Spread Spectrum Communications and Jamming
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Lecture - 32
Performance Analysis of PN Code Acquisition System – Part I

Hello students . In continuation of our discussion on code acquisition mechanisms and the circuits involved for the spread spectrum communication systems and jamming, we will today go in detail of such one such kind of code acquisition mechanism and we will try to do the performance analysis of this code acquisition circuits and basically we will try to evaluate the performance in terms of the detection probability and the false alarm probability.

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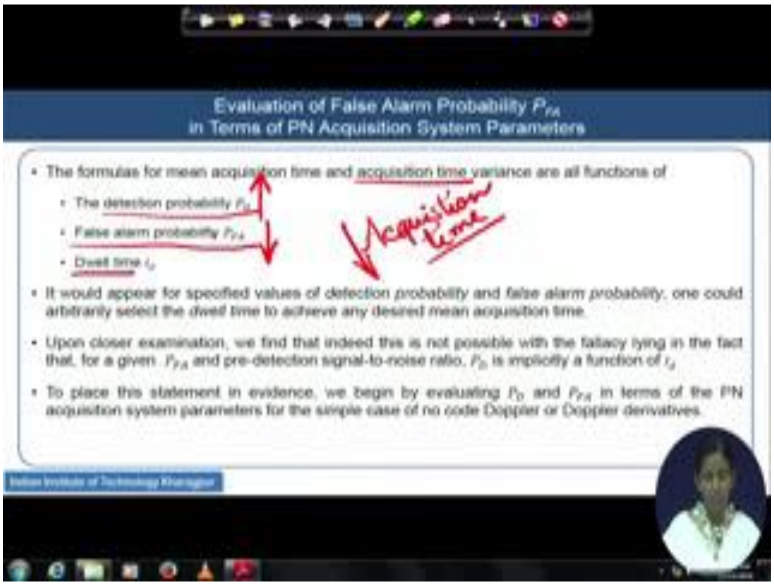

Remember, when we are talking about a code synchronization there are few terms involved related to the performance analysis of the synchronizer and the performance measure is largely involved the term called detection probability which basically is telling how many times you are successfully detecting or achieving the synchronization and in your detecting the threshold is properly crossing the level and hence you are declaring that you have you have acquired with the synchronization. If you are detecting then also it involves how many times you have missed and how many time you have given the false detection.

So, falsely detecting something is will be quantified by the probability of the false alarm and why this false alarm is coming into picture is in your whole detection process and the synchronizing process is a random process, is the stochastic involved because it is a function of, it is having the large dependency on the Doppler key or the Doppler frequency change or the Doppler derivatives as well as the different delay profile changes of the wireless communication channel. So, hence the fall and also some other issues are there circuit issues are there of the front indirect circuits.

Dual time is the time that we have learnt in the previous module that involves the time for your integration process inside the detection, apart from all these three we have a very important parameter called acquisition time.

Remember the acquisition time and the dual time are not the same thing, dual time involves the integration time involved inside the detector, and acquisition time is the total time you are taking to declare the synchronization is achieved. See the dual time can be having multiple time cpr if it is a multiple dual time based detector and at the age the total dual times that you are taking plus the decision, the time that is involved to apply the verification algorithm and at the time declaring that the verification at the end of the verification the synchronization is achieved, this total time is involved inside the acquisition time.

As we understand that the detecting process itself is a stochastic process and hence the acquisition time is also a stochastic process, and finally it is a random variable which is having a means and a variance. And we are really interested to know what is the acquisition time or basically whether the acquisition system is good or bad it depends on the several things. One important part is the acquisition time it is taking to convert and declare the synchronization is achieved. Always try to device the acquisition system which is having the very low acquisition time, it means very fast actually the synchronizer is able to synchronize and hence you need a very high detection probability to achieve with the high acquisition at the same time the probability of the false alarm should be very very low. This is the ideal situation that we always dream of and which is a target of designing synchronizer in practice.

Remember one more thing that whenever we are talking about this acquisition time and if we see the expression of the acquisition time we later on we will discuss on it. If we see the expression of this acquisition time we will see it is the function of this P D the detection probability the false alarm as well as the dual time.

So, one may ask like this that I know the detection probability and given the false alarm probability. So, I can arbitrarily choose the dual time t D in such a way that I can achieve the mean acquisition time, but the answer is not so easy. If you see the expression of the detection probability given the false alarm probability and the signal to interference noise ratio at the detector input we will see that detection probability itself is a function of the dual time. So, it is the complicated relation and it is not so easy to pick up, comment anything on the acquisition time.

We, today our task is to develop the expression for this the detection probability as well as the false alarm probability. We will try to do the false alarm probability expression, derive the false alarm probability expression first and to see actually how these relation is coming in to picture and finally, whether we can comment something on the acquisition time.

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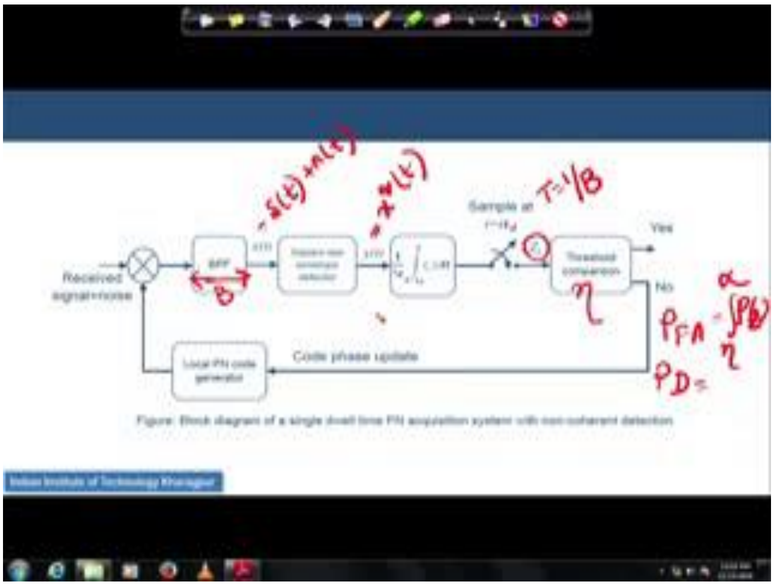

We will consider a system to derive the expression. This is a system block diagram of the single dual time PN acquisition system and it is non-coherent detector based. Remember as discussed in last module, if it is the non-coherent detector you will see you do not know anything about the phase ambiguity of the carriers.

So, carrier synchronization is not done and hence you do not have any clue about the phase ambiguity of the carrier frequency and hence the non-coherent detector is coming into picture and the non-coherent detector involves the band pass filter and the squalor envelope detector. Output of the squalor envelope detectors fed into a intricate and dump circuit it will be sampled and this parameter that is entering into the threshold device is basically a decision device where the verification algorithm is running he is taking the input of the Z. Remember this Z he is the input to the decision device we will do lot of derivation based on this Z.

If the threshold output is saying that the synchronization is achieved then you are here in this line and there is no feedback going into the local PN code generator, otherwise if it synchronization is not achieved then there will be some correcting information should go inside the PN code generator and based on this information coming from the decision device the wave form that is generated inside the PN code its space and its timing as well as phase and timing it will be changed. Basically the phase is changed and in order to reduce the difference of the phase between the received signal PN code as well as the locally generated PN code. So, that is the target and over the several iterations we believe that actually the decision algorithm is perfectly running and if it is correct, then over the several iterations we will be closely achieving the synchronization at a particular time. So, that is the target, this is the way the whole circuit operates.

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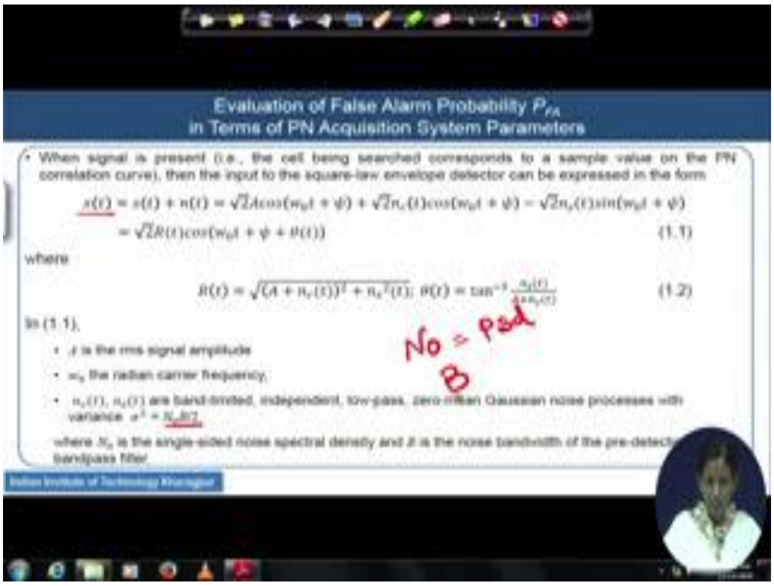

We will now enter into the signal model. If the signal is present, if the signal is present then remember x t is here in the block diagram. So, that the input of the squalor envelope detector if x t is present you will receive a signal who is the combination of the intended signal plus the noise. So, we will start from here in the next slide.

So, once my signal is present my input signal to the squalor detector is s t plus n t; s t I can express as square root of 2A cos omega naught t psi and n t is having cos as well as sin component because it is the complex noise getting added and the omega naught is a the carrier frequency in radian, carrier frequency of transmission. In radian A is the rms value of, rms signal amplitude and then psi is actually the related phase ambiguity related to the transmitted signal as well as added with noise component, and n c t and n s t they are both the independent, band limited, low pass, 0 mean Gaussian noise processes and as the Gaussian noise processes they are having variance which is given by N 0 B by 2, what is the N 0? This N 0 is the single sided power spectral density of the noise, capital B is the bandwidth of the pre detection band pass filter.

Remember before this squalor envelope detector we had a band pass filter. So, the B, capital B is the bandwidth of this band pass filter here. So, the noise is heavily affected by the bandwidth of this band pass filter. If I substitute this expression by this lower one 1.1, we get 1.1 where this R is nothing but the square root of A plus and n c this square plus n s square t and this theta t is the coming because of the tan inverse for this noise component plus the A plus density. So, this is the total equation with which we will deal from the now onwards and this is remember, this is the signal at the input of the detect squalor detector. So, the output of the squalor detector what will be is our first important, next important part to see.

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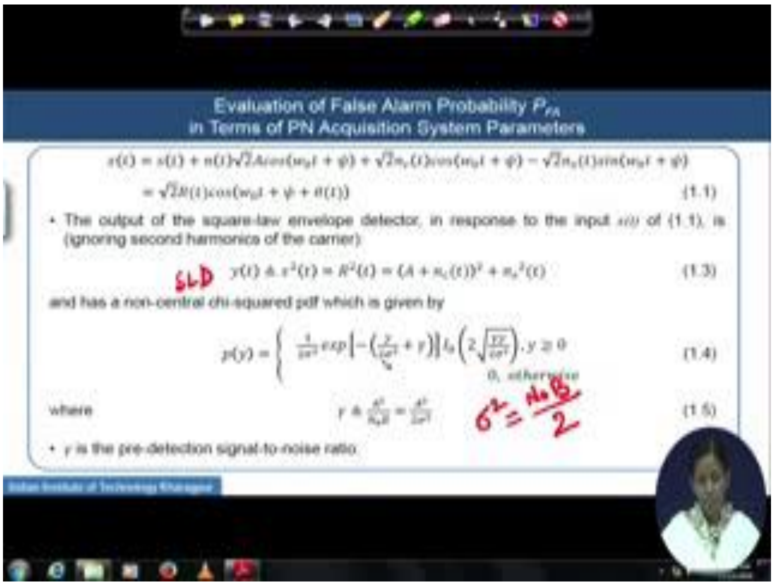

Here you transmitted this that we have seen in the last slide and the output of the squalor envelope detector definitely if the output of the squalor detector is written as y t it will be square of this input and hence it is finally, the square of R square input into t basically and we will be ending up with the expression of 1.3. And it can be easily proved also that this y t will have a central non central chi square distribution and it will be having a pdf governed by the non central chi square distribution which this non central chi square distribution is written as this where your I 0 is the Bessel function of the first kind and gamma is the signal to noise ratio. So, gamma is written as the energy of the signal and divided by N 0 into B.

Remember we have already seen that the sigma square which is the variance of the noise added it can be given by N 0 into B by 2. So, N 0 into B by 2 is two sided power spectral density of the noise and B was the bandwidth of the band pass filter who was residing just before your squalor detector and we have just replaced it here and this is the total expression in terms of the noise variance and the incoming signal we are getting.

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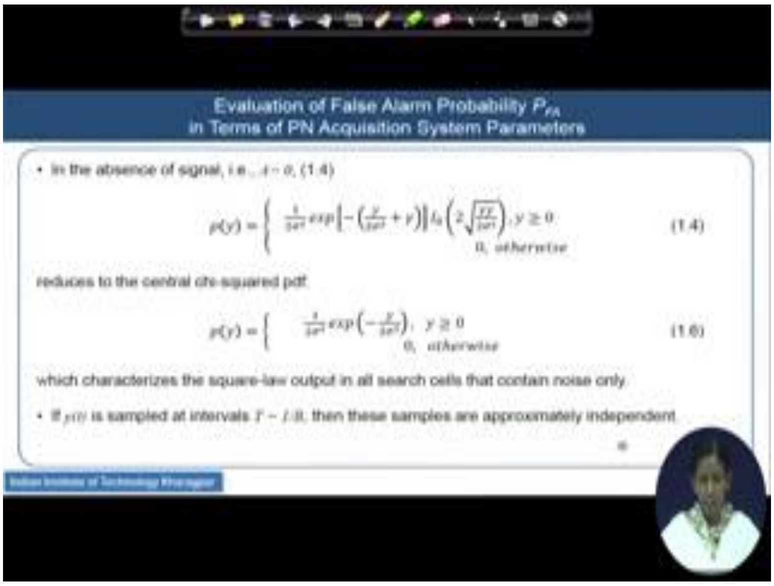

So, this expression is valid when signal is present remember. What will happen if my signal is not present? Means the rms value of this signal A, which is given by A that will be now 0. If I substitute the value a is equal to 0 in the upper equation this non central chi square distributed pdf we will get the pdf of a central chi; we get the pdf given by equation 1.6 which is fundamentally the pdf of a no signal condition and it is obtained by substituting A is equal to 0 in the equation 1.4 and obviously, that it will come to a central chi square distributed pdf.

Remember that we were we are right now let us go back and revisit. We started with the x t we have squared it, we have had entered into the; we have got the expression for y t and we have tried to see what is the distribution of this y t. We have realized that if the signal is present it is having the non central chi square distribution and if it is the signal is not present it is having a central chi square distribution pdf.

Next part is to enter into the intricate and dump receiver. Remember not only in the intricate and dump receiver from y t we will jump here at the point Z i. So, not only the intricator is there, intricator and the sampler both are involved in the process. So, here we are taking a assumption that if we take, we understand that capital B is the bandwidth of the signal because that is the signal band that was the bandwidth of all band pass signal, band pass filter. So, band pass filters allowed the signal having the bandwidth of capital B only if I take the sample at the interval of capital T is equal to now 1 by B. So, this y t which is achieved which is achieved at the output of the squalor detector if now we sample it at the rate of capital T which is nothing, but 1 by B and then those samples as it is sampled actually for the following the Nyquist test sampling rate then all the samples will be approximately independent to each other.

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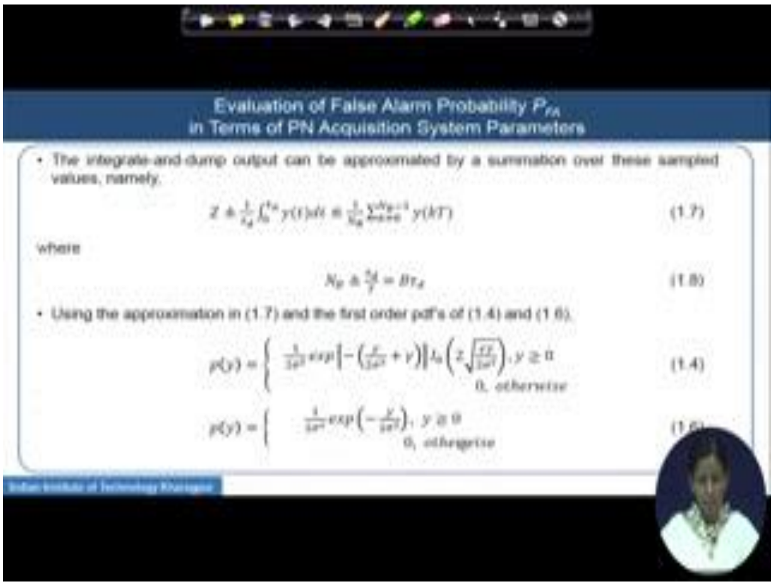

So, how are you getting the value of Z? So, Z is approximately will be given by 1 by the T d, T d is the dual time that over which you have done the integration and then you are doing the samples. So, approximately if I take the samples in the sample domain this expression can be equivalently be written as y of k capital T, where k is varying from 0 to N B minus 1, what is this N B? N B is telling how many number of the samples you can get, the samples approximately that you can get is the what is the sample duration? What is the duration over which you are sampling it is a tau d divided by the sampling total sampling time.

So, fundamentally it is the bandwidth multiplied by this tau d the dual time over which you are dealing with. So, fundamentally this N B, capital N B will be large or small or the moderate in number based on this we will see later on that the pdf of the Z can be approximated to some known distribution actually. Otherwise based on the number of the N B available we can actually simply a lot the distribution of the Z that we will see later on. But currently using the approximation of this 1.7 we can actually revisit the pdfs that we have earlier derived for the signal present and signal not present and they can be written as like this.

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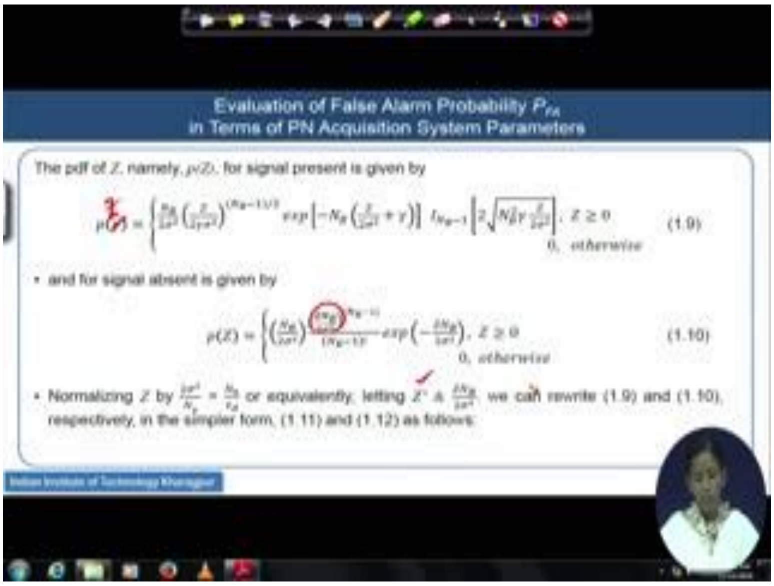

And the if I utilize actually these two earlier generated pdfs, sorry, if we generate this earlier distributed pdf and if we utilize this approximation finally, we can ending up with the probability distribution of the Z and which is given by this p Z basically and p Z will be this is not y this will be probability of this p Z. And the probability of this Z will be given as this. So, it is also a chi square distribution going on, but it is not actually having a Bessel function of first kind definitely and when the signal is absent if I substitute the capital A equal to 0 which is equivalently gamma is equal to 0 we will be ending up with the expression here.

Now, see here we have a term called Z N B by 2 sigma square which is basically nothing but normalizing the Z by the term of the two sigma square by N B as if. So, we will substitute this expression by say Z square, Z star, if I do it then this expression both the expression with the signal present and signal absent both the situation we will be ending up with two more expression given in 1.11 and 1.12.

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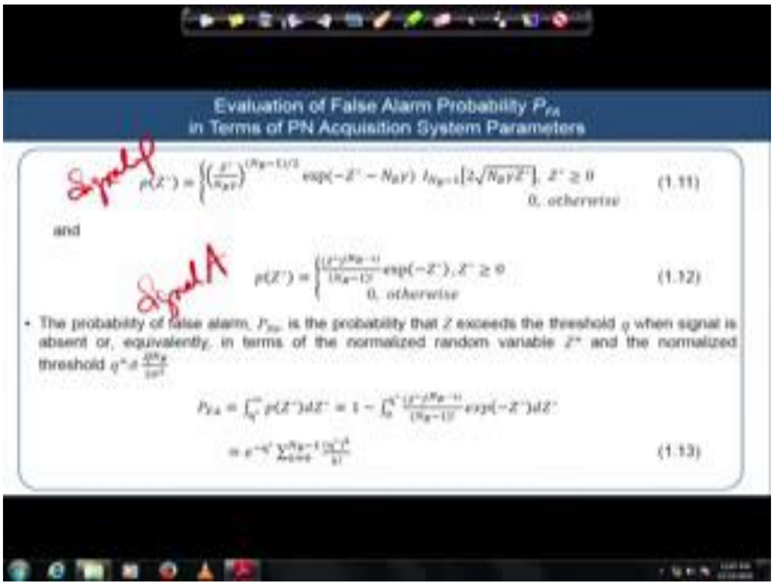

Remember to avail 1.11 and 1.11 is obtained from 1.9 by substituting Z N B by 2 sigma square by Z star and 1.12 is obtained from 1.10 by the same substitution. And remember that the first one I mean 1.11 with this normalization 1.11 is talking about the pdf of the Z, I mean in put signal to the decision device when signal is there, signal present and here signal absent.

Now, we are almost actually in the situation where we understand. So, remember, let us go back once more to the block diagram and revisit what we have already understood. We started with a single dual time PN acquisition system we understand that it is a non-coherent detector. So, the signal that have entered here at the front end which was the data signal over which the PN sequence was super imposed, target is to acquire synchronization between the locally generated PN code and that transmitted PN code which is super imposed on the data. And as it is the non-coherent detection process, so we are having a band pass filter in the forward path, followed by the squalor envelope detector, intricate and dump circuit totally and then a threshold comparator.

And, over the multiple iterations the whole circuit is trying to get synchronized with the minimum error sense. So, with the received signal PN code and we saw that the input to the squalor envelope detector we started with x t is equal to, x t is equal to the signal plus the noise, contaminated with noise. Squalor detector output y t was given as x square t.

We saw that the distribution here of y t was in presence of the signal it gave us the now central chi square distribution and it is largely dependent on the signal to noise ratio. And remember the noise that we have added here in this process here the bandwidth of this noise is restricted by the bandwidth of the band pass filter standing means preceding the squalor detector which we actually thought to be the capital B. And once this is the; and the noise is having a one sided power spectral density of capital N 0.

Hence in presence of the noise when the signal is present we see this y t is having as non central chi square distribution depended upon the signal to noise ratio and if the noise, only noise is there see that; that means, the signal is absent then this distribution will look like a central chi square distribution substituting the value of the signal section equal to 0; that means, the SNR value will be down to 0 substituting that we got the both distribution here.

Next we look it into looked into the signal that is entering into the threshold comparator device or the decision device. Here we had an assumption that this sampling is going on at the rate conforming the Nyquist sampling rate; that means, at the rate of 1 by capital B, which B was the initial bank pass filter bandwidth and hence the now the signal bandwidth or x bandwidth of x t bandwidth of y t all are limited by B. So, if I go by the capital T is equal to 1 by B we saw that this can be nicely approximated and we could also find the probability distribution function of this Z when signal is present and signal is not present. So, now, central kind of the chi square distribution also we found here when the signal is present for Z and central chi square distribution closely with the different Bessel basis function, we got with different Bessel function we got for Z when signal is absent.

So, why we are so interested about the distribution of the signal here is because this is a very important signal part who based on which actually now the decision will go on. What I stared with is the performance analysis of a code acquisition technique means you have to define or you have to derive the detection probability and the false alarm probability right. So, based on this input and its probability distribution we will now take the discussion inside it. So, let us assume that inside the threshold comparator there is the verification algorithm running where there is a threshold in mould as eta. So, the algorithm will tell that whether the algorithm will really try to understand how many times the incoming signal this pdf signal is crossing the threshold.

Remember that there will be the correct detection, if actually for all the cases when this guy this Z i will cross the theta i. So, theta i to infinity to up to that zone there will be a detection probability going on and rest of the term actually when the signal is definitely present for the detection probability correct detection is going on. The same interrogation will run when the detection, when the incoming signal is falsely crossing this, this eta when signal is absent. So, the probability of false alarm and probability of detection both actually integrate the incoming signal pdf Z for the interval that pdf is crossing eta and reaching to eta to infinity times that the pdf of Z is getting integrated. But remember that for the probability of false alarm only difference is that this pdf is in this is the pdf in absence of the signal and in case of probability of detection this integration will run with the pdf that we have obtained when the signal was present.

So, it is basically substitution of the pdf of the Z in presence of the signal and in absence of the signal. Presence of the signal will tell you the detection probability, absence of the signal the false alarm probability.

Now, let us go the last slide where we were. So, here we are and we with the explanation that I have given with respect to the block diagram, now instead of Z we have normalized Z known as Z star. So, we will also normalize the threshold by the same amount 1 sigma square by capital N B and eta star is given as the normalized threshold. So, the probability of the false alarm will be as I told it will be the interrogation of n star to infinity of the probability of Z star p Z star that these probability of Z star is the probability when the signal is absent, I mean the equation 1.12. And it means that signal is basically not, false alarm means what? False alarm means the signal is not present, but you are only detecting that the signal is present.

So, given this probability distribution how many times actually you are detecting the signal is present when signal is really not present is the question and as interrogating up to infinity is the tough ask so we will change to their interrogation as 1 minus 0 to eta star and if you compute it try it at home, if I compute it I will be ending up with the expression like 1.13 where the probability of false alarm will be exponentially given by the exponential of eta star with the multiplied with the summation of A to 0 to N B minus 1 n star to the power k by factorial k.

So, it is the exponential expansion of this guy who is coming into this pattern and with that we will try to end up the probability for this module; we will try to end up this module with the understanding of the probability of false alarm. Our next important understanding we will be to derive the expression by the probability of, derive the expression for the probability of detection in the next.