Spread Spectrum Communications and Jamming

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Lecture - 21
Despreading with Matched Filter

Hello students. Today our topic of discussion will be Despreading Mechanism using the Matched Filter. We have already seen there is your architecture for direct sequence spread spectrum communication system. We have also discussed that in the receiver synchronization is a very vital component needs to be performed before despreading. If the synchronization is imperfect I mean the carrier as well as the code synchronization. So, the direct faithful retrieval of the number of chip pulse symbol will not be visible. And it will directly have hampered the snr gain that is expected at the output of the despreading process as well as data detection.

Now remember that for the different kind of the length of target, length of the spreading sequences that we are using for spreading in the transmitter the different time period is involved and different hence different acquisition system as well as the tracking system for code synchronization is there in the receiver. So, there is a time involved by processing involved a prior to the despreading in the receiver in the synchronization block. Now the question is there any solution or is there any relation of the choice of the wood length, such that inside the receiver the despreading can be done easily and code acquisition also code synchronization can be automatically achieved.

The answer is yes. If you are using a pretty short length code then using the matched filter architecture, we can have a despreading operation which involves the automatic code synchronization. So, today our topic will be to realize the matched filter architecture such matched filter architecture for our low length or short length code sequences, and try to see how much amount of the loss will be happening at the end whether it is a negligible. So, that this architecture can be practically implementable or it is unacceptable, such that we have to go back to the traditional architecture which will have a dedicated synchronization law, followed by the quadrilateral based architecture for despreading.

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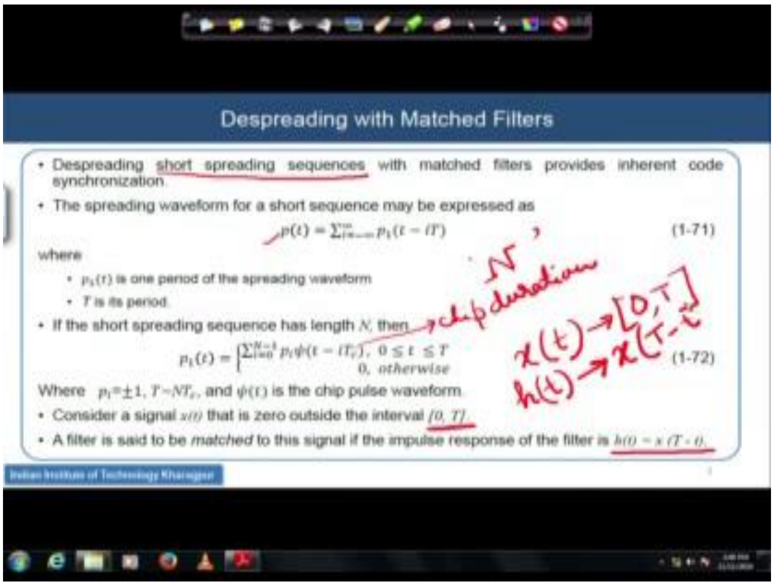

So, as I already mentioned, this despreading with the matched filter will be specially designed for the short spreading sequences and the waveform as usual. So, for long length sequences or the matched filter architecture would not work at all. Let us start with the consideration of a short length sequence, then p t is that short length sequence for us, given by this known architecture already we have discussed several time.

T minus I into T and capital T. Capital T is the period of this sequence and p i p t is a p 1 t is the one period. It is just 1 period of this spreading waveform. So, now, if the short sequence is having say capital n number of the chips involved within that short length or it is length itself is equally equal to capital n, then this upper limit does not go up to infinity, and you will get 0 to capital n minus 1 number of the chips involved in the spreading sequence. So, p 1 t will be given by summation i is equal to 0 to capital n minus 1, p 1 psi t minus i into T c, that T c is our chip duration.

So, I have short sequence whose length is equal to capital N. The duration of each chip is equal to T c and the expression of p 1 t is given like this. And p 1 t is a 1, 1 typical period of this short length sequence. Remember there is no existence of this sequence beyond the zone of 0 2 capital T. Now consider a signal who is and as I told you earlier, that is p i will be plus 1 minus 1 like the earlier cases that we have considered. Now let us see there is a signal x t, this signal is time bounded within the zone of 0 to capital T. Going by the matched filter fundamentals, a filter will be matched with this signal x t, if and only if the filters transfer function or impulse response is equal to x capital T minus t.

So, I am having a signal x t who is time bounded within the time duration of 0 to capital T. The definition of the matched filter says the any filter they will be called matched to this signal x t if and only if I am having this expression x capital T minus t. We have discussed I hope you have go through the digital communication classes and the analog communication classes the design of this matched filter.

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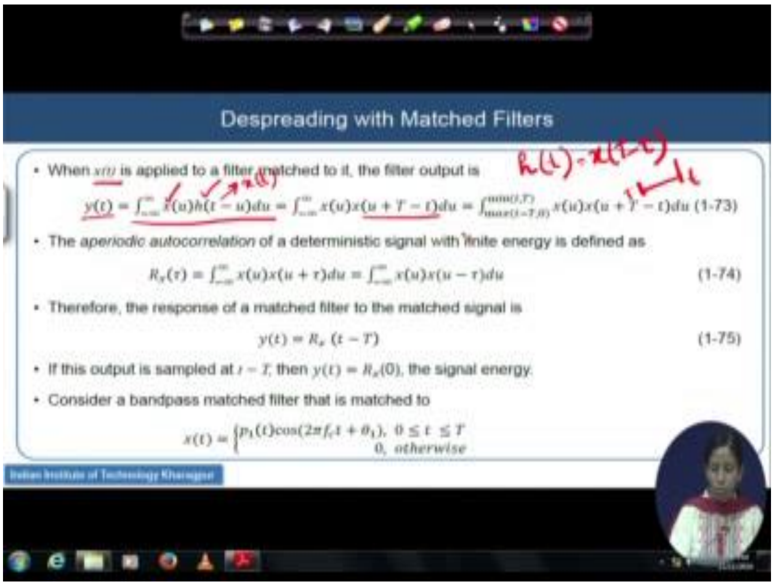

We discussed there and this property is a very famous property; fundamental property of matched filter that we start within the matched filter design. And for the normal con conventional communication system design also. Now if I apply so, now, what I said I have a signal x t, and I have a filter to transfer impulses from this x t. If I apply this x t now on this filter h t, the output is h to be same y t, then output will be given by this I mean this h u is equivalently to the it is a typical instance value of the x t. Then x t minus c u and this is integrated over d u, and the integration is run over minus infinity to plus infinity.

So, this is a very standard equation written as the output of the natural deterministic signal is path 2 through the filter. So, it will be simply all the coefficients of the filter will give will be the will be multiplied with the delayed version of that will be multiplied with the signal and it will be integrated over a point of our interest. And if we are writing we are substituting the value of we understand the value of x t, there is the relation between x t minus u to your x t. So, if you wish to replace x t in terms of h x x t, then it will be like u plus capital T minus t; obviously, going by the expression that x t is equal to your x capital T minus t.

Now as I understand also that the lay the pulse itself and the we are having an interest over the duration of 0 2 capital T. So, either you can start with t minus capital T this is the instant 2 t, or you can start with the 0 to capital T whatever it is. So, maximum of capital T minus t or 0 whichever is a maximum value, from there with the minimum of t or capital T over these actually this interrogation is of our interest. And hence this expression will be integrated over that interval, because that is the interval over which actually the filter is having effect on the incoming signal.

Now, the aperiodic signal, think of a aperiodic signal and it is aperiodic autocorrelation function. From the classical definition of the autocorrelation function R x tau of an aperiodic deterministic signal, is given as x u into x u plus tau d u. And when this is there, you can also be it can be also being written as x u x u minus because it is true for both the delay positive delay and the negative delay. And if I try to now design matched filter the response of this matched filter who will be matched with this signal now, will be given as R x t minus capital T.

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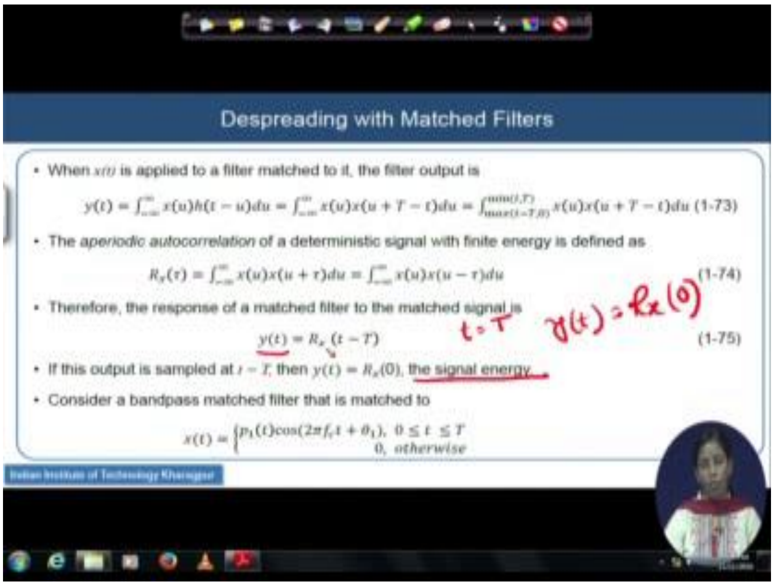

So, that response of the matched filter output of the matched filter basically then it will be given matched to the signal will be given as R x t minus tau. Now if I put this small t is equal to capital T, then it will give you the output will be matched filter output will be just equal to your autocorrelation function of 0 which is nothing, but the energy of the signal. So, at this moment when actually the matched filter is design such that, such that actually their filter output it is a response of the matched filter will be given by the aperiodic autocorrelation function and if it is properly sampled at small t is equal to the capital T. So it will; you are going to get the peak of it and the energy of the signal.

These are common basic understanding and let us now consider band pass matched filter. So, the band pass matched filter is, let us assume that the band pass matched filter is a matched to our signal, band pass signal. X t is a band pass signal, and it is given by p 1 t cos 2 pi f c t plus theta 1. F c is our transmitting frequency p 1 t is short length sequence with which actually which is spread usually by which is multiplied with the rf frequency of f c. Theta 1 is the random phase associated with it and this is our transmitter signal and the signal of our interest. Now this f c, remember this is a desired carrier frequency.

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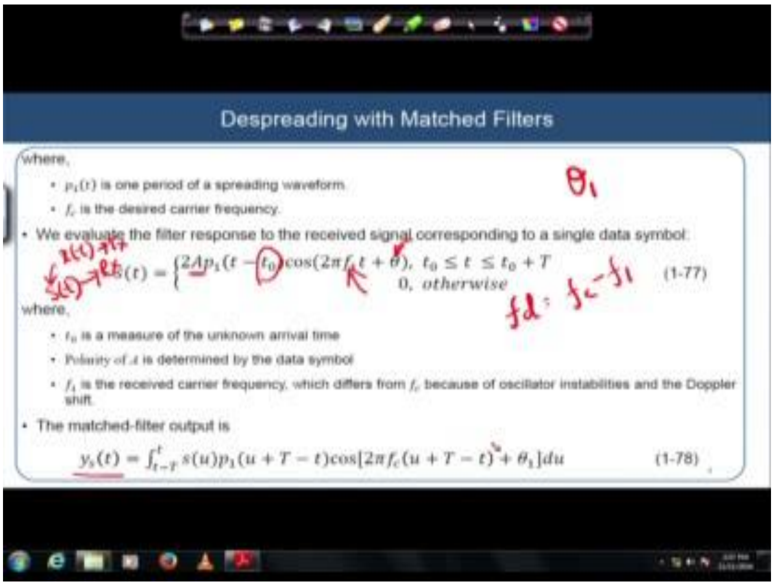

Now, we are interested to see what are we going to receive and what will be the filter response in the receiver. In the receiver corresponding to x t was our transmitted signal from the transmitter. And the received signal corresponding to x t after passing through the tunnel, you are getting a receiver response equal to R x. So, now, p x to R x if you are t x to R x is the transformation x t to s t, in the x t we are going to get to this response where your f 1 t is not exactly equal to f c. So, there is a deviation from f c, but why it is something new. Because there is a Doppler shift associated always in that transmission, and also there is a oscillator instabilities between the transmitter and the receiver, even if you purchase 2 different oscillate same neck oscillator from the market and from the same batch even.

Even then you cannot actually guarantee that the frequency stability, frequency operate it genetic frequency by those 2 oscillators will be exactly same. So, there will be always a deviation that why actually in the oscillator deter manual we mention the range of operation or it is tolerance limit. And because of the mismatch of the frequency of between the 2 risk 2 oscillators siting once in the transmitter and other is there in the receiver, we will get a difference. So, the; obviously, the transmitter frequency is never expected to be exactly equal with the receipt frequency and recipients. So, that is why with the receipt frequency we have mentioned here as f 1. You may also think that f d that is the difference frequency who will be indicating this difference; I mean either f c minus f 1 or f 1 minus f c whatever whoever is actually higher than that.

What is this t 0? T 0 is a unknown arrival time. So, transmitter receiver the moment that you are transmitting that can ever be coincided in the receiver received signal. Because there is a transmission delay involve always in any kind of the transmission to incorporate that there is a delay from the transmitted movement that delay we have entered here as a terms of t 0. Capital A is signifying the polarity of the data symbol. If that data symbol is plus 1 the value of capital A will be positive like that it is. So, this is a finally, this is a structure of the received signal.

Remember the phase associated with the received signal also is having some different notation called theta. In x t there was theta 1. So, the random phase associated with the transmitter as well as the received signal they are also going to be different. Now matched filter output y s t, the matched filter output is always now if this is my signal and I understand; what was this transmitter’s signal. And the matched filter output following the earlier slides expression you will be able to see that this is nothing, but my s multiplied by the locally generated that pulse and then cos 2 pi f c u plus capital T minus t plus theta 1 into d u.

This is a matched filter output. Where from we got? We got it from the matched filter expression that we have derived in the last slide. So, now, we are ready to substitute the value of this s from the expression 1.77, because this is our received signal. This is a very general one any kind of that received signal. And remember as we are always interested over duration of capital T hence this interrogation is running from t minus capital T to capital T. And rest is already you know. So, in the next slide we will see out what is the sub further derivation on this matched filter output.

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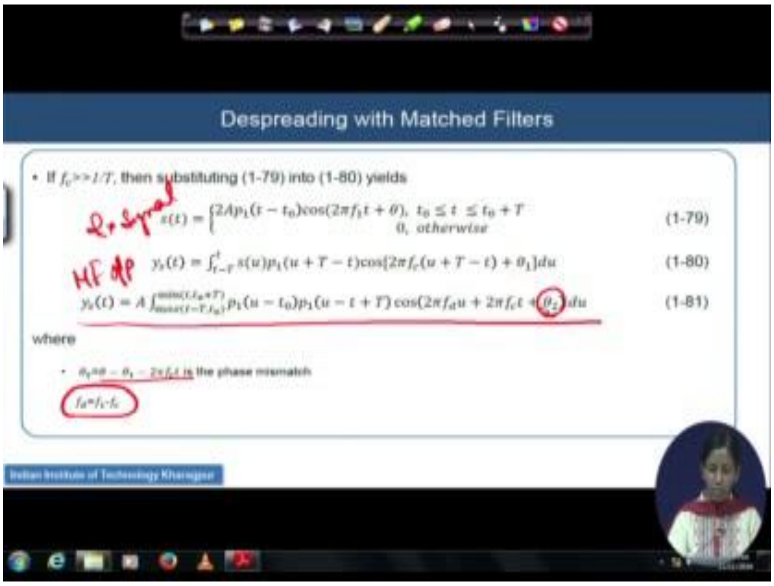

Remember if you consider that the transmission frequency is high very high compare to the chip rate that is the common consideration for all the derivation that we are doing during this session. And if it is to really too much high the transmission frequency I mean the rf frequency is really very high compared to the chip rate. Then I told that this is our received signal and this was a matched filter output. So, I can now substitute this s t instead of here in the s u. If I try to do that and if I go back and once again we see the expression of this integration boundary, the lower to upper written as the mini max to min. Then you will be finally, ending up with this. Because we have actually now one received signal that was that was that came actually the sequence came from the transmitted one, basically transmitted sent it and this is a locally generated one and finally, will be ending up with this equation where actually we have got a new term called theta 2 in a phase term.

This phase term is nothing, but the substitution of theta minus theta 1, minus 2 pi f c t. This is basically stating the phase mismatch. Phase mismatch between the transmitted as well as the received filtered output. And f d I told already that f d will be either f 1 minus f c or f c minus f 1, whoever is high whoever is greater. So, coming here this is the final matched filter output where we are concentrating in. And we can see here that it is not only a center frequency f c, we are having 2 other components to be considered here. One is a different frequency component another is the phase associated with it.

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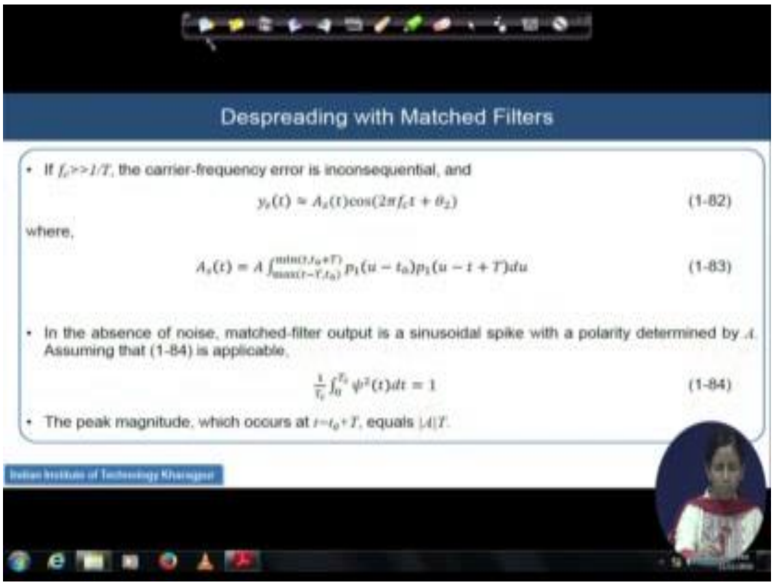

Now, see one thing if my f c is too much greater than one by capital T the chip rate the center frequency transmission is too high than the chip rates. Then look at this expression this is a whole expression can be approximated by A s t into cos 2 pi f c t plus theta 2. So, this approximation holds good before certain limits that we will discuss little bit later. And to hold this good inside it we have substituted value of A s t which is fundamentally the A I am at this is actually the integration of the 2 pulses that was once we have transmitted. And once there is another one which is locally generated inside the receiver over the integration interval of our interest. Then this A s t multiplied by cos 2 pi f c t plus theta 2, this guy will be this will be approximated with the matched filter output.

Now, remember in the absence of the noise and any other interference to the matched filter input and to the receiver input I should say, and then the matched filter output will give you some sinusoidal peak values, sinusoidal spikes rather to say. And whose polarity will be governed by this capital A. And remember if we assume that the energy is of that of the signal I mean energy of the spreading sequence it is can be non-normalized to be 1, then the peak magnitude that you are finally, getting that we will be ending up with at t is equal to t 0 plus t means at the end of the duration. You will be ending up with mode a into capital T. So, it is finally, governed by the peak of the sinusoidal peak multiplied by the time duration, it is the peak magnitude that we are ending up with and that is your coming from the equation 1 hyphen x t 2, following the consideration here and followed by the explanation given here.

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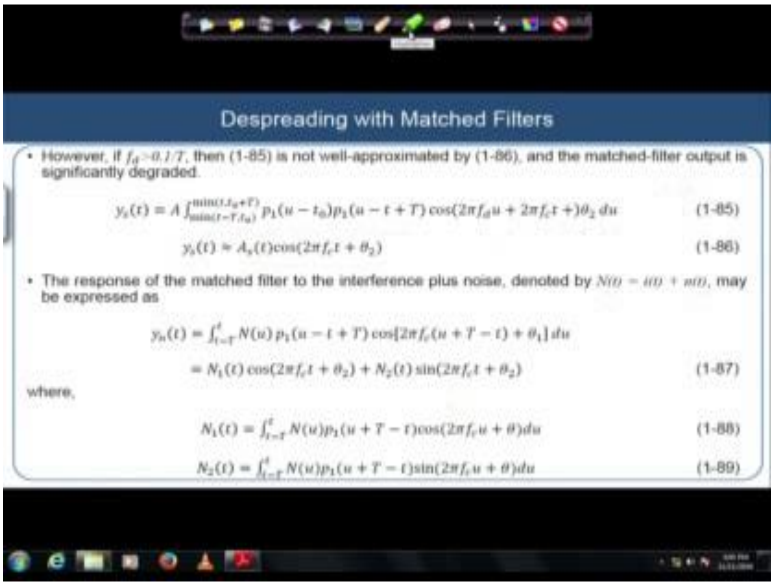

Now, remember that I told that in the last slide that the consideration that y s t will be approximated with this expression A s t into cos 2 pi f c t plus theta tau, it will be done it is really true, if and only if my difference frequency f d which was the difference frequency, difference frequency was received carrier frequency minus the transmitted carrier frequency, the gap between the received transmission frequency and the transmitted carrier frequency. If this one is less than 0.1 by the 0.1 by 0.1 by t, but if it is greater than that I mean if f d is greater than your 0.1 by capital T. So, I mean it is 10 percent of it I mean 0.1 by capital T then 0.1 percent of the frequency at which actually we are considering the signal, in our signal of our interest. Then actually this bigger expression which we got at the output of the matched filter, they can never be approximated by this expression, which we were discussed in the last week, last slide.

In this situation response of the matched filter to the interference plus noise which is denoted as n t plus I t into n t, see this is actually giving ask the indication how matched filter output will be for the demanded signal. What will be the matched filtered output for the interference section as well as the noise section?

Because of the total signal few interests plus the noise plus the interference it is passing through the matched filter. So, for this section, we let us revisit the matched filter output and in this situation. We have mentioned it as y n t and if I substitute here that n u is the combination of that i t plus n t. So, fundamental if I substitute the value for each of them and it is try to expand, I can simply separate them out into 2 halves. And this 2 halves are n 1 t into n 2 t finally, can be written as this expression as 1.88 and 1.89. Remember they are not separated out in terms of the combination do turns of the contribution from i t and n t. It is not like that. It is actually the cosine component and the sine component involved which is coming by the trigonometric identity after expansion and during some application of the trigonometric identity on the top of the bigger expression.

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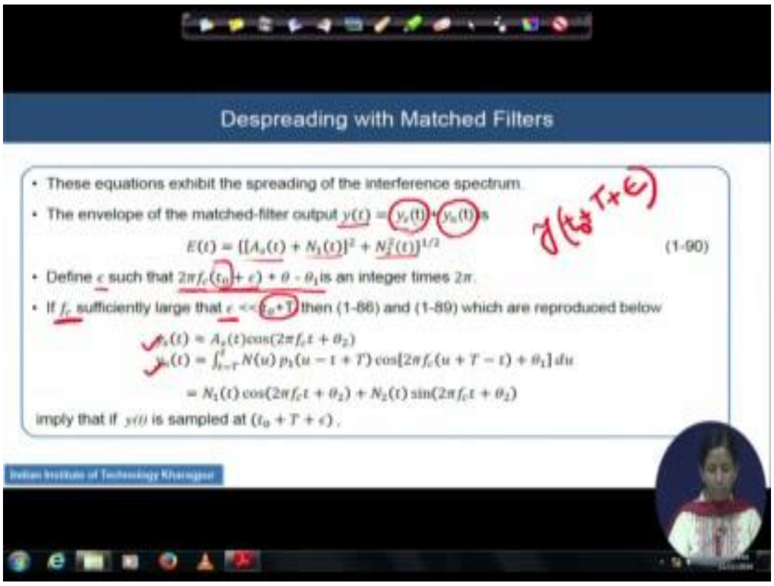

Now, that point to be discussed here is this equation. What we have seen in the last slide here, see remember that we started with n t is equal to i t plus n t that got added, but the matched filter output I am seeing actually they are getting multiplied. They are getting spread basically by means of this locally generated short sequence. So, now, the situation is as if the whole as it is, the whole process is becoming equivalent traditional or conventional blocks where we separate out. The spreading sequence, the synchronization block and the matched filter block separately we separate them out. So, these 2 equation fundamentally for the noise. The concept that the noise portion will be spread in the receiver both the noise and interference will be spread in the receiver because of the despreading process, that holds good in the matched filter architecture also.

Now, then I know the expression for the intendant signal section. I know I am now ready with the noise plus interference section contribution from there so the total envelope at the output of the matched filter. Output y t should be governed by the both of them. And if I try to find out the envelope of it; so he what it will give it? Will give the all the meaningful components the A s t plus n 1 t square as well as the sine component governed by the into square. So, it is the cos, it is a meaningful energy as well as a orthogonal energy, on the orthogonal part on the orthogonal side of it.

So, it is the combination of the both and the square root of it will be giving you the envelope of the matched filter output. Now let us do some small experiment. Let us define epsilon is a quantity. Epsilon is such that this 2 pi f c, I mean the delay plus the tau 0 delay plus this epsilon, plus this theta minus theta 1, will be getting controlled in such a fashion that it will be always the integer time of 2 pi. So, this whole block I mean 2 pi f c t plus this all the whole portion, it will be always the integer time software 2 pi. In such a situation if your carrier trans frequency is sufficiently large such that this epsilon will be heavily low, I mean too low than the delay plus the total time of our interest, then this earlier derived expression of y s and y n, and this 2 if we imply that this expression which are reproduced below, implies that if y t is sampled at t 0 plus capital T plus epsilon.

Then we will get the equation that is shown in the next slide. So now, t is getting sampled at the delay some capital T the duration of the symbol and as well as the epsilon. What is this epsilon? Epsilon we have zest control in such a way that always this full term will be the integer times of the 2 pi.

So, now this plus this we will be the combined effect will be now sampled at the rate of t 0 plus capital T plus epsilon.

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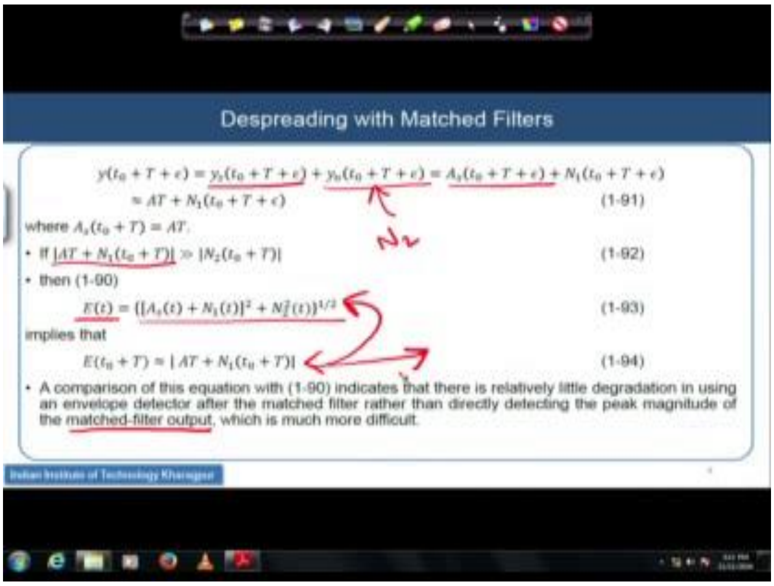

Let us see; how does it go in the next slide. So, the basically these sampling on the t 0 plus capital T plus epsilon will finally, give you the signal component at that instance and the noise component as that instance. And basically we understand that this, from here, we boil down to the point where A s plus n 1 is approximately there. And this term is actually the point to coming from into section, and that can be neglected because such contributing will do some the meaningful energy, and this 2 again can be approximated the mainly the first term can be approximated with A into capital T because this the one we have already seen earlier. There is A s t 0 plus t will be approximated as A T ideally because into considering that the t 0 is negligible, and no and again this section will be have some importance here. Because this is actually the total effect of on the detector output, on the envelope detector output we will see little bit later, because this is a whole part that is contribute into the envelope detector output.

Now, situation is such that from this section we will get a term of N 2. And if now I think if I have a situation where a t plus N 1 t 0 plus tau, if this guy is becoming too high than the N 2 section which is N 2 into t 0 plus capital T. Then this expression which is already derived earlier envelope of the signal envelope detector signal which we derived, that should be should be looking like that. It implies that at t 0 small t equal to t 0 plus t it will be approximately written by this term. So, this guy can be neglected completely.

And if I compare now this 2 then what we will get it is and also we bring the consideration of 1.90. So, this 2 comparison will tell that there is a little degradation definitely. And the output in using the envelope detector output, after the matched filter. Then directly if you try to detect the peak magnitudes of the matched filter output which is more difficult also to implement in the practice. So the conclusion is that matched filter followed by an envelope detector is a very good situation to afford your choice for the practical implementation.

Because it shows that there will be a very little degradation in the envelope output, compare to the situation where you are only using a matched filter and you are trying to pick up the peak of the matched filter output. And it is implementation wise also matched filter combined with the envelope detector is much easier than the matched filter alone detecting the peak.

But finally, it is proved that dispreading with a matched filter output is heavily possible heavily possible for the small spreading sequence. There is no need to put another synchronization block.