Spread Spectrum Communications and Jamming
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Lecture - 18
Performance Analysis During Generation Tone Jamming

Hello students, in continuation of the last module, we will now consider the performance analysis respect to the general tone jamming. By the term general tone, what I meant to say is here actually the jamming signal, the presence of the jamming signal is considered to be anywhere close to the centre frequency of the intended signal transmission, which is equal to f c. It needs not to be exactly coinciding over f c. It is any general tone or any general carrier frequency, where the jamming signal is present. How far actually the symbol error probability calculation for the intended signal will be now affected by the presence of the jamming signal on any general tone close to it, but not exactly coinciding with its own transmission frequency, that is the point of consideration for us.

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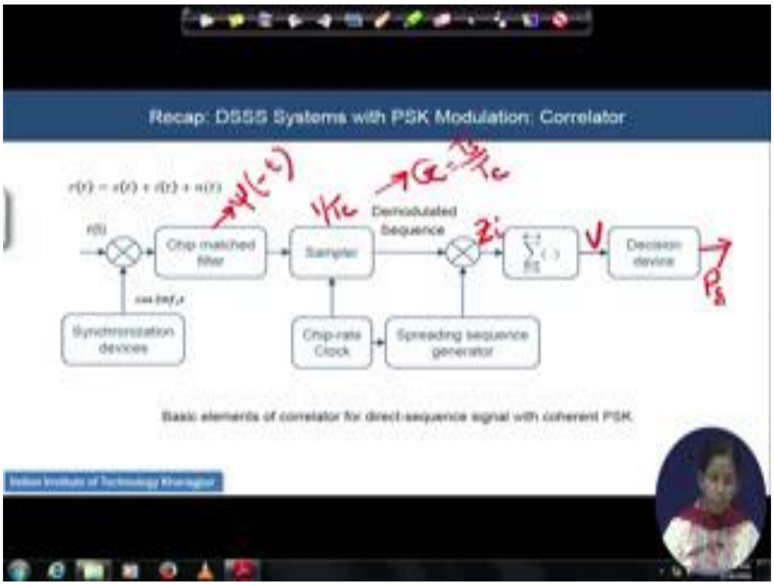

We will refer again the same block diagram, which is for the matched filter based coherent PSK transfer coherent PSK receiver we have discussed to several times earlier. And we also understand again that that receive signal model is given by r t is equal to s t plus i t plus n t; where s t is the intended signal, i t is the jamming signal, and n t is the noise associated with the incoming signal. After receiving the incoming signal, receiver has the synchronized to the carrier frequency; and basically it has brought it down to the base band. The chip match filter is designed such a way that the transfer function of the matched filter is matched with the chip waveform psi t, so its transfer function is psi of minus t.

Output of the matched filter is the sampled by the sampler; remember the sampler and the spreading sequence generator, they need to be in sync which is taken care by the chip-rate clock. Sampler is sampling the incoming matched filter output signal at a rate of the chip-rates that is equal to 1 by T c rate. And his target rate is to produce the capital G number of the samples, G is given by the T s by T c. Basically, G is the number of the chips available within the symbol duration.

And there is a demodulated signal sequence at the output of the symbol, output of the sampler is getting multiplied with the locally generator spreading sequence. And here we saw earlier that we have generated the chips, which is called the z i after multiplication. And this multiplication combined with this adder is equivalently jointly actually offering de-spreading operation in the receiver. And this is a quadrilateral database architecture also has a whole, and this z i when summed up over the G number of the samples I mean i is varying from 0 to G minus 1, total G number of the sample, you will be after addition you will be ending up with signal v, which is entering at the input to the decision device.

And decision device output we are interested to see the form of a V and basically the mean and the variance of the V as we have seen in the last module in presence of the jamming signal. And he will be this mean and the variance of the V will be greatly actually affecting the competition of the symbol error probability P s at the output of the decision device. So, decision device is basically identifying what taking the decision over the transmitted signal and whatever the error is coming out at the output of the decision device, we are computing the symbol error probability based on that.

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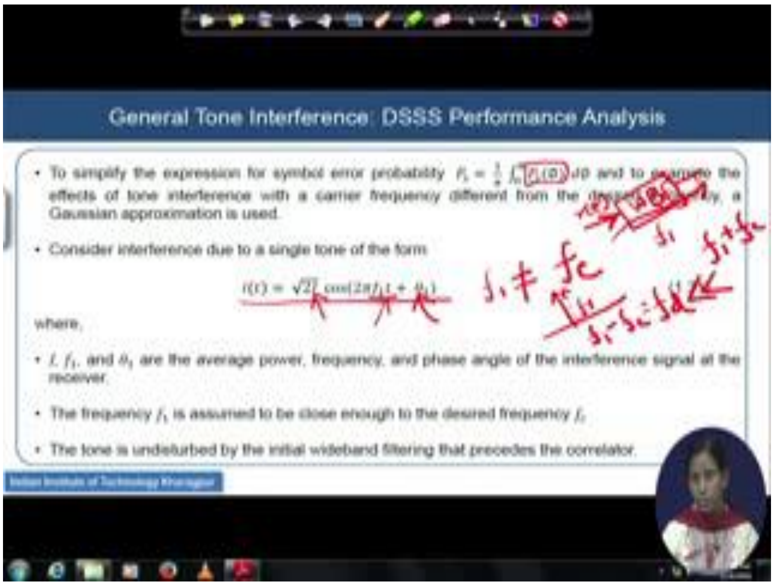

So, with the understanding of this, let us proceed. Remember in the last module, we have understood that symbol error probability, we will take a form of like this P s is equal to 1 by pi uniform integrated over 0 to pi P s phi, where P s phi was the symbol probability conditioned on the phi. And phi was the phase relative phase of the interfering signal relative to the desired signal. And in continuation to this equation, we will revisit this equation basically for the new consideration of the interfering signal. And remember that for the effect to examine the effect of the tone interference with the carrier frequency which is different from the desired frequency now. We will utilize a Gaussian approximation for the easy tractability of the mathematical derivation.

And this is the new interfering signal of our consideration given by equation 1.31, where like the earlier days our I will be the average jamming power remember now the signal is transmitted by the jammer at the at f 1; and f 1 is not equal to my intended signal transmitted at f c. And theta 1 is the corresponding phase of the jamming signal; remember theta 1 is measured with respect to the desired signal, the phase of the in jamming signal with respect to the desired signal. Though f 1 is not exactly equal to f c, we will think that if this is my transmission frequency f c, f 1 is very, very close to f c. In such a way that the difference of this f 1 minus f c which we will define later on as f d is much, much very, very less compared to your f 1 plus f c.

So, centre frequency of transmission of the jamming signal, they could not it is given by a f 1 here. And assumption is the estimation process of the transmission signal is not done correctly by the jammers and that is why actually because of the estimation errors you here is transmitting and the frequency of the frequency of f 1.

Another consideration is that see this when f 1 is entering in front actually we have seen that R t is received by the receiver antenna. And it is usually for all kind of the receiving antenna the receive signal is passed by a wideband filter, and then it is amplified also pass to the low noise amplifier, and then fed to the matched filter. But remember that when this wideband filter was filtering the signal the wide location of a f 1 is such that because of the closeness of with f c this f 1 was also passed through the wideband filter. So, wideband filter could not cut it out.

So, in what sense it will cut it out is if difference between the f c and f 1, if it is large it may happen that f 1 is falling outside the wideband filter bandwidth. Then in that situation the effect of the f 1 would not be at all in this kind of the mathematical analysis that we are worried about. And also in the symbol error probability, it would not have any effect on that. And hence in this kind of situation that can be neglected. But here in our analysis we are considering that f 1 is so close to the centre frequency transmission f c that wideband filter is allowing that f 1 to enter into the baseband. So, it is completely undisturbed. So, f 1 is also completely undisturbed like our desired signal frequency f c.

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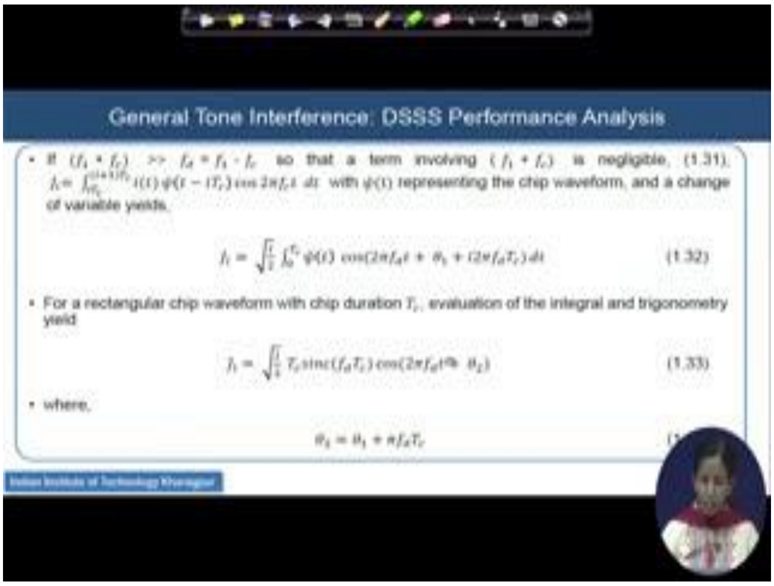

And we have also this already discussed that this consideration hold goods, the difference frequency is very, very less compared to the f 1 plus f c. So, that all the terms in that derivation which involved f 1 plus f c, we will been neglecting them. We will refer once again that expression of the J i, because J i is a part of this Z i we have saw in the earlier module that Z i constituted of a psi plus J i plus N i, it is a input to the adder circuit. This J i expression was given by this, because it is the interference multiplied with the locally generated PN sequence waveform and cos 2 pi f c t was contribute from the frontend of the receiver and we were integrating it over one chip duration. And psi t was the chip waveform.

And we will vary the chip waveform from rectangular to the sinusoidal in the later slides, but fundamentally we will now replace the value of this i t given in the previous slide into this equation to proceed further. So, J i if I substitute the value of i t from where from the previous slide that we have discussed this expression. And if I substituted there and you do the actually some change of the variables especially substituting the small t as t plus i T c we will be ending up here. And where actually this two is contributed because we have considered the 2 cos a cos b in order to consider 2 cos a cos b we have entered inside the integral two and that two came out also. So, we need two divided by another two.

And fundamentally that point where we are ending up with is interesting part is here where the contribution from the difference term of the two frequency is playing a good role. We have another term called theta 1, we will see little bit theta 1 is the phase relative phase of the interfering signal with respect to the signal of our interests. And here another x term is expected to come which will be the contribution for the every shift multiplied with the chip duration, and for the ith chip it is a function of i also. So, this section will be keeping on increasing when the i is also increasing kind of. So, fundamentally the if I integrate this section if I evaluate this integration, and we sum simply apply some trigonometry, this one we further solved by equation solved two equation 1.33. Where this is the same function and this cos 2 pi f d t plus theta 2 will ending up with the theta 2 we will be replacing by this theta 1 plus i 2 pi f d into t c.

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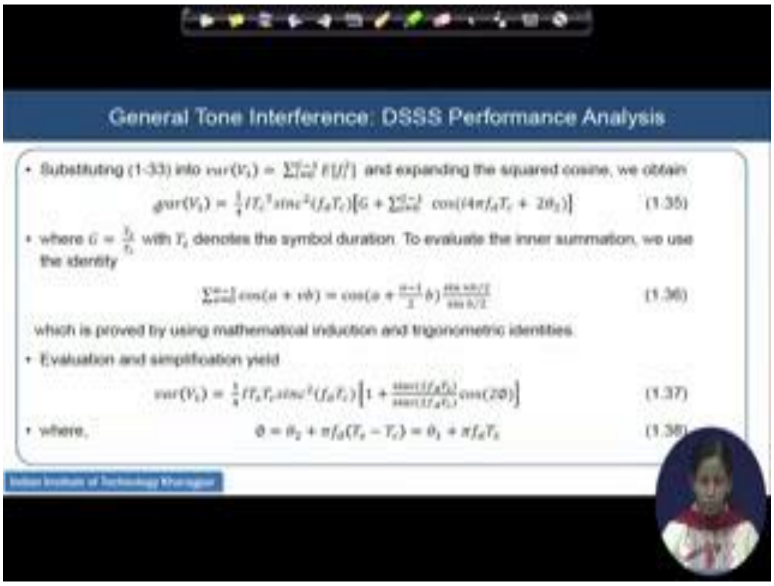

In the next slide, we will be interested to compute the variance of this V 1. V 1 is the contribution, V 1 is where V 1 if you recall after the adder circuit. So, Z i was the input of the add circuit where we computed the J i part contributed by the jammers, output of this adder network there was a decision device, our v was here where like J i, your z i. Your v will be will be also contributing three part, one is the contribution from the signal then is the contribution from the jammer part and the contribution from the noise part. V

1 is the contribution from the jamming part. So, we are interested to check the variance of V 1 and the mean part of the V 1.

In order to compute the variance of V 1 as you have crossed the adder, so that J i that you have computed in the earlier slide that square of it, and the mean value of it, definitely this is a definition of the variance. It should be summed up over the G number of the samples. And now if I substitute the value of J i from the previous slide, it will boil down here where what we have done is inside that you have seen that we have a cos term cos of theta 2 plus that 2 pi T c. Let us go back.

Remember here we have a term of cos 2 pi f d t plus theta 2 term and then here squaring up that term will lead you to cos square theta and if I add 2 cos square theta, it will be 1 plus cos 2 theta. So, it will be based on that and summation over the G that one will be turn to G, and here we will get the summation left over the cos 2 theta section. So, simply and the remaining part is squared up nothing else is has been done. If I replace now this G by T s by T c, where T s is the symbol duration; and if I utilize the following trigonometric identity to evaluate this section, the summing section where cos of a plus nu b is given by cause of a plus n minus 1 by 2 b into sign n beta by 2 by sign beta by 2 it is a trigonometric identity will be utilized to change it to compute this in a summation, we will be ending up this from equation number 1.35.

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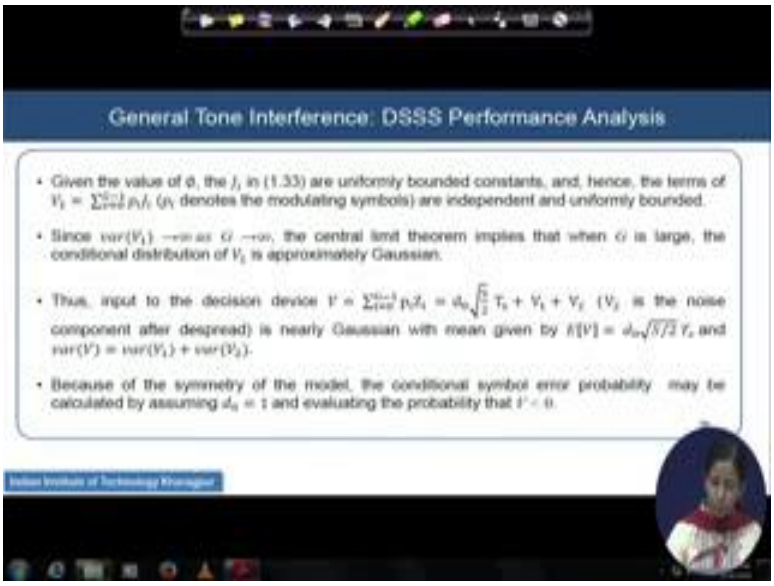

We will be ending up from the equation number 1.35 to 1.37, where actually this summation T s is coming because of the summation output sending over all the number of the chips that is c the sum will end up with that T s. And sinc square f d T c will be remaining, and this because of the addition of this trigonometric part, we this contribution will come down. Remember we here for the substitution has been done when mentioned by phi, because this phi is now the whole part this theta 2 plus pi f d into T s minus T c. Where this if I substitute the value of this theta 2, which is we understand that it is nothing but theta 1 plus phi f d into theta 2 is equal to theta 1 plus phi f d into T c we will this whole phi we will be ending up with theta 1 plus phi f d into T s. So, this is the final variance term which where we are our interest is lying.

Remember, if I give you the value of this phi then the value of the z i the J i sorry the value of this J i given by this equation or given by this 1.32 or 1.33 wherever you go, this is bounded this is uniformly bounded constant, they are the uniformly bounded all the J i is the uniformly bounded constants. Hence, whenever we are computing this v I adding up actually over the multiplying with pi and adding up over the i number of the samples, remember this would not be nu this o will be i; i is varying from 0 to G minus 1. And then they are the independent and uniformly bounded it they will be also uniformly bounded because of the property it says so. Since, the variance V 1 can goes to infinity as G standing to towards infinity, so it all will keep on increasing.

By applying the central limit theorem, we can see that when G is really very large, so V 1 can be approximated by a Gaussian distribution. And thus if the input device the V which is a combination of these three terms this three terms we find that actually that this whole distribution will now go towards the Gaussian distribution. So, anyway this noise is having a Gaussian distribution the mean value of it, and this guy is also now showing the Gaussian distribution for when G is really very large.

So, we can say that this V will be also Gaussian distributed, where actually the mean value of this V will be approximated to this and the variance will be this plus this. Remember that the mean value of this jamming part will be again 0, because with the constraint discussed in the earlier module, the V 1 is having the mean 0 value and you see having some variance, here we have whatever we will be interested next is the computation of this variance of both of them and checking actually the effect of them on the error probability computation. And as we understand that there is symmetry in the model then hence the conditional error probability may be calculate by assuming d 0 equal to 1. So, whatever the conditional probability we will be ending up with considering d 0 equal to 1, it will hold good also for the d 0 is equal to minus 1.

And we will try to evaluate the probability that v is less than 0 because that is a situation that you will get an error given transmitted data was 1. So, your decision device will give a wrong indication when you have transmitted one, but it has seen that the input is less than 0, it would not be able to detect d 0, you will decide that as if the transmitted data symbol was equal to minus 1 which is wrong. So, hence the symbol error probability will keep on going.

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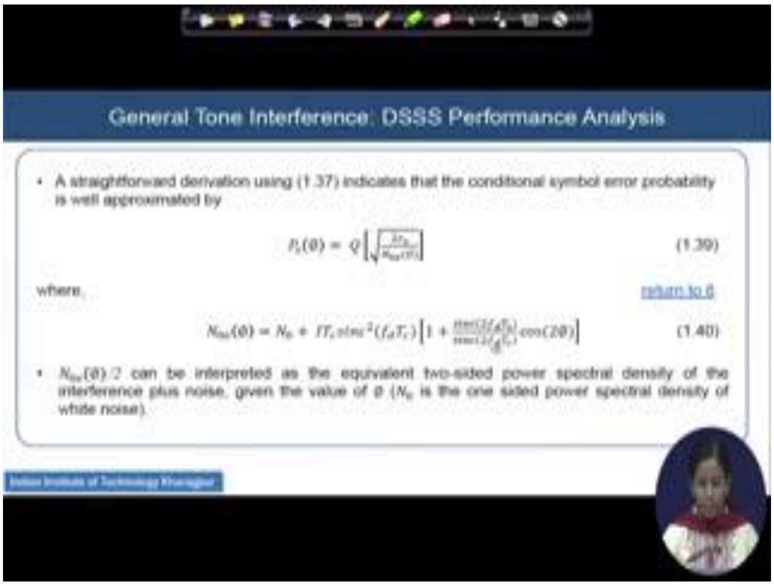

So, a straight forward derivation, if I we understand the P s phi that conditional error probability is given by a Q function, where actually the symbol energy by the total noise power spectral density is given. And if we utilize this equation 1.37, which we were actually we have ended up with the computation of the variance from the jamming signal. If I substitute, I will get some expression of the power spectral density of contributed by the combined effect of the noise as well as of the jamming signal, so which was given by N o e phi, but N o e phi can be expressed like this. Where N 0 is a one-sided power spectral density of noise, N o e is one-sided power spectral density combined power spectral density of the noise as well as the jamming signal. And this equation is fixed it forward carry forward from equation number 1.37.

So, here interest is and from using this equation 1.39 finally, we would like to see what is the expression for P s, but before entering there, remember when we decided to when we ended up with this kind of the variance calculation, we thought that the chip waveform will be a rectangular type.

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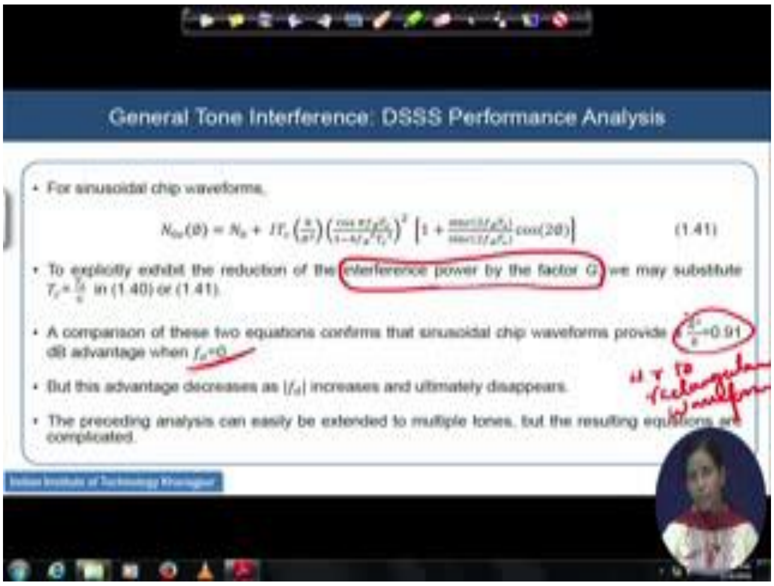

Remember whenever we have come from here to here, we consider here definitely that the chip waveform will be substitute by the rectangular one. Apart from the consideration that it integration will be done on some trigonometric knowledge will be applied to solve the equation 1.32, so that is why all the expression that we have derived till this variance computation it is a hold good only for the rectangular chip waveform. So, let us revisit little bit before proceeding further on the computation of the P s. What will be the situation if I utilize a sinusoidal waveform?

Going back to the immediately big module that we have discussed last we have shown what will be the chip waveform designs in case of the sinusoidal case. And in straight forward plugging in the expression for the sinusoidal waveform here in the equation, we are ending up with the competition of one-sided power spectral density of this interfering signal in case of the sinusoidal chip waveforms. So, the expectation this 8 by pi square the contribution will be actually very important to consider, and we will see the effect of these two different chip waveforms on the error probability calculation graphically here in this module later on.

Now, if we consider this capital G is equal to T s by T c, where T c is actually fundamentally by T s by T c. So, if I substitute this value say then you will be able to see that there is exhibition of reduction of the inference power by a factor of G. So, from where the interference power will be totally substituted by will be reduced by a factor of G compared to the symbol, when we are putting only the value of the T s. And you are computing the noise power spectral density, and if you put T c equal to T s by G, so this power spectral density will be directly divided by capital G. So, the processing gain that you will have in the receiver design in a spread spectrum communication system that will help you to bring down the effect of the interference by the G times or by the processing gain times that is proved also by both the equation 1.40 and 1.41 compared to the conventional system. In conventional system, the same equation hold good, but you consider T s instead of the T c and here you are whole period is getting divided by the period of capital G.

So, we claim that this kind of spread spectrum communication system has high resilient they are high resilient with respect to the jamming signal compared to the conventional communication systems. And if I compare this 1.41 with the previous 1.40, I mean the power spectral density compared to comparison is carried down by the sinusoidal chip waveform and the rectangular waveform. You will see because of this pi square by 8 f x that is coming here because of the chip design itself, you will be gaining around 1 dB advantage, when your difference is equal to completely 0 and this over what over what this rectangular chip waveform. So, this gain is coming with respect to your rectangular waveform.

So, this observation was there in the earlier module where when actually f d was equal to 0, exactly this what this portion we derived in the last module. So, here this is repeated just. And remember this advantage of getting of 1 dB will be gradually decreasing if we increase the value of f d equal to 0, we increase the value of f d from 0 value. So, as f d is increasing slowly, so this advantage of getting from the pulse shaping, whatever the advantage you are getting slowly it will decrease and ultimately it you would not be able to see any advantage if the f d is pretty large. As following the same path we can actually continue the analysis with the multiple tones. Here we had the tone where the interference is there we have considered is a single. So, we can keep on adding actually that tone interferences based on the how many number of the certain jammers are present following the same frame work, but it will be little bit complicated.

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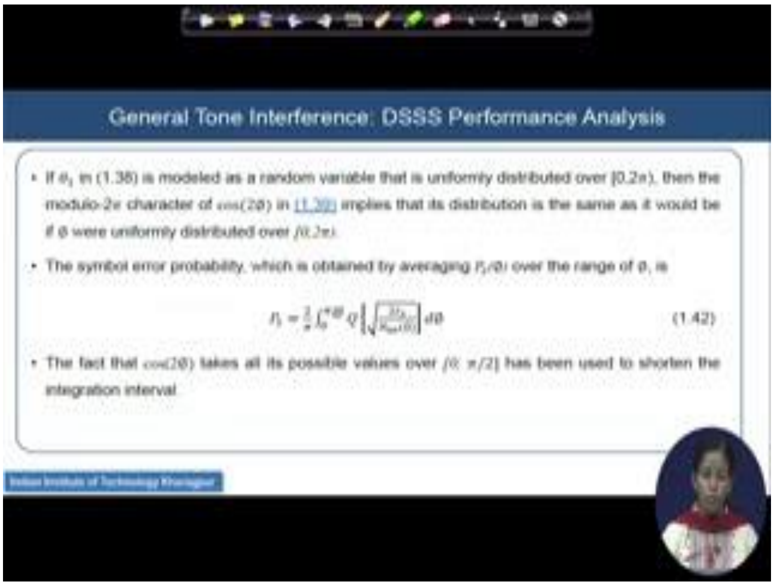

And that theta one that we have considered the phase, relative phase of the jamming signal that we have considered in this equation. We modulate as a random variable, which is spread over the duration of the 0 to 2 pi as we have seen, and it is uniformly distributed over that. If theta 1 is like that then the modulo-2 character of cos 2 cos 2 phi, it implies that its distribution will be such same as theta 1. So, phi can be considered to be uniformly distributed over this 0 to 2 pi also. If so then meaning is that if theta one is distributed over 0 to 2 pi, because of the modulo-2 character 2 pi character of this cos 2 pi hence phi will be having some uniform distribution of this whole duration.

So, then, down the going by the symbol error probability calculation and averaging it over the whole duration of our consideration 0 to 2 pi; basically it is same to the 0 to 2 pi also, so it can be good it can be going on from 0 to pi also. But remember we understand that all the positive values cos 2 pi takes all the positive all the possible values within the duration of 0 to pi by 2. So, to shorten the integration interval, this integration can be equivalently run over 0 to pi by 2 what we have shown in equation 1.42.

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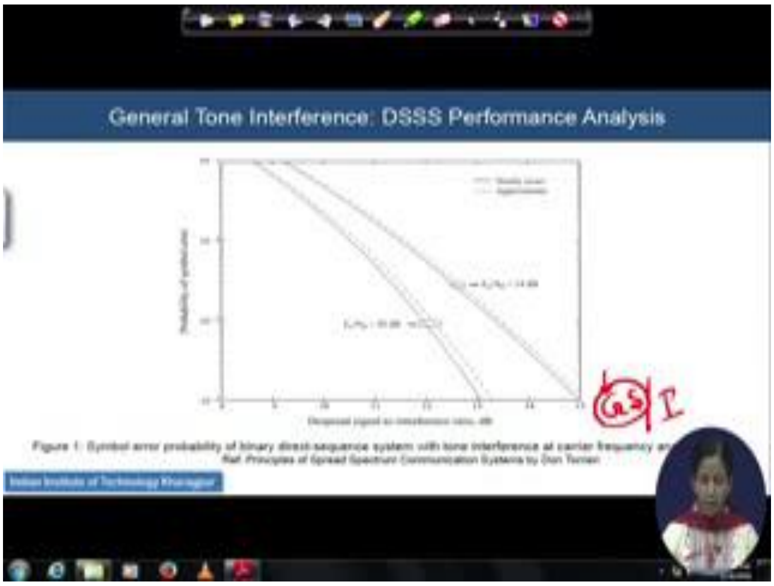

Now, this is the time to compare the performance of this nearly exact and the approximated values and the effect of the symbol error probability when we are dispread signal interference noise ratio with respect to the dispread signal to interference noise ratio. The signal to noise interference ratio is given by your G s by I, because your dispread signal is having the gain of processing gain of G a, and this was the interference power actually at the receiver frontend. If I considered the value of G is equal to 17 dB, and the other values are considered like this, where your difference was equal to 0.

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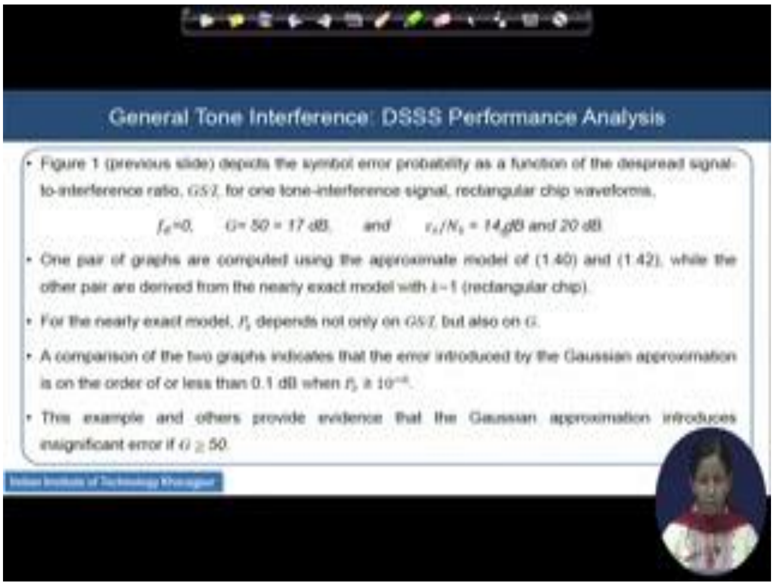

And E s by N naught was given by 14 dB and 20 dB considered both. So, we are here. What is the understanding is understanding is when the bit error rate was equal to target bit error rate was equal to 10 to the power minus 5. The approximate one was derived following this equations, whatever we have computed in 1.42 considering the corresponding pulse shape. And the pulse shape even it either 1.40 or 1.41, you have considered; and put the values of the P s phi here to compute P s if you go by that then you will be ending up dotted lines approximate one.

And nearly exact whatever we have already discussed in the last module the fundamental equations if you follow all that where actually tone is exactly on the f c and we have derived all that. If I put that value and if we keep the E s by N naught varying from 14 dB to 20 dB, then we will be seeing this kind of the comparison between the approximate model that we have developed here and the nearly exact model that we have developed in the earlier case.

So, what is for observation? Is for 10 to the power minus 5 target bit error rate for the data communication where we are mostly interested in. We will be able to see that this approximate model and the nearly exact model defuse by an amount of approximately 0.1 dB. The difference is really small even smaller in case of your lower SNR values. So, we can actually confirm that the model that we have developed here in this discussion matches nicely with the close to the exact solution. And this example provide evidence that the Gaussian approximation will introduce actually severe error as we will introduce actually severe error if actually the value of the G m in the processing gain is less than 50. So, if for high value tremendous high value of the G, this difference that you are seeing actually it would not be there. See, where we have considered this G, we have considered the G to approximate the vary variance of this V 1 to be Gaussian.

So, if the G is not attending towards the infinity, the J value of the G is really not high central limit theorem does not hold good. And if it does not hold good, so this approximation that this V 1 will be Gaussian does not hold good. And if this is not Gaussian, then the V would not be having a Gaussian distribution and so the difference here that he will be seeing that approximation that we have done over capital G, and hence the error probability we have derived that that will have a much larger difference compared to the nearly exact situation. Experimentally it could be found that for the G greater than equal to 50 values this graph will be close to both the graphs will be close to each other.

So, to the conclusion is that to hold the Gaussian approximation chose the processing gain at least greater than 50. And another important part is that one part the rectangular chip corresponding to the gain due to the rectangular chip waveforms corresponding to the sinusoidal chip waveform, all the gains that we have learned in the last module. The gain or from the chip design itself also hold good here, another 1 dB improvement is possible maximum if you choose a sinusoidal waveform. So, this is actually considering the graph that we have shown here considered for both of them are using the rectangular waveform.

So, if you utilize another one is a rectangular and with respect to the sinusoidal waveform actually if we utilize we will get another 1 dB improvement here. So, it will come bellow the nearly exact, but in that situation you should also develop the nearly exact graph with respect to the sinusoidal one. In both the cases, approximately the gap will be if your G is pretty large, the gap will be within 0.5 dB for a target near of 10 to the power minus 5 which is observed.

So, here is the solution that if we choose a large processing gain then even if actually our tone interference is not exactly on the centre frequency, but it is really close to the centre frequency of transmission, the deviation on the error probability will be hardly affected by 0.1 dB to say 0.15 dB which is not that much important. But remember if G is less or your f d difference is increasing between the f c and f 1. So, in both the situation that approximation does not hold good. And if f d is equal to if so large that wideband filter can simply cut out your jamming signal; in that situation, also you will be letting down there is no effect at all once again and you will be governed by the symbol error probability calculation that is shown early by the classical at the conventional communication system.