Spread Spectrum Communications and Jamming

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Lecture - 39
Detection Probability Analysis of Code Acquisition for FFH/MFSK

Hello students, we were discussing the code acquisition mechanisms serial code acquisition mechanism in FFH MFSK system. And in continuation of that in the last module, we have seen the mathematical model of the code acquisition techniques, I mean the different signal models of the incoming signal locally generated sequence in presence of the timing error and the frequency error, how the detector output - non coherent detector output is generating the signal.

Today, we will try to see the acquisition how the detection probability is coming out in this system. And also we will focus in calculating the acquisition time and the mean time of this acquisition and the variance of the acquisition we will calculate today. And also we would like to see some graphical representation of this the mean acquisition time based on the number of the code chips used for this kind of acquisition. So, detection probability will be our first importance and first target to calculating this module.

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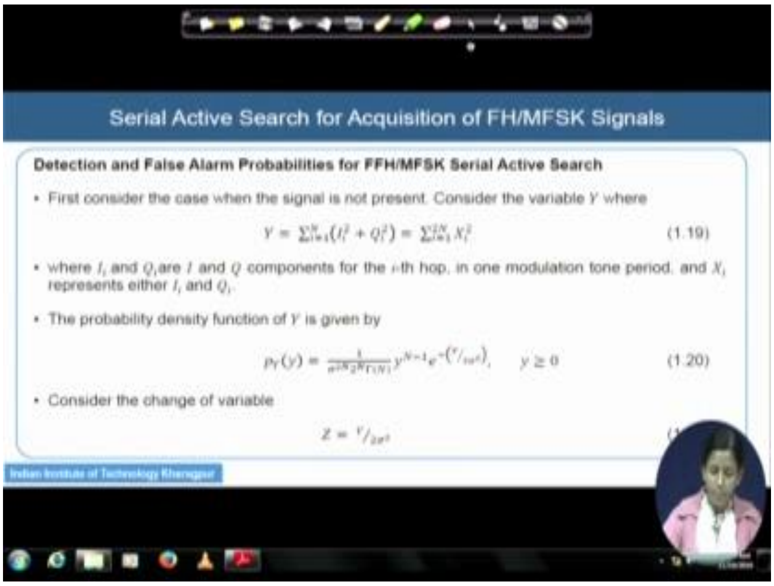

We will start. And remember the last block diagram that we have discussed in the last two modules. And where actually we will we have designed where most we explained all the terms like this I i and Q i I t, I and Q terms. This I and Q are the signals that are coming on the Ith path of the detector and the Qth path of the detector.

We will start with the consideration that when the signal is not present, we define the variable y which is nothing but the sum of all the I is equal to 1 to N number of the hops and summation of all that and I square plus Q square. We are discussing the fast frequency-hopping system. So, within the symbol duration, we have capital N number of the hops. And we understand that small i is now denoting the number of the hops within the symbol duration.

So, Y is a variable who is defining the I square plus Q square and summation over all that. Remember basically this capital Y is nothing but the output of the detector we have added up the I th signal with the Q th signal right you hope you remember. So, when this addition is going on, the added output is we have mentioned as capital Y. And basically it can be consider also as capital X i square where I is varying from I to 2 N. This I i and Q i, they are the for the Ith hop they are the Ith component and Qth component as I told. And whenever I am talking about one symbol duration basically within one modulation tone period part period actually this hopping is going on. And this X i represents either I i or Q i any one if I try to look that is why the summation weighing for the I is equal to 1 to 2 N.

Now, with respect to this, the probability try to see the probability density function of this added output I Q added output then it will be written like this, it will be obtained like this. And if I now change the variable by substituting this y by 2 sigma square as z, then the whole expression will be shifted like this.

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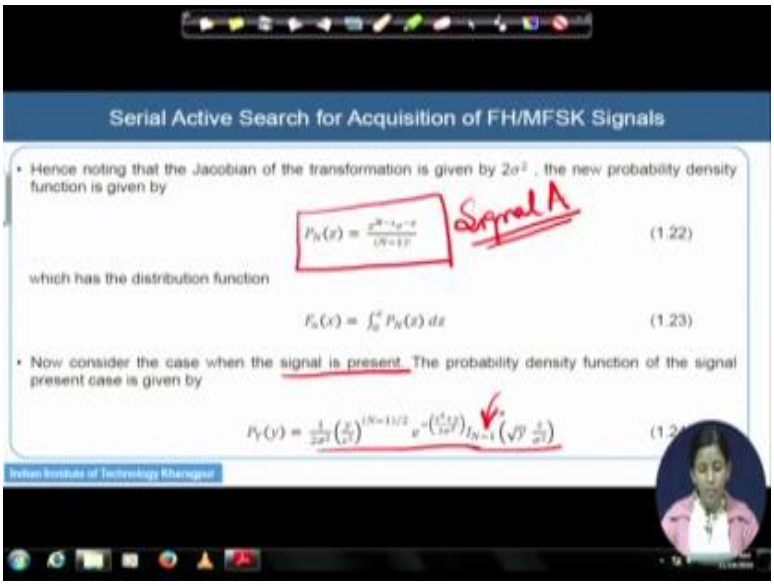

Remember in the previous equation this 1 by 2 sigma square, this 1 by 2 sigma square or this transformation if we consider Jacobian, it will be a Jacobian of the transmission will be given by this 2 sigma square. Because if you substitute this guy here in this equation and try to do this transformation with the Jacobian of the transformation will be given by this 2 sigma square. And the probability density function will be finally, boiled down here. And using this probability density function, we can now calculate the distribution function. The distribution function is nothing but 0 to x and then P N Z dz. And if I try to go ahead with that then the probability density from this is can be calculated further definitely, but this is the probability distribution density function when the signal in absence of the signal we have computed.

Remember actually we did this kind of the calculations earlier when we were dealing with the match filter based acquisition technique. We calculated the detection probability (Refer Time: 05:21) and I am considering the signal present and signal absent. Similarly, here the signal absent situation, the probability density function is looking like this. When the signal is present now, the same expression that was given by 1.21 in the earlier slide, he will be now transformed here like this. And the look this is having this expression the whole expression is having some similarity with the match filter based detection also where we also saw that when the signal is present, we could find a portion in the probability density function where the contribution was there portion can be represented by the Bessel’s option of some kind. It is non-centric chi square distribution equivalently.

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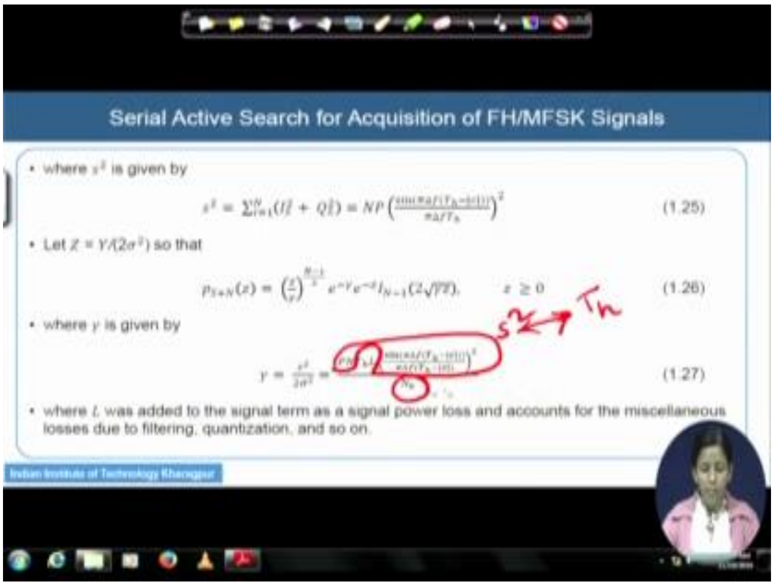

And we could also find that now this changes here that we are having due to the presence of the signal where we have measured here one quantity and introduced one quantity called s square. This s square will be basically nothing but the addition part of this in phase and Q phase signal component, which is getting added over all capital N number of the hops. And that can be actually we saw this I s square Q s square expression in the last module, and which is given by the p into sin of pi del f th minus mod f whole square by this.

Remember this equation we derive with the condition, what was the condition? Condition was that that mod that EPSA mod EPSA is less than T h is too less than I mean the error was too less than the hopping duration, and also this del f multiplied by this error the timing error this section this whole section is very less compared to the unity. So, close to the it is very, very less I mean we also consider hence del f into T h is also less than unity and we consider that this EPSA is to less than the T h and then actually we derived up to this expression. And finally, by substitute that here, so that p multiplied by that whole term pp is the power of the transmitted signal and it will be summed up over N number of the cases, so N times that expression is now the s square.

And like the earlier case if I now substitute like the earlier case where we substituted Z by Y by 2 sigma square similarly, if you substitute here we will be ending up with an expression of S plus N Z given by this. Why this N came into picture because here the signal plus noise power both are introduced in that expression. Here actually the sigma square plus the noise power will be also introduced sigma square actually is the contribution from the WGN noise and this is the guy from the signal power contributing the signal power. And we will be substituting that expression in terms of the gamma, gamma will be the signal to noise ratio. And this gamma substitution of this gamma is done like this, where this you understand that P into N multiplied by this whole section is the signal power contribution coming from the signal power, and divided by the noise spectral density one-sided noise spectral density here. We have multiplied the guy with the if this whole signal power we are interested over the duration of our observation that is equal to the hop duration. So, the total power signal power is multiplied with T h.

What is a signal power? This section is the signal power s 0 or s square, sorry, this is the signal section S square and this is the single certain power spectral density T h we are interested about the signal power over the duration of our interval observation interval which is the hopping time T h. So, it is multiplied with the T h. And then L is coming as an additive factor, L is we understood when we were expressing the terms and realizing all the equation of the match filter that the tone the modulation tone it is also the signal is also modulated.

So, by because of that, whenever the signal is passing the fountain filters, there will be attenuation of that amplitude section, and there will be also the power loss associated with the miscellaneous activities going inside the filtering, quantization, some other losses are accommodated. To add all those losses to incorporate all those losses we have introduced the factor L by which the signal power will be diminished. So, it is the factor less than 1 by means of which you we will be correspondingly decrease our signal power and this is a total noise power. So, hence your gamma is a function of all this.

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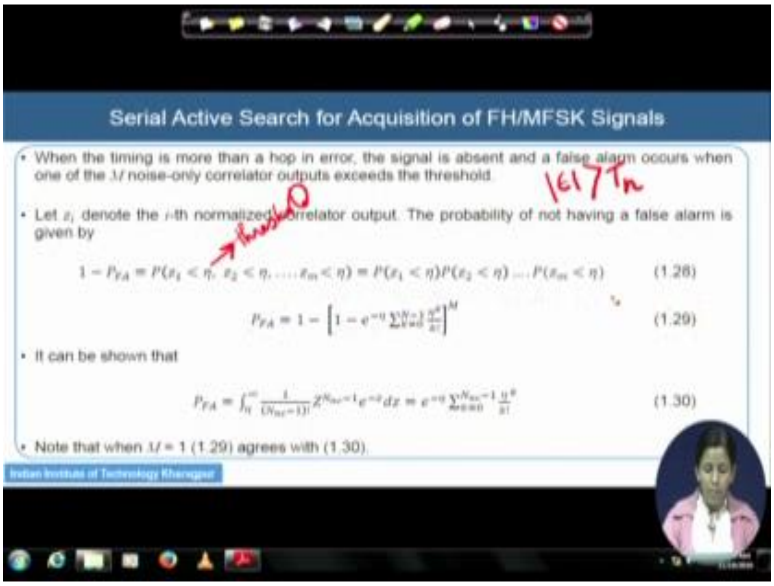

Now, when the timing is more than one hop in error the condition that now we are talking about is the mod EPSA if it is greater than time hopping the signal is absent for suppose in that situation, and there is a false alarm occur then situation in that situation a hop false alarm can occur. Because the accumulative now correlated noise terms for capital M noise terms, they can actually call cause a threshold, they can cross the detected threshold. And hence it can be falsely considered to be a detected stage which is basically we call it as false alarm.

And now the whenever we are actually we have seen earlier also whenever we derive the detection probability, we initially really very very scared and we are very very serious about competition of the false alarm. So, here also we will first consider into the false alarm probability let this Z i denotes the ith normalized correlator output. So, hence the probability that false alarm is not given is equal to 1 minus false alarm; if false alarm is not given the rest of the part the detection will be done.

So, one if the total probability was one. So, the cases when the false alarm is not given should be 1 minus probability of false alarm and which will be further given by the probability that z 1 is coming less than that eta. Eta is our threshold inside the detector, where actually with respect to which all the output-squared envelope added squared envelop of the I th and the Q th part is getting compared with. So, hence we consider that the correlator output is z is all the instances from 1 to small m if we all of them are less than eta. False alarm means all of them will be greater than e eta; 1 minus square root of false alarm is all of them should be less than eta. So, they are independent of each other none of this detection is or threshold crossing is dependent on the previous or the future samples and future test cases. So, we can write down as a multiplication of each and every probabilities.

So, now the probability of false alarm finally is coming, it can be written in the form of like this where from it came you know it can be shown easily that this probability of false alarm is nothing but crossing that eta and going up to infinity. When but which quantity will be crossing it is the probability of receiving the signal that is crossing the probability density function, which is crossing from eta to infinity in the absence of the signal. Where is this N C, this N C is actually the number of the non-coherent detector stages that is consider to take the decision. So, if this is the situation and this whole equation and the centebration can be further simplified by this exponential term and by eta to the power k by k factor. We have seen actually this expression when we deriving the match filter based acquisition techniques, and we have utilized this expression here. So, the probability of false alarm means is represented by this 1 minus f a term is given by this to the power M. Remember if I put this capital M is equal to 1 then basically this expression 1.29 will be equal to 1.30.

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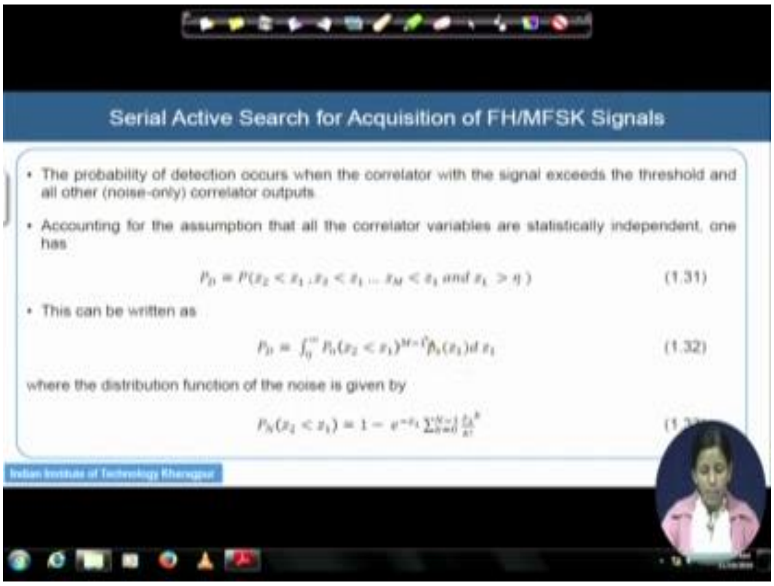

And now the probability of the detection will occur when the correlator with the signal exceeds such threshold. And all other noise are only the correlator outputs. And we consider this by writing by expressing the term by equation 1.31. And see that in such situation the if the situations are z 2 is one instant, so z 2 will be less than z 1; z 3 is less than z 1 to z n; z 1 and z 1 is definitely greater than eta. So, all the cases or the correlated variable outputs, they are coming statistically independent.

So, we gain actually can be equivalently written as a product term and hence it can be raised to the power of M minus 1. So, this expression can be further written as the probability that each of them coming with the consideration that the z 2 is less than z 1. So, z is output of the correlator again as shown in the last slide. And this probability should be multiplied with what is the probability or that the signal and in signal density probability in the presence of the signal there whatever the density Bode density function we have derived. And where the distribution function of the noise will be given like this that we have seen in the last slide.

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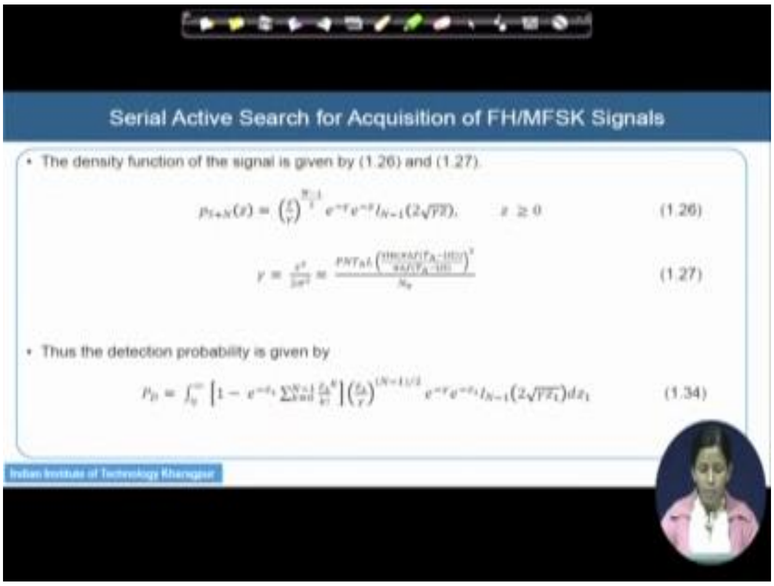

So, finally, where are we ending up is the density function of the signal given by this guy whereas actually gamma we saw that gamma can be written as this. The detection probability will be finally written by this expression. In terms of the gamma, we have written it where gamma should be substituted by this guy, and this is the contribution from the noise section mainly coming out. And the function this detection function is also non central chi square distribution this is the Bessel function of this is the Bessel function I N minus 1. And remember that this detection probability whatever we have gone we have received here it is very hard to compute, but it is easy to give some upper boundary and then lower boundary, calculating some upper boundary and lower boundary of it that we will show in the next.

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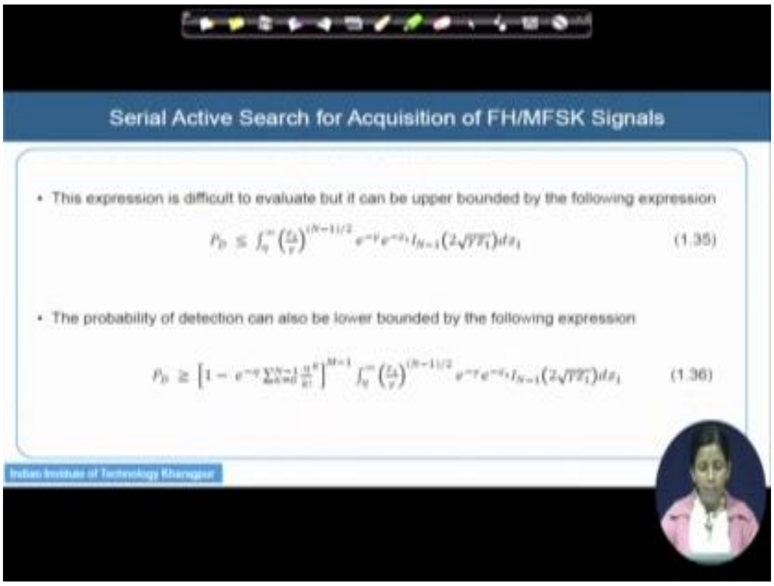

So, the detection probability that is given by equation 1.34 that can be having that can have a lower boundary of like this. It is calculated like this 1.36 is expected the detection probability the minimal detection probabilities will be wall bounded by this guy. And it will be upper bounded by the expression of 1.35. So, at least you can see that the detection probability can never go beyond this or can never be lower than this value. So, lower than this guy and upper by this guy is not possible. So, with between these two limits, the detection probability is expected to get whatever be the acquisition mechanism and then algorithm you design, but detection probability should be in between these two, if you are take a limits in case of your FFH MFSK signal.

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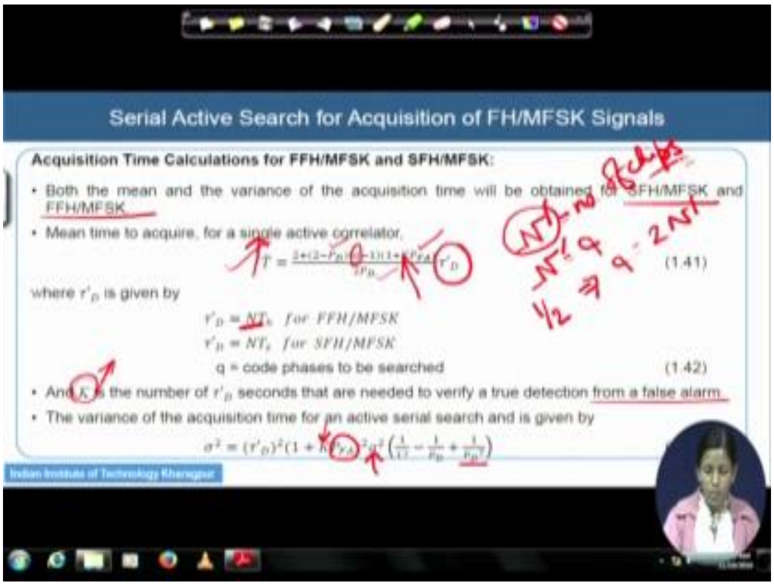

Now, this is the time to look into the computation of the active search time. And what will be the mean time of the acquisition, and the mean time of and the variance of it, and we will take example upon example and try to see whether impact of it, this acquisition time for both the slow frequency-hopping as well as the fast frequency-hopping only defaults in the terms of this tau dash D. This tau dash D will be given in the different form, but otherwise for both the cases SFH MFSK and FFH MFSK the mean time equation will be same.

Now, remember this a mean acquisition time is a function of few stuffs, where the detection probability plays a important role that we have computed in the last. Probability of false alarm is a playing a role that also we have seen this expression. The number Q, Q is a parameter that is specifying that how many code phases you are requiring to be searched. What is the meaning of this? Suppose I have capital N number of chips. So, if I do a code phase search per chip then I will have capital N number of the code search only. So, the value of the Q will be equal to N, that if I do the code search at half of the chip duration then I need to do double the amount of the code search. So, the value of the Q should be there twice N.

So, if you do four times a search per chip then the value of Q will be 4 of N 4 N like that it is. And K, capital K is a number of the tau dash D seconds that are needed to verify that acquisition time. So, before declaring we understand that there is a dual time and how many such dual times you are considering if it is a multiple dual acquisition mechanism. So, how many multiple dual times, how many number of the dual times you are considering to reach to the final conclusion of competition of this acquisition time is the consideration. So, K is basically taking care of how many number of the dual times you are considering for computation of the acquisition. And tau dash D is basically our dual time for one complete search.

And now I told as I told that for SFH to FFH, only the difference is that there is a different expression for this tau dash D or the dual time. And this tau dash D will be given as a capital N times the T H for FFH MFSK signal whereas, the T H is equal to symbol time we understand for the slow frequency-hopping. So, they will be given by capital N in to T H in the case of our tau dash D to compute tau dash D N into T H will be the expression.

So, actually this from the false alarm, if you try to really increase the detection from the false alarm, it is not advisable also that you declare the acquisition within one dual time observing intervals over one only one dual time observation you do not declare it. You please repeat it over the multiple dual times. If we keep on repeating over multiple dual times to verify or to detection is done or not over the fall then you can minimize actually the false alarm from the detection techniques that is why we do not prefer to keep the value of the K is equal to 1. We increase the value of the K to certain extent.

But remember as you are increasing the number of the K value the mean acquisition time as it is directly proportional to K, so T will also be increasing. If T increases definitely actually your synchronization process is the lengthy process in the receiver, I hope you understand that. And as the acquisition time is that you are taking more time to acquire the synchronization information, so the data detection is getting delayed and it is not a fast acquisition that you are now talking about, but you are talking about a very reliable synchronization mechanism.

So, choice of case is very crucial in that sense, you have to do a compromise how fast you wish to do the acquisition versus actually how good or what is the quality of the synchronization you are looking for. So, you can compromise actually in between the quality of synchronization to the time required for the acquisition by properly selecting the value of the K.

So, now the variance finally, the variance of acquisition time for an active serial search finally, we can obtain by this expression. As likely the mean the variance will be obviously, a function of this parameter K, the detection probability P D, and the false alarm probability PFA and the Q. And remember here the capital N, when I am taking about the Q with respect to capital N, the capital N here is actually the number of the hops that you are doing, but I explained this it is actually the number of chips that I was talking about. So, you better map it as N dash N dash let N dash be the number of chips over which the acquisition is going on, and capital N is the number of the hops associated with the hopping mechanism. So, Q is associated with the number of chips, this is not the hops. So, hop is different than the number of the chips associated.

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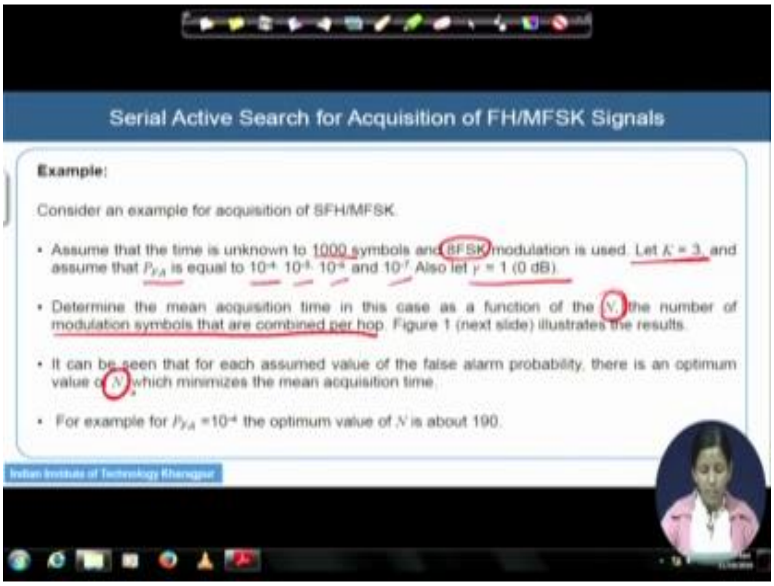

So, let us take an example to get an idea how the acquisition time of SFH and MFSK signal is related to these number of the chips. And this expression can be actually rerun with the fast frequency-hopping mechanism also. Let us assume that we have assume that the time is unknown to 1000 symbols and we are considering a 8 FSK modulation scale. Let k is equal to 3. So, 3 dual time is elapsed to reach the, declare the synchronization is acquired. We are considering the false alarm probability will be 10 to the power minus 4 or minus 5, minus 6, minus 7. So, for different level of the false alarm probability let us actually recalculate the actual acquisition time and let gamma is equal to 1 is equal to 0 dB consideration

So, problem formulation is such that we have unknown 1000 symbols, 8 FSK modulation is going on, value of k is equal to 3, we understand the probability of the false alarm and gamma value is equal to 1. Now, you define the mean acquisition time. And we have in this case as a function of the N we will find out and the number of modulation symbols that are combined is a function of N and N is here is the number of the modulation symbols that are combined per hop. And it can be seen that for each assumed value of this false alarm probability there is a typical value of the number of the symbols per hop this minimizes this acquisition time. For example, let us start with probability of the false alarm the optimum value of the N we will be seeing something. Let us see where are we.

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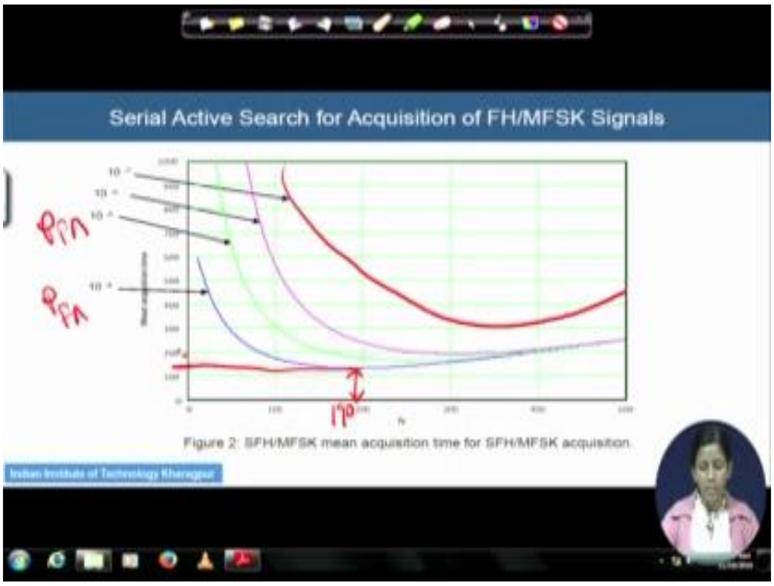

So, this is the graph where N is plotted on the x-axis number of the symbols and this is a mean acquisition time, and the different graphs are plotted for different amount of 10 to the power of minus 7 is missed here, it will go like this. So, this 10 to the power minus 4 minus 5 minus 6 minus 7 are the probability of the false alarm values going on. So, when you are increasing, I mean probability of false alarm is 10 to the power minus 4 to 10 to the power minus 5 to 10 to the power minus 7, actually remember that with the number of the acquisition and the number of symbols that are taken for giving a mean acquisition time they are different. We are actually here with the value of approximately 190 if the probability of false alarm is approximately 10 to the power minus 9, 10 to the power minus 4, we need at least actually 190 number of a symbols to declare the minimum number of the acquisition time.

So, y-axis will give you the number of the; what is the acquisition time with the typical probability of false alarm; and this x-axis is giving you how many number of the symbols you are minimally required to avail that mean acquisition time. The any side of this graph the acquisition time is increasing.

So, you can actually give target false alarm, this graph actually will guide you to understand given a typical false alarm. How many number of the symbols actually you are minimally required to get a minimum acquisition time for a frequency of MFSK signal or with a modulating tone of frequency-hopping 8-FSK signal it is. For an 8-FSK signal and if it is so low frequency-hopping going on where your symbol time and the hopping duration is exactly same and in that situation you require at least 190 symbols approximately for getting a probability of false alarm of 10 to the power minus 4. Similar kind of the graphs can be also generated for the fast frequency-hopping and the simulation.