Spread Spectrum Communications and Jamming
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Lecture – 20
Performance Analysis of a Quaternary System

Hello students. Today our topic of the discussion will be the continuation of the performance analysis for spread spectrum communications systems. But today we will not considered the derive spread spectrum system diagram that we were considering for the last three modules, rather we will shift little bit the receiver, transmitter receiver architecture and the modulation technique. And let us consider today a quaternary system. What is the quaternary system and how the performance of a quaternary system will be changed with respect to the system module considered? And will be our consideration and we will see today.

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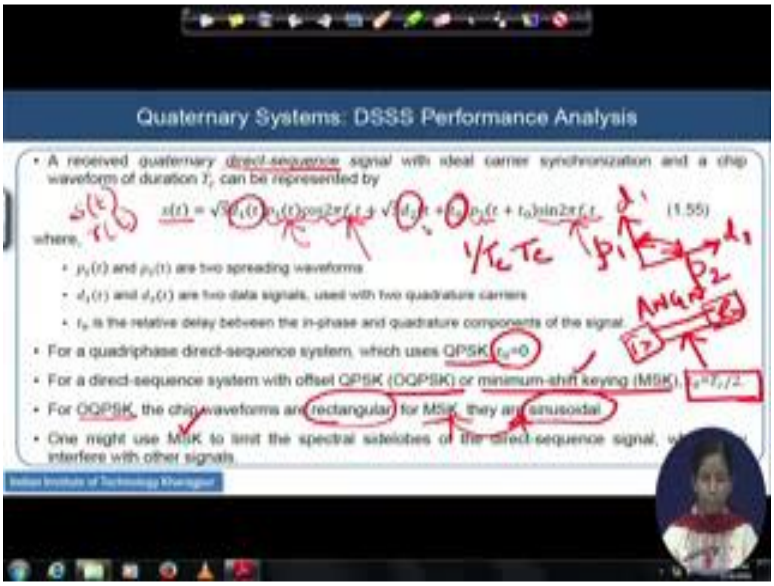

Quaternary Systems: DSSS Performance Analysis

- A received quaternary direct-sequence signal with ideal carrier synchronization and a chip waveform of duration T_c can be represented by

$$s(t) = \sqrt{S}\,d_1(t)\,p_1(t)\cos 2\pi f_c t + \sqrt{S}\,d_2(t+t_0)\,p_2(t+t_0)\sin 2\pi f_c t \quad (1.55)$$

where,

  - $p_1(t)$ and $p_2(t)$ are two spreading waveforms
  - $d_1(t)$ and $d_2(t)$ are two data signals, used with two quadrature carriers
  - t_0 is the relative delay between the in-phase and quadrature components of the signal
- For a quadriphase direct-sequence system, which uses QPSK, $t_0 = 0$
- For a direct-sequence system with offset QPSK (OQPSK) or minimum-shift keying (MSK), $t_0 = T_c/2$
- For OQPSK, the chip waveforms are rectangular for MSK, they are sinusoidal
- One might use MSK to limit the spectral sidelobes of the direct-sequence signal, w
interfere with other signals

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Remember received signal of a Quaternary Direct Sequence Spread Spectrum Signal receive quaternary direct sequence spread spectrum signal, if we considered there is a perfect carrier synchronization availed and then chip waveform duration is considered to be T c. I mean exact code synchronization has happened exact acquisition may be happening. Core synchronization as well has carrier synchronization is done kind of. In such a situation that received signal, which is also equivalent to mine transmitted signal because wireless channel is not considered in between. When we are writing this equation that will be given by this expression.

Remember when we talk about a quaternary signal; the situation is such that we will have a two different carrier signal which, are orthogonal to each other and hence they are device from the orthonormal basis functions, basically involved in this equation. In the quaternary system itself and we have also two different symbols transmitted over these two orthogonal basis function.

And that within that actually once we are using the direct sequenced spreading. So, the spreading signal that is involved to spread the symbol d 1 is p 1 and the sequence that is utilized to spread the signal d 2. Suppose it is p 2. So, the situation is such that; we have two orthogonal carriers to transmit the signal d 1 and d 2. And in each of these carriers the spreading sequence involved to spread the data signal d 1 and d 2 are correspondingly; let us considered to be p 1 and p 2.

We have loaded the power in such a way in the both the orthogonal direction in such a way that the total power, they are equally shared. The power of the both the signals are equally shared and given by the square root of the S. And remember what is t 0, t 0 will be the relative delay the between the signal received on the same cos axis or the one of the carrier with respect to that actually what is a delay, effective delay available on the other carrier? The signal received in one carrier and the signal received over the other carrier, if we consider that there is a relative delay between these two. So, that delay will be symbolized in this derivation as t 0.

So, I repeat in this quaternary system, we have two carriers; simultaneously carrying two different data symbols. The data symbols are mentioned here symbolized here has d 1 and d 2. Remember both of the signals are having dedicated sequence p n sequence or any kind of the code sequence that is of your choice to spread the signal and there is a time delay also considered in between the signal received over one carrier to the signal received over the other carrier and that delay we have modeled to be equal to t 0.

So, the received signal and power that we have actually sent both are having the equal power before transmission. And combining all the effects so we are here given by equation 1.55 where, the receive signal s t is equal to given by this. Why am I saying that receive signal is s t? Because usually we mention the transmitted signal as s t and the receive signal has r t. In the situation s t and r t are exactly same; considering the fact that you do not have any wireless channel. Suppose in between, you have in between the transmitter and the receiver as if you have AWGN channel (Refer Time: 05:37) Gaussian kind of the noise channel which will add Gaussian noise, but it would not actually give any other kind of the effect of an wireless channels like the dispersion or the change in a phase frequency of the signal.

So, that is not expected to happen and. So, there is no attenuation also the signal components will be effect will be visualized on the received signal. So, no effect of any other kind of wireless or any kind of the guided channel is considered and we are thinking that the direct signal is received. An even in 1.55, while evaluating the expression for 1.55; we have not considered even the effect of the noise added to it. So, it is purely the signal generated in the transmitter and received to at the fondant of the receiver in absence of any kind of the channel, in absence of any kind of the noise.

And in our consideration when we have considered that the p 1 and p 2 are the two different spreading sequences. Remember the sequences are different, but the chip duration for both the sequences are equal to T c and the rate at which the receiver is sampling the signal is hence 1 by T c. So, with this consideration let us understand little bit more about the kind of the modulation schemes that we are going to utilize here. If we utilize the QPSK: Quadrature Phase Shift Keying for this modulation of this d 1 and d 2. Remember in that situation you are delay between these two will be exactly 0, but if you utilize the offset QPSK or you utilize the minimum shift keying MSK, that is a very popular scheme for the frequency (Refer Time: 07:22) spread spectrum communications then this value of the t 0 will be equal to T c by 2.

So, for QPSK it is 0, but for MSK it is T c by 2 and orthogonal QPSK also it is T c by 2. So, we would like to choose QPSK communication over others but remember the chip waveforms usually that we refer to utilize, let us finish that also. If the kind of the chip waveform we are referring to utilize for OQPSK and MSK are like this. For OQPSK, the chip waveform will be a rectangular type and for MSK usually we prefer to use the sinusoidal signal. So, if you see the septum of a MSK signal, you will see that the spectral side lobes for this MSK signal is actually very very minimal of the radar sequence signal as compared to the QPSK or OQPSK.

So, that is why actually the loss in the signal power, if you have high spectral lobes actually then you have to capture those lobes also inside the receiver to extract the signal. So, if the less power we will be having on the side lobes, the main lobe power the main part of the power signal, power of the modulated signal power will be confined inside the main lobe and in that situation actually your receiver processing will be actually easier and your processing involve the capturing of the power from the main lobe of the modulated signal only. In that sense MSK will give you the plus point because most of the signal, modulated signal power will be confide within the main lobe of your MSK and the side lobe power side heavily decreased when we are spreading is done compared to the orthogonal QPSK or the QPSK.

So, we prefer the MSK signal to utilize and remember there as another plus point over MSK because MSK signal by default in the MSK signal we prefer to use a chip waveform of sinusoidal type. And we all understand now the plus point of using a sinusoidal chip waveform over a rectangular chip waveform in the main plus points, we have already discussed in the last three modules and the plus point and the advantages for this choice of this chip waveform comes from the fact that you will get an extra dB band, extra dB gain on the symbol error probability because the variance that is contributed form the interference that can be heavily diminished by an amount of 1 dB around.

And it also this sinusoidal chip waveform that is why, in combination with the MSK because MSK is giving a confined power within the main lobe and sinusoidal is allowing you to have a extra gain over the interfering signal at an amount of your 1 dB. So, the combination of MSK and the sinusoidal is the best preferred pair for utilizing in the direct sequence spread spectrum communication. So, in quaternary system, we will proceed with MSK, sinusoidal signal and considering this understanding that the double carrier is coming in the receiver and with two different independent symbols.

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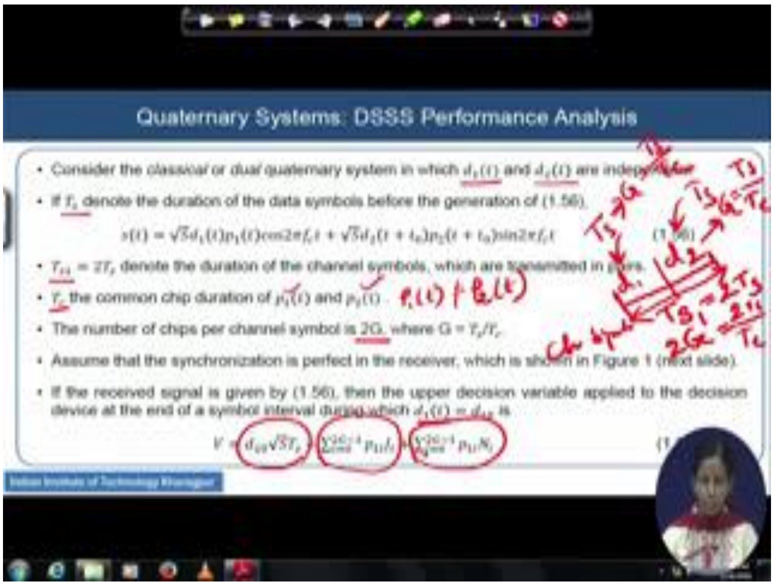

Quaternary Systems: DSSS Performance Analysis

- Consider the classical or dual quaternary system in which $d_1(t)$ and $d_2(t)$ are independent
- If T_s denote the duration of the data symbols before the generation of (1.56),

$$s(t) = \sqrt{S}d_1(t)p_1(t)\cos 2\pi f_c t + \sqrt{S}d_2(t+t_0)p_2(t+t_0)\sin 2\pi f_c t \qquad (1.56)$$

- $T_{s1} = 2T_s$ denote the duration of the channel symbols, which are transmitted in pairs.
- T_c the common chip duration of $p_1(t)$ and $p_2(t)$
- The number of chips per channel symbol is 2G, where $G = T_s/T_c$
- Assume that the synchronization is perfect in the receiver, which is shown in Figure 1 (next slide).
- If the received signal is given by (1.56), then the upper decision variable applied to the decision device at the end of a symbol interval during which $d_1(t) = d_{10}$ is

$$V = d_{10}\sqrt{S}T_s + \sum_{i=0}^{2G-1} p_{1i}I_i + \sum_{i=0}^{2G-1} p_{1i}N_i \qquad (1$$

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Let us consider a classical dual quaternary system. Dual quaternary system is having this d 1, d 2 independent as discussed in the last slide. And T s if we denote the data symbol duration and before the generation of this signal that we have seen in the last slide. And then actually T s 1, will be the duration of the channel symbol. So, there as two different definitions: the channel symbol duration consists of the two symbols d 1 as well as d 2. And each of them symbol duration is considered to be T s. So, the channel symbol, this is the channel symbol duration. So, channel symbol is considered with both d 1 plus d 2 and the channel symbol duration hence we will consider as T s 1. The channel symbol duration we will be given as twice the T s.

And as I have already mentioned that T c is the common chip duration for both the P 1 and P 2, but P 1 is not equal to P 2 t, P 1 t is not equal to P 2 t. We have given different quotes for the modulation. This is a code king also you can think of going on. The number of the chips per a channel symbol is now 2G because G is the symbol number of the chips you are getting after spreading the number of the chips that you are getting per symbol.

So, G is given for T s by P c for d 1, corresponding to d 1. Also there is another gain involved with respect to T s by T c related to d 2. So, if I see from the channel symbol duration. So, the total gain we will get as a 2G and over the gain of this 2G during the channels symbol period, you will be getting a total 2S number of the stuff divided by T c.

So, that is why T s by T c is equal to G n. Hence you are getting twice the gain involved in this kind of the communication.

Let us consider that in the receiver the synchronization is perfect. So, the carrier synchronization and the code acquisition both are perfect. And based on that we will discuss the receiver architecture immediately in the next slide, and let us also considered that we are transmitting the symbol d 1 t and which is equal to and applied the decision based on that. And this 1.56, this equation this once we are receiving at the end of the symbol interval, there will be a signal generated as V and V will be combination of the intended signal and then it will be a combination of the interference contributed and again the portion will be contributed by the noise part.

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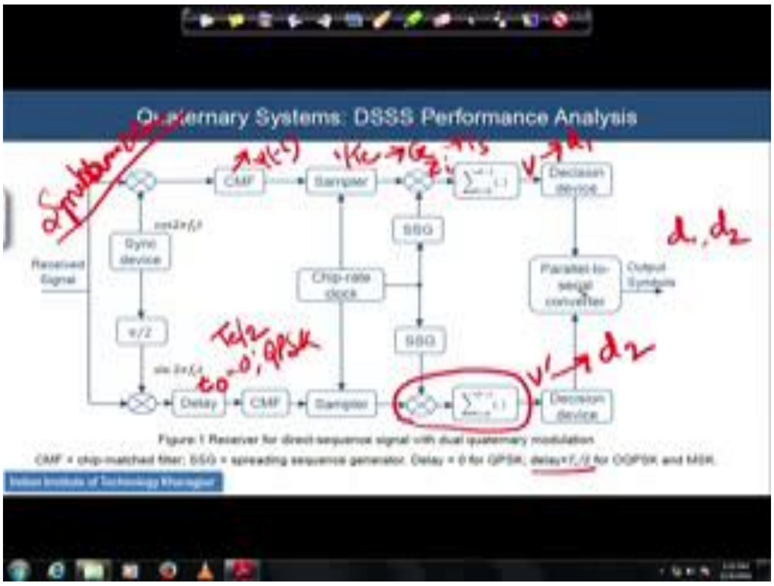

Quaternary Systems: DSSS Performance Analysis

Figure 1 Receiver for direct-sequence signal with dual quaternary modulation

CMF = chip-matched filter; SSG = spreading sequence generator. Delay = 0 for QPSK; delay = $T_c/2$ for OQPSK and MSK.

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Let us first look into the receiver architecture. See we are familiar with the match filter base coherent PSK receiver architecture where, actually the single carrier transmission was going on. We have seen that after entering the signal, (Refer Time: 14:36) signal to the receiver was r t and it was then synched by the sync device. Basically this is a operation that is going on to bring the signal from the r f to the baseband and. So, you need to synchronies it with f c, and finally this is the channel matched filter or the chip matched filter sorry this is a chip matched filter where the we saw earlier that the matched filter transfer function was match to the chip waveform that you are transmitting.

It followed by this match filter was the sampler, that sampler was sampling the incoming signal at the rate of the chip rate. Sampler and the locally generated your spread spectrum communication, spread spectrum signal they were actually in synched by the chip rate clock and sampler also was responsible to generate the G number of the samples over the duration of symbol duration of T s.

And next comes actually the sections of the dispreading where, actually you are having a multiplier followed by a adder following the correlator architecture. And we have done our discussion in the earlier few modules that input to the adder circuit was Z i and here is a V, the combined signal after the addition and the dispreading the signal V generated which was entering into the decision device and. So, here we will have has we are having two different carriers and the carriers are actually one on the signed another was on the cos. So, we will have two independent paths parallelly processing in the receiver

The received signal is hence in this quaternary system will be multiplied with the cos 2 pi f c t as well as sin 2 pi f c t generated by the pi by 2 shifting of this synched signal and. So, here also you will, here actually see from the top to the bottom you are having an extra block called delay where, we understood we have seen in the expression that the another career is delayed by an amount of t 0 with respect to the earlier one. To bring both of them in sync here you have to introduce another delay of t 0. And then you do the match filtering, sampler is doing the same job that the earlier one was doing and remember now from here onwards has you have delayed the signal another carrier by an amount of t 0 to sync in with the other carrier.

So, sampler can be driven, the same clock circuit can drive the sampling of both the path and the same SSG, the SSG the spread spectrum generating say generator signal can be applied to both of them. The orient, the addition circuit collide the upper one they are having there identical in both the paths and like the upper part this multiplier plus the adder he is contributing to generate the spread spectrum, generate the dispreading the signal.

So, V if I call it V it will be also V dashed. So, they are expected to be a corresponding to the data signal d 1 and it is the corresponding to the data signal d 2. So, individually the decision is taken on the d 1 and d 2 on the upper path and the lower path. And once the signal detection decision and the detection is over, now here is a combiner circuit who will be doing the parallel to serial conversion and these are the outputs symbols. So, in the output symbol you will get d 1 and the d 2 at a same time. So, it is a parallel to serial, parallel processing to serial edition of the detected devices. So, will be going on in the parallel to serial converter and you can actually have a output. Remember this delay will be set equal to 0, if you are following a QPSK modulation.

And if you are having a MSK modulation or orthogonal QPSK modulation we understated that these delay is equal to be set with T c by 2. So, and in the whole process we have considered that, there is perfect carrier synchronization. So, synchronization is a perfect; synchronization by the term synchronization I mean to say that it is a carrier synchronization followed by the code acquisition is done. So, there is no misalignment and hence the sampling, the SSG generation and your addition dispreading, hence the dispreading operations and all are the time synchronized and you are there is no offset also available in terms of the frequency between transmitter as well as in the receiver.

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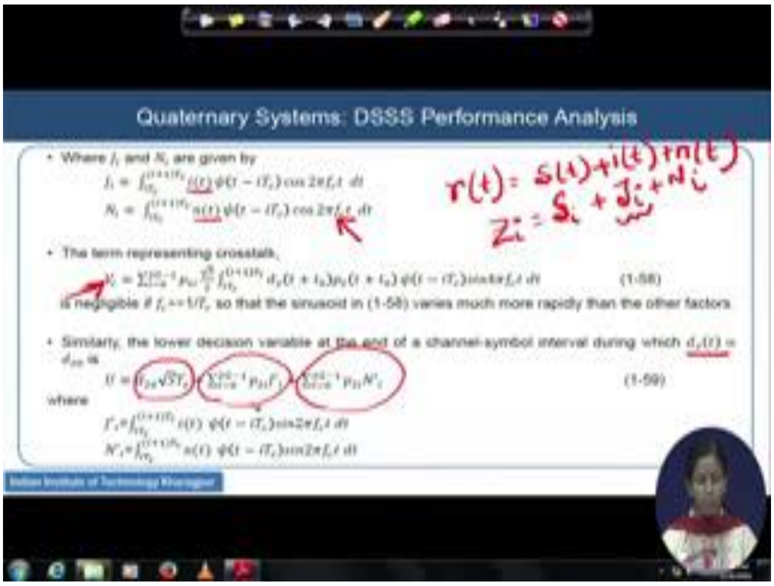

Quaternary Systems: DSSS Performance Analysis

- Where J_i and N_i are given by

$$J_i = \int_{iT_s}^{(i+1)T_s} i(t)\,\psi(t - iT_s)\cos 2\pi f_c t\; dt$$
$$N_i = \int_{iT_s}^{(i+1)T_s} n(t)\,\psi(t - iT_s)\cos 2\pi f_c t\; dt$$

- The term representing crosstalk,

$$V_i = \sum_{l=0}^{G-1} p_{1l} \frac{\sqrt{S}}{2} \int_{iT_s}^{(i+1)T_s} d_2(t + t_0)\,p_2(t + t_0)\,\psi(t - iT_s)\sin 4\pi f_c t\; dt \qquad (1\text{-}58)$$

is negligible if $f_c >> 1/T_c$ so that the sinusoid in (1-58) varies much more rapidly than the other factors

- Similarly, the lower decision variable at the end of a channel-symbol interval during which $d_2(t) = d_{2\nu}$ is

$$U = d_{2\nu}\sqrt{S}T_s + \sum_{l=0}^{G-1} p_{2l} J'_l + \sum_{l=0}^{G-1} p_{2l} N'_l \qquad (1\text{-}59)$$

where

$$J'_i = \int_{iT_s}^{(i+1)T_s} i(t)\,\psi(t - iT_s)\sin 2\pi f_c t\; dt$$
$$N'_i = \int_{iT_s}^{(i+1)T_s} n(t)\,\psi(t - iT_s)\sin 2\pi f_c t\; dt$$

$r(t) = s(t) + i(t) + n(t)$

$Z_i = S_i + J_i + N_i$

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So, this is the close architecture of a quaternary system. It is just scaled up version of a single carrier communication matched filter base receiver architecture for a single carrier communication that we have observed in the earlier modules. From here onwards, we will carry, we will recap that we understand I am going back once more because in order to recap we know that here was our sample values Z i, after the multiplication here was our value V dashed. Remember inside Z i has we are talking about continuously the interference, there was a contribution from the signal part and there was the contribution from the jamming part and contribution from the noise part. And inside V 1, so corresponding terms will be some contribution from the signal part and the contribution because of the jamming part and the contribution because of the noise part was involved.

The corresponding expression for J i and N i we will recall from the previous understanding. We have several times seen this both of these expression in the previous module where, our i t is the jamming signal, n t is the interfering signal and hence like the earlier modules r t is having a module like signal module like s t plus i t plus n t. And inside Z i the corresponding terms are called S i plus you are J i sorry capital S i plus J i plus N i. The expression of this J i after dispreading operation we have seen we can write it like this. And after dispreading at the dispread noise can be written like this and f c is our transmission frequency, carrier frequency.

That is an important term here involved that we have not it is discussed in during the single carrier transmission. The term is called the crosstalk; this crosstalk is coming by approximated the expression that will be the contribution on the first one, i mean the P 1, on the P 1 due to the P 2. So, P 1 t and P 2 t are the two dedicated spreading sequences given to spread the data symbol d 1 and d 2 though we understand. But finally, you see where to and after the final decision the both the signals are getting combined, the decision is combined.

So, there may be a influence of, if the choice of the codes are such that they will be having some interference, some overlapping during the dispreading procedure. Then the codes will, if they are not perfectly orthogonal in that sense I mean to say that they have chosen is such a way, they are partially correlated and cross correlation function between P 1 t and P 2 t is such that it is the cross correlated output is not perfectly 0, even if when they are perfectly synchronized. In that situation you will be a there will be a term generated called the crosstalk which is given by like this.

And this crosstalk will play an important role on the decision device. And for the lower decision variable like the upper one, we can write down the equation like this because if we are transmitting that for lower one the d 2 symbol is transmitted and d 2 t is equal to d 2 0 say symbolized axis. So, we will have a signal component like this and we will have the contribution from the jamming part and we will have contribution for the noise part.

Remember one thing, though you are processing parallely for two different carrier signals the contribution of the jamming signal and the noise signal will be equivalently added the on both the path. So, the contributions from the jamming and the noise power can be equivalently, can be treated as a way that if I know the contribution and the expression on the over the one path that will be simply replicated over the next the contribution section from the combined effect of the jamming plus the noise.

And J 1 dash and [noise] 1 dash that is N i dash see be actually having some resemblance on these. They are basically the same, the expressions are basically the same that we have written for the upper path called J i and N i.

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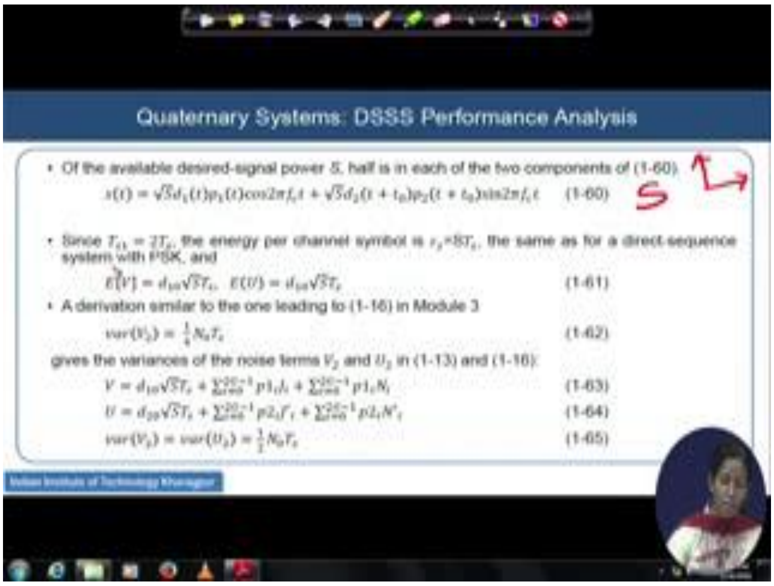

Quaternary Systems: DSSS Performance Analysis

- Of the available desired-signal power S, half is in each of the two components of (1-60)

$$s(t) = \sqrt{S}d_1(t)p_1(t)\cos 2\pi f_c t + \sqrt{S}d_2(t+t_0)p_2(t+t_0)\sin 2\pi f_c t \qquad (1\text{-}60)$$

S

- Since $T_{s1} = 2T_s$, the energy per channel symbol is $\varepsilon_s = ST_s$, the same as for a direct-sequence system with PSK, and

$$E[V] = d_{10}\sqrt{S}T_s, \quad E(U) = d_{20}\sqrt{S}T_s \qquad (1\text{-}61)$$

- A derivation similar to the one leading to (1-16) in Module 3

$$var(V_1) = \frac{1}{4}N_0T_s \qquad (1\text{-}62)$$

gives the variances of the noise terms V_2 and U_2 in (1-13) and (1-16):

$$V = d_{10}\sqrt{S}T_s + \sum_{i=0}^{2G-1} p1_iJ_i + \sum_{i=0}^{2G-1} p1_iN_i \qquad (1\text{-}63)$$

$$U = d_{20}\sqrt{S}T_s + \sum_{i=0}^{2G-1} p2_iJ'_i + \sum_{i=0}^{2G-1} p2_iN'_i \qquad (1\text{-}64)$$

$$var(V_2) = var(U_2) = \frac{1}{2}N_0T_s \qquad (1\text{-}65)$$

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So, there is no difference between these two. The same interfering signal, interfering the both path same noise signal is interfering and it is getting applied on both the path. So, of the available desired signals and the signal power S if I understand that S will be the total transmitted signal power, as we are having two different carriers. The law is that you will divide the transmitted power equally over both the carriers and that is why actually the component, this was a transmitted signal a half of the power will be loaded on the I phase and half of will be loaded on the Q phase signal.

And we understand that T s 1 is the total channel symbol period which is equal to twice the symbol period and we also understand that the energy then the path channel symbol will be given by capital S into t s because each of them are getting S by 2. And hence it will be getting totally it will be getting, totally for the channel symbol you will be getting S by t s. And hence for the direct sequence system with the PSK, the mean value of this V will be coming with your d 1 0 square root of S into t s and mean value of the other part. So, it is the mean value of the V is, V is considered here.

V by means of V, we are thinking that we are in the in phase part. V is sorry V is considered here and the upper part and U is considered here in the lower part and so the mean value of the V and mean value of the U where, we are interested in because we understand that finally we will be interested to calculate the symbol error probability. The mean value of this V and the mean value of this U both of them will be given by this expression. And the derivation of this noise component following the way we did for the single carrier communication, we can compute the variance of thus, we can compute the variance of the V 2 and the variance of U 2. Both of them will be governed by half of this N 0 by T s because the channel duration time and all because of channel duration time over which the interrogation is going on that will play a role.

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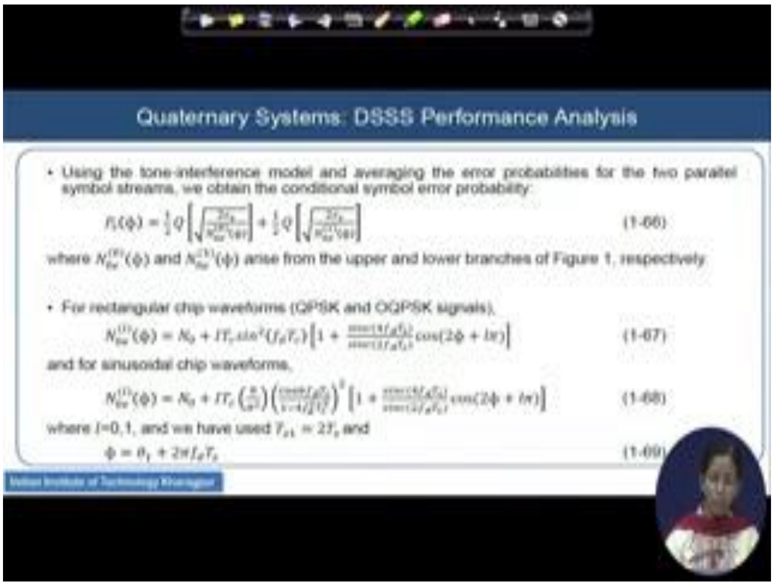

Quaternary Systems: DSSS Performance Analysis

- Using the tone-interference model and averaging the error probabilities for the two parallel symbol streams, we obtain the conditional symbol error probability:

$$P_s(\phi) = \frac{1}{2}Q\left[\sqrt{\frac{2E_s}{N_{0e}^{(0)}(\phi)}}\right] + \frac{1}{2}Q\left[\sqrt{\frac{2E_s}{N_{0e}^{(1)}(\phi)}}\right] \qquad (1\text{-}66)$$

where $N_{0e}^{(0)}(\phi)$ and $N_{0e}^{(1)}(\phi)$ arise from the upper and lower branches of Figure 1, respectively.

- For rectangular chip waveforms (QPSK and OQPSK signals),

$$N_{0e}^{(l)}(\phi) = N_0 + IT_c\,\text{sinc}^2(f_d T_c)\left[1 + \frac{\sin(4\pi f_d T_s)}{4\pi f_d T_s}\cos(2\phi + l\pi)\right] \qquad (1\text{-}67)$$

and for sinusoidal chip waveforms,

$$N_{0e}^{(l)}(\phi) = N_0 + IT_c\left(\frac{8}{\pi^2}\right)\left(\frac{\cos(\pi f_d T_c)}{1-4f_d^2T_c^2}\right)^2\left[1 + \frac{\sin(4\pi f_d T_s)}{4\pi f_d T_s}\cos(2\phi + l\pi)\right] \qquad (1\text{-}68)$$

where l=0,1, and we have used $T_{s1} = 2T_s$ and

$$\phi = \theta_1 + 2\pi f_d T_s \qquad (1\text{-}69)$$

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Now, coming to the using the tone interference module that we have done in the last three module. We can calculate the error probability of the two parallel symbols streams. As we understand the total probability at the output of this decision device, output of the parallel to serial converter; here if you try to consider the total symbol probability P s, it will be a combination of the individual branches error probability. And that the contribution should be such that half of the time the error is equiprobable in both the branches. So, it will be half plus half and actually the as they are having a Gaussian distribution because of the understanding of all this expression. So, they will be governed by a Q function where, actually your E s is the symbol energy given by the earlier slide and N o e will be 0 or 1 I mean it is something l, defining actually upper path or the lower path. And this is the one sided power spectral density, combined power spectral density of the interference plus the noise.

And we know that the expression for all this, if we utilize a rectangular chip that will be governed by this and this has a direct relation of the expression that we have derived in the earlier modules where, l is coming extra because to capture the effect of the in phase and the quadrature phase component in the cos component and you will put at the 0 and 1 for the value of the l to get actually either the cos 2 pi or the cos 2 phi plus the pi 180 degree out of phase expression for the lower value for the lower path.

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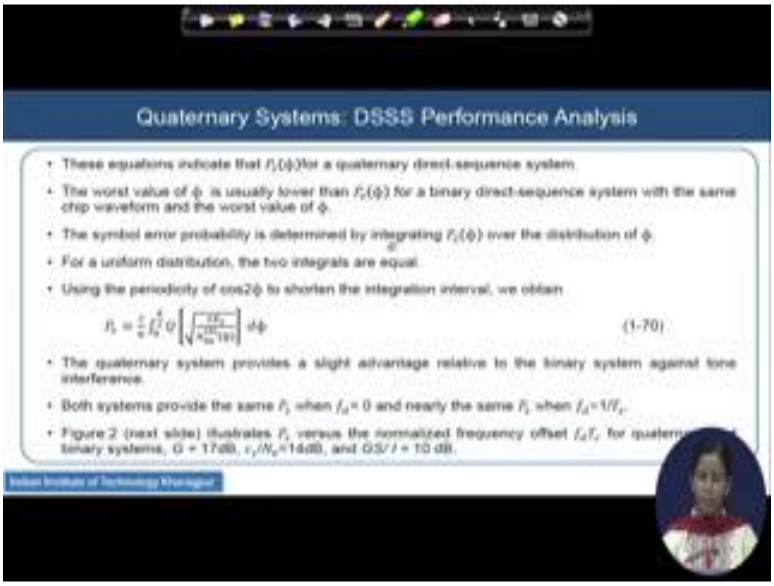


Quaternary Systems: DSSS Performance Analysis

- These equations indicate that $P_s(\phi)$ for a quaternary direct-sequence system
- The worst value of ϕ is usually lower than $P_s(\phi)$ for a binary direct-sequence system with the same chip waveform and the worst value of ϕ.
- The symbol error probability is determined by integrating $P_s(\phi)$ over the distribution of ϕ.
- For a uniform distribution, the two integrals are equal.
- Using the periodicity of cos2ϕ to shorten the integration interval, we obtain

$$P_s = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} Q\left[\sqrt{\frac{2E_s}{N_{0e}(\phi)}}\right] d\phi \qquad (1\text{-}70)$$

- The quaternary system provides a slight advantage relative to the binary system against tone interference.
- Both systems provide the same P_s when $f_d = 0$ and nearly the same P_s when $f_d = 1/T_s$.
- Figure 2 (next slide) illustrates P_s versus the normalized frequency offset $f_d T_s$ for quatern binary systems, $G = 17$dB, $\varepsilon_s/N_0 = 14$dB, and $GS/I = 10$ dB.

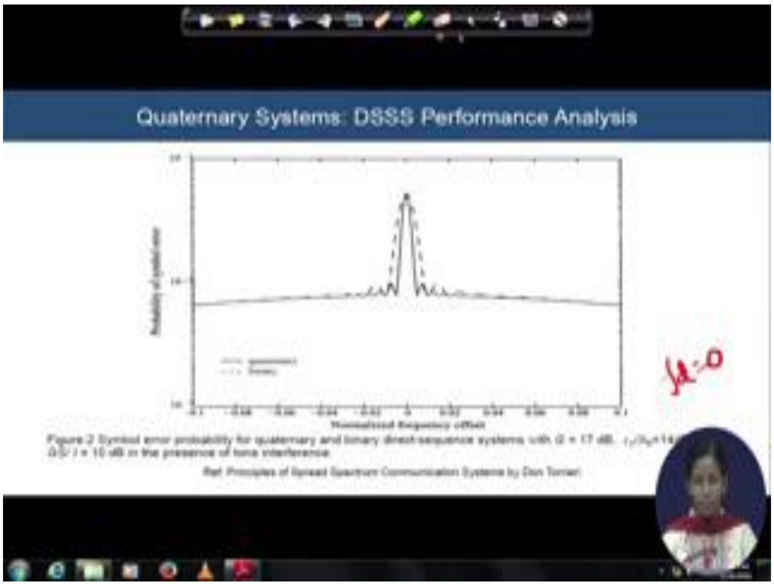
Here, phi if you remember the phi was given by theta 1 plus 2 pi f d t into T s. And if you have to utilize that T s 1 is equal to 2 by twice the T s. So, all this expression should follow that. And the equation indicates what? The finally, the equation indicates that this P s phi for quaternary system will be given by the total expression of the upper path, error probability of the upper path and plus the error probability of the lower path and they are equal probable. So, it is half plus half and the worst value of the phi is usually the lower than the P s phi for the binary direct sequence system, with the same chip waveform and the worst value of the phi.

So, if it is usually lower means the phi of the quaternary system i mean the QPSK system or the any quarter system which will be lower than this both of the n cases where, the binary the spread spectrum communication system will be utilized. The symbol error probability will be determined by finally, integrating this P s phi by integrating P s phi over the range of the phi and we will consider that this range of the phi is again will be uniformly distributed over a zone of 0 to 2 pi and we also understand that cos 2 phi will be having almost all the values within 0 to pi by 2, I am going by the logic that we followed in the last module. Here also the integration will be shortened over the duration of 0 to pi by 2.

And we have substituted the value from the Q side here, from the Q function here. And quaternary system in that sense it gives a slightly little bit more advantage related to the binary system. When the tone interference is present and we will compare the performance of both the symbol error probability for the quaternary system as well as the binary system in the next slide.

When actually you are f d is will be in 0; that means, the tone interference is been given exactly on the carrier frequency of our interest and it will be also and nearly the same. Almost actually if you see that, if I carrier frequency tone interference is exactly and the carrier frequency then only the binary system and the quaternary system will be providing almost the same kind of the performance.

(Refer Slide Time: 32:05)



Quaternary Systems: DSSS Performance Analysis

Figure 2 Symbol error probability for quaternary and binary direct-sequence systems with G = 17 dB,
DS/I = 10 dB in the presence of tone interference

Ref: Principles of Spread Spectrum Communication Systems by Don Torrieri

And next case what we have done is? We have given a symbol error probability versus the normalized frequency offset for both the quaternary and the binary system. What we tried to see? That this symbol error probability, what we have derived here in presence of the consideration assuming that the there will be a perfect synchronization between the transmitter and receiver and carrier frequency is perfectly transmitted. What will happen which is not a valid assumption for the practice. If I consider that there is a actually in the system implementation, there is a normalized frequency offset in between transmitter receiver and with the increment of these offsets from the 0 value.

From 0 value means the perfect synchronization situation, how both of them will vary? If this is our consideration; we did the this simulation or the experiment is done when the gain view is equal to 17 dB and your signal to noise power spectral, signal power to signal energy to noise power spectral density is given fixed at 14 dB and this dispread signal to the interference power is considered to be 10 dB. Tone interference is present and f d is considered to be 0; that means, tone interference is present exactly on the centre frequency of transmission.

What we were wearing now we are not considering that, we are normalized. The normalized frequency offset is exactly 0. If it is 0, then both of them will have the exact performance, but if we are having some amount of the normalized frequency offset, we find that the quaternary system has some better performance. It gives a little bit better performance compared to the binary system. And this effect of this improvement basically it vanishes if you are increasing the normalized frequency offset further.

So, with this we will try to end up the whole analysis, on the whole performance analysis of a direct sequence spread septum communication. So, in (Refer Time: 34:22) over the last four modules. So, we have learnt how actually the jamming signal a placed on the centre frequency, close to the centre frequency with having some Gaussian interference affect our symbol error probability? In the last module, we have learnt that if we are shifting from the binary spread spectrum communication system to a quaternary spread spectrum communication system whether, we are gaining somewhere in terms of the error performance.

We have seen that compared to the binary systems, in the quaternary system; the performance of the quaternary system will be more resilient with respect to the normalized frequency offset, which is a very often you will face when you will design a practical spread spectrum communication system. Every time you cannot get the perfect synchronization, perfect estimate of the carrier frequency in the receiver. So, in that situation it will be preferable to go ahead with the quaternary systems compared to the binary system design.