

**Spread Spectrum Communications and Jamming**  
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**Lecture - 35**  
**Rapid Acquisition Using matched Filter –Part I**

Hello students. Today we will go ahead with continuation of our discussion over the code acquisition systems the performance analysis of those code acquisition system related to direct sequence spread-spectrum communication systems. We will continue today over the same; and specially, today's discussion topic will be the rapid acquisition mechanisms over through the matched filter techniques. And we understand that the in code acquisition followed in the synchronization specially in the code acquisition, acquisition time is a very important and very critical issue with respect to the system design concerned.

And we always try to minimize the acquisition time. The acquisition time is the measurement of how fast you are getting the asynchronous situation or synchronization point, how fast you are detecting the synchronization point and you are declaring the signal is locally generated PN sequence is now synchronized with the incoming sequence signal. So, that actually the data recovery can start now. So, getting as fast as possible the synchronized point is of very, very important issue to us, and hence the rapid acquisition mechanisms are really very interesting for the system engineers. So, let us see today how the match filter technique can expedite the acquisition technique that we have already discussed in the last three classes.

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The slide is titled "Rapid Acquisition Using Matched Filter Techniques". It contains a list of bullet points and a block diagram. The diagram shows a signal entering a multiplier block, which also receives a "locally generated PN reference" signal. The output of the multiplier goes to an integrator block, which then feeds into a detector block. Handwritten red annotations include a bracket around the first two bullet points, a bracket around the last two, and a label  $M = r_d T_c$  with arrows pointing to the integrator and detector blocks.

- In DS acquisition techniques the measure of PN correlation is produced by an active correlation of the received signal with a locally generated PN reference.
- For example,
  - In the single dwell serial search system, the received PN signal plus noise was multiplied by the local PN reference and subsequently integrated for  $r_d$  seconds.
  - The result was used to make an acquisition decision by comparison with a threshold.
- Such a multiply-and-integrate type of correlator/detector structure is typified by the fact that the local PN generator is running continuously.
- Hence, a completely new set of  $M = r_d T_c$  chips of the received signal is used for each successive threshold test.

The techniques till now we have discussed is something like that in the direct sequence spectrum acquisition mechanism, the measurement of the PN correlation is produced the measurement of the correlation is the fundamental part. We saw in the block diagram that the incoming sequence is getting correlated with the locally generated PN sequence. And then its output is correlated output is fed into a detector cum threshold decision device, which is based on the threshold and finally, we are declaring the output of that threshold device which is also having a verification algorithm is finally declaring whether you got the synchronization point or not.

So, fundamental part is to get the PN correlation measurement and then have a detection some decision on that correlation measurement, whether it is very high whether it close whether it is not close to each other and then what to do about if they are not close is the total task under the synchronization. The acquisition techniques that till now we have discussed, there we have seen that this acquisition measure is obtained by the active correlator circuits, where the incoming signal is continuously entering into the receiver and as if the PN sequence is the continuously getting multiplied with the incoming signals.

So, both the incoming signal as well as a locally generated PN sequences are completely running. So, both of them are running. So, they are the active correlator, the way the whole structure is called the active correlation mechanism. So, for example, the way we

did is we understood the single dual search mechanism, whether the search will be done and decision will be made based on the single observation interval. And the received PN signal plus noise, we saw that it is getting multiplied with the local PN reference and it is integrated over a duration of  $\tau_d$  seconds. So,  $\tau_d$  was the dual time single dual time to us.

And this result was then asked to make an acquisition decision and by comparison with threshold; this was the through two steps already we have discussed in the last module. And we have done the probability of the detection probability of the false alarm calculation based on this philosophy. When such this multiply and integrate, so fundamentally two blocks were running in those acquisition systems, one is the multiplier and then is an integrator. So, integration was running over duration of  $\tau_d$ , and here is the acquisition declaration. So, this multiplier and integration type of this correlation detector architecture, it is typified by the fact that this local generator who is fed into this multiplier circuit in the multiplier block, they are continuously they are the running PN sequences.

So, as if it is running continuously, and the incoming signal  $r(t)$  is also a running fact continuously the signal is coming, and here it is also a running one, none of them are static then it is happening something like this. Every set of the  $r(t)$  and I mean if I consider this is a length over which the multiplication is going on and this is the set over which is the correlation whole correlation operation will go on. So, this the number of the chips over which this multiplication and addition is going on, every time it is going like this. For every new set of the  $r(t)$  values, sampled values, a new, new set of the PN sequence is getting multiplied.

It is not like that you have a selected set of the PN sequence, and you have continuously this is moving, and you are trying to compare the selected set of the PN sequence with the piece of the  $r(t)$ . When the  $r(t)$  is changing your PN sequence is also continuously running. So, over the dual interval over the single over the dual time, every dual time if you try to compare it is heavily possible that the PN sequence values that are coming in within the dual time interval they are not constant the it is continuously running. So, the phase is also expected to change also after certain interval.

And if I quantify that a how actually this new set of the chips are coming into picture, what are those number of the set of the chips that is getting compared with incoming set of the or the portion of the  $r t$ . It will be given by capital  $N$  is equal to we understand  $\tau_d$  divided by the  $T_c$  number of the chips. So, these are the new number of chips over which have getting multiplied with  $r t$  that every set then  $M$  is same, but those number of the number of the chips are same, but the type of the chips that the values of the chips that are coming inside that set, they are varying.

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**Rapid Acquisition Using Matched Filter Techniques**

- This poses a basic limitation on the search speed since the local PN reference phase can be updated (steered) only at  $\tau_d$ -second intervals (assuming the threshold test fails).
- Thus, if the search is conducted in  $1/N$ -chip increments, the search rate  $R_{1D}$  of the single dwell serial search technique is
 
$$R_{1D} = \frac{1}{\tau_d} \quad 1/N - \text{chip positions per second.} \quad (1.5)$$
- The search rate of a DS acquisition scheme can be significantly increased by replacing the multiply-and-integrate operation with a passive correlator device such as a matched filter (MF).
- This device can be implemented either as a continuous time or discrete time operation, and with such candidate state-of-the-art technologies as
  - Charge Coupled Devices (CCDs) ✓
  - Surface Acoustic Wave (SAW) correlators ✓
  - Discrete Time Correlators ✓

So, if this is the situation for active correlation, so fundamentally the time after which you are getting a decision about the estimated phase difference between the locally generated waveform with respected to the incoming PN sequence waveform that gap and that phase of set only you get after an interval of  $\tau_d$  seconds. Because this is the dual time, at the end of this only you will be declaring that this is the offset happening and this is the amount of the offset we have got. And at the end of that  $\tau_d$  seconds interval only, you will be going to update the phase value of the local generated waveform.

If I now consider the rate at which in the single dual system, I am updating the phase of the locally generated waveform is given by  $R_{1D}$  basically the rate at which you were updating the phase interval will be given by  $1/\tau_d$  into  $1/N$  chip position per second. Because  $1/N$  chip increment, if we are thinking that we are continuing the search in  $1/N$  chip increments, and the search rate is then giving by in terms of

these 1 by N chip intervals. So, 1 by tau d is the rate at which the tau d is the time duration after which you are taking the decision. So, rate at which you are going to take the decision is 1 by tau d of 1 by N chip positions. So, search is running at the rate of 1 by N chip duration, and decision is coming at the gap of tau d interval. So, the rate is in terms of your 1 by N chip positions per second and this is given by equation 1.1.

So, the search rate of this DS acquisition system is now one by tau d of one by chip duration and we are really not happy with this kind of the time involved to acquire the search to complete the search. So, decision needs to be much more faster in practice. And we need search for other techniques mechanism and methodologies which can improves this rate at which the synchronization is obtained that is why actually the passive correlated device that involves the matched filter basically is invoked.

And these multiply integrate that mode of the acquisition mechanism which is going on by multiplication, and the integration that block will be completely removed by a match filter now. And this match filter device will be implemented by the state of the technology for example, charge couple devices, surface acoustic, wave convolvers or discrete time correlators we will see the structure with discrete time correlators to implement the matched filter methodology in the rapid acquisition mechanism today.

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**Rapid Acquisition Using Matched Filter Techniques**

- In the continuous time case, the received PN waveform plus noise is convolved with a fixed finite segment of the PN waveform corresponding to, say,  $1/\tau$  chips, and the continuous time output is tested against a threshold to determine when acquisition has occurred.
- In this configuration, the input continuously slides past the stationary (not running in time) stored PN waveform replica until the two are in synchronism, at which point the threshold ideally would be exceeded and the local PN generator enabled.
- In DS/SS systems, the PN spreading waveform is typically bi-phase modulated, the carrier whose phase is as yet unknown at the receiver.

Block diagram showing:  $r(t) + m(t)$  (with handwritten  $r(t) + m(t)$  above) entering a multiplier block, which is also fed by a local PN generator. The output goes to an integrator block, which then feeds a threshold detector. A 'Tracking' block is also shown connected to the integrator output.

So, we understood the need; we understood that the way the acquisition system using the architecture of multiplication or integration was going on that is not giving a very good

rate of acquisition. So, we are searching for a new architecture. And we believe that match filter based technique can give us that, but what amount the acquisition will be improved if i implement map filter based technique that is the question till now, that we will answer at the end of this module.

Remember in the continuous time case the received PN waveform plus the noise I mean the  $r(t) + n(t)$  this received PN waveform plus noise. It will convolve first with some in match filter techniques your  $r(t) + n(t)$  will be first multiplied with some known set or prefixed set of the PN sequence, replica of a part of the PN sequence. And say capital M number of the chips we have taken and for in capital M number of the chips actually the number of the chips will be keeping constant, and this will be continuously getting multiplied with the incoming signal.

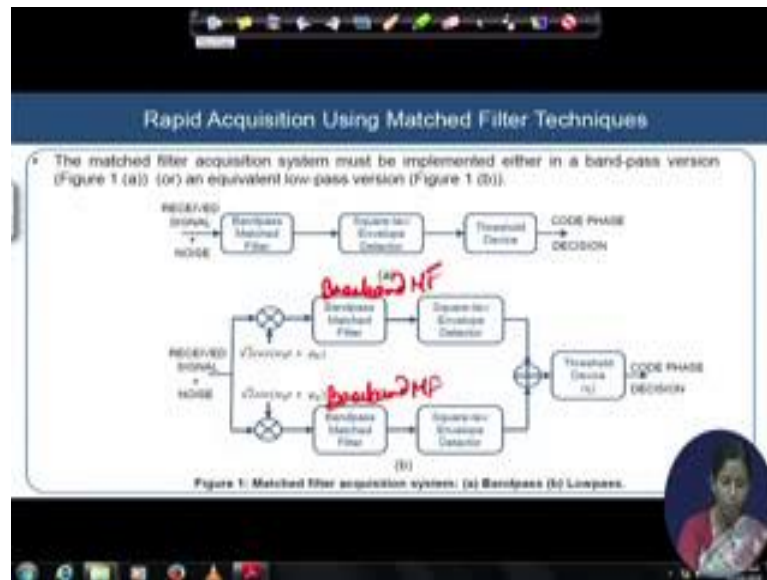
And then the continuous time output is will be also continuously getting tested against a with defined threshold. So, there is a detector who is having a threshold  $\eta$ , and this continuous time multiplied output or correlated output, it will be continuously checked by this detector against that threshold value. And when acquisition has occurred that means, that multiplied value is crossing the threshold value, it will be declaring a hit, there is a synchronization point achieved.

And in this matched this is the matched filter architecture, and when this where actually the key difference from the earlier architecture is that the PN sequence segment is not moving for a typical kind of a test. Once still you are not getting one hit you are not moving if you taking any decision to change the PN waveform change the chips of the PN segment. And once the input is continuously sliding actually input is sliding fast and this is the stationary, so the stored PN waveform replica which is actually now getting compared continuously when the synchronization point is got and you are declaring it is done.

So, even after that you are typically enabling the tracking mechanism, s, after getting that you are getting the tracking mechanism. And for a certain period of time you are actually continuously multiplying with the same state of once the synchronization is a achieved you are continuously multiplying with the same set of the incoming signal. But once actually there is not a hit then you are changing it, and trying to check it over with a new set of with a new set of the PN sequence. And we understand that in a typical direct

sequence spread-spectrum system this incoming PN code will be typically bi-phase modulated on a carrier, and again that carrier phase is yet unknown to the receiver. So, that concept is still preserved that you do not know anything about the phase of the carrier. So, carrier synchronization is not done yet.

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Given the structure and basic understanding of this match filter based implementation of the code acquisition mechanisms, the match filter can be implemented either in the analog domain or in the digital. I mean it can be in the base band or in the band-pass. In the band-pass version and they are other two options which are given here in the figure. Figure 1 is, 1 a is band-pass version of this matched filter implementation; and b is the low-pass equivalent or baseband version of it in. A band-pass version what we do is the receive signal plus noise has entered and this is the band-pass matched filter. What is the meaning of it, we will see in the equations later on.

And this is the band-pass matched filter where which actually we will take care I mean the filter function the transfer function of the filter will be designed in such a way that it is matched with the received signal, definitely that is the function of the match filter. But the signal will be band-passed and the band-passed signal is now put into the square-law envelop detector and threshold device is will be the next block to follow and code phase decision you are getting at the output.

Now, what is the difference of the low-pass equivalent circuit of this band-pass circuit is this receive signal plus noise after receiving in the receiver frontend, what we do is either we use the fact that we have estimated the frequency of transmission - sender frequency of transmission which is written here as  $\omega_0$  which is the radian of the carrier frequency expressed in the radian. I mean either you are estimating this value of  $\omega_0$  I mean equivalently the sender frequency  $f_c$  or you are utilizing the a priori known value of the  $f_c$  of transmission. But remember the associated phase is not known to you. So, there is some ambiguity over the phase. And the carrier you have generated with the a priori knowledge of this  $\omega_0$  or the estimated value of this  $\omega_0$ .

And once you understand  $\omega_0$  with some approximate phase associated with that you are dealing with then we can definitely actually generate in phase as well as out of phase two carriers here. And hence actually incoming signal once will be multiplied with the in phase as well as the out of phase carriers, so that we can bring both the signals into the base band. So, inside the base band actually we then utilize the matched filter, and low-pass version and the we utilize the matched filter as the same thing and square-law envelope detectors are there, and then threshold devices are going on.

Remember one thing, sometimes we this  $\omega_0$  may not be actually selected to be exactly the  $f_c$ , we can sometimes bring it down to the some IF value also not purely at the base band value all to 0. But we sometimes actually bring it down to the IF also in that case actually you also need a base-band matched filter which will be not centered around the IF frequency of your down conversion.

So, now the i phase matched filter square-law detector output and the q phase square-law detector output they will be added up here to get the threshold to feed to drive the threshold device, so that actually you can have a decision on the code phase. Remember here we never put actually the two-two different threshold devices one for the i and one from the q that is there is no meaning of it. Because synchronization demands actually there should be one phase offset with the incoming signal of compared to the locally generated with the globally generated one and that phase should be corrected. So, decision should be done over the combined signal of the in phase and the phase and the threshold device will be helping us to do that.



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Rapid Acquisition Using Matched Filter Techniques

- In the former case, a band-pass matched filter is used whose maximum output is detected by a square-law envelope detector.
- In the latter case, inphase and quadrature carriers with arbitrary phase but known or estimated frequency is used to demodulate the received signal, followed by baseband matched filtering of each demodulated signal.
- The matched filter outputs are then non-coherently combined to produce the desired correlation measure for threshold testing.
- Conceptually, the implementation of a matched filter for a finite length PN waveform is most easily visualized in the form of a tapped delay line followed by a passive filter matched to a single PN chip waveform (Figure 2 - next slide).
- In the former case, a band-pass matched filter is used whose maximum output is detected by a square-law envelope detector.

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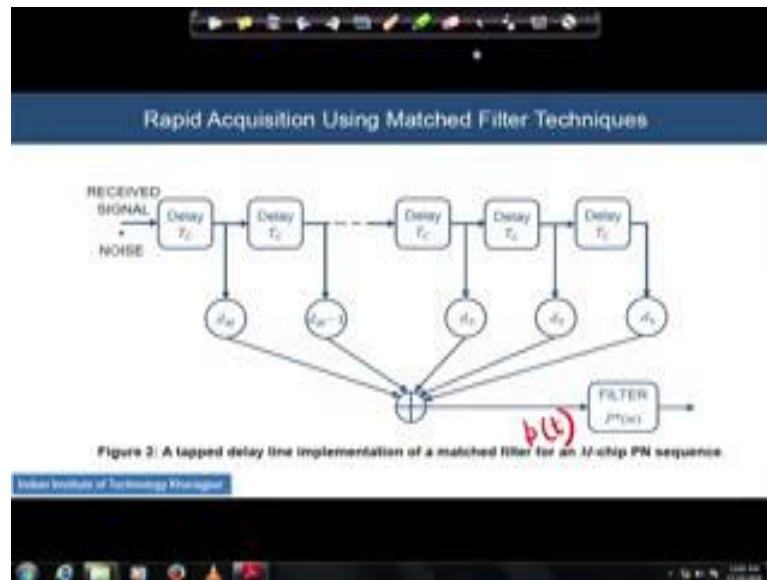
Now, little bit inside the architecture. So, as I was explaining in the last slide, same thing is written here that the band-pass matched filter is used whose maximum output will be detected by the square-law envelope detector, like the other cases we have seen. In phase and quadrature phase carriers we are either generating or we are estimating with the generating with priory knowledge of the carrier or we are estimating the carrier and then we are generating. And the arbitrary phase, we are picking up because we do not know about the; and if you do not have actually any apriori information about the phase and we are fine with that gate we can go ahead with that, and then we will proceed with the base band matched filters.

So, then the matched filter outputs, remember one thing as I have already told in the previous slide that if you are bringing it if it is not the center, if it is not an IF frequency then it would not be a band-pass matched filter once again then it should be replaced by a base band matched filter. It will be a base-band matched filter both the i and q section. Because in that situation, you are bringing it down directly to the base-band that is no IF frequency involved. If  $w_0$  is related to the IF frequency then you may need another base band-pass matched filter which is a actually centered around the IF frequency.

So, based on that you are using it and designing the filter; and at the output of the next we are doing the threshold testing. Actually it can be very easily visualized; its implementation of this match filter will be very easily visualized by the captive align

filter design and that we will see actually in the next slide, how you are going to do that. And square-law detector in the earlier case the output of the square-law detector whose is giving the maximum output, when the filter is really matched with the input signal that follows a fundamental theory of the matched filter operation.

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Now, this is the structure that we will be able to see, if we are using the matched filter architecture that are delay line architecture. The delay line architecture means actually there are the delays chip delays, all the coefficient of the filter that you are designing they are basically delayed by a chip - one chip duration. And they are all of them are weighted by some value of the  $d$ ,  $d$  is can be actually having different value; in our case we will consider it is a plus and minus 1. And output of all these weighted filter taps will be added up here, and then they pass through a filter whose filter function transfer function will be given by  $P^* w$ . And what is this  $P^* w$ ,  $P^* w$  is the Fourier transform version of our  $p(t)$ ,  $p(t)$  is the spreading sequence waveform, base with waveform for the spreading sequence. So, it is a Fourier transform will be given by  $P^* w$  and here is the output of the filter matched filter.

So, what I meant here is we will come back to the this figure once more, what I mean here is this structure is now getting fed will be fed inside this filter base band filters in the matched filter. So, whatever I have written here in the box if you now try to zoom inside it that box will be the implemented like this architecture. Why actually it is like

this, we will see actually once we are writing the equation of the matched filter techniques in the next slide.

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**Rapid Acquisition Using Matched Filter Techniques**

- A matched filter is generically a passive device that maximizes the signal-to-noise ratio of its output when the signal at its input is embedded in additive white Gaussian noise.
- Mathematically speaking, for an input signal  $s(t)$  of duration  $T_0$  seconds, the impulse response  $h(t)$  of the matched filter is given by the reverse of  $s(t)$  in its  $T_0$ -seconds time slot, i.e.,
 
$$h(t) = \begin{cases} s(T_0 - t), & 0 \leq t \leq T_0 \\ 0, & \text{otherwise} \end{cases} \quad (1.2)$$

or in terms of Fourier transform

$$H(\omega) = S^*(\omega)e^{-j\omega T_0} \quad (1.3)$$

- Suppose now that  $s(t)$  corresponds to an  $M$ -chip segment of a PN waveform, i.e.,  $T_0 = M/T_c$ , then,
 
$$S(\omega) = \sum_{n=1}^M d_n p[\omega - (n-1)T_c] \quad (1.4)$$

where

- $d_n$  is the polarity ( $\pm 1$ ) of the  $n$ -th chip
- $p(t)$  is the basic chip pulse shape

*Handwritten notes on the slide:*  
 - "2. impulse response" written in red above equation (1.2).  
 - "Transfer function" written in red above equation (1.3).  
 - A red arrow points from the text "where" to equation (1.4).

So, we will revisit the fundamentals of the matched filter. Matched filter is basically a passive device and we understand that it is maximizing the signal-to-noise ratio at its output. When the signal at its input is embedded in AWGN or additive white Gaussian noise provided the filter function filter is perfectly matched with the incoming signal. So, if this is the fundamental of the matched filters, it has taken example in terms of mathematics suppose I have input signal  $s(t)$ , its duration is  $T_0$ . Then the impulse response of the filter that that we should design to declare it as a matched filter should be such that the response will be just reverse of  $s(t)$  in its  $T_0$  seconds time slot. And mathematically it should be written as  $s(t)$  equals  $s(T_0 - t)$  over the duration of  $0$  to  $T_0$ ; otherwise the response of the filter should be equal to  $0$ .

If I take the Fourier transform of each this is the impulse response of the filter; in the time domain, we call it impulse response. And in frequency domain, if I come then it will be the transfer function of the filter. This simple just a Fourier transforms of this  $h(t)$ , and hence you will get the  $S^*$  with the furrier transform of  $s(t)$ . And now the time delay will be reflected as the phase on the phase of the signal and of the filter function and then this is the  $T_0$  by amount of which you would be able to see the reflection on the phase of that filter taps.

Now, suppose we have a  $s(t)$  in our case direct sequence spread spectrum communication we have an  $s(t)$  which correspond to capital  $M$  chip segment of a PN waveform. Because remember in our case  $s(t)$  that is this is the general expression of any matched filter. Now, we have to look inside what  $s(t)$  we are dealing with; in the  $s(t)$  that we are dealing with looks like this equation 1.4. We have a spreading sequence PN waveform where  $d_n$  is the value of the it signifies the polarity of each and every thing chip we are having capital  $M$  number of the chips. So, the  $s(t)$  is constituting of the capital  $M$  number of the chips value of each and every chip may be either plus 1 or minus 1. And  $p(t)$  is the basic pulse waveform and this is the  $T_c$  is the duration of that chip. So, this is the signal  $I$  mean PN waveform now we are dealing with instead of our small  $s(t)$ .

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**Rapid Acquisition Using Matched Filter Techniques**

- For a baseband matched filter, we would have
 
$$p(t) = \begin{cases} 1, & 0 \leq t \leq T_c \\ 0, & \text{otherwise} \end{cases} \quad (1.5)$$
- whereas for a band-pass matched filter,
 
$$p(t) = \begin{cases} \sqrt{2} \cos \omega_c t, & 0 \leq t \leq T_c \\ 0, & \text{otherwise} \end{cases} \quad (1.6)$$
- Taking the Fourier transform of (1.4) and substituting its complex conjugate into (1.3)
 
$$H(\omega) = S^*(\omega) e^{-j\omega t_c} \quad (1.3)$$
- $$S(t) = \sum_{n=1}^M d_n p[t - (n-1)T_c] \quad (1.4)$$
- results in
 
$$H(\omega) = P^*(\omega) \sum_{n=1}^M d_n e^{-j\omega(n-1)T_c} \quad (1.5)$$

Handwritten annotations on the slide include:  $T_c = MT_c$ ,  $S^*(\omega) P(\omega) Z d n$ , and  $n=1$ .

Now, if this is the situation and if I substitute this value in that the filter functions the target is to see what will be the output of the filter, but the filter function should look like. For a base-band filter, matched filter now that  $p(t)$  will be plus 1 or 0; otherwise it will be where as for the band-pass matched filter there is  $p(t)$  should be square root of 2  $\cos \omega_c t$  over this duration or 0, otherwise. So, if I take the Fourier transform of this signal. So, target is what we have to take the Fourier transform of this signal and substitute inside your  $S^*(\omega)$  to get the frequency domain interpretation of this filter to the filter  $H(\omega)$ .

So, if I take the Fourier transform of the signal, I will be ending up with what this pt will give me the  $P$  star  $w$   $d$   $n$  summation of this term will be remaining as it is. And instead of your and this whole shift over the time, it should be reflected to me as an  $e$  to the power minus  $I$  should get, so  $s$   $w$  to me  $S$  star  $w$  to me is becoming as  $P$  star  $w$  summation  $n$  is equal to  $1$  to capital  $n$   $d$   $n$  then  $e$  to the power minus  $j$   $\omega$   $n$  minus  $1$  into  $T$   $c$ .

And remember in this  $H$   $\omega$  term inside that each  $H$   $\omega$  done in that  $H$   $t$  earlier you please go back you see that in inside  $H$   $t$  you have another  $e$  to the power minus  $j$   $\omega$   $T$   $0$ . This  $T$   $0$  is nothing, but now to ask now it is capital  $M$  into  $T$   $c$  because within the  $T$   $0$  was the duration of the signal. Inside the duration of the signal, we have capital  $M$  number of the chips each of the chip is having the chip duration of  $T$   $c$ . So,  $T$   $0$  is equal to capital  $M$  into  $T$   $c$ . When I substitute this value of  $P$  star  $w$  inside it of  $P$  star  $w$  inside this expression of  $H$   $w$ , so will be ending up with this expression and here we will be having one term  $e$  term from minus  $j$   $\omega$   $T$   $0$  if I replace it will be minus  $M$  into  $T$   $c$ . And here we are having another term of expression  $e$  to the power minus  $j$   $\omega$   $n$  minus  $1$  into  $T$   $c$ . And if you just add them up finally, you will be ending up with equation 1.7.

So, fundamental conclusion is that transfer function of this matched filter for your direct sequence spread-spectrum signal and for which the PN sequence actually you should be looking like this it should be looking like this. So, it is the Fourier transform of the base-band basic pulse and with the governed by the polarity of each and every chip and multiplied with this amount of the delay, this amount of the phase changes involved actually for each and every frequency based for each and every filter tap.

And with this understanding, we will continue in the next module some further detail of this matched filter technique, where we will reuse this filter function, and we will try to see Fourier transfer function and we will try to see how the acquisition time is getting reduced if I utilize this matched filter technique.