver generator is now trying to estimate the value of m i. So, one is the detector and second is the decision device and the estimator decision device who combinely work to estimate the value of m i in a receiver.

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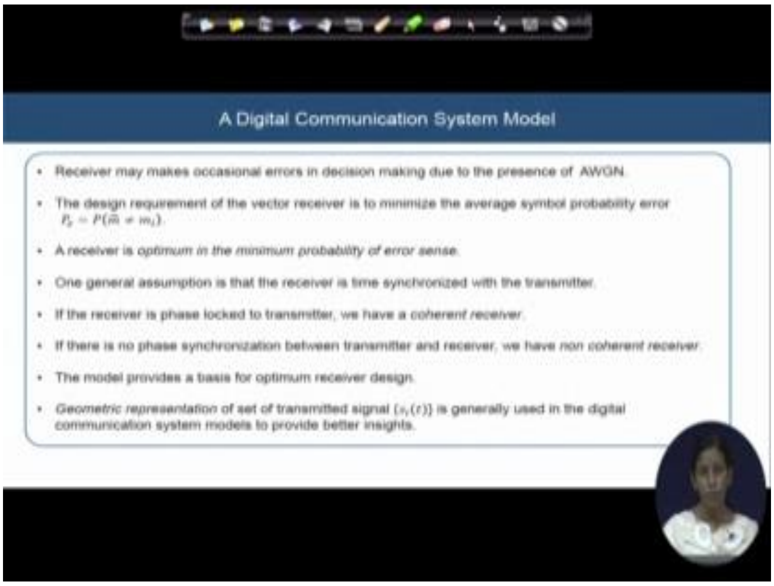


So, this is the normal receiver architecture that we deal with the digital communication system. And remember that receiver can do the occasional errors because white Gaussian noise is present, and noise is completely random in nature. And error probability will be defined if we see that estimated value of m is not matching what was transmitted from the transmitter. This architecture what we have discussed right now this gives that optimum receiver design; in the sense that the error probability will be minimized. And in that the general assumption that we have done in this discussion that transmitter and receiver are well time-synchronized that means, that receiver understands that at what moment m i changes.

Also if we are having an understanding that receiver is completely phase-locked with the transmitter then we call that receiver is a coherent receiver; and if the receiver is not phase -locked with the transmitter, we will call that kind of receiver is a non-coherent receiver. And this model provides the very basis assumption of the optimum receiver design, but we wish to zoom inside the graphical representation and the graphical representation of this transmitted signal s i t, because this graphical representation of s i t will be very much important for us to analyze our direct sequence spread spectrum transmitter receiver. So, now onwards we are zooming inside this s i t and trying to see the geometric representation of this s i t. Very famous method of having a geometrical representation of this s i t is the Gram-Schmidt orthogonalization procedure which will be discussed next.

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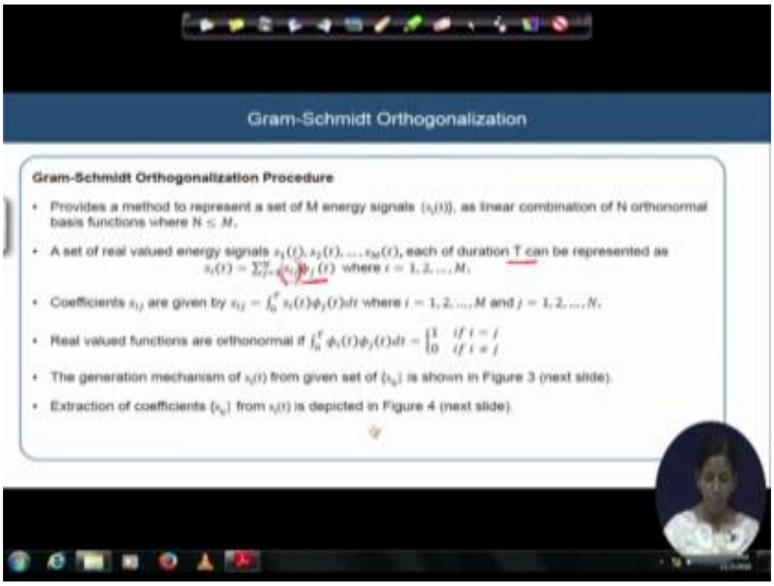


This orthogonalization procedure is basically telling the method how will you represent capital M energy signals s i t as some linear combination of capital M orthonormal basis functions. We have seen in the vector transformation that we are having capital M dimension for each and every ith symbol that is coming from the message source, and we saw also that inside the vector transformation vector representation, the value of the capital M chosen it is less than equal to the total number of the symbols present in the alphabet set. So, N will confirmly it should be less than equal to capital M.

Now, we are having a set of some real-valued energy signals. So, it is varying from s 1 to capital M because that we have started with. Capital M is having the values of this s 1 to your s M, capital M was a set of s m. So, here it is not the m, here it will be the value of capital S, where actually s is now s 1 t to varying up to capital S m t, where m was actually related to the message bits where message bits were varying from m 1 to capital M.

And remember that real valued energy signals we have confined to have a duration of capital T. And by Gram-Schmidt orthogonalization procedure, each of this signal s i t now can be represented by this equation, where s i j is the coefficient, phi j is the orthonormal basis function. So, each and every energy signal, we are representing by in terms of some coefficient and orthonormal basis function. Remember this basis function is varying over 1 to capital N, definitely value of M will be less than capital M, and i the symbols they are varying over one to available number of the symbols capital M.

Each of these coefficients can be given by can be constructed by this equation. So, it is actually the original signal, the energy valued signal if I multiply with the corresponding basis function and integrate it over the duration of the T, it will regenerate the each and every coefficient. And why this basis function the property of this basis function is such that if I integrate it integrate two basis function of ith interval and the j th interval, and integrate it over the interval of 0 to T, what is the symbol duration time. I will have the property that this integration value will give me equal to 1 if both are exactly same; and it will turn to 0, if it is not equal to they are not the instances and not same.

What does it mean is this phi i and phi j is telling that there will the first condition that there will be 1, it is stating something like you are forcefully normalizing the power to be to be 1. And when it is 0 that means the second condition is signifying that they are perfectly orthogonal to each other. So, the property of orthogonality and property of normalization is there, and hence these basis functions are called orthonormal basis functions. So, we are now interested to see that how s i t can be constructed given a set of s i j, and how s i j can be constructed given s i t. So, both of the generation mechanism are described in the next slide.

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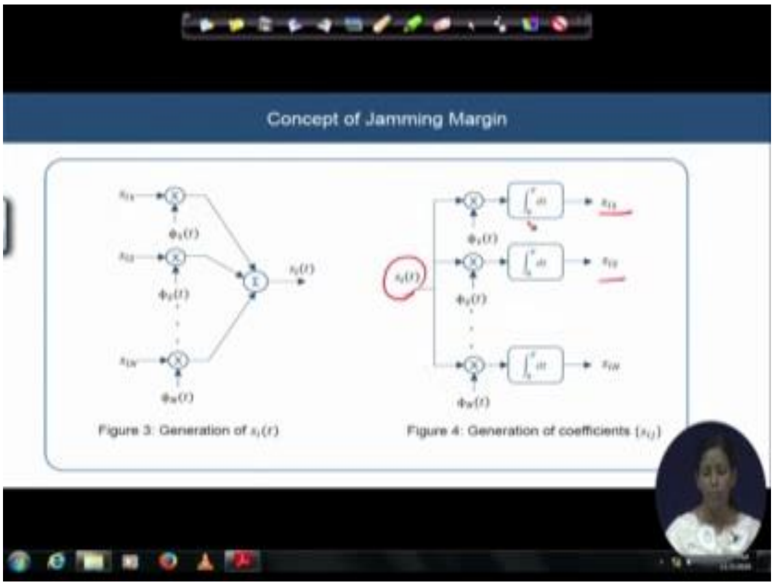

Let us see. So, we know that if we are having a set of the coefficients from s i 1 to s i n and if we are having the basis functions orthonormal basis functions set, so their corresponding multiplication with them and addition will generate my s i t following the expression shown in the last slide here. And oppositely, if I am having the incoming signal s i t, I will generate each and every coefficient by multiplying with their corresponding orthonormal basis function and integrate it over the symbol duration 0 to capital T which is coming from the expression given in the expression here in the earlier slide - this one. So, the generation process is ok and understood clearly now.

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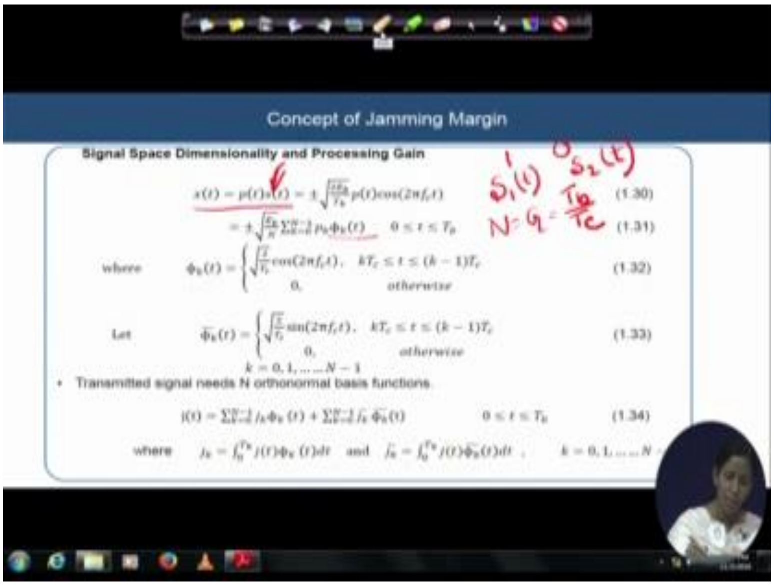


Now, with this understanding that means, we are now thinking that in a in a signal space we can actually represent any transmitted signal by combination of the orthonormal by means of some orthonormal basis functions. And hence if I try to revisit BPSK modulated signal in terms of the discussion just now we have done. We understand that our transmitted BPSK modulated spread spectrum signal will be given as x t equal to p t into s t. If I consider this s t is the BPSK modulated signal, which is spread by the sequence p t, s t will be given by plus minus square root of twice E b by T b into cos t2 pi f c t.

Actually the BPSK signal, you need to transmit the signal value 1 and signal value 0. For that we need to transmit over the two real-valued signal s 1 t and s 2 t. We transmit them in such a way that they are out of phase to each other. So, square plus minus 1, if 1 is transmitted for the plus square root of twice E b by T b cos 2 pi f c t then s 2 t will be minus square root of twice E b by T b into cos 2 pi f c t. Basically two signals are 180 degree out of apart pi plus cos 2 pi f c t will generate you the minus of the same thing.

This analysis is based on the BPSK. So, here we our p t into s t will now lead now to plus minus square root of twice E b by T b p t cos 2 pi f c t. The thought processes should be aligned with the discussions we have done that we have done that in the signal space dimensionality. We have two real valued signals associated with the BPSK transmission. And if I now define the orthonormal basis function associated with this BPSK transmission is given by square root of 2 by T c cos 2 pi f c t. And phi tilde k t is equal to 2 by T c sin 2 pi f c t over the duration of k T c to k minus 1 t means one chip duration. And if it is not existing anywhere value is becoming 0; other case you remember this cos and sine they are perfectly orthogonal to each other and hence they constitute the orthonormal basis function for representing the BPSK modulated signal here.

We can reorient this equation 1.30 in such a way that we need square root of 2 by T c if I divide by T c and multiply the numerator and denominator by T c and take square root of 2 by T c into cos 2 pi f c t out. And if I substitute that value by phi k t, I will be ending up with square root of E b by N is coming, what N is equal to my G which is equal to the width duration by the chip duration, this is nothing but the gain. And p t is now the summation of k is equal to 0 to N minus 1 p k because here we are considering the chi k is equal to one for the rectangular chip waveform. So, p k will be basically value of the p k will be either plus 1 or minus 1 continuously going on. So, the equation 1.30 will turn down to 1.31 for the bit duration period of T b.

If this is the condition for the s t, then we will like to see that how many number of the orthonormal basis functions now we are dealing with. If I look inside the equation 1.31, see that as k is varying from 0 to m. So, you need at least capital M number of the orthonormal basis functions to represent the value x t, so which is not same according to the conventional communication system. There we were not requiring all this because of spreading you are in the requirement of capital N number of orthonormal basis function to represent x t in the signal space.

Come back to the jamming for jamming signal j t, I have that jammer will jam me on both the incoming in phase on both the phases. So, jammer circuit will be phi k t plus j tilde phi k tilde t. And in both the situation, we I can re compute the value of this j k by the expression of this. Remember here one thing jamming has no information about your; it has information about your bandwidth correctly, but it has no information about the phase.

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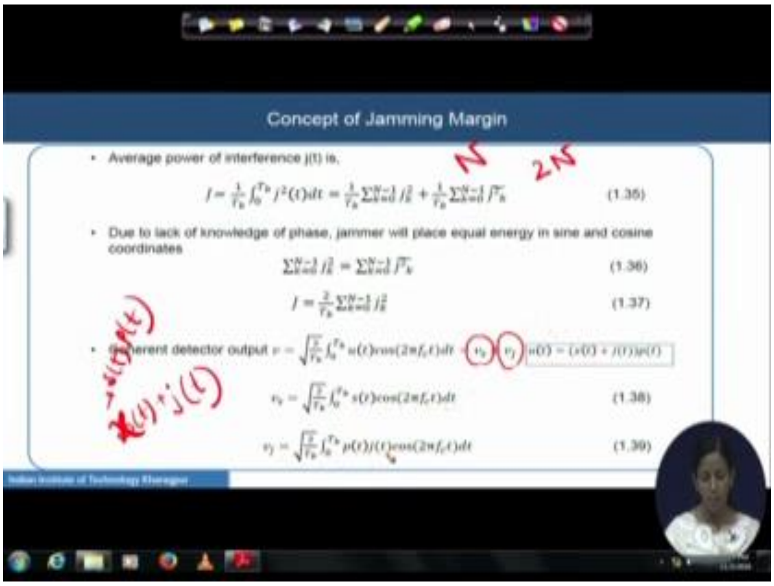


So, it does not know whether the signal is transmitted in phase or only on the q phase. So, what he will try to do, he will try to means load the power equally over the in phase as well as the q phase and if as I am interested over the computation of the average power of the interference jamming power it is. So, the jamming power will always be given by the square of this j's, jt the jamming signal integrated over duration of the bit. And utilizing the previous equation I can now write down it like this.

Now, another thing is as we as I have told that jamming has no information about your phase, so he will load the equal power on the on the in phase as well as the q phase. And equating both of them finally, the jamming power is nothing but the twice of any one of this power that I have represented here. But interesting thing to note is that to jam, a jammer needs to spread its signal on the 2 N orthonormal basis, because for him it is N number of basis functions is associated with the in phase n number of the basis functions is associated on the q phase. So, jamming is going on over 2 N dimension 2 N dimension and signal is getting spread over capital N orthonormal dimension.

Suppose, with this understanding, let us enter into the coherent detector. Finally, this jamming signal plus s t plus j t is getting detected inside the coherent detector, inside the coherent detector. If I see at the output of the coherent detector, we understand that inside the receiver the received signal x t plus this j t now which is basically this s t this is not s t now, it will be x t for you as we have seen that x t is equal to x t is given by my s t into p t. So, you have received the spread signal x t. So, x t plus this j t in this receiver is now getting multiplied with p t. So, p t square it will be for the signal component and but j part will be multiplied with the p t. So, you are having at the output of the detector, we will be always be seeing two different components; one is contribution from the signal one is the contribution from the jamming part. Signal part will be given by s t into cos 2 pi f c t because p square t is equal to 1 and jamming part is having the p t associated with it.

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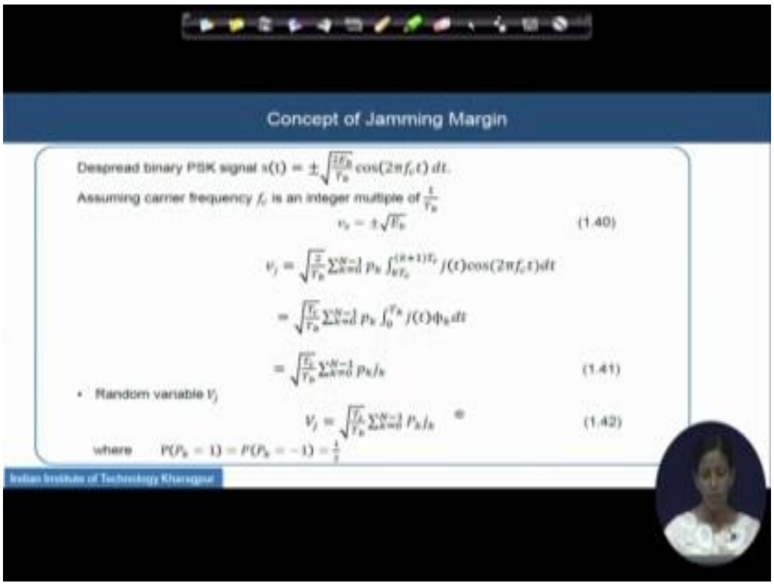


Now, if I proceed further, and we understand that the PSK signal the dispread PSK signal is given by this that we have considered at the beginning. And if I substitute this value in the equation 1.38, s t value if I substitute, and if I complete the integration, then I will see I will end up with this v s is equal to plus minus square root of E b considering the fact that the carrier frequency is an integer multiple of 1 by T b. And there also we would not consider the higher frequency terms and for b j interestingly, if I substitute the value of now the j k we understand that it will be a summation and j t is there integration is running over a period of T c.

Now, if I utilize the integration of the j t, and the j k if I substitute from the previous equation then I will finally, end up here it is square root of T c by T b. Please try to do this derivation by your own and please check whether you can come down here or not. Substitute the value of the j t and j k from the equations here. This is the value of the j k, and this is the value of this j t. And substitute here and you will be coming down here.

Now, if I see what is the random variable capital V j, if I constitute the random variable V j, where small v j is a sample value of that. And now it will be given by this P k into k k; small p k is also sample value of this capital P k. We have now remember that this capital P k is a proportion that is coming from the P i and its valued to be plus 1 or minus 1 or equi-probable and this probability is hence equivalent to half.

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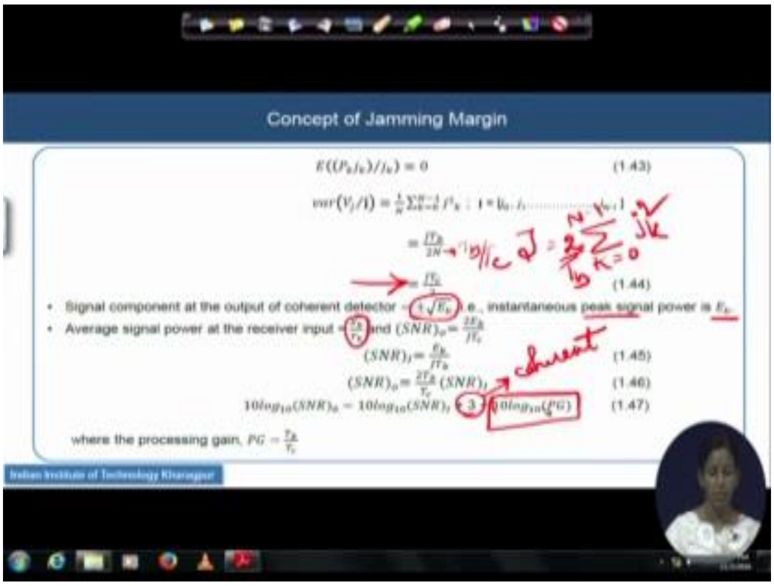


And. So, if we are trying to compute the p k into j k given the value of the j k, the mean value will be equal to 0 because this p k are equi-probable and the value of probability is equality to half. So, half minus half it will give you the value half of the time the value is equal to plus 1 and half of the time the value is equal to minus 1. If we add up we will end up with the mean value of 0. So, given the value of j k, p k into j k will always give the mean equal to 0.

If I now constitute a vector j, where actually j is the samples of the jamming signal starting from j 0 to j N minus 1 overall in orthonormal functions and given that value j there is a random variable this v j variance of v j given that the value of the j now that can be computed as 1 by N square of this j k. And now I wish to refer the equation where we derived the value of capital J earlier. Recall the equation 1.37 where we established that capital J is equal to 2 by T b it is the power jamming power; it is equal to 2 by T b summation of k running from 0 to N minus 1 j k square. We will utilize this equation in the next slide here to compute the value.

See, here we understood that J the total jamming power will be equal to k is equal to 0 to N minus 1 j k square and it will be also having a power of 2 associated with it. And this jamming power let us go back it has also it is a power. So, it has a component of divided by 2 b. And if I substitute now in place of this, if you try to substitute then you will be ending up with j into T b by 2. See, this J into T b by 2, and this capital N divided by N will be giving you the variance and given the value of capital J of course. And if I substitute the value of capital N as T b by T c, you will be finally ending up with the variance of J T c by 2.

So, what is this is the approximate power that you are getting the calculation of the power that you are getting at the output of the coherent detector after dispreading operation in the receiver contributed by the jamming signal. So, this is the power contributed by the jamming signal output of the coherent detector after dispreading operation in the receiver.

Now, let us revisit. We have seen that output of the coherent detector, the signal component is plus minus square root of E b. We have seen it in the previous slide. So, the instantaneous peak power will be equal to E b. So, the average signal power at the input of the receiver was E b by T b, this was the power computation. And output of the SNR, if I now write the output SNR, the output of the coherent detector, now it will be given by what, it will be given by this peak power divided by the noise, the noise is equivalent to my jamming power. So, I will divide E b with J T c by 2, hence it is coming it is coming twice E b by J T c by 2. E b was the peak power of my signal at the output of the coherent detector; and J T c by 2 was the contribution from the jamming power at the output after dispreading operation and at the output of the coherent detector in the receiver.

But what was the input section, input section was input SNR was E b divided by T b that was a signal part of the input of the receiver. And it was also the jammer power jamming power was only J. If the power became different because of the spreading operation only, so the power at the input and power that you are getting at the output that is differing because spreading has happened in for the jamming, for the jamming signal.

If I now try to find out the establish the relation between the output of the coherent detector and the output of the receiver and the input of the receiver, I will be ending up with the expression that the output SNR is a function of the SNR input to the SNR, SNR at the input of the receiver multiplied by 2 T b by T C. In the dB scale, the dB value at the output of the SNR will be the value of the input dB value input SNR dB value plus the processing gain because T b by T c is equal to the processing gain plus 10 log to the base 2 who will give you the value equal to 3. Remember this additional value of the 3, you are getting not because of the spreading, this is getting because you have utilized the coherent receiver.

This is the it is the property of the coherent reception, you have got this extra value of dB value, but we are only interested here because of the spreading process, you are gaining here the 10 log to the base 10 of the PG where PG stands for the processing gain T b by T c. So, output will be some 10 log to the base 10 PG dB higher compared to the input SNR value that is why we continuously was telling that output value of the coherent detector, you will have the higher value of the signal to noise power compared to the input SNR values. And this extra value is viewed is contributed by the processing gain that was coming from the transmitter.

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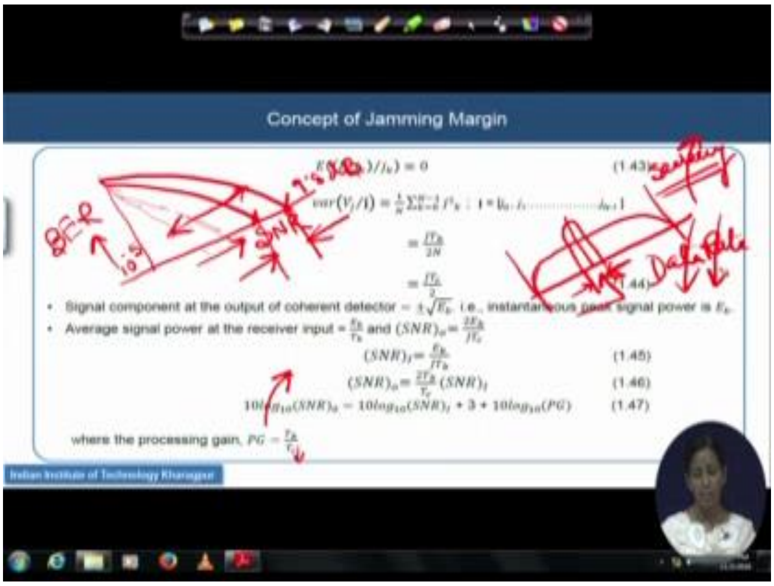


This because of this gain, you will be able to see that if the conventional receiver I have the plot of this signal to noise ratio, and I was suppose plotting the bit error rate where we are interested in. If a conventional receiver shows over the AWGN channel that the 10 to the power minus 5 bit error rate, we are touching over a BPSK transmission over AWGN as at 9.8 dB value. Then for this processing gain based on the processing gain you are computing the value related value of T b by T c definitely actually to get that same 10 to the power minus 5 value bit error rate value, you would not need actually to put that much amount of the SNR. You need to put a lower value of that lower value of the SNR, you will automatically get the value that same targeted 10 to the power minus 5 bit error rate value.

So, this gain the processing gain for a typical error computation, a typical error rate the gain is reflected the processing gain 10 log to the base the processing gain, it will be reflected over the gain on the x-axis gain on the bit error rate graph, the performance analysis graph. So, how do we actually express the gain of this processing? Gain reflected on the system performance is like this and this is true actually and the gain this gap with respect with the conventional communication system. It is not fixed, it is a function of this design of the sequence itself, because if we lower down the T c, the processing gain will increase and the graph will keep on proceeding keep on actually coming like this. It will go towards the left and your processing gain will keep on increasing, but remember when you are increasing the processing gain somewhere, you are having a some loss at the cost of something you are doing it.

But remember whenever we are telling that we will be spreading the sequence, so spread spectrum is spreading the sequence, but remember we cannot spread a sequence beyond the available channel bandwidth. So, always the spreading is restricted by the bandwidth you are dealing with. So, whenever I am trying to have a higher spread, so the bandwidth that we have the original bandwidth that we are trying to utilize that bandwidth will be shrink. So, more the value of the lower the value of T c, you are starting with more actually your bandwidth will be there is no doubt about it, but the actual signal bandwidth that you are really getting this with which actually we will start with that will keep on shrinking. So, the message bandwidth you are going to be reduced and the reduction of the message bandwidth that you are playing with will be a very big hit on the data rate that you are transmitting.

So, increase in the processing gain will give you increment on the bit error rate performance with lower value of SNR, you can get the same bit error rate compared to the conventional system. But the cost is that you cannot transmit the same data the data rate of the transmission that actually is decreased, but however, with the almost of the same data rate, you need actually the higher bandwidth for the transmission for the minimum data rate for any kind of the data rate whatever is the minimum bandwidth requirement that for the conventional system compared to that you are using a much more higher bandwidth to get the privacy or the message secrecy incorporated in the communication system.

So, you are gaining in terms of secrecy you are gaining in terms of the quality of the service, because in terms of the there is a gain in terms of the SNR involved, but there is a very big hit on the data rate, your data rate is getting damaged like anything. So, heavily fall will be there on the data rate.

So, we will be ending here today with this concept of the jamming margin. And next class, we will try to enter into another kind of the frequency; another kind of the spread spectrum communication system which is also widely used in practice which is called the frequency-hopping spread spectrum system.