Spread Spectrum Communications and Jamming
Prof. Debarati Sen
G S Sanyal School of Telecommunications
Indian Institute of Technology, Kharagpur

Lecture - 03
Performance Analysis of DSSS

Hello students, we were discussing the direct sequence spread spectrum communication systems. In continuation of that today we will discuss the performance analysis of this DSSS system.

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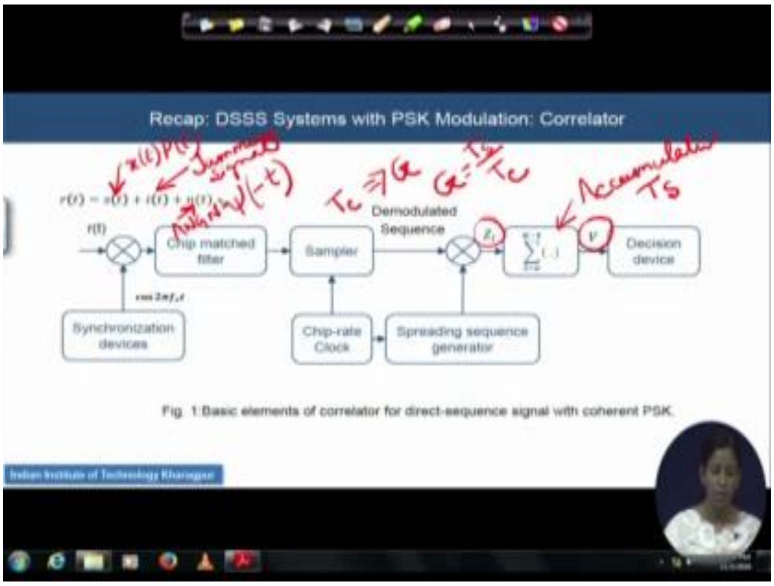

Hope you remember in the last module, we discussed about the receiver architecture two different kind of the receiver architecture we have discussed the last one was matched filter based receiver and where the demodulated circuit was a correlator based. And this is the same figure that reproduced here in this module from the last module, where the chip matched filter is basically matched with the chip form chi t, hence its impulse function I mean sorry it is transfer function will be is equal to the chi of minus t if chi t is my chip waveform. r t signifies incoming signal that is entering into the receiver frontend and this is a synchronization device who is helping basically to sync the incoming signal with the receiver circuitry. And it is also bringing down the signal from the R f to the baseband.

The matched filter is basically selecting and then selecting the signal and maximizing the signal to noise ratio at the output of this filter provided the input signal is properly synced. And output of the matched filter will be now be sampled at the rate of the chip duration T c to provide the capital G number of the samples. Why? Because we saw in the transmitter that G was the processing gain and basically G was telling about the number of the chips available over the symbol duration, and G was given by T s by T c. And once actually the sampler is sampling the G number of, it is producing the G number of the samples, and if the chip clock rate is properly synchronized, it is properly synchronizing the sampler with the spreading sequence generator; spreading sequence is now helping you to dispread the demodulated sequence. And Z i represents the output of this dispread signal.

The accumulator here is now accumulating the number of the samples. It is accumulating the number G number of the samples that you have obtained over a symbol duration of T s; and decision-device is finally, taking the decision about the demodulated signal to identify what was the message information transmitted from the transmitter. We will revisit on the waveform to the part of the retransmitted of the transmitted signal as well as we will try to look into the detailed mathematical expression of all these points of this Z i, V. And we will try to identify finally, how this whole system is working today the work is mathematical analysis of this whole receiver architecture to have a detailed knowledge of this process - receiving process.

Remember this r t when we were receiving at the front end of the receiver that r t is now the combination of the intended signal transmitted. This is the spread signal which is transmitted s t is the combination of our x t into p t, where p t was our spreading waveform and x t was the modulated signal. And x t was and i t was, the now jamming signal or interfering signal that is getting added with the transmitted signal on air. N t signifies the additive white Gaussian noise and this is coming from the thermal noise component of the transmitter receiver front-end circuitry. So, the system model with which we will start our analysis will look like this the s p, the intended signal intended spread signal in presence of the jamming signal and the additive noise we are receiving. We will come back to this receiver block architecture whenever will be required in the subsequent slides.

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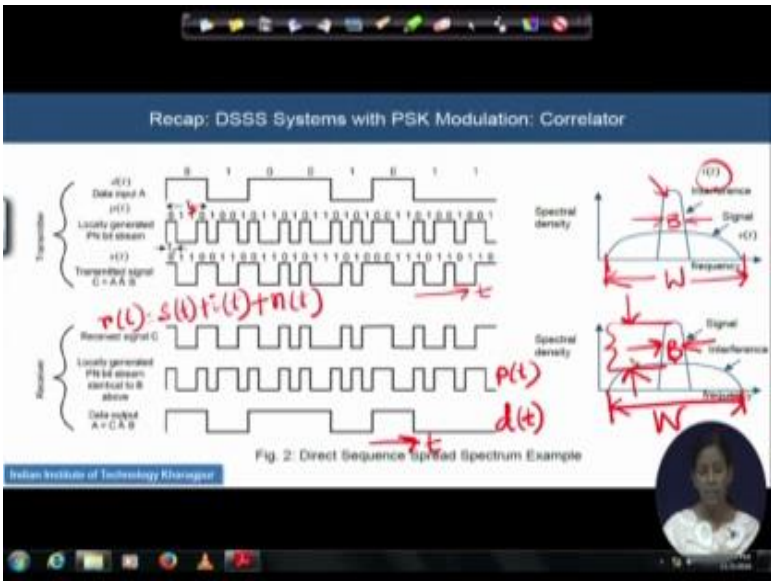

So, this the recap of the transmitted waveform, generated waveform and the spreading and the dispreading process. In the transmitter, we took the signal d t like this. Remember I have utilized for the 0 transmitted signal as a plus 1 and transmitted signal 1 will be denoted here as minus 1. The transmitted signal had the signal duration and symbol duration of T s and locally generated PN sequence bit stream was having a duration of chip duration of T c, the chip waveform is governed by the figure number d. And that spread signal was generated by the XOR operation of these two; and as we understand that in the XOR operation when the data signal value was 0, it was allowing the XOR operation will allow all the PN sequence chip; and if the data value is equal to 1, he will invert all the PN sequence chip. So, you go by that logic and you will be able to regenerate this spread signal. So, this is my d t, this is my p t, and this is the s t - the spread signal of transmitted from the transmitter.

This is the time diagram. If we look this is the time axis based on which you are getting, if I look into the frequency axis, so we after the spread signal we have got the spread waveform. This is my signal now s t with the spread spectrum signal, where the signal bandwidth is now coming W as we discussed earlier. And the interference now that is getting added on air we are considering it is a narrowband signal who is having some bandwidth is equal to B. So, this is the way now the relative spectrum of the signal as well as the interference with which we will be dealing with in the receiver. In the receiver after receiving the signal, this signal is s t here which is now basically coming to us as r t, where r t is equal to not only s t, s t plus my interfering signal i t getting added to it as I have seen. And with that actually the noise signal is also added that is shown in the previous figure in the block diagram.

So, let us consider that n t is negligible. So, basically you are receiving s t plus i t and which is shown here as a received signal. You have locally generated PN sequence V because we have apriori knowledge of the sequence with which you have spread in the transmitter. And once you are regenerating that signal and if we consider that they are exactly time synchronized, so we can by another XOR operation in the receiver, we can reconstruct our transmitted signal d t. So, final diagram here is reconstructed signal then as well as the dispreading signal, d t recovered from the received spread signal c in presence of the interfering signal.

If we try to see then this is again the time axis interpretation, if we try to see on the frequency axis. So, after multiplication of the received signal with my spread signal p t, spread signal p t we will because actually the dispreading operation the signal will come back and then the signal bandwidth will again will be compressed to the original bandwidth B. And now as the interfering signal was not spread earlier, it was newly added on the air and now because of this and inter this demodule dispreading operation uniformly is operated both on the s t as well as i t.

Now, i t multiplied with p t will expand the bandwidth of this i t, and hence the bandwidth of the interference now will be spread over the bandwidth of the p t, which is now W. And going by the gain bandwidth product constant whenever my bandwidth is expanding the power of that corresponding signal is coming down. So, now, you can see that the technique gives an inherent resilient with respect to the interference because power level of the dispread signal in the receiver is much much higher than the power maximum power level of the interference. So, this is the margin over which your signal is your receiver is operating your signal SNR signal power is this much higher compared to the noise power or the interference power. So, it will based on that you will have to detect your signal. So, this is the fundamental of the transmitter receiver architecture of your direct sequence spread spectrum system that we discussed in the last module here we have a recapped it.

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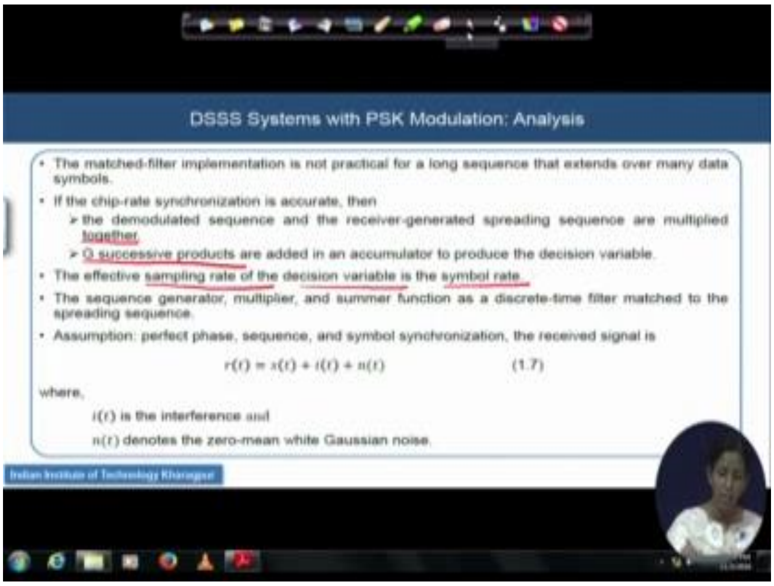

From now onwards if this was the concept, and if we look back the block diagram which was there in the first slide, we will be seeing that the demodulated sequence as I have explained here the demodulated sequence in the receiver-generated spreading sequence they are multiplying getting multiplied together. And G successive such products then we are adding up and considering that there is a perfect synchronization getting maintained. And remember the effective sampling rate of the decision device now it will be the symbol rate. So, there is a change in the sampling concept in this spread spectrum receiver.

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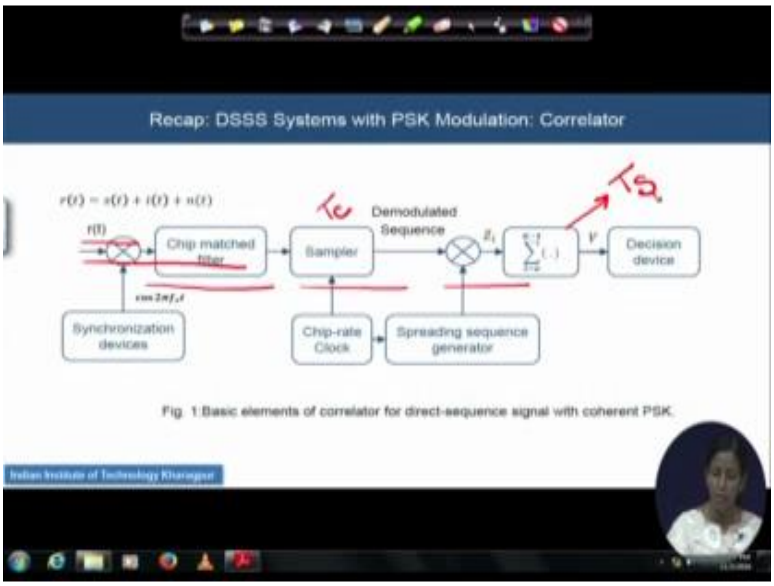

Remember that sampling here you are doing is at the rate of the chip because that is the way that timing diagram is of the received signal timing information is in the received signal. But once you have actually added all the samples, so you will be ending up with after the accumulated you will be having the time notation which is over the notation which is over the symbol duration t s. So, decision device will be working on the symbol duration where as the initial sampler will be working over the chip duration.

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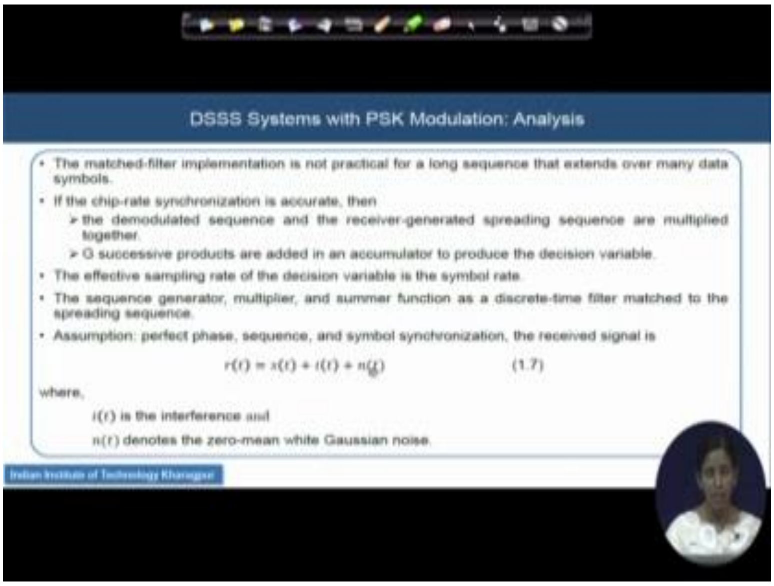

So, that was the case and we understand that all the sequence generator, the multiplier, the summer function they all can be implemented as the discrete time filter and matched to the spreading sequence. Remember one thing this matched filter implementation would be practical for a very long periodic very long kind of the spreading sequence because that will be extending over the multiple number of the data, very large number of the data symbols. If that is the situation then match filter implementation is not practical. However, we will come back to the mathematical model we started with the r t is equal to s t plus i t plus n t, which we now understand this integrated model we are receiving at the front-end of the receiver. We have modeled n t as a additive white Gaussian noise that is mentioned already.

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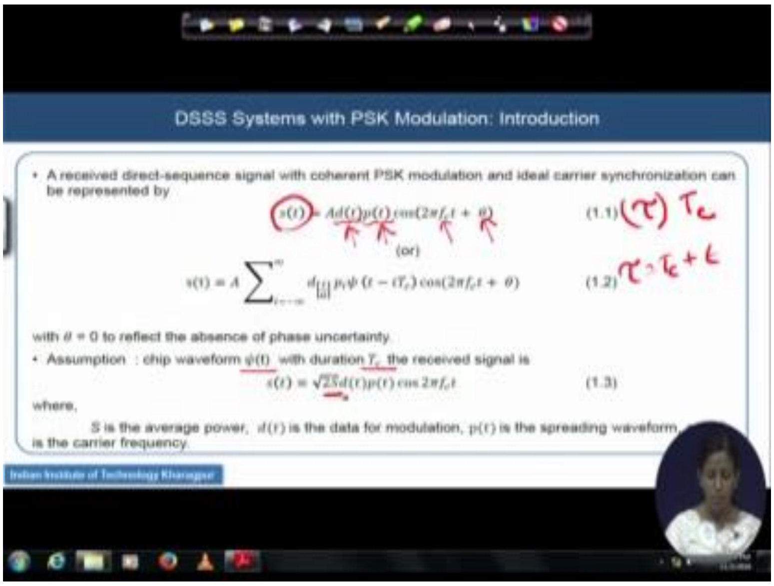

The s t was form of my a d t p t cos 2 pi f c t plus theta. We understand now f c is the transmission frequency, theta is the random phase associated with it, d t is the modulated signal, and p t is the spreading waveform. All that part we have also shown currently seen in the timing diagram as well as in the frequency diagram, what will be the spectrum of s t after this operation. Remember this equation 1.1 holds good if actually your delay tau is auto correlation delay that tau is exactly integer multiple of your if it is integer multiple of the T c that chip duration T c. If it is not that actually the tau is greater than actually it is not an integer multiple it is T c plus some exa if tau is something like that we have already seen in the first module. If it is T c plus exa then the transmitted signal module will look like this we had discussed it earlier.

And if we assume that this theta value is equal to 0. So, there is no phase uncertainty inside the received signal. So, we can omit the presence of theta in the received signal. And moreover if the chip waveform chi t with the duration is T c in the received signal is present, and which will be modeled later on as a rectangular waveform, we will come down to the equation 1.3 where theta is not present. See this A is now replaced with square root of 2 S, S is the average power of the received signal; and rest part we have already saw the symbols we have already discussed.

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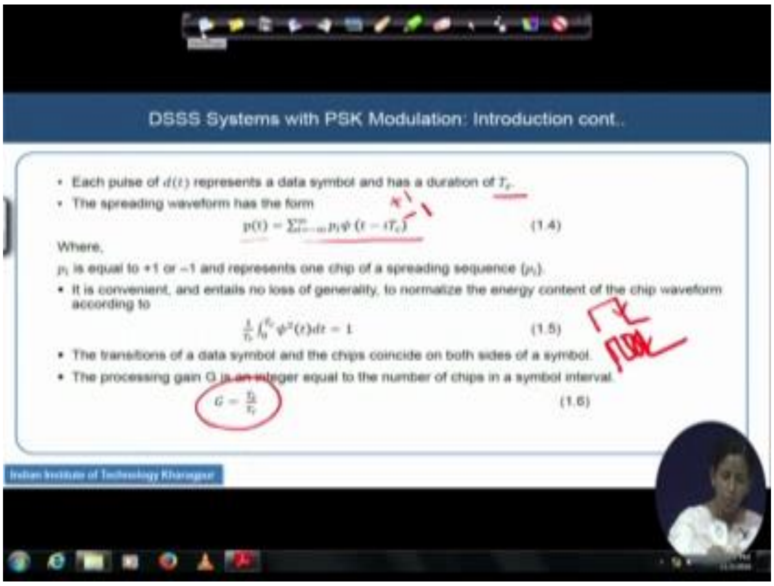

Now, this each pulse d t it is having data duration of T s that already we know. The spreading waveform p t is given by this that we have seen already. Eta is an infinite sequence coming, and chi t is the waveform; p i can be either equi-probability to get either plus 1 value or minus 1 value. And hence actually it is a part every p i that sum of all the p i's is constituted in the spreading sequence. Without any loss of generality of this discussion, we can consider that this chip waveform is having a normalized to unit energy. Such that if we integrate it over a T c, and normalize it over the duration of the chip, I will get the total energy equal to 1. And this transition of the symbols that we discussed that the transition of the symbol and transition of the chips, they are coinciding actually exactly in the same exactly in the same timing duration that means, they are in the sync for our discussion. The processing gain will be given for the spread spectrum communication is T s by T c.

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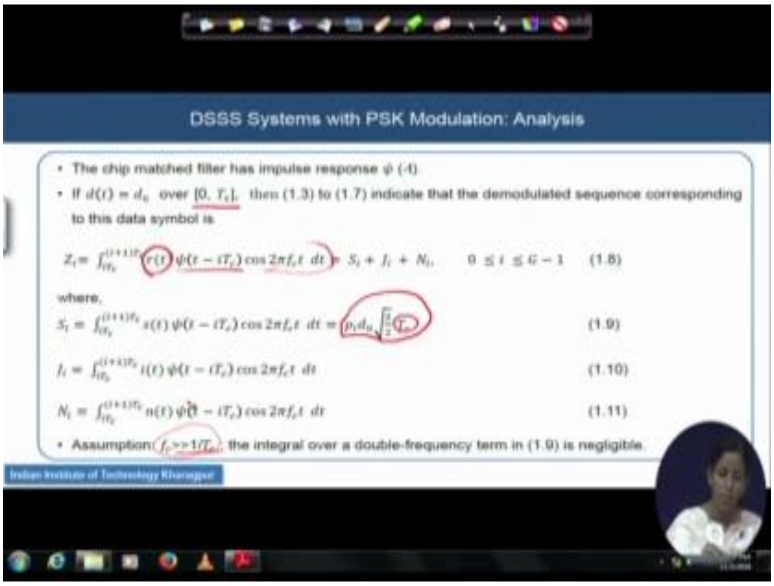

So, with all that consideration, let us start with the understanding that we have transmitted d t is equal to d 0. Let us think that the whole explanation will be continued now for the data value d t transmitted is equal to d 0 over the time of interest. Then remember that Z i is the location, Z i is data symbol that is coming after the demodulated sequence getting correspondingly multiplied by locally generated PN sequence, this is here. So, we have considered that p then we have thought inside as s t the condition of the d t is equal to d 0 over the 0 to 2 s, we have sampled it, we have multiplied with locally generated sequence and then we are here inside Z i.

So, Z i, the equation of Z i we can rewrite as this. We received the signal r t. Now, the locally generated PN sequence is multiplied with it. And it has a cos 2 pi f c t multiplication because you are having the cos 2 pi f c t synchronization at the frontend. And this introduction is we are interested to faster chalk out the received signal over the chip duration, a single chip duration. And remember whenever you are running such kind of integration there after Z i there is a accumulator. So, you are bound to derive the Z i's which were over 0 to G minus 1 number of the samples.

If i look inside the zoom inside the circle r t understand it has three components the intended signal component, the jamming component and the noise component. Whereas, the signal component is given by the s t, so each of these component will be now investigated separately. The value s t is will be multiplied with the locally generated PN sequence, it will generate the sample s i. And this is expression for the s i. Similarly, the jamming part when we will be multiplying with the local component PN sequence component, it will generate the sample value of the J i.

And similarly the n i for the noise part also we will get the n t getting multiplied with the spreading sequence, and then we are ending up with the value n i. Good part is if I see inside this s i, you see that this s t is now will be substituted by the summation of this p i and the d 0. And remember as we are interested only over the i th chi there is no summation coming up over all the p i's. So, it is only just the p i and multiplied with the value of the data who is considered here to be d 0. And there is a summation because we considered here to be the s to s is the total amplitude of this signal and that is coming out; and there is portion left inside the integration which is a cos 2 pi f c t d t that is getting multiplied with chi t.

Remember inside your s t also, this chi t is basically inside s t also there is a component of the chi t; and this 2 chi t will be multiplied and will be generating chi square t minus i T c, and we have considered that it is having unit energy in the last slide we have seen. So, you would not get any component left for the waveform. Psi we will be left with cos square 2 pi f c t. And this integration cos square 2 pi f c t if I am considering with then and again considering that this f c is having it is really too too high compared to the one by T c. Then the higher component the second order or the higher component value of the cos would not be good enough to contribute anything inside the component of the s i, we will neglect all those terms.

So, we will be ending up here with the cos square 2 pi f c t where by adjusting the two here. We will get inside the 2 cos square 2 pi f c t we will be further by trigonometric identity can be boils down to 1 plus cos 2 theta and that were cos 2 theta term we are neglecting because of this logic. And hence you will be ending up only with the integration of i T c to i plus 1 T c d t, and who will bring you down to the computation of to the value of T c. Finally, with all this logic we will be ending up with t i t 0 s by 2 into d c in the intended signal component. We now are interested to see what is there left how the component Z i and N i will be computed next.

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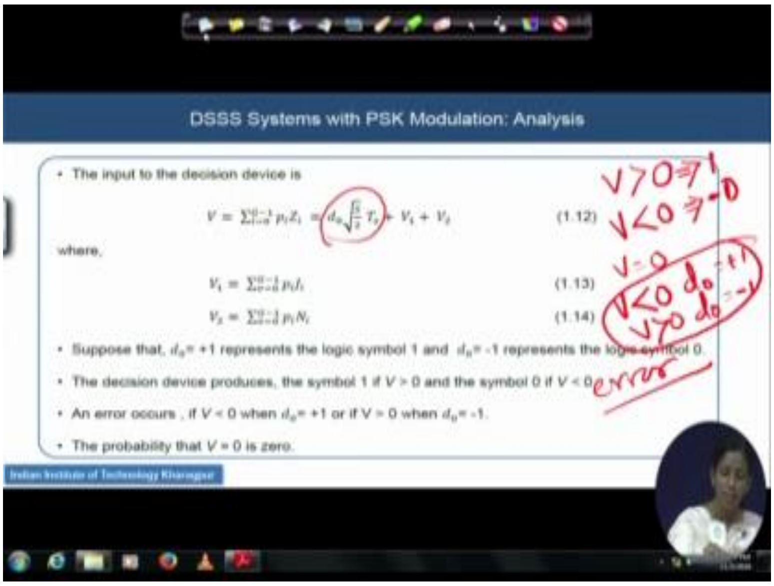

Remember the input to the decision device; decision device input is now having three components again like Z i. He is having the component from the intended signal that we have already calculated d 0 square root of s by 2 into T s, because now it is becoming T s because you are having addition over the G number of the samples. So, you will be ending up after accumulator you will be ending up over the sampled signal duration, there is no chip duration concept left and there are the components from the jammer and the jamming signal as well as from the noise signal.

From the jamming signal, although J i parts, all the J i's are getting added over and this p i into J i that is the section you are left with all the p i's they are corresponding J i's are getting multiplied and added over. Now, remember one thing that suppose that d 0 is equal to plus 1 represents the logic symbol 1, and d 0 equal to minus 1 represents the logic symbol 0, then the way the decision device works. If the signal value is 1 then the decision device will give you 0 greater than 0 and will tell that symbol is equal to 1 if the value of this capital V is greater than 0. So, if value of the V input to the decision device is greater than 0, the decision will be there that transmitted symbol was 1; and if the decision device input value V is less than 0 then the decision device will tell that the transmitted signal was transmitted as minus 1 basically it is 0.

So, if this is the situation then there is no chance that V can be exactly equal to 0. So, V value will be either greater than 0 or V value will be less than 0, because we have modeled with the transmitted signal as either plus 1 or minus 1. So, probability of getting V equal to 0 of no chance and when error occurs, error occurs in a situation when if V is less than equal to 0, when transmitted value was actually plus 1; and V is greater than 1 when transmitted value V 0 was exactly minus 1. So, whatever you transmitted, but because of the some problem inside the because of the high impairments from the jammer as well from the noise components, if it brings down the value of the V in such a way that it goes below 0 when actual data was transmitted one, then you will get an error in the decision process.

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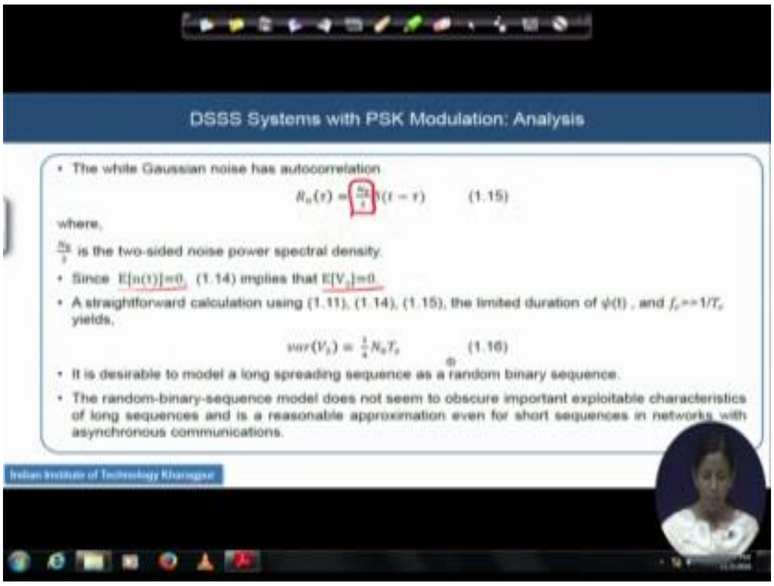

The white Gaussian noise that we have considered. So, we have to now think little bit about the variance part, the variance as well as the mean value of the V we are interested in because that is the part we understood that there is randomness in the jamming plus the noise section who will actually will greatly interfere in the decision process itself. So, we have to understand as it is a random process about the jamming plus noise it is a random process. So, it is statistical property, second order statistical property; that means, the variance will be very important part for our consideration. And which will from where actually we will take care of the fact that how much error probability we are expected to get in such kind of the scenario.

To start with we will think that we will see inside the white Gaussian noise process as it is a Gaussian noise, it will have the autocorrelation function given by this classical expression. And we understand that this N 0 by 2 is a two-sided power spectral density of the noise and eta 0 by as it is a white Gaussian noise, and it has a mean value is equal to 0. So, we see that the expected value of the V 2 is equal to 0. So, the straightforward calculation, we will be ending up with to compute to the variance of this guy how.

If we substitute the value of N i into my V 2 and then try to compute the square of it to compute the variance where this squaring of this N i basically will end up with the squaring of this N t which is nothing but the autocorrelation of N t with itself where we can substitute the value of this autocorrelation function and taking care of the fact that psi t if psi t is equal to rectangular function to us. And we will be ending up the variance computed as 1 by 4 N 0 into T s. Why this T s because we are at the input of the decision device where your psi t will be the p t inside will be substituted by the addition of all the points over 0 to G capital number of capital G or the number of the sample points.

And this N 0 by 2 is the contribution from the autocorrelation function, another half is coming from the fact that we have to take a 2 cos square theta inside the integration and the higher order terms again will be neglected and that half will come as a additional part multiplied with this N 0 by 2 leading to the 1 by 4 N 0, T s is coming because there is addition over the accumulator over the number of the samples equal to capital G.

And see it is desirable that this we will try to do the modeling over the long spreading sequence as a random binary sequence. And this random binary sequence model does not seem to be obscure the important exploitable characteristics of the long sequence. And it is reasonable approximation even for the short sequence also in the networks with this asynchronous communication. And then with this hold good this computation of this V 2 its variation its variance there is no effect of this because they are the jammers and the noise they are the completely independent and identically distributed two quantities we are dealing with.

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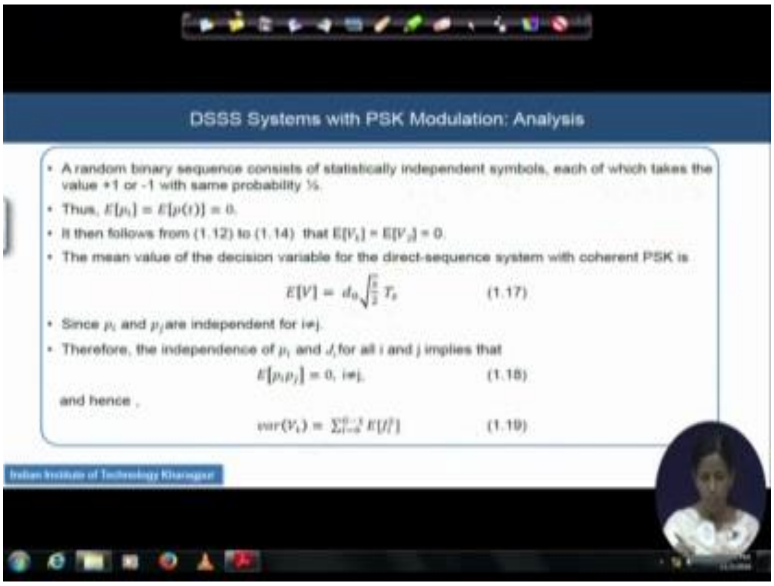

Now, we have to look inside the another part left where actually V 1 is there; V 1 is the jamming section, but in order to understand the jamming contribution the power contributed from the jamming section, we have to understand that the jamming is not now alone standing here. In the receiver the jammer the jamming signal is already got spread by the spreading sequence it in. So, it is whole spectrum it is spectrum is getting dominated by the spreading signal.

The property of this p t now we have to look into. We understand that this random binary if we utilize the random binary sequence then it has a statistical property we have already discussed that this will have a statistical property such that it can take a plus 1 value or minus 1 value with equal probabilities equal to half. Thus your mean value of this p i which is equal to also p t, it will be equal to 0. And then we come to the value that is the mean value is equal to 0 and then the mean value of V 1 and mean value of V 2 in V, in V we had the component from the signal then the value V 1 and V 2. We already seen that the mean value of the V 2 is equal to 0. Now, we have proved that if you use a random binary sequence it will lead the mean value of V 1 is also 0. So, both are equal to 0, so the mean value of V you will be ending up with is the expression d 0 square root of s by 2 T s.

Now, if I look into the variance computation as i and j are independent to each other, because each and every random because of the randomness involved in this binary sequence in the PN sequence, your i th and j th component of that sequence they are independent to each other. And hence we can actually also see that p i into p j that mean value into p i and p j that will be also 0 and the variance will be straightforward computed by the individual component J i square summed over G number of the samples.

So, finally, the variance of the V will be sum of the variance you were computing here plus the variance of the noise that you have we have computed in the earlier slide here. So, they are the two variances that we will be finally important for us to compute the signal to interference noise power for the decision device, which will largely actually control the decision of the decision device. We will see in the next module, how actually these powers corresponding powers are playing the role in the decision device.

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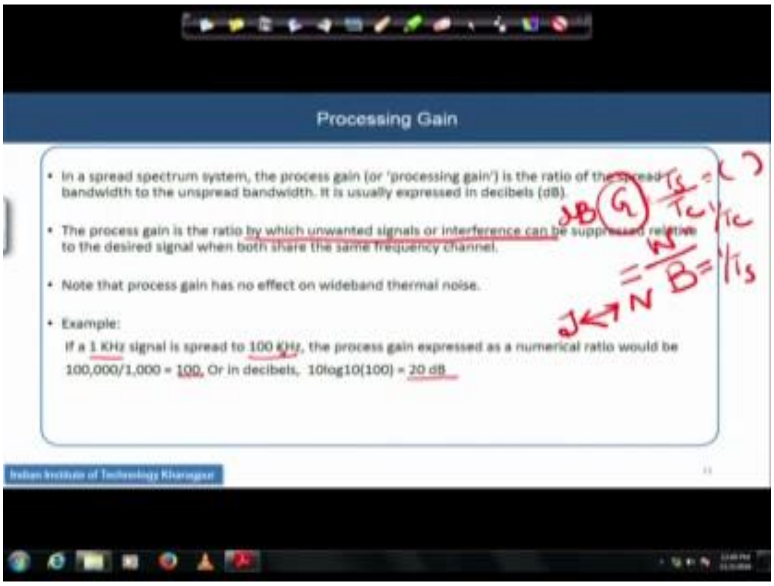

Before ending this talk, we would like to highlight little bit and reframe our understanding of the processing gain that we are continuously talking about in the direct sequence spread spectrum system. Remember that this processing gain was declared as a ratio between our ratio between the transmitted symbol duration by the chip duration which is also given by the spread signal bandwidth W by the original signal bandwidth B. Where W was 1 by the chip duration, and the signal bandwidth was one by the symbol duration. Usually we express this processing gain in terms of the decibels or dB. This processing gain ratio I mean the dB value is fundamentally it tells that by which amount the unwanted signals or the interference signals can be suppressed relative to the signal relative to the signal itself.

So, if this factor always this factor will have some integer values and that factor the value we are computing here directly tells by this amount the interference can be suppressed with respect to the signal. When both of them will share the same frequency channel and the same frequency channel for the transmission. And remember this processing gain has no effect on the wideband thermal noise. So, the jamming process as I was mentioning in the earlier slide, the jamming process and the noise process they are completely independent and completely independent to each other.

For example, here is a short example is given here for your understanding if I am having a 1 kilo hertz signal and I spread it to 100 kilo hertz then the simple processing gain will be equal to 100 or in the terms of decibel it is 20 dB. So, its computation of this processing gain has no effect, it is never actually getting influenced by the competition of any noise power.