Spread Spectrum Communications and Jamming

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Lecture - 19
Performance Analysis in Presence of Gaussian Interference

Hello students. In continuation of our discussion related to the performance analysis of the direct sequence spread spectrum communications in presence of jamming. In this module we will continue the same with the consideration that now the property distribution of this jamming signal will be considered in the calculation, and we will consider in this module that this jamming signal or the interfering signal is having a Gaussian distribution.

In the last 2 module we have learnt, we have seen, we have derived also the symbol error probability, in presence of different kind of the chips we have seen the effect of the choice of the processing gain, we have also seen over the symbol error probability verses the detected signal I decided signal, signal power or receive signal power after the decision device after the spreading also with respect to the interfering signal be plotted, and we could see actually that when the what was the effect of will be the effect of if the tone jamming is exactly on the centre frequency of our interest, and it is having actually a small it is present on the another tone frequency which is close to the centre frequency of our interest, that we did not consider anything on the distribution function of the signal.

So, now with the understanding of the P s that we have derived we will slowly proceed towards the considering will proceed and we will revisit all those symbol error probability calculation, in presence of the Gaussian distribution of the interfering signal.

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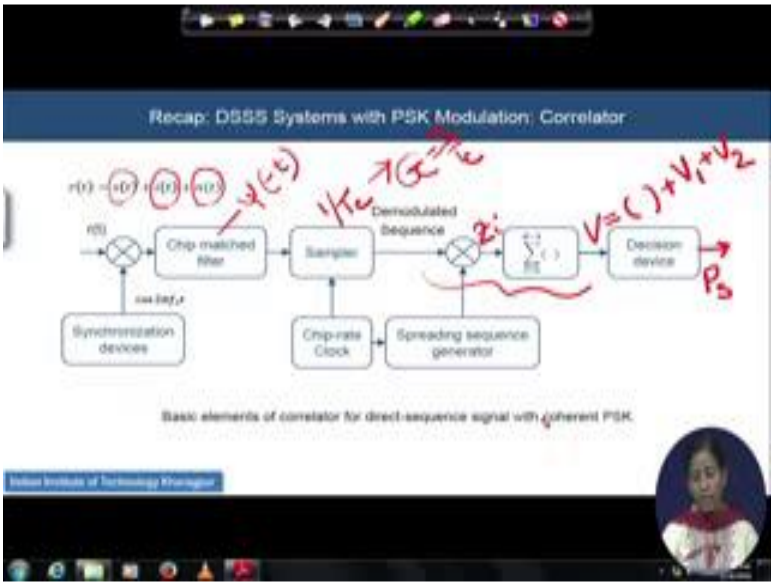

Refer the same block diagram that we have referred in the last two. So, we are I am not going in detail we understand that the receive signal is modeled as a combination of the transmitted intended signal, the interfering signal, and the noise signal. The match filter here at the front end of the receiver is a transfer function is match to the chip waveform; and after coming down to the baseband and after the output of the match filter is getting sampled at the rate of the chip rate which is 1 by T c, the sampler and the spreading sequence generator of the receiver they are in sync in this consideration, and their sync is carried out by the clock chip rate clock.

The output of this demodulated sequence with the spreading sequence the multiplied signal of this 2 is we are mentioning it as Z i and once they are added over the capital G number of the samples, this samples are generated by the sampler; because if it is sampling at 1 by T c rate is targets is to give you back, the capital G number of the chips who are present over the duration of symbol; and this combined effect of this multiplication class addition he will give you the dispreading operation and we are really very interested to know the output of the adder circuit or the dispread circuit, where this V is having basically 3 section; one section we saw is a signal contribution, another part is the contribution from the jamming signal, and third part was the contribution from the noise section.

And we were kindly interested that we understand that the noise as well as the jamming as well as a noise signal both are of random nature and as they are having the randomness, and they are means basically determining the value of this V the statistical probe they are defining the statistical property of V, I mean the mean and the variance of V is largely dependent upon all of them and hence the decision we had property calculation at the output of the decision device we carried out, where we are whole discussion is ending up. And with this understanding we will proceed again in this module. So, we are talking about a Gaussian interference now.

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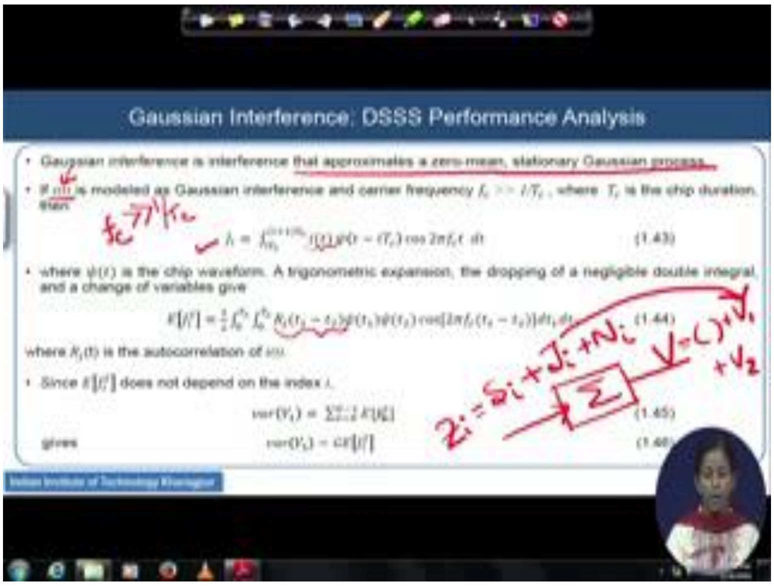

So, Gaussian interference can be defined like this; it is an interference that approximates a 0 mean stationary Gaussian process, and I t will be symbolized for this Gaussian interference in our analysis in this module, and we also considered the same and assumption will continue with the same assumption that we have done in the last 2 module.

The consideration was that transmission frequency of interest f c is definitely much much larger than a chip rate. So, that actually we can omit all the higher frequency terms, and we understand that this J i we saw this expression of this interfering signal at the input at the input of this adder. So, J i is a part like this, we saw that input of the adder in the last block diagram we saw the input of this adder was given by Z i, and Z i had 3 parts one is my S i another is my J i and the last one was my N i for the i th chip.

So, this J i expression we have seen earlier; that J i can be express that is i t multiplied by the locally generated this psi t minus i T c, cost 2 pi f c t was contributed from the front end of the receiver, which synchronize it and brought it down to the baseband. So, fundamentally the contribution of the J i over a one chip interval will be given by this. And our target is what? Our target is to now replace the i t replace the computation replace the i t by this consideration of the germ Gaussian interference.

We consider that now this i t is a Gaussian interfere and if this is psi t is definitely the chip waveform for consideration, and then if we are trying to compute now the J i square, then definitely you have to compute the relative autocorrelation of this Gaussian interference i t. If I come try to compute that in the expression would look like this 1.44, if I try to compute the variance of this Z i square finally, I will boil down here and then inside that this R j T 1 minus T 2 is coming because there is an i square t which is basically the autocorrelation of this Gaussian interference, coming into picture in this derivation and this autocorrelation the expression for this autocorrelation we will see in the next slide.

But finally, see this value of this expected value of the J i square which is a variance is not affecting on the he has no contribution, it is not dependent that all on the index i see in the 1.44 items are there is no presence of small i. So, the variance so will be basically if I now talk about the output of this adder, where the V is residing and where actually V 1 will be the corresponding portion of the jamming section at inside the V.

So, J i is mapping to this V 1 in the V section, where V is V 1 is actually that this interfere this variance of the i th chip, and that is getting added up I mean added up over the g number of the samples, if this is the that is the situation we happening at the output of the add up then as this I is G i J i is not dependent on i; that means, when is you are calculating for each and every I sample they are independence. So, I can add up the variance. So, finally, it we will end up with the G times the variance of these contributed by this expected value of the J i square. So, finally, 1.45 balls down to 1.46.

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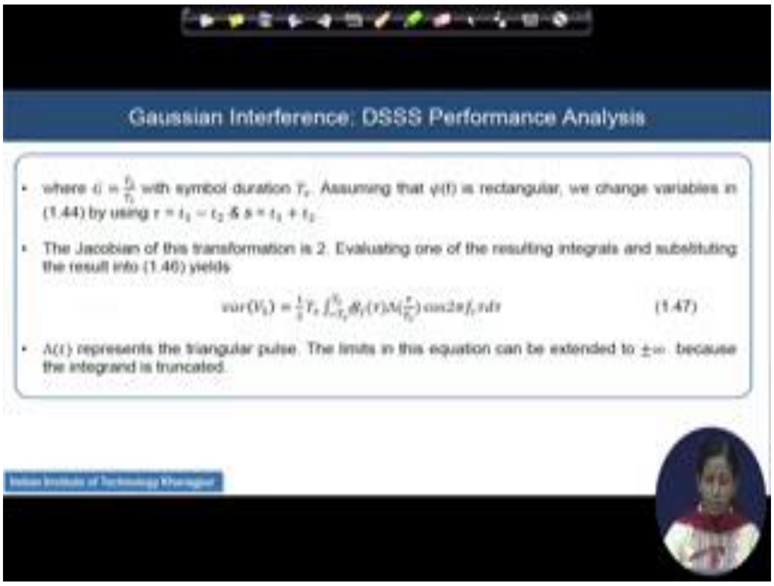

Now, we know that G v the processing gain is given by T s by T c, and we assume again first that the psi T is a rectangular chip waveform; we also substitute that t 1 minus t 2 and is equal to tau and capital S will be t 1 plus t 2. So, here actually this substitution will be by tau, and capital G we understand that will be given by T s by T c and this psi t now we will continue with rectangular chip waveform, and another parameter is coming called as small s we will be giving the t 1 plus t 2 value.

And this transformation the Jacobean of this transformation is equal to 2 and if you try to evaluate the integration of resulting of the resulting integrals of this (Refer Time: 09:51) coming from 1.44, if I try to substitute the value and if I try to do the integration, then this 1.46 so will turn down, and if I do G into this whole integration and if we try to solve it, it will come down to this point. I remember that I told that this is a autocorrelation section of contributed by the interfering signal, and there is a triangular waveform coming into picture associated to it, which will is a function of this t 1 minus t 2 the difference between the time delay between the 2 pulses involved in this autocorrelation process and T c the chip duration and remember the limit of this equation minus to c 2 plus T c it can be extended to plus minus infinity also because this integration is all the integrand is there truncated already.

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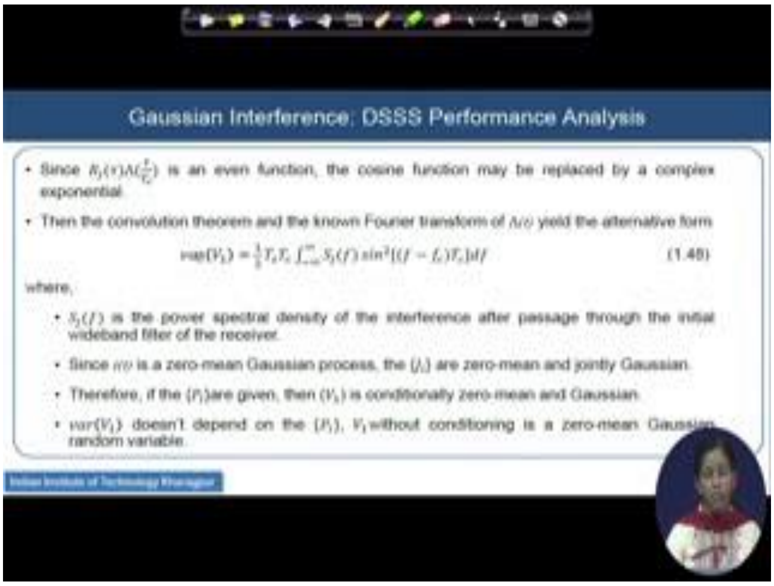

Now, the function that we have seen here this R j tau and dell multiplied with the triangular function this is an even function. If this is a even function given here then the corresponding this cost 2 pi f c tau, this cos term itself they can be also substituted by the exponential term. If I do that and I understand that the Fourier transform of this triangular function will be giving a sync function again, we will be boiling down once again another form of this variance we can write down like 1.48, but this S j f is nothing, but the power spectral density of the interfering signal after passing through the wideband filter of the receiver, and the i t which was considered in the earlier slide, which is 0 mean Gaussian process for with which we started hence actually if i t is we are starting with a 0 mean Gaussian process.

So, Z i will be also having a 0 mean and jointly Gaussian. If this P i are given in the earlier slide whatever we have seen here. In the definition of all these this J i square will be the summation of this P i multiplied by Z i also, if the P i is given and then the V i is conditionally 0 mean over and then it is also Gaussian. This variance of the V 1 it does not depend on the P i because we have seen the actually that variance V 1 is the contribution of J i, and J i is actually not having any contribution from the I itself and. So, P i is not playing approximately not having a lot of roll on it and then. So, V 1 without conditioning it is a 0 mean Gaussian random variable blankly it is following the statistical property of i t. I t is Z i J i is was following the property of i t and J i is influencing the V 1. So, the variance of V 1 is also getting largely dominated by the i t itself fundamentally.

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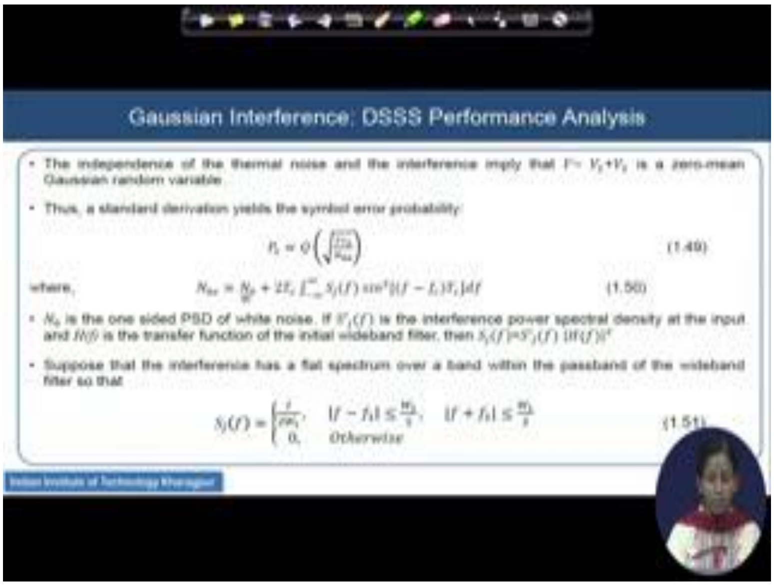

So, it is actually 0 mean Gaussian random variably finally, we are ending up with; we understand that the thermal noise and the interfering signal they are completely IID process I mean the independent and identically distributed and hence the total noise that we will be considering here V 1 plus V 2 and if I try to see actually the mean value of both of them, they will be actually this capital V will be V 1 plus V 2 and it is a 0 mean Gaussian random variable finally, and the standard derivation yields the symbol error probability will be a Q function.

Because it is a Gaussian random variable finally, ending up with, where it will be given by the symbol energy to the noise power spectral to one sided noise power spectral density, where this noise power spectral density will have the contribution from one sided noise power spectral density, and this n o e is the combined power one sided power spectral density of the noise plus the jamming signal, which will have a part of noise one sided noise power spectral density, plus actually the one sided power spectral density contributed by the jamming signal, and this equation is directly brought from the expression here of 1.48.

Now, see the consideration see let us look inside it that if; we consider that this S j f can be given by the interference power spectral density multiplied by the transfer function of the initial wideband filter, because noise will be always having a will be band limited by the wideband filter itself. So, if this is the situation. So, this interference power spectral density S j i f that will be basically we can shift with by means of your S dash to j f into h f square and if it is has a flat spectrum suppose over the whole band of their interest within the pass band of this wideband where the wideband filter is operating, so then this S j f can be given by I by 2 w 1.

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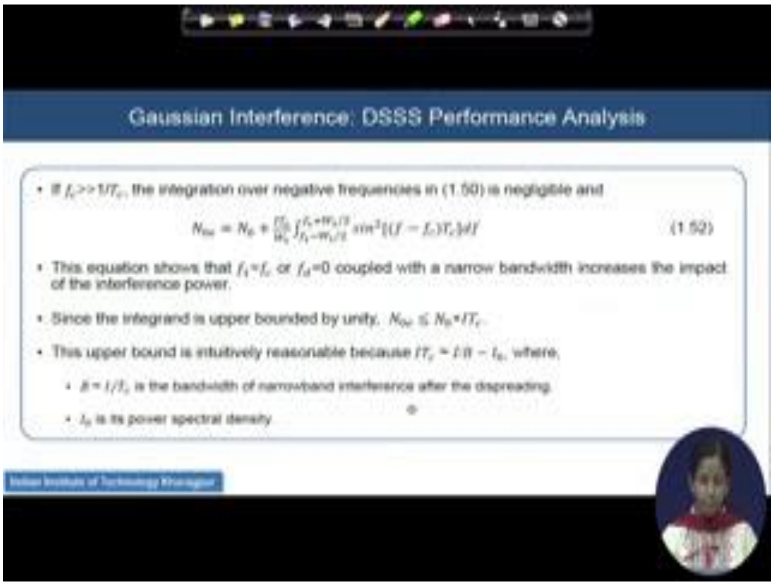

Where the when the mode of this frequency is less than equal to w 1 by 2 and it will give you the 0 otherwise. So, I by w 1 substituting the value of I by w 1 in the expression, and considering the fact that the transmission frequency is much higher than your chip rate, we will be coming down to this expression of 1.52 we are substituting the value into 1.50 using 1.51 we are down to 1.52.

So, now this equation shows you some nice thing actually; if my f 1 is equal to my f c or your f d equal to 0. So, then actually meaning is that that they are now the coupled with some narrow bandwidth coupled with the narrow bandwidth increases narrow bandwidth, and then it will in keep on increasing the impact of the interference power. So, question is actually if we are having a exactly coinciding interference on the transmission intended transmission frequency, that is why your f d is equal to 0, the first situation that we have considered. And then even if actually f 1 and otherwise actually f 1 is equal to your f c and.

So, and then actually you are having a very narrow bandwidth of this interfering signal, then the impact of the over the interference power its keep on increasing; and since the integrand upper boundary is bounded by unity. So, this effect of this one sided power spectral density will be always less than equal to this one sided power spectral density of the noise, plus some contribution coming express like this. This I into T c is nothing, but this I by the total bandwidth which we are taking as 0, and this bandwidth is equal to given by 1 by T c is the bandwidth of the narrow band interfering signal after the dispreading and this is a power spectral density of the power spectral density finally, because it is a with the power of the incoming signal and it is divided by the bandwidth of this interfering signal after dispreading.

So, if I compute this guy. So, I 0 is the contribution from the power spectral density contributed by the jamming signal. So, N o e can be having either less than the both of them or it can be actually the equal to that and this equation is finally, showing and proving you that if you are having a very narrow bandwidth interference, very close to the centre carrier frequency of your interest or close to that or exactly on the centre carrier frequency of your transmission, you are effect on the impact of the interference power how actually it will be varying.

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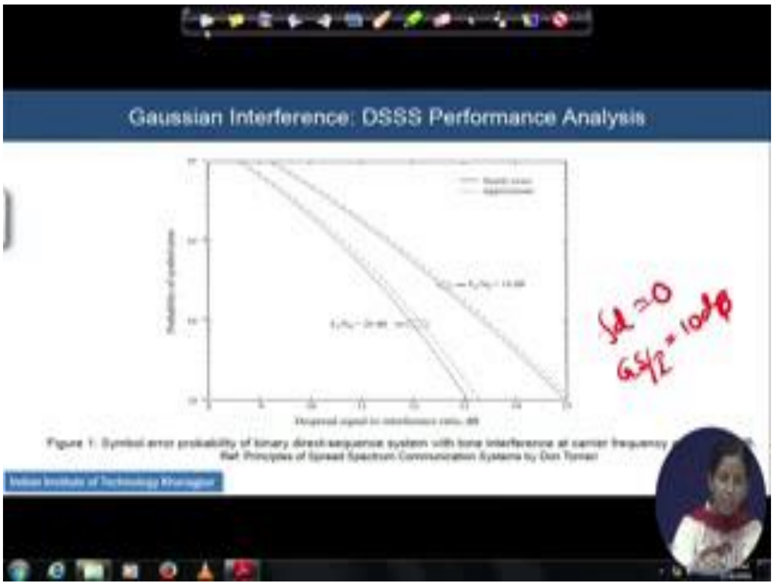

We have seen this graph in the earlier module, where we discussed that for if you are having a interference close to the centre frequency, and if you are having a very high processing gain. So, you can control the deviation of the model develop similar probability model developed in the last module with compared to the nearly exact consideration, and at the target bitrate of a data communication of 10 to the power minus 5 that approximated model is proven to be close to the nearly exact model and they are deviating around 0.1 d B. If I consider all the other parameters which we are considered to generate this result for example, that G value was equal to 17 d B, G s by i equal to g s by i; that means, the dispreading signal dispread signal to the interference ratio we considered to be 10 d B, we consider that f d will be equal to 0. So, for all that consideration if we hold good same way and now we are considering now that the distribution of this interference is equal to Gaussian.

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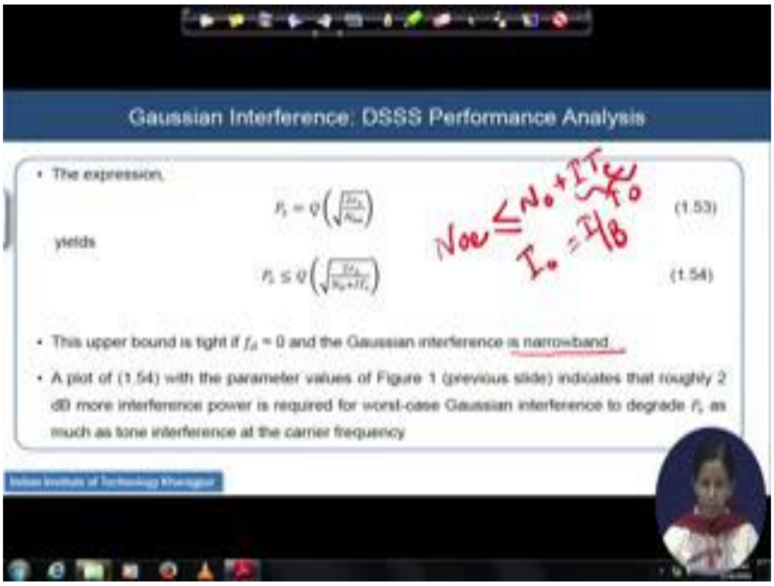

Then we will see that the error probability that you are ending up with here the error probability was given by that square root of twice e s by N o e that we have seen earlier and which is basically less than equal to the N 0 plus I T c because we have already establish the expression that N o e will be less than equal to always the N 0 plus I T c and where my I T c is basically I 0 where I 0 was given by I by capital B or T c equal to 1 by capital B it is a power spectral density of the interfering signal and hence it will be the symbol error probability will be always less than equal to this Q function, and this upper bound is really very very tight.

If and only this will be really very type tight if f d is closed to 0 and the Gaussian interference is a narrowband interference, and if I plot the same graph with the parameter that I have shown earlier with the parameter that we have considered for the last module case, then there is a very nice observation the re plot of this expression or this 1.54 basically 1.05 expression 1.54 with the parameter that we have considered for the last slide, and the last module which is equal to my gain is equal to 17 d B and your GS by I, I mean received signal to the interference power to be 10 d B and my f d is equal to 0. Only consideration is that the power is I mean distribution of the interference signal is Gaussian in that case what we find that there is roughly around 2 d B more interference power you will be required for the worst case Gaussian interference to degrade the symbol probability, the symbol error probability as much as a tone interference at the carrier frequency does.

So, situation is such that worst case for worst case Gaussian interference 2 d B more interface power you have to put. If you wish to degrade the symbol error probability as much as the tone interference at the carrier frequency does. You have a gap of around 2 d B, 2 d B value in terms of the interference power if you consider the Gaussian distribution is going on. So, worst possible a Gaussian interference also needs 2 d B more power to degrade this symbol error probability equal to the tone interference exactly happening at the carrier frequency.

So, we will go back and revisit we will go back and revisit; the whole equation and the target what we have try to do in this module. We thought that we will do the analysis for a Gaussian interference; target is that the Gaussian interference was a 0 mean having stationary Gaussian process. So, by means of each now the interfering our system, and we also considered that the transmission frequency waves having a much much higher value compared to our chip rate, and we have re computed the received signal I mean receive insert the received signal what is the contribution of this jamming signal.

We saw that actually with this Gaussian process we can nicely boiled down to some known autocorrelation structure; and that autocorrelation structure will followed as with a triangular kind of web shipping, triangular function involved inside that variance computation. Because of the involvement of this a triangular function we saw that if we are applying the standard convolution property and we are introducing the known Fourier transformation for this triangular function.

So, the variance will be the function of given by a power spectral density involved inside the interference or interfering signal, and followed by the other parts of the analysis, other parts of the element and we considered here few things to be remembered when your interference is having a Gaussian distribution, and if you are considering that the Gaussian distribution or the signal having a Gaussian distribution is having a 0 mean and that we have considered it is considered to be followed when we have entered after the sampler, we are trying to consider the contribution of the samply; contribution of the jamming part inside the sampled signal, and we carry forwarded the 0 mean; and the Gaussian distribution consideration jointly inside that also.

And we have seen nicely that this variance computation of this J i, which is the interference part acting role playing some role on the incoming or sampled signal it is independent of i. So, that is why actually the various furthermore the variance computed from this Z i, which is actually the variance of interference or the jamming signal entering into the decision device, he is also having no effect on the I and basically he is also not dependent on the P i, is P i is the sequence of values plus 1 or minus 1.

And as the variance is this is a one direction, and in other direction we saw that the variance computation total variance of the V which is contributed by the variance of the noise signal and the variance of the interfering signal, we saw that as the noise sample and the interfering sample they are independent and identically distributed, and there is they are mutually independent process also enhance that is no coupling in between and they are the 0 mean Gaussian, Gaussian on random variable each of them are and finally, leading the distribution of the capital V to be a 0 mean Gaussian random, and that is why we could apply the classical standard derivation for the symbol error probability which is governed by the Q function.

But inside that Q function we saw that there is noise power spectral density, but noise power spectral density here is governed by the one sided power spectral density, and plus of contribution from the interfering signal. Inside that interfering signal contribution we have a power spectrum, interference power spectral density, interference the power spectrum of the interfering signal and that can be further simplified considering the wideband filter bandwidth. And given that actually he is giving the waveform and the interference is having a flat kind of the flat spectrum over the bandwidth of our interest wideband filter into bandwidth interest, then we could find out that this N o e what is a one sided power spectral density of a combined effects; we could compared and we could finally, see that this one sided power spectral density.

Now whatever we are getting it is basically less than the power spectral individual power addition power spectral density of this N 0 plus I into T c and finally, the symbol error probability the expression for this Gaussian distribution we could find that it is always less than equal to the q function of square root of twice e S by N 0 by I T c because it is upper bounded by 1; and to get a symbol error probability equivalent to the tone interference that equivalent to an tone interference, the you have to with a Gaussian interference we have to put a 2 d B more interference power to get to degrade the symbol error probability up to the previous situation when the tone is exactly present on the centre frequency of our interest. So, degradation would not be that much with respect to the Gaussian interference, if we consider interfere to be a Gaussian interference that is the solution that is a final conclusion that we have learnt from this module.