Spread Spectrum Communications and Jamming
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Lecture - 17
Performance Analysis of DSS in Presence of Tone Jamming

Hello students, in this module we will start doing the performance analysis of direct sequence spread spectrum system in presence of tone jamming. By the term tone jamming, I mean to say that in RF carrier frequency, the jamming signal is present in one of the RF carrier frequencies. And the analysis will be going on with the continuing module, when the presence of this jamming signal will be exactly on the carrier frequency of your interest, and may be close to the transmission frequency of your interest RF carrier frequency of your interest or and also on the distribution of this jamming signal. We will start today with the in general format and the general framing up of the general mathematical frame format of this error property calculation namely in presence of the jamming signal and then we will keep on adding one-by-one the typical characteristics of the jamming signal.

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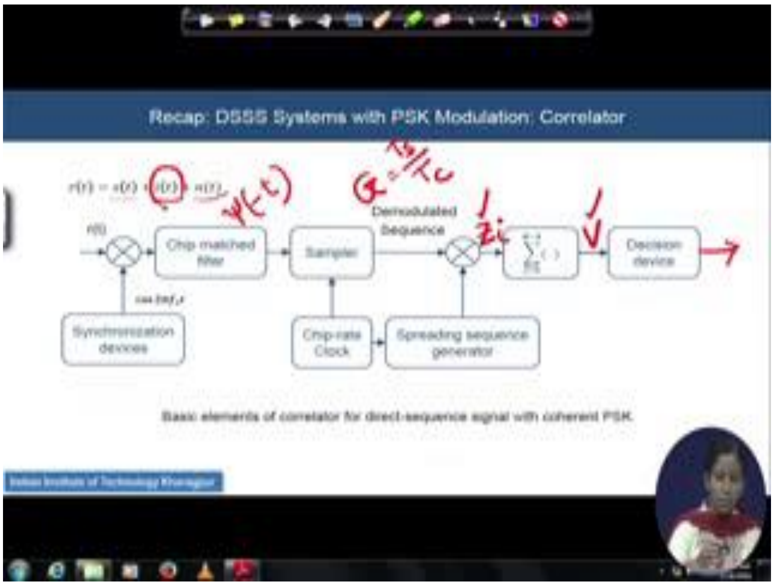

Let us refer this known block diagram of matched filter based receiver architecture for direct sequence spread spectrum communication. We have referred this block diagram earlier also. Hope you will remember I am repeating once again the block diagram architecture that we have planned earlier for PSK coherent PSK communication. R t was the received signal at the receiver frontend; r t was modeled as the intended signal transmitted added with the interfering signal or the jamming signal and this is a noise signal. So, this combined signal for intended plugged with the interference as well as the WGA noise has entered into the frontend of the receiver. Receiver has synchronized it to the carrier frequency f c and next chips matched filter is matched it is a transfer function is matched with the chip waveform psi t. So, it should be its function will be psi of minus t.

The sampler is running at the chip rate next after the match filter output the target of the sampler is to produce G number of the samples where the G will be symbol duration to the chip duration, the number of the chips present for symbol duration. And it should be exactly with G to for the next adder circuit to work up on it. The demodulated sequence here obtained here, I mean the G number of the samples will be now this spread with the locally generated spreading sequence. And here the samples that we will get after the spreading are denoted by Z i such Z i samples after it is told me total mechanism of the spreading occurs by this multiplication and this edition finally.

So, the samples that we are getting after the immediate multiplication by the spreading sequence is denoted as Z i. And after that the after adder, the signal that is entering into the decisions device for taking the decision about the message signal transmitted is written as V. We will actually start our analysis from this Z i then we will try to find out the expression for this V, so that the decision device output whatever we are getting will be utilized for the error probability calculation. So, we will vary the structure of this i t from module to module from now onwards. Let us first see how this error probability framework can be obtained.

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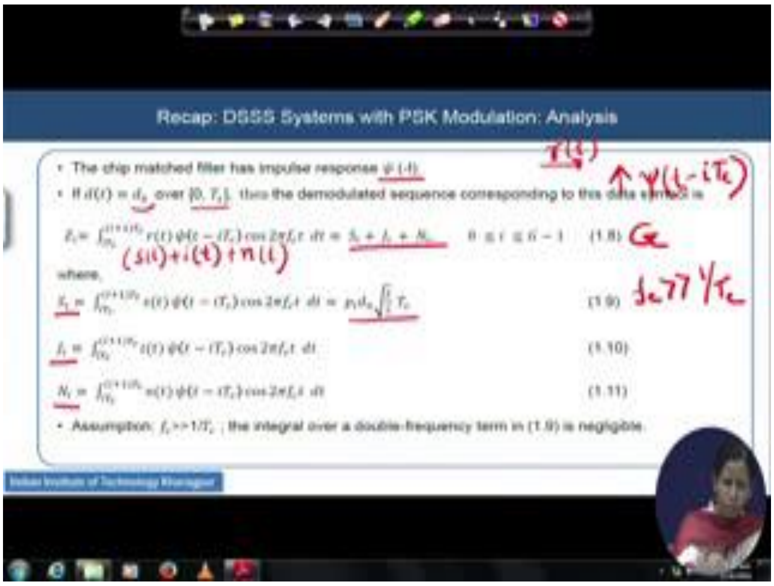

So, as we understand when I have mentioned that the chip matched filter is already matched with the impulse response using impulse is psi of minus t is match with the chip waveform. Let us consider that we have transmitted d t is equal to d 0. So, we understand that transmitted signal can be either d 0 or d 1 corresponding to the message signal of 0 and 1 message bits of sorry 0 and 1. And this d 0 is transmitted over the symbol duration of T s.

And hence then the demodulated sequences Z i, what did we get this Z i, Z i is a input of this adder. And it is also the output of this modulated signal multiplied with the locally generated spreading sequence. Hence we will get our input was r t and the psi t minus i T c, it is actually the locally generated sequence. And these are getting multiplied, so we are ending up with this with the cos 2 pi of c t they it was there with multiplied with this r t beginning of the match filter to bring it down to the basement. And this integration with this as it is we are interested on the ith chip, so the integration is running over 1 chip interval let us assume that it is ith chip, so the duration will be i T c to i plus 1 T c.

Remember this r t is a combination of as I have told earlier that it is a combination of the intended signal plus the interfering, this is the jamming signal and also the noise. So, obviously, this integration will have three parts. The parts contributing to the intended signal will be a s t psi t minus i T c and cos 2 pi f c t d t which we will mention here now s i. i t into psi t minus i T c cos 2 pi f c t will be the contribution from the jamming part, and n t into cos 2 pi f c t will be the contribution for the noise part.

So, finally, this whole expression or the Z I can be expressed in terms of S i plus J i plus N i and this i is varying from G to capital G minus 1 because we have total g number of the samples in hand. And remember when we will compute these individual integrations circuit is to find out the variance and the mean and variance of each of these parts in order to find out final E b by the signal power to the noise power, so that we can utilize that in the scenario condition of the symbol error probability.

And in order to proceed when we are computing disintegration of the first one equation 1.9, remember that this frequency f c which is the carrier frequency of your transmission. This f c is considered here to be too high compared to the frequency of the chip. So, the rate of the compared to the rate of the chip our transmission carrier frequency is too high as such that the higher frequency terms of the cost which is coming when we are solving this integration can be avoided.

If we proceed little bit by avoiding that higher order terms, we will be ending up with this p I d 0 square root of s by 2 T c. And this expression we have also seen earlier, and I have explained about how did we arrived s p i into d 0 square root of s by T c in the sum of the earlier module. So, we will proceed with the consideration that this integration will boil down to p i into d 0, where your p i is the value of the ith chip, d 0 was the transmitted signal, s is the power associated with the transmitted signal, T c is the chip duration. And with this understanding, in the next slide, we will try to say that what will happen now if this interference i t tone interference i t is exactly coinciding on the transmission frequency of our interest which is equal to f c.

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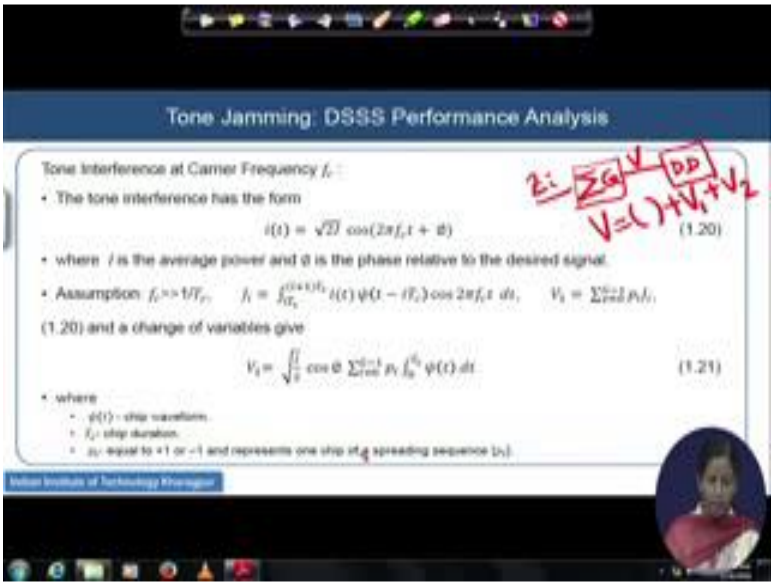

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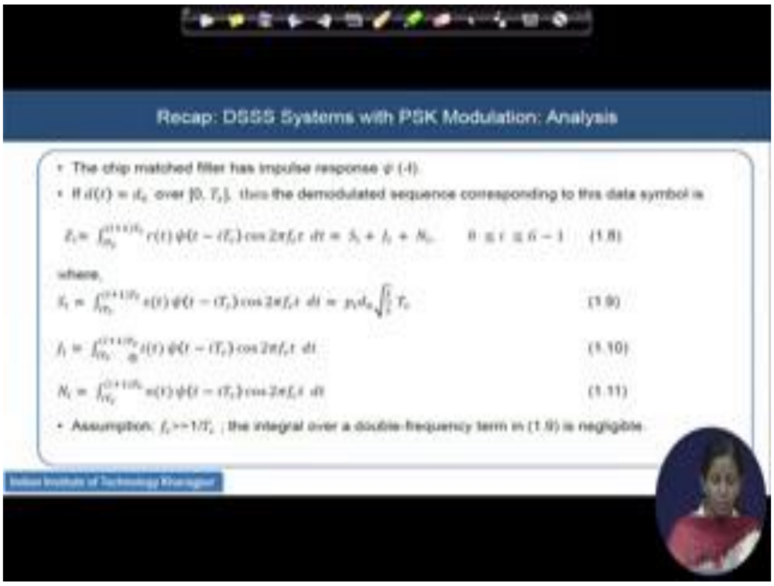

So, what we said is we said that now there is J i, which is actually getting now if you are getting from the contribution of i t, this i t we are trying to see that what will be the effect how to compute the power of the section, if i t is exactly coinciding with f c. So, i t is such that someone have could locate your transmission frequency. And exactly on the f c he has design his signal if that design is possible how i t will look like, i t will look like the equation 1.20. So, it is having this average power called I, and it is having the carrier frequency of transmission exactly same with our intended frequency of transmission and phi is the relative the phase associated with signal i t which is relative to the desired signal. So, it is having some phase of such with respect to the desired signal denoted as phi.

So, if this is the situation, and if this is the assumption hold good that the carrier frequency transmission is to high than the chip rate then we can now compute J i of the earlier equation that I have shown in the earlier slide by substituting the value of i t from this equation 1.20. And remember one more thing that the signal that was entering output of this adder if you go back and check the block diagram I have shown earlier, there was an error that is adding over G number of the samples and then output of the adder was entering into the decisions device. I wrote in the front that the input to that device is decision device DD is V. This V is also having three components because we have seen that the Z i is having three components S i plus J i plus capital N i, so interference signal and the noise.

So, here also we will be able to see three components one contribution will be there from the intended signal part and another contribution will be from the jamming part which we are defining as V 1, and the third contribution will be from the noise part which will be will be defining as V 2. So, once we are able to calculate this integration of z i. So, V one here will be nothing but the sum of all them multiplied with the p corresponding p i values, because we understand that once in the Z i component all the i t is now spread by this spreading sequence locally generated spreading sequence. So, when this addition will go on all the contributed all the corresponding values of that sample should be multiplied with the Z i J i.

So, if I do this substitution and continue the integration, we will be ending of with the equation 1.21, where actually some change of the variables will be definitely required to come here because of the substitution instead of i T c to I plus 1 T c have done it over the 0 to T c. So, that shifting of the change of the variables is essential. And finally, the V 1 first you please substitute the value of i t in the examples of in the expression of J i and then you add them up.

So, finally, we will be coming of V 1 with this expression where you have utilized the after substitution we have utilize the fundamental of the trigonometry by taking 2 cos A cos B this is will be the cos A and this is the cos B section. So, 2 cos A cos B will lead to cos A plus B into cos A minus B, and then finally, will be ending up with this because you understand that cos of A plus B is there giving a very high order terms and we are not ready to accept all that. And inside this expression, we know that the psi t is the chips waveform, p i is having values either plus 1 or minus 1, phi is the relatives phase of the interfering signal with respect to the desired signal, and T c is the chip duration, I is the average power of the jamming signal.

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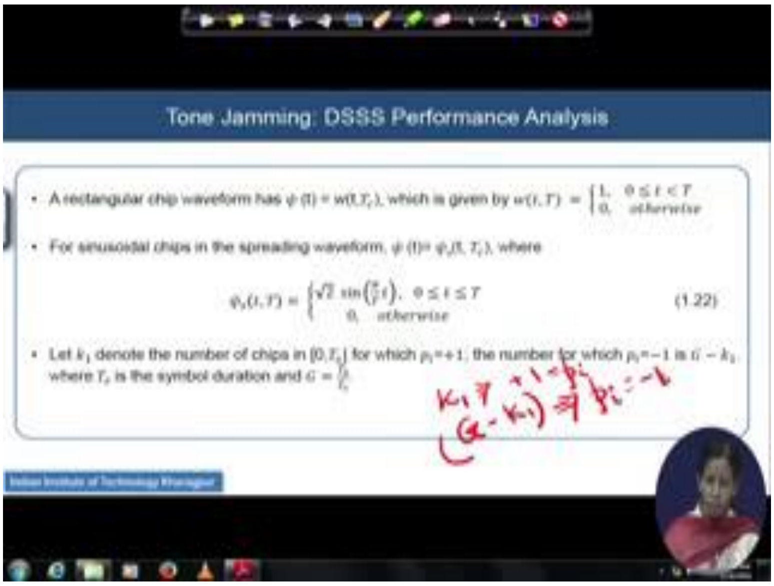

Once we have calculated V i then we understand that the shape of the waveform that we have considered here for the chip design for the spreading sequence. We consider since beginning that there are two different kind of the chip waveforms are possible. In our consideration one is the rectangular chip wave form and another is a sinusoidal waveform. For rectangular chip waveform, we know that the waveform will be having a value of one within from when that t will be varying from 0 to the capital T; and otherwise it will be 0. Whereas the sinusoidal chip waveform will look like this. And let us also consider that out of this G number of the sample that the sampler has given us; let us consider that for k 1 number of the chips with the value of the chips are equal to plus

1. For k 1 number of the chips value of the chips are plus 1 that means the value of the p i in the earlier equation you can put equal to 1. And hence for the rest number of the chips, which is given by G minus k 1, you will get the value of this chip is equal to minus 1, with this consideration and if I substitute the value in the earlier slide, here if I substitute the value.

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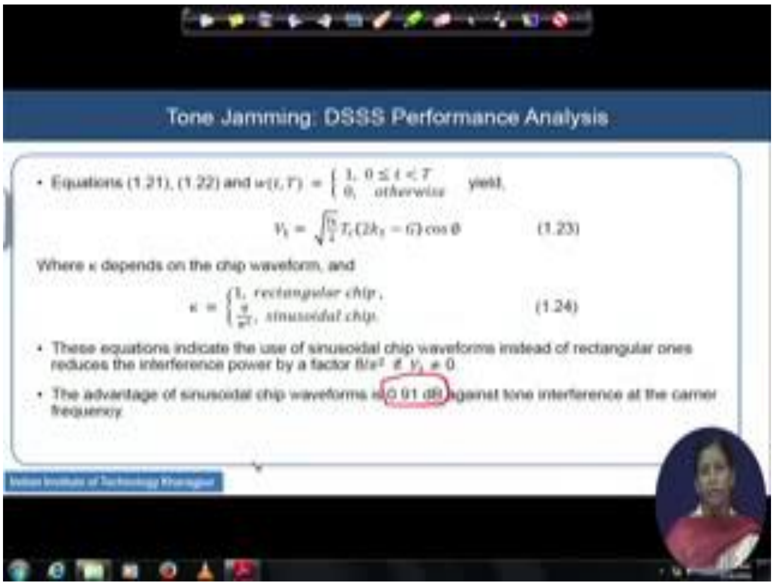

So, now, what is a suggestion going on I will substitute to the value of psi by the first by rectangular wave then by the sinusoidal wave. And then I will considered substitute the value of the p i by some expression because the total number of the chips that I will get will be k 1 plus G minus k 1 actually. And then G minus k 1 plus actually your k 1 will be the total number of the chips available. And if I keep on substituting all that and finally, we will be coming we will be converging to the equation given by 1.23, we have evaluated that integration there.

Remember this 2 k minus 2 k 1 minus G is a contribution of the total number of the chips that you are having and this kappa is a new term introduced in this expression, because it we wanted to keep the effect of both the waveforms rectangular and sinusoidal chip by some common factor. If you consider for a rectangular chip than this value of the kappa will be 1 and expression will boil down to square root of y by 2 and T c 2 k minus 1 G into cos phi, but for the sinusoidal chip, the value of the kappa will be 8 by pi square.

So, good part to observer here is if I choose a sinusoidal chip then it is true that the contribution of the V 1 will be some amount down. And if I calculate that in terms of the dB you we will get 0 0.91 dB gain against the tone interference which is designed by the rectangular chip. So, if you are a jammer, and you are designing your signal, and if you consider the signal to be spread by the rectangular chip or by the sinusoidal chip, so if you actually sitting in the intended receiver you design the chip wave form by sinusoidal chip, so it is expected that the effect of the jamming can be actually 8 by pi square or 0.91 dB down. So, you will get an additive advantage of 1 dB around over the rectangular chip designs. So, by designing the chip using the chip waveform, you can have a gain over the jammers you have a gain over the jamming signal or jamming interference due to the jamming compared to the rectangular chip.

And another importance observation if my k 1 is equal to if I chose k 1 equal to say half. So, sorry k 1 is equal to G by 2, then this contribution whole V 1 will boil down to 0. So, if you choose the sequence in such a way that half of the time the sequence is giving you plus 1, and half of the time the sequence will be minus 1. So, if you are having a even length sequence, and if you are having half of the sequence values is always plus 1, half of the total half of the sequence values is equal to minus 1. So, your interference will be 0.

So, when you are designing the code for your spreading code, you are choosing the spreading code for your design, so keep it in your mind. So, one is related to the number of the plus 1 and the minus 1 that should be in the codes to minimize the effect of the interference and also what should be the choice of the psi t, I mean the choice of the pulse shape that you should be considering in the code design itself. So, in the both way, this is the this equation give you the indication how actually you can minimize the effect of the jamming signal in the receiver. So, code plays code and the chip waveform for that code plays very important role in this regard.

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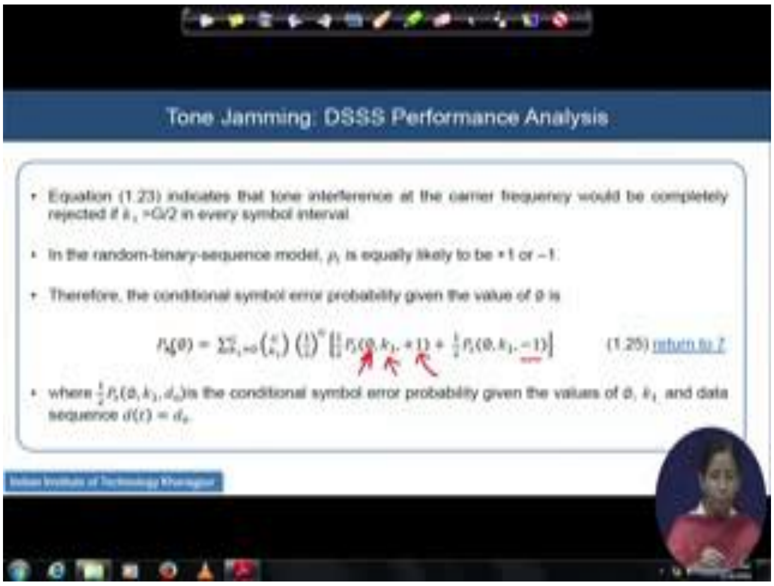

So, coming back again, if the random binary sequence we are considering for this sequence designs. So, we understand that p i will be equally likely to be plus 1 and minus 1. And in such a situation the conditional symbol error probability can be given by equation 1.25, where actually this conditional error probability is also the combination of a conditional symbol error probability when your k 1 is given k 1 number of the times you are transmitting the symbol to be plus 1 and given the value of the phi and given the data symbol to be either plus 1 and minus 1, if they are equally probable.

So, it should be the additional symbol property will be the combination of the both the conditions, when your symbol error probability will be the conditional symbol error probability will be half of the times equal to plus 1 governed by plus 1 transmission, and half of the time will be governed by the minus 1 transmission. We can actually compute each of their value and then take the effect of the combined effect of both the situation to compute this P s phi.

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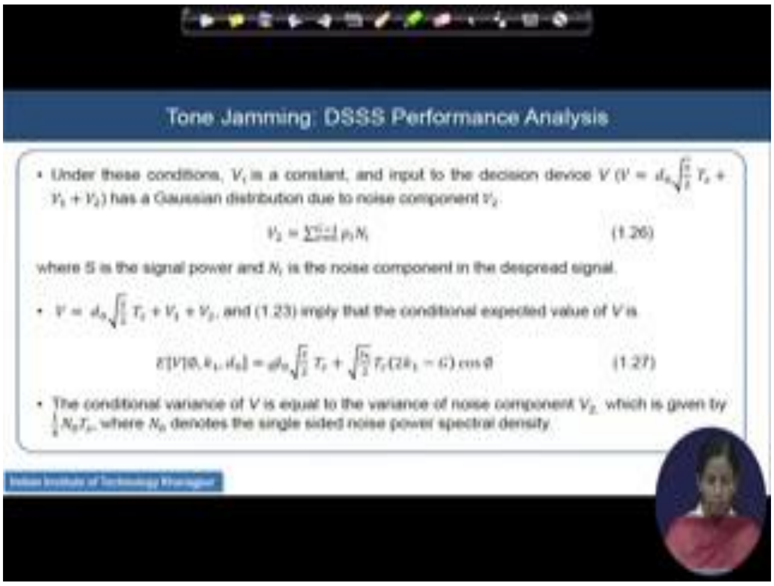

Remember one thing, once we are considering that situation of equi probable case in such situation this V 1 is approximately constant. And hence if I go back and see the input condition of the decision device V, which is the combination of the known sequence contribution from the known sequence mean value of the known sequence part, and mean value of the V 1 plus the mean value of the V 2, constituting the mean of this V then we can see that this guy will have a Gaussian distribution now. Because this is almost deterministic and V 1 is a constant and V 2 is additive governed by the additive wide Gaussian noise Gaussian process is associated with it. So, definitely the V will be now having some Gaussian distribution.

So, V 2, the way V 2 is will be expressed similarly the way we express the jamming signal at the input of the decision device following the same thing it will be given by this. And this S we have understood the signal power and this is a symbol duration. And N i is the noise component in the described signal. So, we can write down the V as like this and if I take the consideration of 1.23 the equation we have explain last time; 1.23 is explaining about this d 0 section and also the V 1 expression for the V 1 is obtained from the 1.23.

So, using the fact using the earlier fact that we have a contribution for the signal section like this given by 1.9, we have the contribution from the V 1 given by this. And mean value of the noise is equal to 0. So, finally, the conditional expected value of this V which is entering into the input to the decision device will be given like 1.27. Basically it is a addition of the signals section mean value of the signal section and the mean value of this V 1. The variance equivalent like the mean the variance of V will be also equal to the variance of all these three terms. And this is a constant parts its variance is having no effect, but finally, the variance of V will be contributed by the variance of both the noise as well as a interfering signal.

We will see next remember using that Gaussian density to evaluate each of the terms of this conditional error probabilities for transmission one and translation of minus 1 will be given by this. Where this E s is a energy park symbol which is written by S into T s. And this Q function is having a classical definition governed given in equation 1.29. Remember that once we are reaching here, so we will be knowing that typical value of the P s this is a conditioned on the phi conditioned on the k 1 and conditioned on the transmitted value d 0, so plus 1 and minus 1. So, what is the total symbol error probability now we are interested? In order to complete the total symbol error probability, you have to know what is the distribution of this phi. See, this we should go back and we should revisit what was the definition of this phi. We defined in the first few slides that this phi is nothing but the phase associated with the jamming signal, and this will be given by the phase relative to the desired signal.

So, this distribution of the phi for the sake of this mathematical analysis and which is quite often true also. The consideration is valid that the consideration should be that phi show is uniformly distributed will be over a period of 0 to 2 pi, definitely it is random variable and it will be its uniformly distributed over a period of 0 to 2 pi. And we understand that cos phi will be periodic, it has a periodicity. So, we can give the final symbol error probability P s equal to it is the integration running over 0 to pi and this P s phi.

So, P s phi is given by the equation here, where it is the function of the total number of the samples considered, and also having the contribution for the conditional symbol error probability. And for this, we considered that the both the sequences are equally probable. So, half of the times the contribution will be from the transmission of the signal 1 and half of the times the contribution will be signal of the minus 1. Please remember that both these commissioner symbol error probability is heavily dependent on the choice of the k 1 also. So, understanding the fact that you are properly choosing the value of the k 1 properly choosing the value of the k 1 such a way that actually that error probability error probability I mean the V 1 value of the V 1 is not coming to 0 and he is contributing on the error probability calculation.

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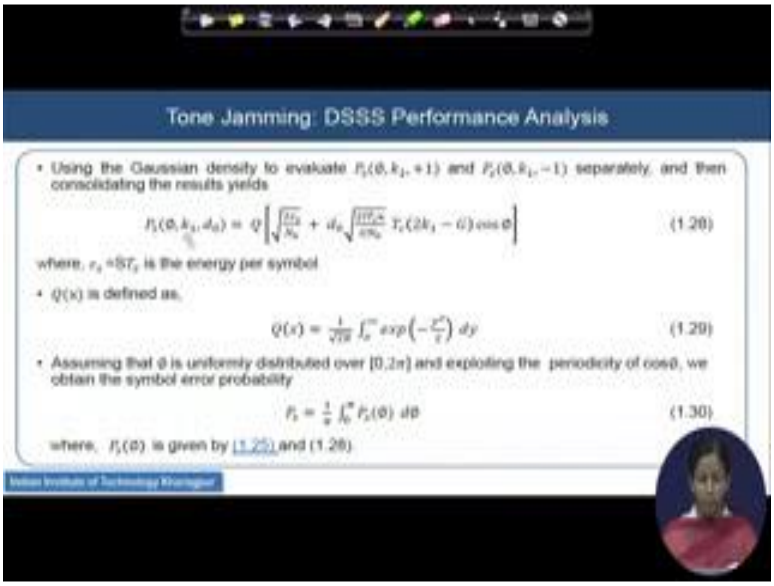

We are coming here and finally, we have reached to the P s specification of the P s. And substituting the value of this P s back to the equation of 1.25, we will be able to compute this P s phi. And now the total symbol error probability will be given by the final integration of this P s phi over the zone 0 to pi, because periodicity of the cos phi there is no need to go ahead with the 0 to 2 pi zone. And averaging over the pi out of that will be ending up with the competition of the symbol error probability.

So, these equations and these considerations will be carry forward it in the next two modules, where we will see that what will happen if we shift the carrier frequency the jamming frequency close little bit from the targeted carrier frequency. But it will be close to the carrier frequency. What I mean to say is you could assume what exactly is the carrier as a jammer, but you could not exactly estimate the transmission frequency or carrier frequency of the intended signal, but you are very close to that. So, whether the consideration whatever we have seen in this expressions, whether how this expressions will keep on changing will be the constitution in the next module. Please remember we will refer once again equation 1.25, where we consider the P s phi, and where the P s phi is the based on the conditional symbol error probability.

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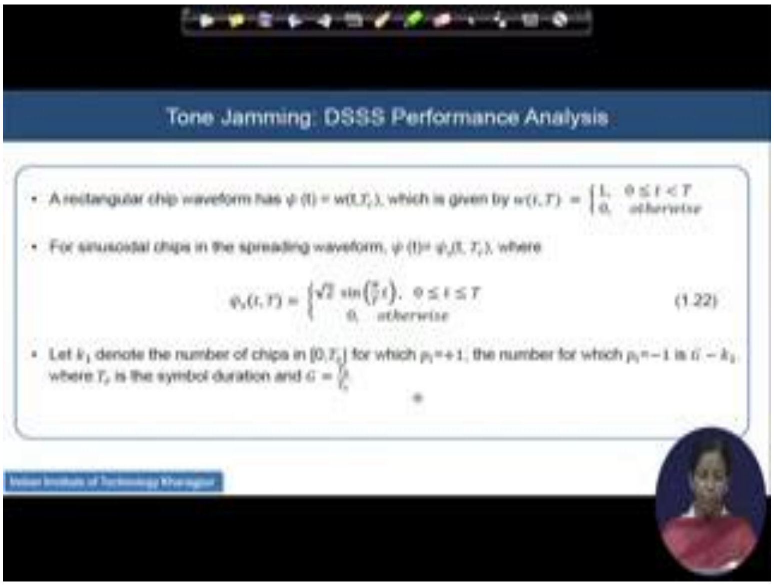

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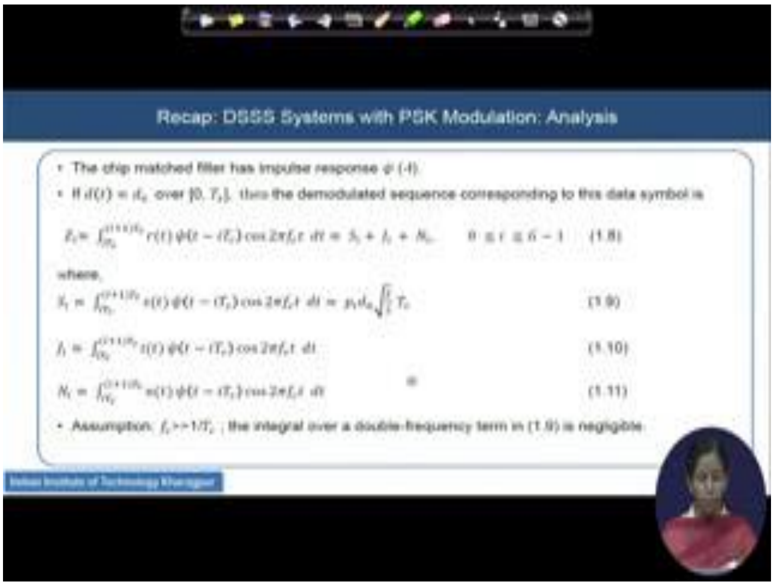

We will recall the pulse shapes here we have utilized to establish the expression both the rectangular as well as sinusoidal chip. And we will also recall the contribution of this symbol section on the whole computation of this computation of the V which is entering into the decisions device. And the concept of this Q function and final symbol error probability in view of the different nature of the interference with different kind of the carrier frequency will be continued in the next.