Spread Spectrum Communications and Jamming
Prof. Debarati Sen
G S Sanyal School of Telecommunications
Indian Institute of Technology, Kharagpur

Lecture - 38
Active Search Code Acquisition for FFH/MFSK Analysis

Hello students, we were discussing the serial active search code acquisition technique for FFH MFSK system. Today we will do some analysis on that code acquisition mechanism for FFH MFSK.

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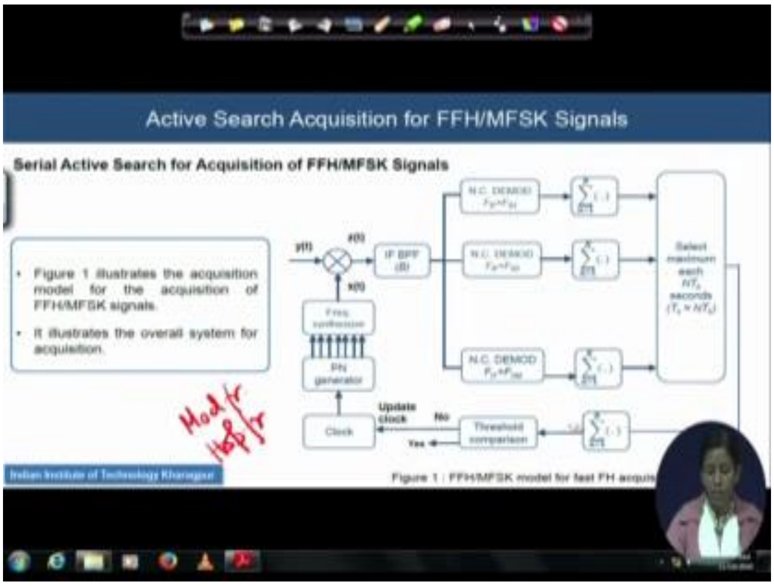

Quickly we will revisit the lock diagram that we were discussing. You have seen this block diagram in the last module, where the incoming signal y t was mixed, getting mixed with the locally generated p n sequence, and which is also dragging the frequency synthesizer. We understand that this is a frequency hopping communication mechanism. So, frequency synthesizer will be generating the hopping sequence, and it is the fast frequency hopping mechanism; that means that within a symbol duration or tone duration, there are multiple hop frequencies over which the same signal is spread, and we have utilized the MFSK signaling mechanism; that means, for transmission of a typical signal. We have a set of that tones from where actually the for a typical symbol a typical tone is assigned. So, we are dealing with now two different frequencies; one is the modulation frequency, we call it modulation tone, and another is the hopping frequency, which is actually the frequency that is getting generated locally here in the receiver, and the pattern of that generation will be governed by the secret key, and this frequency synthesizer will generate that hop frequency.

So, this 2 different frequency terms that are there inside, with the incoming signal we are now multiplying with the mainly the hopping frequency to de-hop the signals. So, this is the spreader or the mixer whatever you say. And the signals are actually then pass thorough a bypass filter, having a band width of B. we saw that then the filtered signal is passed through a correlated, it is non coherent detector architecture. So, this n c stands for the non coherent demodulation architecture, which de modulus at the frequency of the IF frequency plus the symbol frequency, symbol tone frequency, we understand for different symbol we have the different tone associated with it. So, the center frequency of the demodulator, there is a bank of the demodulator and the center frequency of each and every demodulator, is synced with the different frequency stages of the frequency modulation, modulation tones and anyone of the at a particular moment anyone, depending upon which symbol was transmitted hence which modulating tone was transmitted.

Anyone of the demodulator architecture output will give you the maximum or the peak at a particular time and hence this maximum is peaking over a duration of n into th, I mean equal to the symbol duration we will select anyone of this bank of the modulator output, and then it will be threshold compared and will adjust, adopted the clock. That was the block diagram, as we understood last time. Based on this only today we will try to write down the equations of y t x t and then try to find out z t and we will try to realize, how that the output acquisition mechanism is obtained.

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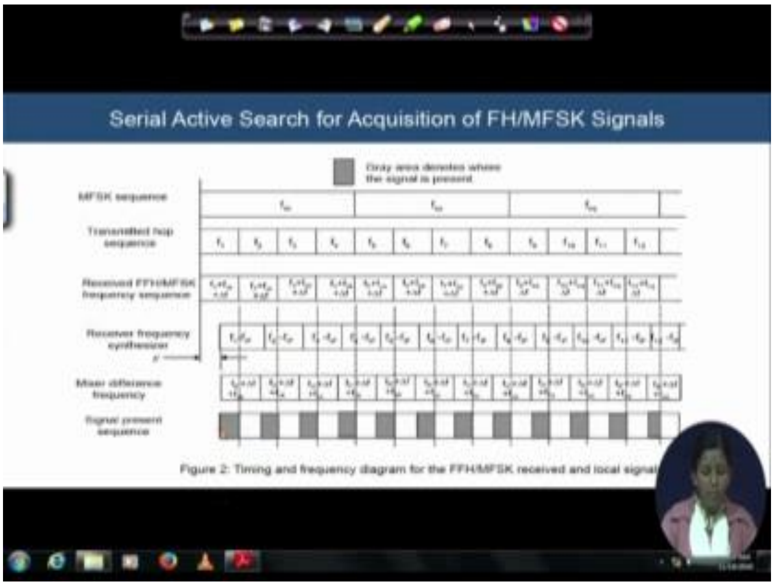

So, from here we also understood that the time frequency diagram, of this fast frequency hopping MFSK signal, and the diagram will be little bit change, if you change the system from fast hopping frequency to slow frequency hopping. The diagram that I am showing you it is basically the fast frequency hopping. What we found that is, which was a MFSK sequence and, these are the frequency of each sequence coming, and the transmitted hop system was given by f 1 to f 4. So, within the duration of 1 MFSK symbol, you have the 4 hopping frequencies. So, the hop frequency is 4 times higher than the symbol frequency, and the received FFH MFSK signal, hence will be governed by the FSK plus f 1 plus some del f. Del f is continuity by the Doppler effect of the, at which Doppler effect comes between transmitter and receiver if anyone of the m r moving or both of the m r moving.

And for that we understand that the Doppler frequency as is just be constant for the receiver or the transmitter, then the Doppler frequency we have assumed to be added as a constant fashion in each and every received sequence interval. And we also assumed, that local synthesizer, frequency synthesizer of the receiver is not properly time matched with the transmitter frequency synthesizer, and hence there is a time error also associated with it. So, this diagram completely talks about that there is a time error associated, given by EPSA, and there is a frequency error associated given by the Doppler.

And hence the receiver frequency synthesizer, because of this error in the time domain, all the frequencies will be shifted in this fashion, and hence you will get the mixer output, the mixer output of the receiver frequency as well as the received signal, and the receive a locally generated frequency synthesizer output, frequency synthesizer output after multiplication in the mixer, and the output of the mixer you will be ending up with this FIF, because your incoming signal is f 1 plus FSK plus del f, and the frequency synthesizer is generating a frequency of f 1 plus del f. So, after biting you will be left with a FIF plus this del f plus this FSK. So, like that all the patterns will come serially. and remember one thing the decoded output will be high, only for the duration over which that the overlapping is happening, overlapping of the difference frequency mixer output and the received signal, whatever we do not know that the time axis overlapping is there, for that duration only the decoder will be able to give you an some output.

In this assumption we have an assumption that it is 4 times remember. The 4 times is the hopping frequency than the incoming frequency. Assumption number 2, we have a Doppler contributed by del f Doppler frequency, and we have a timing error attributed by EPSA. And hence we understand that the frequency previous to this point of the context, the point of our interest, and the frequency next moment and next instant both will be different than the current hopping frequency. So, the hopping frequency at the previous section and the hopping frequency and at the next section they are different, at any moment, and you will be able to get the output of the detector wherever the overlap will be having only. We may come back to this diagram, if we have some confusion in the successive section.

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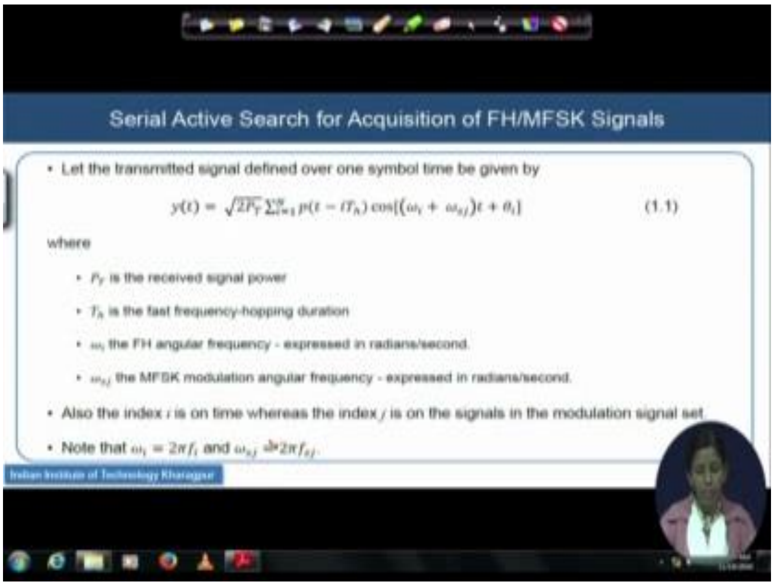

We will start with the signal and the transmitted signal, defined by y t over 1 symbol duration and y t is here. So, we are currently writing the equation for y t, and y t is given by look square root of 2 P T, P T minus iT h summed over i to N and cos of omega i plus omega s j t plus theta i. Remember this P T is the power of the transmitted signal as well as we are considering this is the received signal power, T h is the hopping duration p t is the basic over form chip wave form, omega i the is in the index i corresponds to the hopping frequency at the i th moment, and j is the index of the modulation tone frequency associated with the current symbol. So, i will be telling over which hopping frequency you are in, j will be telling what modulation tone you are dealing with.

So, remember over a particular duration of the symbol, the j will not change, but i will keep on changing, and we are considering that part symbol we are having N number of the hops. So, this hopping frequency will change from i to N, beta constant of s j; that is the meaning of fast frequency hopping we understand.

And these are the frequencies expressed in the radian, and angular frequencies expressed in the radian per second, and theta i is the corresponding phase with respect to the hopping frequency i. Remember we consider that we cannot actually guarantee that whenever hopping is going on, there will be same phase, I mean relative to every hopping; that is impossible. So, every hopping is associated with some amount of the different kind of the phase associated to that. So, that cannot be get on to it; that is why theta is also varying with respect to your index hopping index i. And expression for your omega i and omega s as I expressed, they should be equal to 2 pi f i and 2 pi f s j. So, this is a signal that has entered into the front end of the receiver, and this is actually the signal transmitted also from the transmitter.

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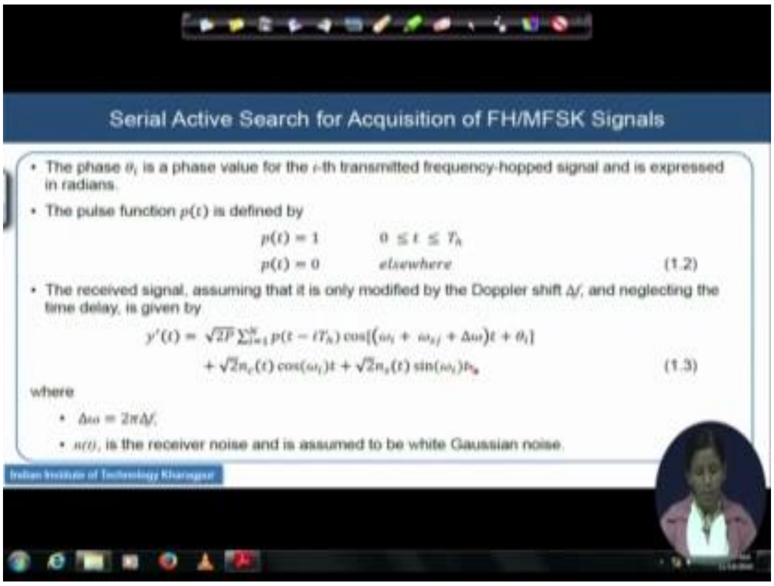

Next slide, we have seen that p t is nothing but the pulse given basic pulse shed, and we will redefine what is the meaning of this p t is here, p t is nothing but within this hopping duration p t's value will be equal to 1; otherwise p t will be 0. And if we assume that the received signal is also affected by the Doppler frequency, and it is having a Doppler shift of del f, and there is no timing delay associated with it, if I first only introduce changes due to the Doppler shift, then they receive signal y dash t will be written as like this.

Remember that earlier w i plus w s j is getting associated with an another shift attributed by the Doppler shift of del f. And now we are also associating the noise, incoming noise associated with that, and remember this is an additive white Gaussian noise, process n t is, and n t is having, it is a complex Gaussian noise. So, it is having a cos as well as sin. I mean in phase and q phase component written here. And remember like others we have added, we have represented Doppler shift in terms of its angular domain until omega will be represented by 2 p i del f. So, now, the equation 1.1 is now changed to 1.3 when Doppler shift is added, or contributed into the receiver signal model, and noise is added, complex white Gaussian noise is added also.

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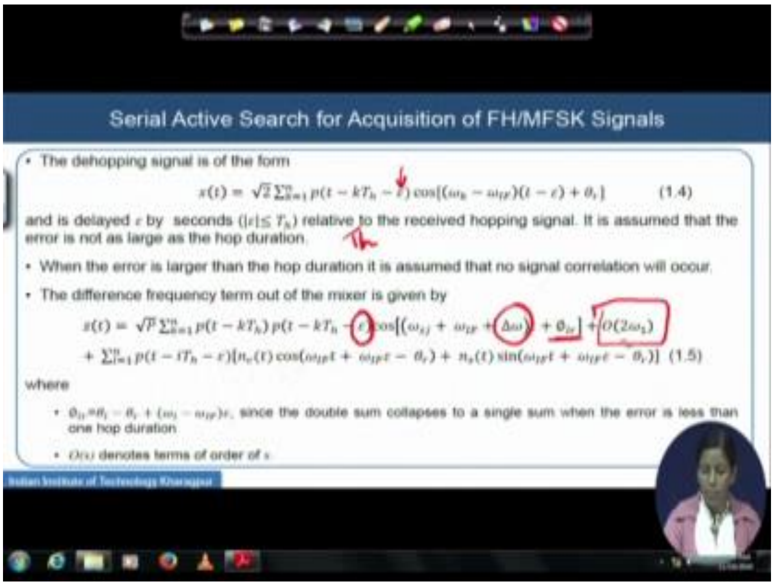

After that when the de-hopping will be happening, the de-hop signal the x t that signal will be looked like, there is a, we understand that in the de-hop signal we cannot omit the error that we have considered in the time frequency diagram. The error was in terms of EPSA, and when the de-hopping will be done, if that synchronization is not done. I mean time synchronization between the frequency synthesizer of receiver, and the frequency synthesizer of transmitter is not established the received signal, de-hop signal will reflect that error definitely in the time. So, t minus k T h will be further delayed by this error, time error, and you will be also getting ending up with the relative term, coming from the frequency section.

And this is the k-th, for the k-th frequency we are dealing with and we are having the intermediate, sorry the IF frequency ended up with after the mixer is multiplied and the band pass filter has done its job, and theta r is the remaining amount of the phase, or the remaining phase associated with the de-hop signal.

Remember if this delay, this relative delay, it is relative to the receiving hopping signal, if it is assumed that this delay, is not large than the hopping interval or hopping duration T h. So, we have consider that we will get the de-hop signal the decoder output will be able to give you something, I mean some output will be coming out only if your error is less than equal to the hopping duration. If this hopping duration, if it crosses the hopping duration then it will be consider that no signal at all. Correlation will occur over the, because there is no common overlap, and you will not get anything at the output of the decoder. So, the perfect 0 is expected to get at the output of the detector.

We understand that the incoming signal is now having, having a Doppler effect, having time error effect and. So, the mixer output z t. The mixer output will be showing if I try to see the incoming signal with this, all this kind of the errors associated with it. We will be able to see that the mixer output is now will be reflecting both the Doppler error, the effect of the Doppler, as well as you will also reflect the error of the timing. And this has got expanded because we have also incorporated the noise complex Gaussian noise term. We have also got a new term here, which is called the o x which denotes the order of the, it denotes the term of the order of this x. And remember this phi i r, what we are seeing here the derivative is the equivalent error in the phase it will be given by the theta i minus theta r, I mean receive associated, phase associated with the i th hopping, and the received phase plus the omega i minus omega f into multiplied by the EPSA term.

And if I consider that the error that is less than this timing duration hop duration, then the double frequency term will be coming up, and they say it will be collapsing to the single sum like 1.5. So, fundamentally this is the term that is coming up at the output of the mixer, because of the presence of the both frequency error, as well as the timing error in a FFH MFSK signal.

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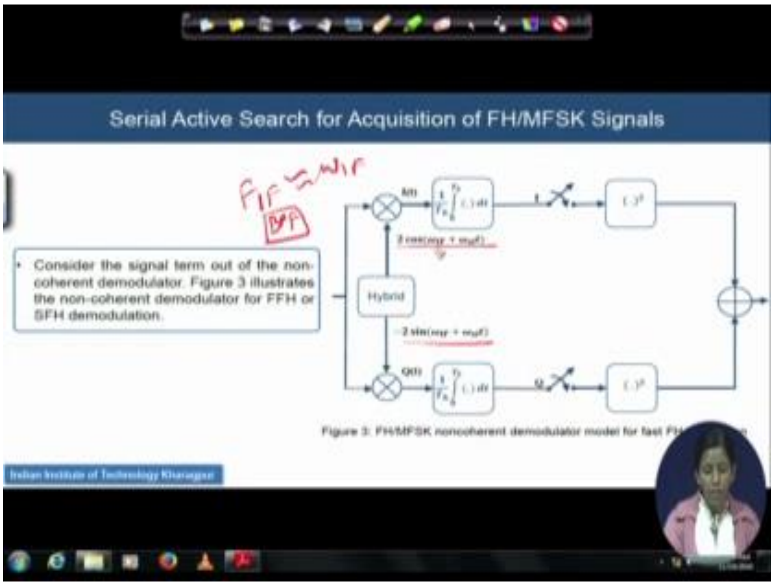

So, now we will enter into the detector section, and we will try to see that if I now detect it, I mean in the detector it will multiply with the signal in the i th part as well as in the q th path, by the in phase and the quadrature phase component. Remember the detector, prior to the detector there was a band pass filter right, and the band pass filter has filtered this incoming signal at the frequency of F IF, or equivalent to omega IF, the term we have seen in the z t. So, if it is used to IF. So, this decoder will also be tuned at IF plus the frequency, angular frequency of the modulating, of the modulation tone, and both the in phase and quadrature phase will be created, and we will be a continuing the in phase and q phase integration and square, it is basically the correlated architecture multiplication and integration and then the squaring it up the envelope detector output we will take, and finally, adding up both the term, we will feed it to the decision device.

So, here remember that, based on the choice of the band pass filter frequency or intermediate frequency, it is intermediate frequency that we are choosing and based on the IF filter, we are here also the synchronization with that IF filter needs to be established by the decoder. So, decoder does what? Decoder takes a incoming signal from the IF filter, and he creates to pass, he creates a, he separately operates over the in phase as well as the q phase part of the incoming signal, and does the integration sampling and scalar overlap detection, it is basically because it is a non coherent detective going on, and we will see actually the mathematical output of this detector. Let us first consider over the upper term, the upper term is the in phase path coming inside the detector.

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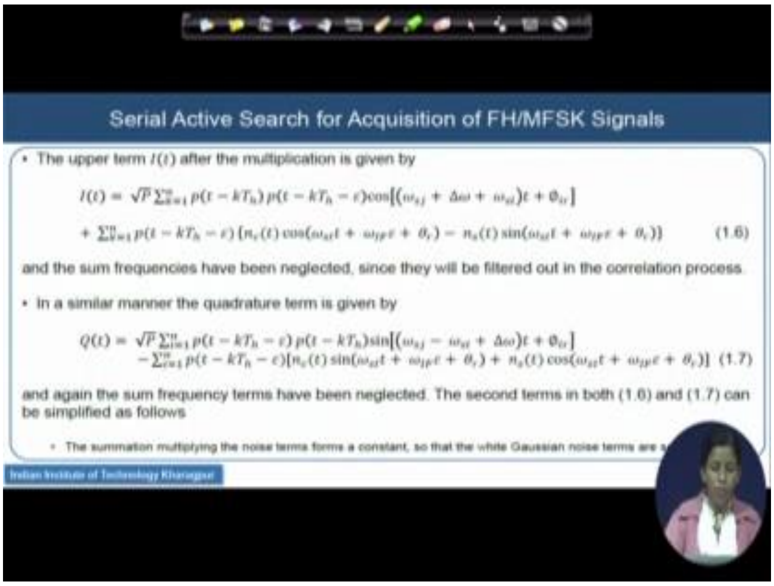

This in phase path will be we understand from the last equation 1.5, that with the in phase path, now the detector signal, I mean the cos that section will be multiplying, and if my cos, let us go back if I multiply with the cos 2 omega if plus omega s l and we had earlier, the terms of omega is assuming IF and del omega, then we will be ending up with omega s j plus del omega plus omega s l t, and omega IF term will be going out, because they will be betting with each other.

Similarly for the noise terms also you will be seeing the same kind of the operation is going on, and the some frequency, again the same frequency in the higher part is neglected since they will be filtered out in the correlation process. Similar to the I th part, in the Q th part also you will be seeing the same kind of the expression that is at the Q phase component or the sine components will be reflected here. And the original signal section, the upper portion which is the signal section, he will be out of the intermediate frequency term.

Remember if you look carefully in that in phase and the q phase term, they are divided in to 2 halves. The first half is basically the signal section, the lower part is basically the noise component. So, in next slide we will write down this I and Q component as the i s class i n. And remember that we can easily show that the summation multiplying by the noise terms, this guy, noise terms multiplied with this sum term actually it forms finally, a constant and it is a, as it is a wide Gaussian noise. So, it will be scaled by a factor of 1 by root 2, it is easily doable.

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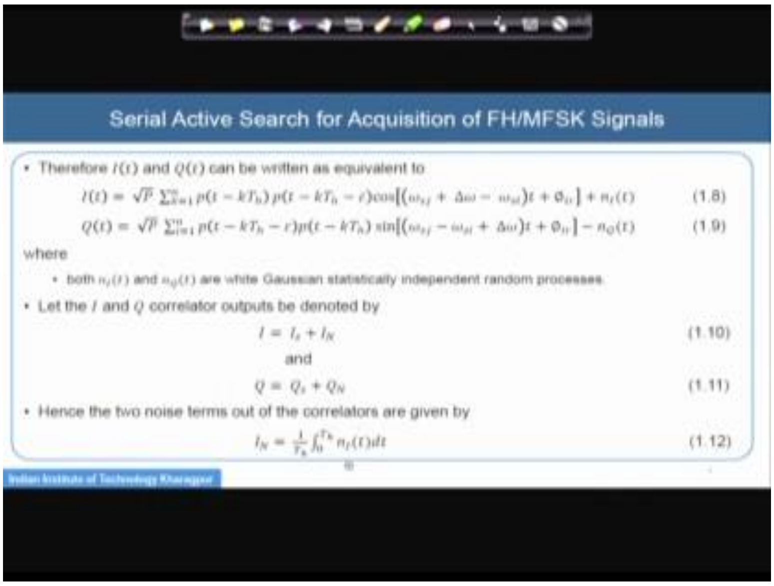

And as I told in the last slide that each of this in phase and Q phase components are combination of the noise section as well as a signal section. For the noise you are associated with a in phase, we will mention it as a nit and the noise associated with a q phase, the whole term will be now considered to be n Q t. If we consider that the first one to be i s then; obviously, you can write down 1.8 and 1.9 as 1.110 and 1.11 simply.

And now, this let us look inside these noise terms, and remember these two noise terms if I see the what is coming for the these noise terms for the output of the correlators. The correlator will take individually both of them, and it will integrate over the interval of our interest, which is the hopping interval and average it out.

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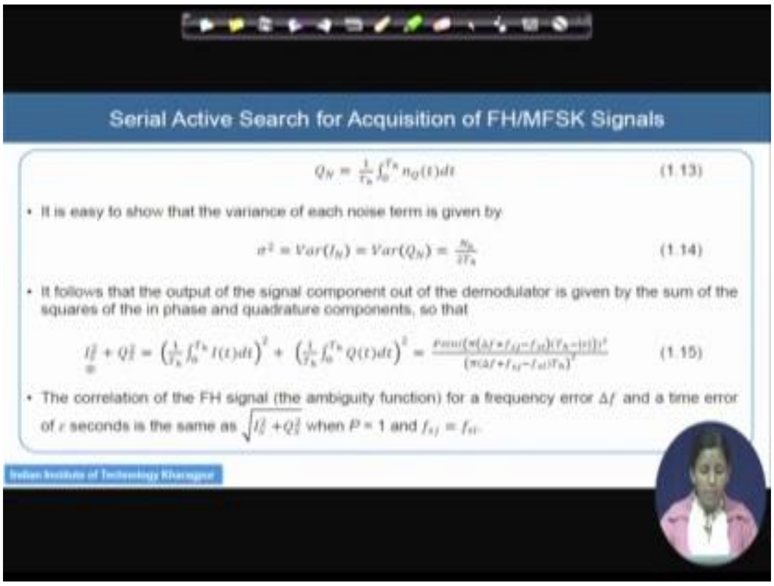

And if we do the same for the I and the Q phase, we will find that, the variance also can be easily calculated and both the variance, so that for the two paths will be equal, and will be given by the eta 0 by 2 by T h, where eta 0 by 2 will be your now two sided pass vector density of the noise, wide Gaussian noise and T h is our hopping duration. It follows also that if I now looking; that was a noise section, and if I now look into a signal section, the signal output, the signal component will have the output.

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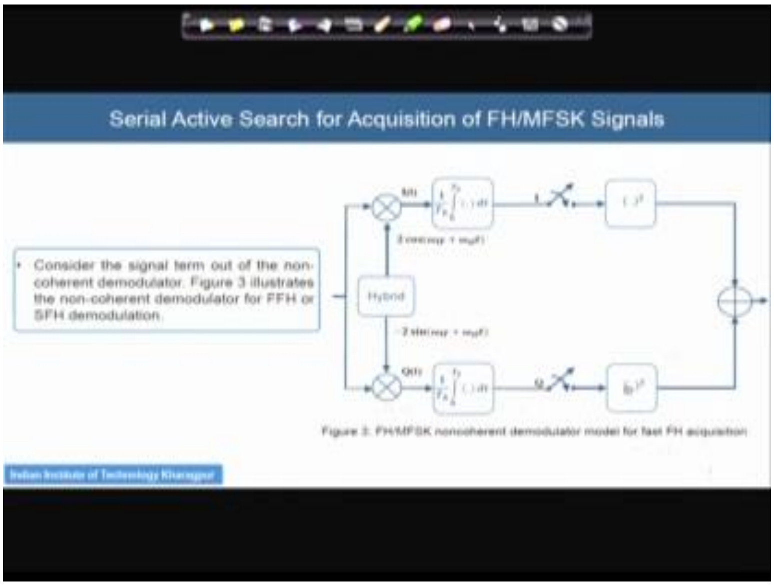

The output of my this adder circuit. So, this will be i s square for the signal section this will be the q s section square, and this will be added up here. So, this adder output now we are looking in to this equation 1.15. And definitely it will be the individual integration and this squaring up. So, correlation and then the squaring up, according to the architecture, it is below first correlation, it will say integration part, then the sample integration part is getting squared up for both the parts. So, the same thing we have done in the equation nothing else. And if you solve it by substituting the values of the I t and Q t from here, equation number 1.8 and 1.9, where we will be ending up. We will be ending up with this equation of the e sin of this guy.

Capital P is a power of the transmitted signal, del f we understand is the contribution coming from the Doppler frequency or the Doppler shift, f s j is the frequency associated with the j th modulation tone, and remember this l is the essence frequency associated with the l th modulation tone. Actually this substitution is coming because of these errors del f error as, you are actually a may get some interference from the neighboring modulating tone and so the difference of these two tones will be reflected in the computation of the I s square plus Q s square. Also you will be able to see that there is a contribution from the timing error section and so it will not get multiplied directly with the t square, there is a difference between the remaining time period, for which actually detector is creating the output. This is the remaining time interval that is coming in to consideration in computing the I s square, the total value of signal component.

And the correlation of this frequency hopping signal for in frequency error del f and the time error EPSA, it is equivalently same as if, when p is considered to be 1, and if FSF and FSL are actually considered to be same. I mean there is no ambiguity and we could properly, select the modulation tone in the receiver, if that is the situation, then this whole equation will look like this. We call this factor as ambiguity function.

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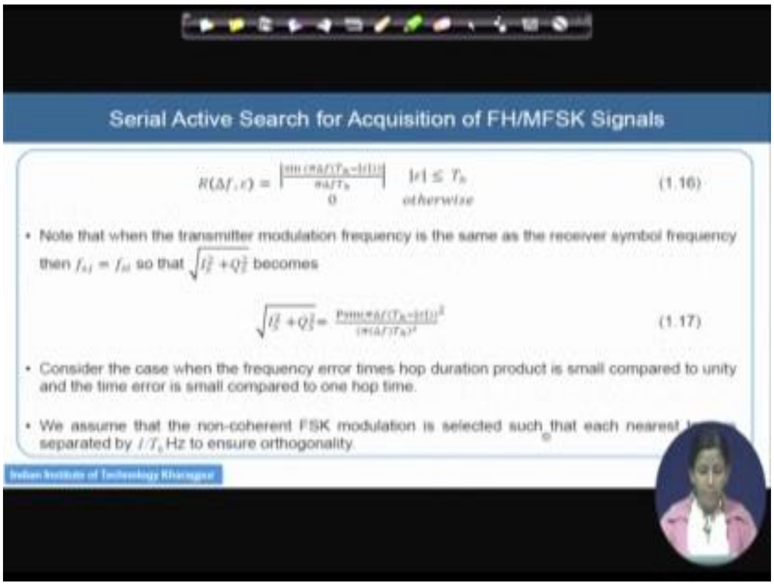

So, we will coming down here, considering that my error, the timing error is much less than the hopping duration, and this ambiguity will come down here considering that my p is equal to 1, and then there is no ambiguity from the fs and fsj detection. This, when this transmission modulation frequency transmitters modulation frequency is same as the receiver symbol frequency, that is situation when fsj and fsi is equal fsj is equal to fsl and so that this square root of this term will now come to this fact if p is not considered to be 1 definitely. So, it will be coming like this.

And now consider a case, when the frequency error time, times hop duration, this we can say that times this hop duration. I mean T h minus this guy multiplied by this frequency error time it shows very small. It is so small that in the; it is a very small compared to the unity, and the timing error is also very small compared to 1 hop time. So, this is close to you, very small closed to unity and this is very small compared to this tau h. if this is the situation and we assume that this non coherent fsk modulation is going on, and we are thinking that the modulation tone are kept n such a way, that the minimum gap between two tone, the nearest tone is, they are separated by 1 by T h hertz to ensure the orthogonality if we consider like that.

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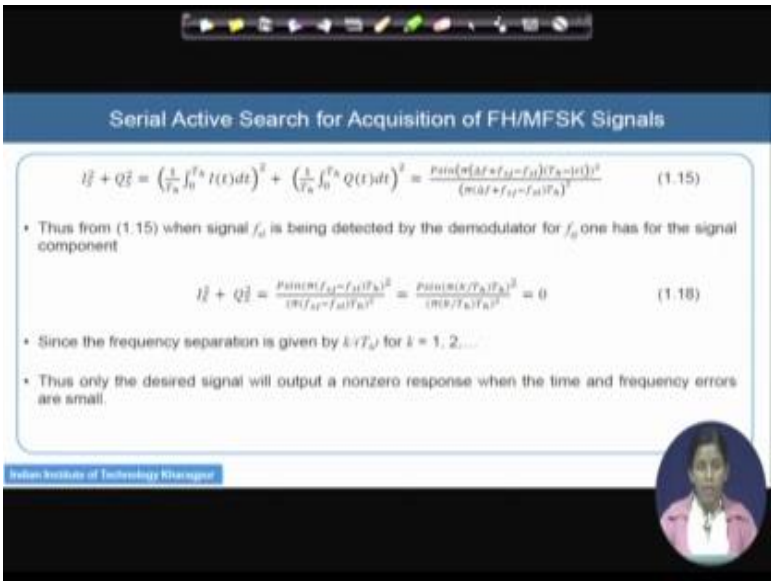

Then this I s square by Q s square the signal component will be boiling down to this guy. And it can be easily understood that when fsl is being detected by the demodulator for fsj, when fsj transmitted, but fsl we have detected. Then we will have at most no component kind of the output at the detector, because this difference will be coming almost 0, and then you will be ending up with the perfect 0 situation, because of sin in p i is equal to 0.

So, basic point is that, the detector is output is coming perfectly zero and you are not getting anything, if the tone frequencies, modulation tone frequencies are not matching, and desired signal we were only getting, nonzero response only you were getting when the term frequency errors are basically. The condition is time frequency errors should be basically very small, and you will be getting at the output with the desired signal only; otherwise actually the desired signal output is expected to be at 0, complete 0 if the a time frequency errors are significantly high. And also the modulating tone frequency that you are developing is not properly chosen in the receiver circuitry.