Spread Spectrum Communications and Jamming
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Lecture - 33
Performance Analysis of PN Code Acquisition System –Part II

Hello students. As indicated in the last module, in this module we will try to drive the detection probability of a code acquisition system for the direct sequence spread spectrum communication.

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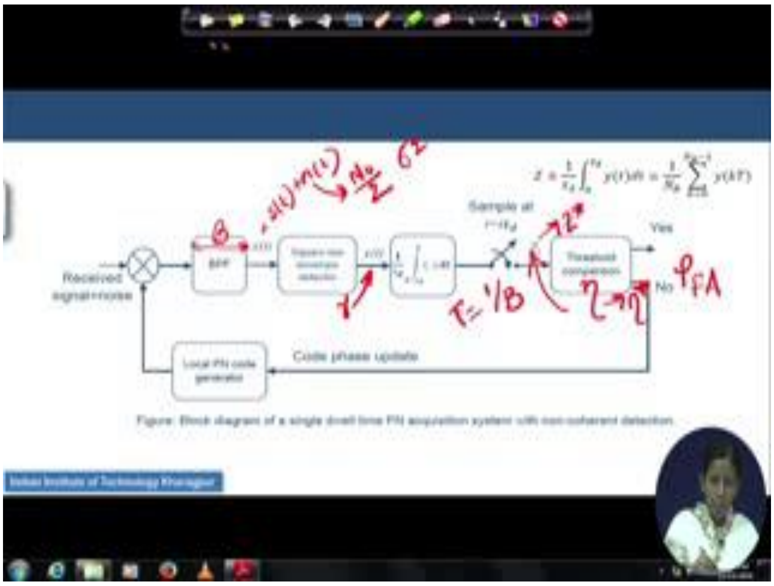

And we have seen we will refer the same block diagram that we have seen in the last module. And this is the block diagram of a single dual time PN acquisition system with your non coherent detection. We understand that the non-coherent detection involves no information about the carrier synchronization; carrier synchronization is not done, so no information about the carrier phase is known to the receiver, and known to this acquisition block at least. And as it is hence it is the non-coherent type detector involved; in a non-coherent type detector, we have a band pass filter which is that limiting the incoming signal up to a bandwidth of capital B.

And x t we saw in the last module that it is having a form of the received signal intended plus the noise. Noise is having a two-sided power spectral density of N 0 by 2. And we saw that the square-law envelope detector output is having a non-central chi square distribution in the presence of signal, and central chi square distribution in the absence of signal. If I put the capital A is equal to 0 in the expression of the SNR, and this gamma actually present or non-present is equivalent to the signal present or signal non-present. And if we feed this y t into this integrated dump receiver, where the sampler we assume that is running at the (Refer Time: 02:10) that means at the rate of one by bandwidth capital B that T is equal to.

And when it is the sampling at that bandwidth, we saw that the distribution the probability density function of this Z is also having a similar pattern of non-central and central chi square distribution with some different basal functions and that also was related to the presence and absence of the signal. And based on that we also derived here; the value of the probability of false alarm, which we taught that inside these threshold compressors. Suppose, there is the threshold defined and if the false alarm is typically same; that means, the signal is not present, how many times the incoming signal z is crossing the value of the threshold and falsely detecting that as if signal is present.

And further derivation what we did is we normalized both z to z star, we also normalized this threshold from eta to eta star normalized it by means of our 2 sigma square by N B; 2 sigma square was the variance of sigma square was the variance of the noise sample. For the reference, I have written here the expression of the z that we could see the time because of the sampling at this Nyquist rate, so it could be approximately written like this, so that was the recap of the last module. And here onwards we are already we understand completely what is the probability distribution function of this z in presence of the signal, in the absence of the signal both. Going by the same philosophy the way we devised in the last module, the probability of the false alarm expression going by the same philosophy, we will now derive the probability of the detection.

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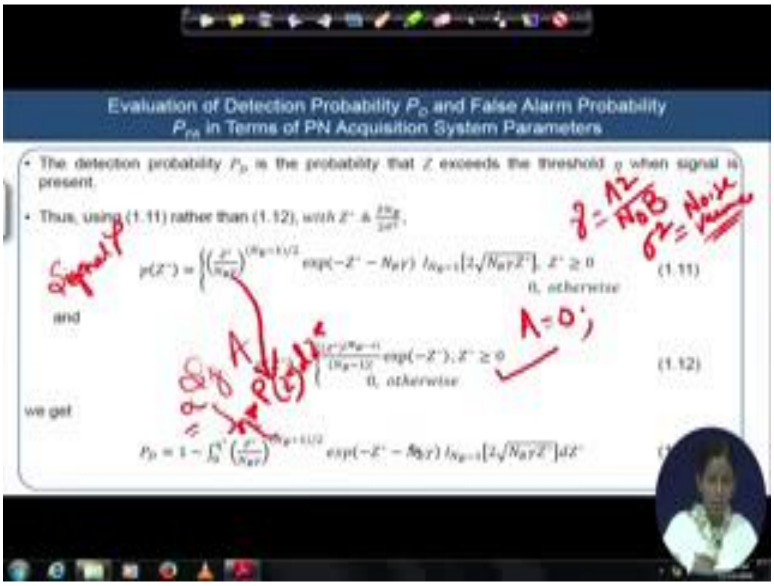

So, we have seen this two expression. This where the expression of the PDF of z when the signal present and signal absent, so by substitution when we substituted capital a is equal to zero in the expressions. And gamma was given by your A square divided by N 0 into B. And as your sigma square was given as N 0 sigma square was given as the noise variance. It was the noise variance. And we had substituted by the noise variance and then we got the sigma success rate in sigma in the gamma. And we finally, got the finally, actually when capital A is becoming 0 gamma is absent, and we can prove the gamma value is zero gamma equals to 0 in the expression of 1.11 to derive 1.12.

And if these are there, we develop the probability false alarm using the expression 1.12. Now, we will do the same philosophy that it was earlier the detection probably is basically given by the interrogation which is running from the eta star to infinity of probability of z star dz star, where this probability of the z star is basically now will be substituted from the signal present PDF of z. As we understand this interrogation is tough to achieve because our limit is a up to infinity, and then hence we have redefined it in the terms of 1 minus 0 to eta star and this, the final expression that we are getting for the detection probability using the expression 1.11.

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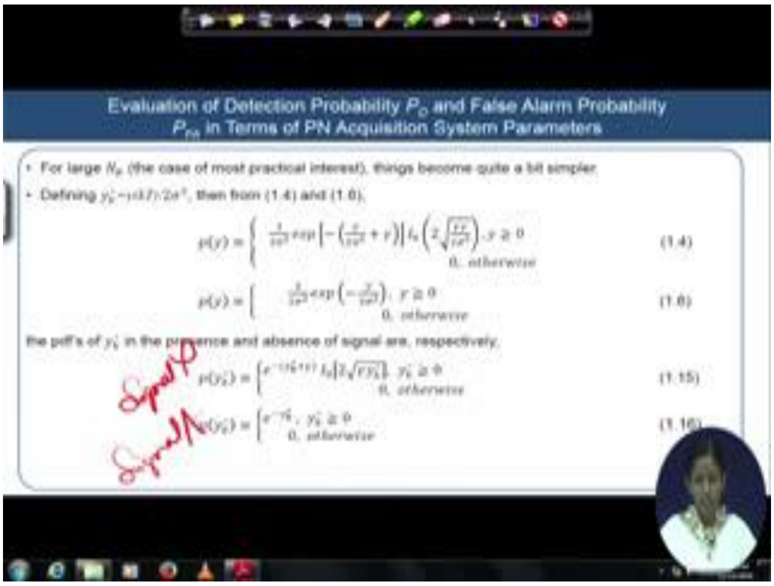

Now, we have to go ahead with some assumption. Let us consider that in N B. What is this N B, N B was there inside the probability distribution of the Z and hence it is here also. N B, if you remember, N B was the number of the samples that you were getting. The number of the samples we wrote given by number of the samples was written as tau d divided by capital T, 1 by T is equal to B. So, basically it is B into tau d. So, see that I told in the last module that how many number of sample are based on the number of the samples you are getting at the output of sampler, the probability distribution of this p Z can be largely approximated.

For example, if the value of N B is too large, then it can have a approximated by an Guassian distribution. We will proceed considering that P z is having the Gaussian distribution, because N B is very, very large. And hence, we can redefine the y k, y k will be refined as y k star such that the y k t we were getting earlier it is normalized with the noise variance once again. And, hence these were the two PDFs for p y, now in terms of y k star, I can get this two remember two-two coming because two-two expressions are coming because one is for signal present and another is for signal absent. Where was this y, this y t was, if you remember correctly, this y t was at the at the output of your squalor detector hence at the input of your interrogate and dump circuit.

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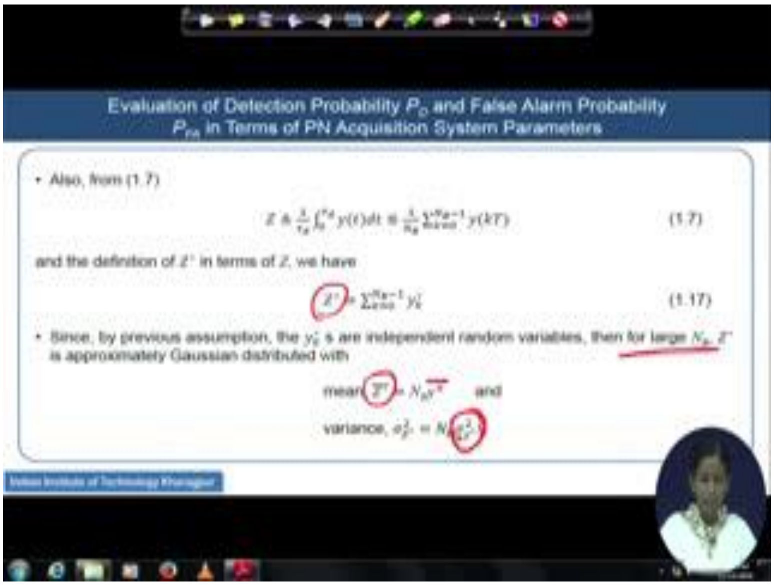

So, now if this is the situation for your y k, I can also get from the expression of the z that we saw in the last module this approximation we did in the last module where my N B is equal to tau d by capital T. And in terms of; why can I rewrite actually this expression in like expression 1.17 by substituting Z as Z start and y k T will also substituting between by y k star by substituting the values of Z by using relational Z to Z star, and Y to Y k star.

And remember if the samples are actually the sampling is done at the Nyquist state and if we hence if the previous assumptions holds good, all this y k s they are all the independent random variables they should be. And if the N B is too large then this y k s will be all the independent random variable, and hence this Z star will be Guassian distributed I mentioned it earlier. If it is Gaussian distributed the mean of this given by the Z star bar, it will be simply the N B y star and the variance will be y star is not y s, so it will be y star or y star bar whatever you write it should be y star bar. And the variance will be the sigma Z square equal to the N B into sigma square of this y. So, it is basically the N B times multiplied with the mean of this y and of this variance of this y. And lots of research papers are available who shows that this means and variance will be easily obtained.

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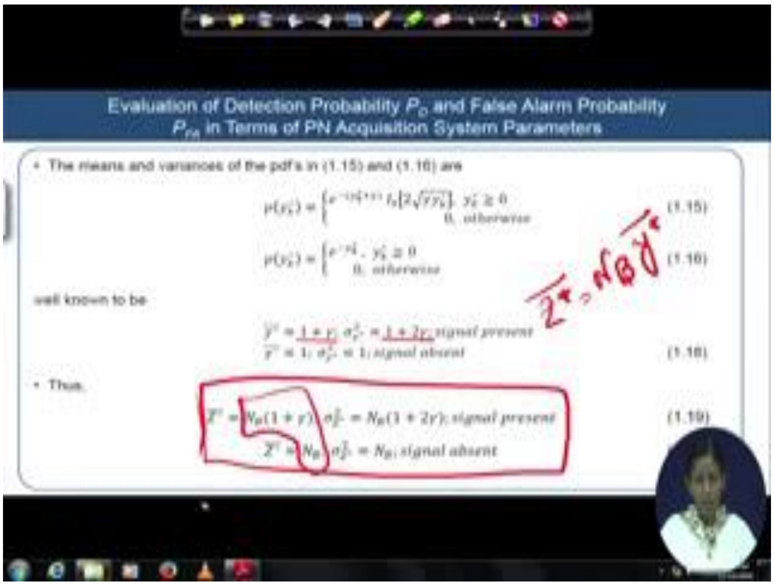

And prior to that revisit the probability of y k star in terms of the mean and the variance, so this two probability equations these two probability equations in terms of your mean and variance would look like this. And from here actually this mean and variance of this PDFs, so we understand that first we have to now get the mean and variance of this y k star, y star and as well as the variance of this y. And that will be brought from the previous PDF expression given and developed earlier. And lot of literature available which shows that the probability mean and variance, when the signal is present for with respect to these two PDFs, when the signal is present then the mean will be given as 1 plus gamma; and variance will be given as 1 plus 2 gamma.

When the signal is absent, so put directly gamma equal to 0, you will be ending up with the mean and the variance both equal to 1. If this is a situation going by the fact that Z star bar is nothing but the mean of this Z star bar is nothing but N B times the y star bar. And hence it will be simply multiplied by N B into 1 plus gamma when the signal is present or only N B the signal is absent, so the mean value is varying from N B into 1 plus gamma to the N B value. So, N B multiplied by 1 plus Z signal-to-noise ratio when the signal is present for the mean of the Z star bar, and it will be simply governed by the number of the samples is absent.

Come to the various section, we substitute actually the value of the variance of the y multiplied with the N B for the computation of the variance in the presence of the signal, and put gamma equal to zero and when we calculate the variance in the absence of the signal for z. So, these are the very important four parameters that will be helping us to derive the expressions further.

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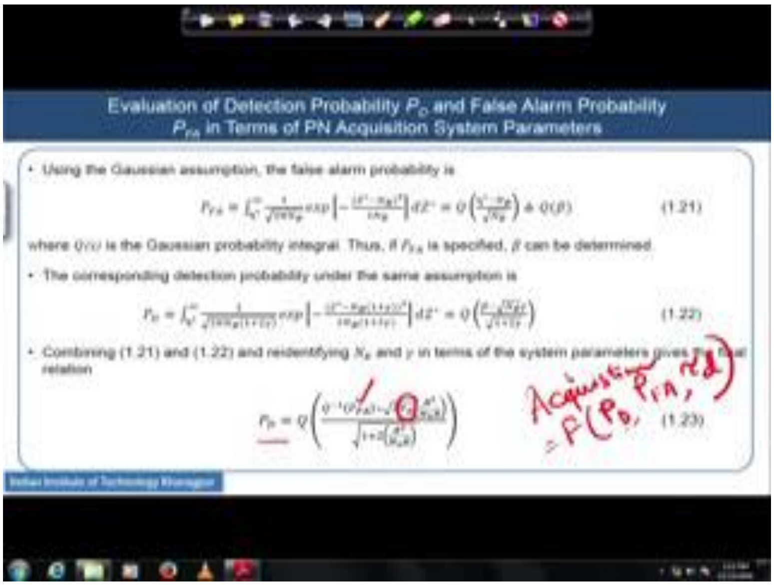

Now, we are ready to redefine our probability of false alarm and probability of detection that was derived earlier. Using the Gaussian assumption, we understand the probability of the false alarm will be governed by the Gaussian distribution give here, and if it is the Gaussian distribution, which can also be approximated by your Q function. And we write actually the expression inside the Q x just the beta. And Q x is the Gaussian probability integral here given; and if the probability of the false alarm is given because of any synchronizer design usually have a constraint on the probability of the false alarm always. Given the minimum availability of the false alarm, this beta can be you can define this beta.

So, there is a combination on relation of this chosen normalized threshold inside the verification algorithm, and the number of the samples that you are having in your hand there is the relation between this two with the probability of the false alarm. And corresponding detection probability, he will be also given by this and here you are representing all the mean and the variance. See, actually the variance was only N B and here it is coming 1 plus 2 gamma like that; and the mean was here and here N B and here I am substituting it by 1 gamma variance is actually 1 plus 2 gamma.

And the new function is similarly you can come up with Q function in terms of the beta represented here, you can actually write in short form of the detection probability; that remember inside the detection probability analysis already not only the gamma the signal-to-noise ratio is involved already the probability of false alarm section I mean it is involved. So, beta is actually a function of this eta square and the N B. So, the threshold is also coming in addition to the N B and signal-to-noise ratio in the expression of the detection probability. And if I substitute try to write the detection probability in terms of the false alarm probability and I substitute all the where N parameters of this detection probability by the system parameters.

For example, N B was equal to capital B into tau d; and gamma was equal to A square divided by N 0 into B. So, if I write like this then finally, we will be ending up with the fact that we were claiming at the beginning that look the detection probability given up false alarm probability, detection probability is the function of the dual time. So, even if actually I am giving you the detection probability and false alarm probability and if it is the function of the dual time and then again the acquisition time is also a function of your probability of detection probability of false alarm and as well as the dual time tau d, then you can cannot actually easily even if you know the detection and probability and false alarm probability, you cannot arbitrarily choose the dual time. So, that the mean acquisition time is achieved, because if you arbitrarily choose the dual time definitely the P D value will be changing. If P D value changes and the acquisition time will be something else.

So, choice of the tau d such that even if you know the P D it not. So, easy to calculate do the optimum do the proper selection in between the tau d without changing the P D. So, it is not straightforward to do the pick up some acquisition time or to do justice with the main acquisition time just arbitrarily picking up any tau d value. You have to have a very close observation between the P D, the way P D is changing with respect to tau d and then accordingly the given problem of false alarm and given gamma value how that is varying and based on that how the acquisition time then the acquisition time will be calculated.

Remember up to this we could define the relation nice relation of the detection probability, we understood actually how it should be calculated taking a very simple example of a non-coherence square law detectors, but this is wherever whatever we have derived here we have several assumption inside that. So, one by one, we will try to see whether those assumptions are valid in practice or they are really not.

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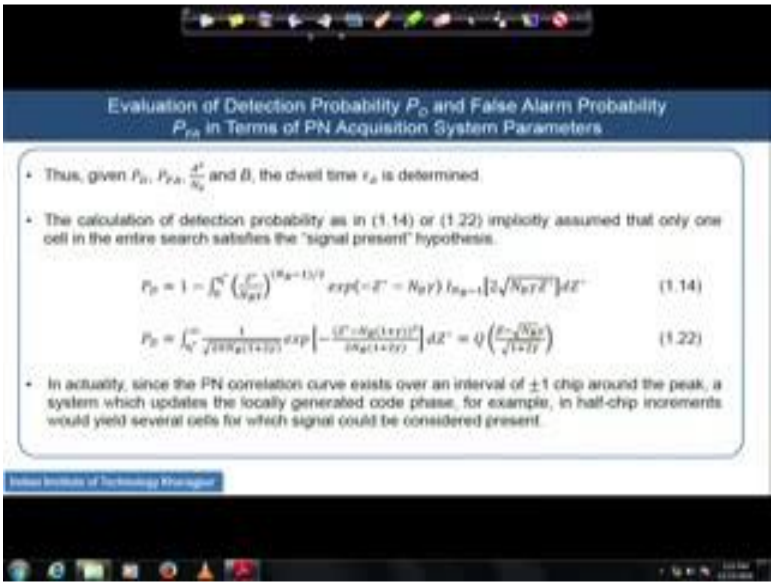

Remember number one assumption that we did that P D and PFA this gamma, these bandwidth tau d all can be determined and all are calculation of this simplicity assume that only one cell in the entire search satisfied the condition that the signal is present. So, out of even if it is multiple dual time based detector, even if actually detection process all stuff we have calculated considering that at least one cell is having the entire search, the entire search process satisfy that the signal is present. So, even if actually one time the signal is present we could detect we are done and then we will define the probability.

But in actual what was done in actual it was done that the PN correlation curve, if you observe the correlation graph actually exists over an integral of plus or minus 1 chip around this peak. So, this you have to actually look into the fact that detection can also get locked here or here, it may not be exactly the peak. So, you may get actually anywhere within the plus minus 1 chip duration and that is why actually you are not always going to get the good peak value, peak SNR value also, because the value of the capital A here is not same that peak value of it. So, hence it will vary the gamma value inside this detection probability. How much that variation will be let us have a look.

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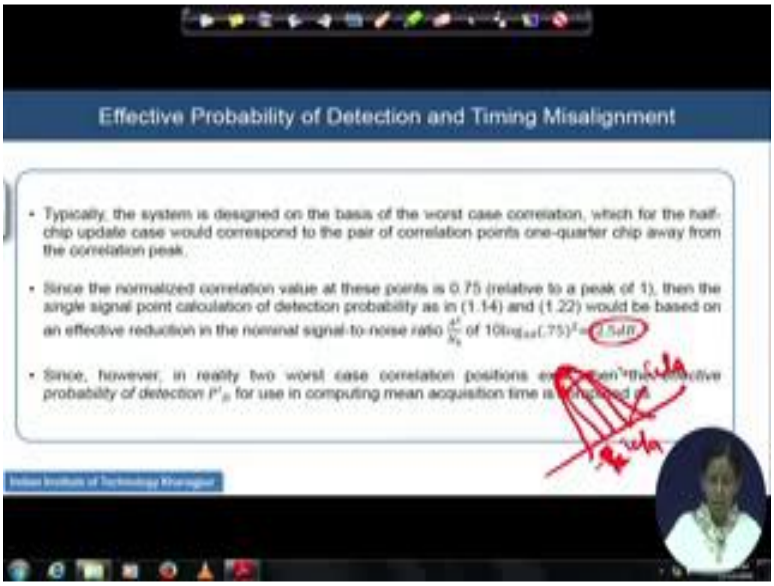

So, typically the systems that we have designed already, they are designed base of the worst case correlation. Actually the situation is such that the acquisition that is getting achieved that will be within half of the period of the chip duration. So, we understand the correlation peak is spread over the plus minus 1 chip duration. And even if you get the code acquisition that will be within a plus minus half of the chip duration and hence the correlation points will be within the one quarter chip duration. So, one-fourth of the chip duration time here and one-fourth of the chip duration time there you will be able to get.

So, after acquisition, you are getting correlation peak spread over one-fourth of the T c by 4, minus T c by 4, it is minus T c by 4 to plus T c by 4 duration. So, over this chip duration, you are really not getting the peak value of the auto correlation function, you are getting actually 75 percent of it. So, if we substitute that 75 percent of the peak value of the auto correlated output, the value of A square, you are not one here, it is only 0.75, and hence you will get a loss of db in the SNR by an amount of 2.5 dB. So, the detection probability will be definitely affected by redefined signal-to-noise ratio that is why we call it is an effective probability detection, we do not call it a true detection probability. So, the expressions that we have deserve we have got in the last few slides P D and PFA, they are the actual detection probability and actual false alarm probability.

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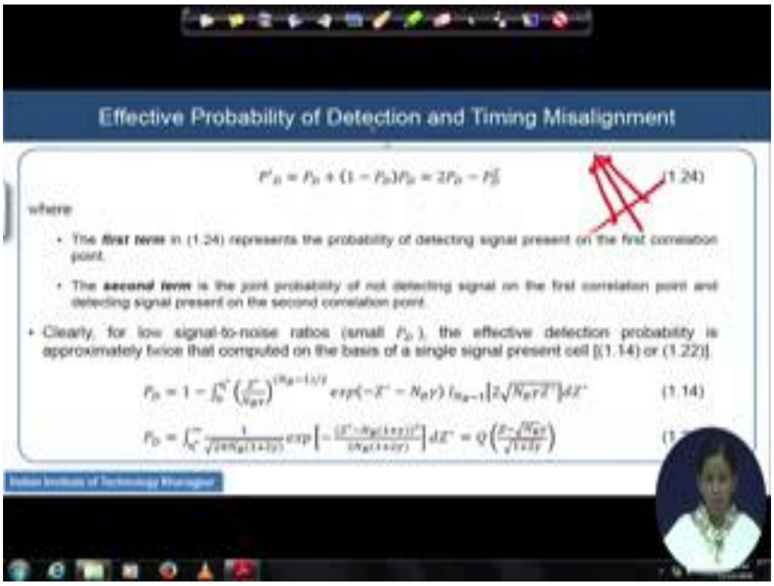

Whereas, in practice we will always get the effective detection probability which will computed as this the expression of this detection probability effective detection probability will be given by equation 1.24. Where the first part is representing the probability that you are detecting the signal present on the first correlation point, by means of first correlation point what I mean is you are at minus T c by 4. So, and in the second correlation, second part of it is telling, what is the joint probability of detecting the signal on the first correlation point as well as detecting the signal on the second correlation point?

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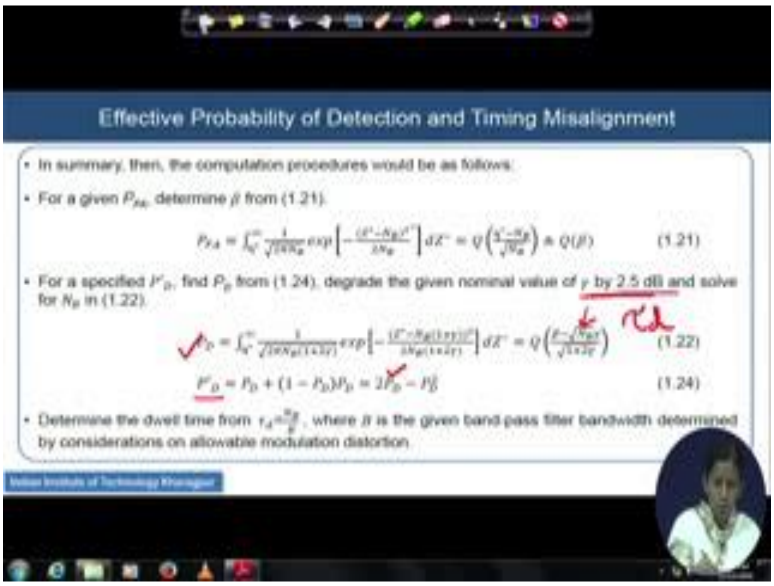

So, you have to take the effect of the both, and hence our earlier detection probability for this when the on the basis of the signal present will be given would not be the only way to declare that this is our detection probability. Now, the computation should go like this. You compute first the probability of the false alarm going by the equation 1.21 which is exactly similar the way we defined earlier. Then you compute use this beta to compute the theoretical detection probability following the expression of 1.22 which was also defined earlier.

You may utilize actually the you may see all the you substitute all the values of the gamma and all, and this for specified P D dash find P D from this 1.24 degrade the given nominal value of the gamma by 2.5 dB and solve for this N B now to get. So, now, you solve for N B to get actually the value of the dual time tau d. And you can get the value of N B also then from there you can get the approximate idea about the tau d.

So, the case is something like this for now you will be getting P D, P dash d finally, from there you can calculate the value of the P D. And once you are getting the value of the P D the value of P D, then you understand actually; what is the value of the beta to get actually the expression of the P D. But you understand the value of the P D. And from there you have the redefined value of the gamma, gamma is not exactly equal to the one exactly effect of the whatever the SNR you were dealing with earlier, you have nominal value of gamma by 2.5 dB.

And finally, from there you can calculate the P D. So, now the P D is actually having the effect of P D and P dash D in that sense. And determine the dual time now once you are getting the value of N B from this expression we can find out tau d and B is given dominated by the given band pass filter bandwidth. So, this is the way we actually in practice should compute P D.

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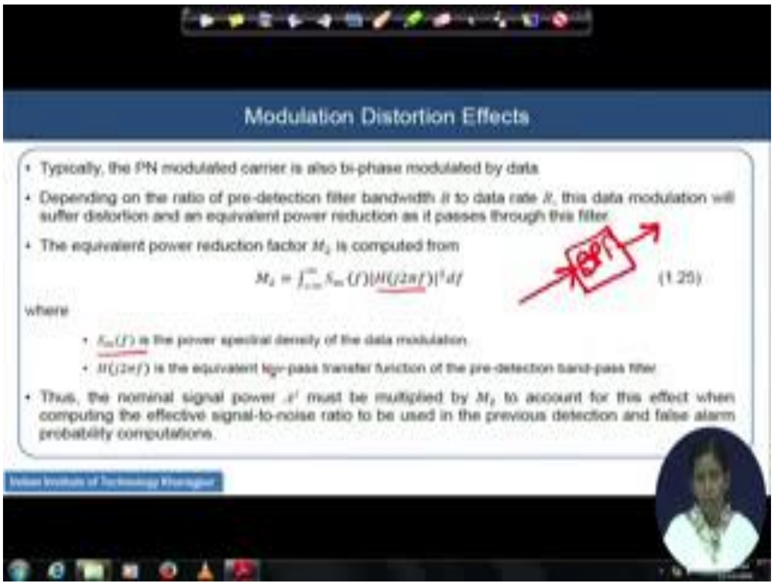

Another effect that we should discuss at last, that is the modulation distortion effect. The modulation distortion effect means the PN modulated carrier, it is also bi-phase modulated by the data. So, when we are using the pre detection band pass filter, which is having a bandwidth of B to the data rate R. So, once you are using the band pass filter and you are passing a signal through it, so it will distort the incoming signal to great extent. And once actually the distortion happens there will be a huge deduction in the power of the modulated signal at the output of the filter. We can quantify that deduction factor by into which is computed by the expression this where this H J 2 pi f, it is the equivalent low pass transfer function of the detection band pass filter and S m f is the power spectral density of the data modulation.

So, define see that there will be an reduction by an amount M 2 which is the function of the power spectral density of the incoming signal and also affected by the incoming signal affected by the equivalent low pass transfer function of the band pass filter, and thus the value of the S square that is used inside the signal-to-noise ratio to calculation that should be multiplied by the same two. So, you are not actually getting S square. Earlier we saw that because of the code acquisition getting within plus or minus T c by 2 there is the deduction in the SNR value. Here is another further reduction in the SNR value because we are not getting there is an effect of the band pass filter residing the square law detector. And its effect is such that it will be the s square the power of the signal will be reduced by an amount of M 2 that we have to capture in the calculation.

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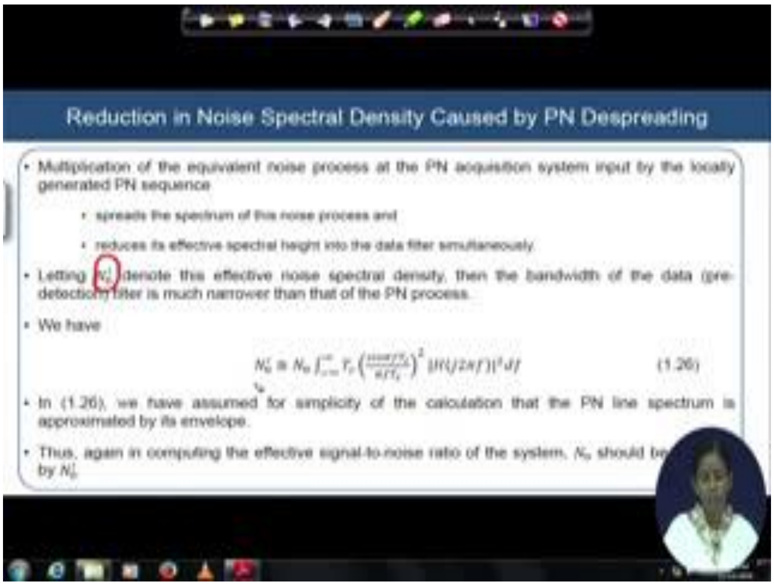

The last effect is not only there is the deduction in on the signal. The last two effects we discussed about the reduction on the signal power. Remember when we do the despreading process inside the receiver frontend, the noise spectral density reduces because noise spectrum it expand noise spectrum expand by the despreading process itself. So, the spreading of the spectrum reduces the spectral height and the effectively actually it reduces the spectral density. So, N 0 you cannot consider as an direct N 0 hence will be an effective noise spectral density like the effective detection probability. And we define effective noise spectral density as N 0 dash which will be now given by the expression 1.26.

So, then you are basically again actually the effect of the band pass filter is coming into picture, and the envelope instead of actually simplicity for calculation we have considered that the line spectrum is approximated can be approximated by the envelope of the noise. And thus again effective signal-to-noise ratio computation will be by multiplying A square with by M 2 and substituting the N 0 by N 0 dash how we will see next.

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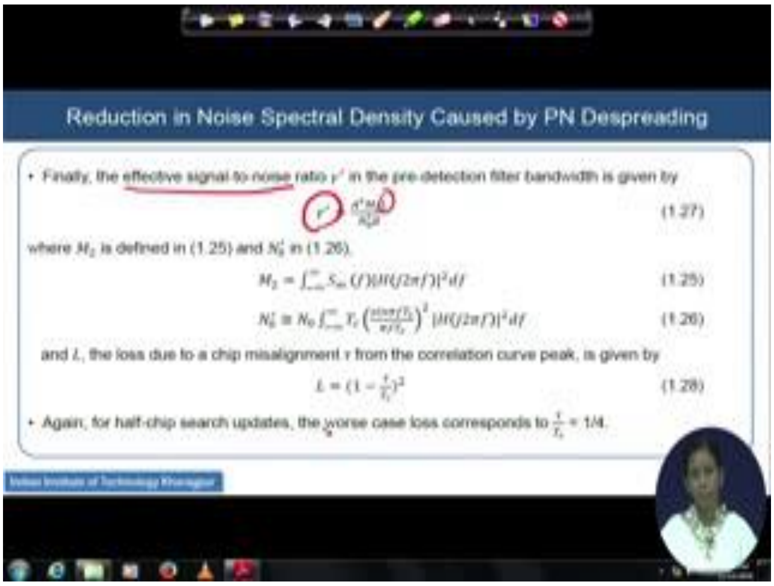

Look that now by the going by the similar terms, we will redefine the signal-to-noise ratio as effective signal-to-noise ratio and represent it by gamma dash instead gamma. And as we understood that because the effect of the band pass filter, the power of the signal will be multiplied by M 2 there is a reduction of the signal power. N 0 will be substituted by N 0 dashed, bandwidth is remaining constant, there is another factor coming as capital L which is also getting which is considered to be a loss. What is that loss we understand M 2 and N 0 will be given by this as shown in the last two slides?

This L is nothing but because of the chip misalignment residual offsets that you are having and that alignment if the alignment is given by an amount of tau with respect to chip duration T c, so there will be a loss because of that remaining residual error. So, the loss due to that residual error is incorporated by the factor capital L in the expression of 1.27; so originally whatever the gamma expression that we have put in the calculation of the probability of false alarm and especially the detection probability analysis.

Now we understand that the if we try to get the true detection probability, we have to modify the computation of the signal-to-noise ratio by the effective signal-to-noise ratio by taking care of the effect of the band pass filter on the signal on the data modulated section and the effect of the despreading process on the noise power spectral density given by the equation 1.27. Then only we will be able to calculate the proper or the correct effective detection probability.