Discrete Time Signal Processing

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Lecture – 05
Graphical Evaluation of Discrete Convolutions

So we continue with that exercise. We are (Refer Time: 00:28) convolution at n equal to 0 first. So, multiplying n is 0. So, h r into g r, for all r we have to multiply and then sum.

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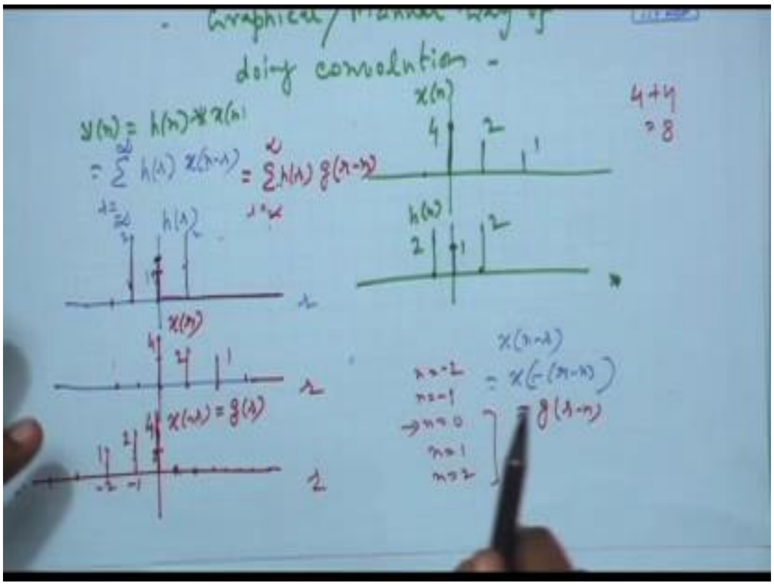

So, this is h r sequence, this is g r sequence, g r is nothing, but x minus r. These are the definition, g r is x minus r. So, it gets x, this sequence gets flipped; that is from positive side it goes to negative side, it is called flipping g r is nothing, but flipped version of x r x r was here 0 is sample depends as it is, but first sample is goes to minus first, second sample goes to minus second. So, right hand side goes to left hand side. If there are left hand side sample they would have gone to right hand side also, because this is minus, so g minus 1 means x plus 1 g minus 2 means x plus 2.

From right hand side they go here g minus 1 minus 2 g plus 1 means x minus 1. So, x minus 1 from here would have gone to the right hand side, g plus 2 means x minus 2, from here it would have gone to 2. Therefore these are zeros, so there why zeros are going. At we have to now carry out the convolution for various values of n, to understand the matter. So, I am starting with n equal to 0. So, n equal to 0 means just g r x r g r for all the r s and add. So, if I just multiply for various r r equal to 0 this into this, r equal to minus 1 this into this, but I equal to 1 0 into something r equal to 2 0 into something.

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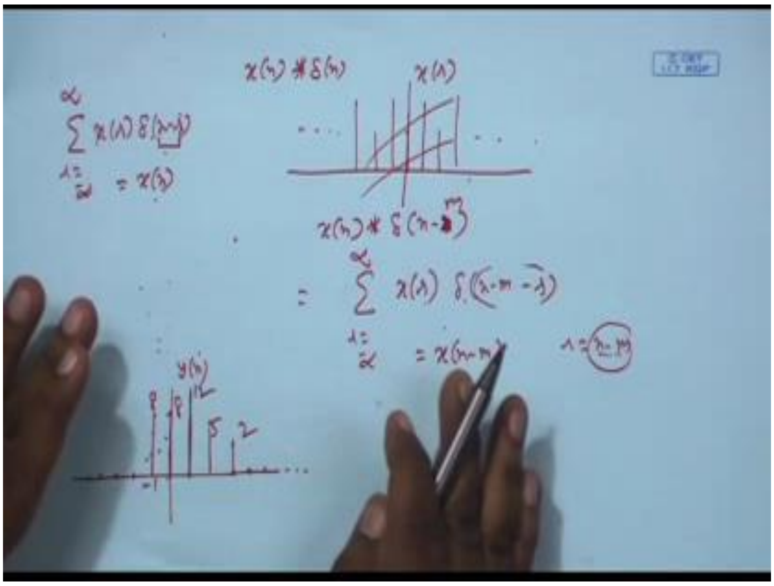

So, they do not contribute r equal to minus 2, so this into 0 r equal to minus 3 0 into 0 r equal to minus 4 0 into 0, they do not contribute anything. We are left with only this two terms 4 into 1 plus 2 into 2 4, so 8. So, output is 8, output, convolution output if it call it y n n equal to 0 1, I mean 8. Then consider n equal to 1, n equal to 1 means is r minus 1. So, g r is to be shifted to the right by 1, then I have to carry out the product so; that means, if you take this g r, if you shift to be a right by 1 4 will go to the 1 2 will go to here. So, it will be like this, which is equivalent to x of 1 minus r 1 is n. So, this will be g r shifted by 1. So, 4 will go to 1. Here the value will be 4 at point number one. Then 2 will go to 0, 2 and 1 will go from minus 2 to minus 1, and then zeros.

Now if you do sample wise multiplication 4 into 2 2 into 1 1 into 2 and then 0 into 0 0 into 0 they do not contribute anything alright, 8 plus 2 10 plus 2 12. So, this value is 12. In a same manner for n equal to 2, n equal to 2 how much will be, n equal to 2, this is shifted to the right by 2. So, minus 2 will come below this, this point will come below this, that will be raise to 1 into 1 this, 2 will come below this that will be raise to 2 into 2. So, 2 into 2 4 and 1 into 1 1 4 will come here, but that as 0, that as 0 hence no contribution. So, only minus 1 will come with this, because shifted by two. So, minus 1 will come here, sorry 1 will come here from point number minus 2 to point number 0 1 will come; jump by 2, it will find 1. Then 2 will move to here it will find 2, so 2 into 2 4 1 into 1 1. This 4 will move here, but it finds 0. So, it is nothing.

So, 4 2 into 2 4, then 1 into 1 it is like this. This is getting shifted from, already shifted one it is now getting shifted further by one. So, this will comes here, these 2 goes here, this 4 goes here. So, if you align with this these with these, these with these that is all. So, you have 1 plus 4 5, then n equal to 2 if we further shifted. So, you will have 1 going here, 2 going here, which means 1 going from 0 to 1, only 1 here 2 here 4 here, 0 moves from left side. So, only these guy into these guy that will survive. Here 0 into this 0 into this 0 to 0 nothing I think and they had 2 into 0 4 into 0 and so on, means nothing. So, 2 into 1 you see this enter sequence shifting gradually to the right and overlap initially I had I mean this much x r g r. So, overlap it was overlapping between these two and these two others where you know either non 0 here to 0 here or non 0 here 0 here and 0 in both places, so these two places.

Then it move further to the right 4 came under this 2 came under this 1 came under this that time I had maximum overlap if I align 1 came under this 2 came under this for. So, maximum also you are overlapping if I shift further four goes out only 1 and 2 they are overlapping with this if I shift further only 1 comes below this. So, overlapping only here and otherwise no overlap if I shift further 1 will also go out. So, there will be no overlap they will be multiplied by 0 from here these will be multiplied by 0; that is if you have 1 moving further that is for next n.

So, 5 into r before we complete this when for this case 1 into 2 only; that means, here we just 2 0 1 2 3. So, n equal to 3 for n equal to 3 also we found out value is 2, but if I go for n equal to 4 then what will happen in this 1 will move here, what will happen is this, for n equal to 3 1 came below this right. So, 1 into 2, it will further move, it will be like this. This one will move here 2 here 4 here, but this is outside, this will find 0, this will find 0. So, no overlap this is meeting 0, this is meeting 0, this is meeting 0, this is meeting 0 this is, no overlap, so I get 0 value.

If it shifts further to the right, continuous we do that the there will be no overlap. So, it will remain 0. So, I will have 0 values, which means after this we have 0 0 0 0 zeros; that is for the positive side of n. For the negative side of n minus 1 minus 2 minus 3, if it is minus 1 it will be g r plus 1. So, g r is to be shifted back by 1, and then I have to see the overlap multiply. This you can you can carry out. If g r is shifted back, this 0 will come here 4 will be here, 2 will go 1 will go. So, a 0 comes here 0, 4 comes here, 2 comes here, 1 comes here and 0 0 all this zeros, only this 2 overlaps 4 into 2, this with 0, this with 0 this with 0 this with 0. So, on 4 into 2 is 8, which means output will be 8 and minus 1. If I go further back n equal to minus 2, it will be g r plus 2. So, g r will be shifted back by 2; that is this will be shifted by 1.

This is already shifted by 1 to the left one more. So, 4 will come here, 4 will come here, at minus 2 with point here, this 0 will come this 0 will move here another 0 here like that 4 2 1 dot, dot, dot, dot, dot. Now you see there is not overlap between h and this, this bit 0, this bit 0, this bit 0, this bit 0, this bit 0 this bit 0. So, output is 0, and as n becomes more and more negative output contributes to be 0, which means 0 0 0 0 like this, this is the output alright. Suppose I have an x n, if you convolved with delta n what is the real. So, you write again x r, x r is (Refer Time: 12:16) dot, dot, dot, dot. If these we can do directly from here, so x r delta n minus r right r equal to minus infinity to infinity. Now, this delta, this is one only if the argument is 0. Argument 0 means r equal to n r equal to n it becomes x n, and if r is not n for any other index non 0 index here, delta value is 0. So, output is 0 so; that means, only term survives is x n into 1; that is x n. So, if you convolved then outputs are any index is same as x n.

So, you get back in your original sample. So, you do not need to do graphical. So, convolution by this delta n is always same as the when. If you say we take a sequence convolved with delta n you get back the sequence, because at any index n if you are convolving you get back the same sample x n. As I told you n minus r only if I did is 0 this delta (Refer Time: 13:28) will be 1, why it is 0 r equal to n, r equal to n means x n into 1 x n, for any other r in this range it will be 0. So, do not count. So, x n will come out of this summation. So, convolution between x n sequence and delta sequence, if you convolve evaluate the convolution at any nth index you get back your x n. So, all samples remain as it is. So, sequence (Refer Time: 13:51) as it is. So, convolution by delta n does not change anything. If you convolve y, so delta n minus suppose 1 or delta n minus r, then what happens n minus r sorry this n minus m.

So, if you write x r delta n minus m minus r r equal to minus infinity to infinity. Here this argument are should be searched that this argument is 0, then only this delta is 1, if that x goes out that into 1, for any other r delta will be 0. So, r equal to n minus m when r takes that value in this when moving in this way from minus infinity to infinity, when it takes these value we may have got x n minus m that goes out that into 1, all other values are 0. So, if you convolve x n with some delayed delta, delta n minus m convolution output at anything the x will not be original x n, it will be shifted version, delayed version by m, which means every sample is shifted by m. So, sequence is shifted by m, is positive, if m is positive shifted to the right, m is negative shifted to the left. So, convolution with delta n minus m, these basically shift your sequence.

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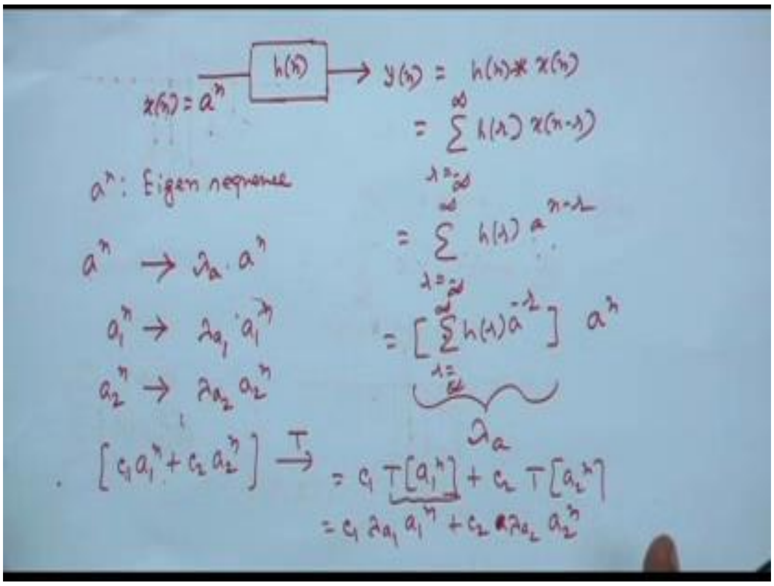

Now, before I conclude this section today, one interesting property I have to show, that suppose there is a linear shifting (Refer Time: 15:47) is impulse response to the h n. You give as sequence x n of the form some number a real or complex to be general is complex a to the power n, not causal n positive if n negative for both you get values, what is the output. Output will be convolution and I prefer h convolve with x that form, x n is a to the power n. So, x n minus r is a to the power n minus r. Now the beauty is this is coming in the power. So, a to the power n minus r you can separate out, is a to the power n into a to the power minus r, and a to the power n can go out as common, because that does not depend on r. So, it will be, into a to the power n out as common, what is it, this summation part over r.

Now, you see this summation does not depend on index n, it is a global, it is a constant, you are just varying this summation you know value of a, you know value of h carry out this summation of offline say is a constant as to the power n of n. So, for any index you have to find out y n you have a to the power n by this common constant, which means a to the power in 35 sequence that if it is input at to a this linear (Refer Time: 17:40) output is same a to the power n, just multiplied by a common constant for any n. For every index n if it is a to the power n it is the same a to the power n, only thing is it as got a common amplitude factor constant for any n.

This sequence is called Eigen sequence, because if you input you can give it a name lambda as a function of a, a is your choice for lambda is a function of a times a to the power n, this is a constant, this is not a function of n, this is true for any n. So, this if you input a to the power n you get the same a to the power n, all samples get multiplied by a constant factor amplified or at (Refer Time: 18:34) by constant factor, it is called an Eigen sequence. This is the very important thing, and it comes from these concepts of Eigen value Eigen vector of matrices.

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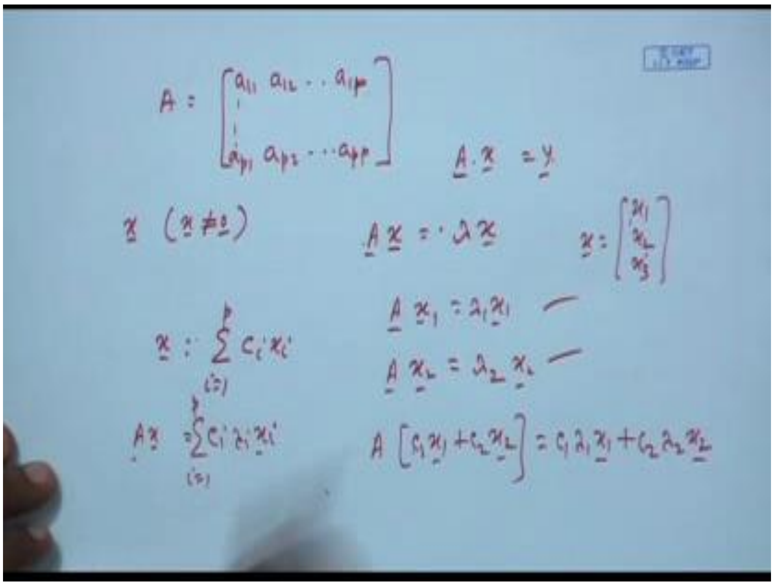

Suppose I give you a matrix a alright, square matrix. Now if you multiply a with any vector x you know how to do this you will get another one, but y and x is not in general that y is just x all sample multiplied by a constant is not, some x some y, but for this matrix is there exist a class of vector called Eigen vectors x, where x must not be 0 of course, by definition, underline this vector, underline this matrix, upper case later underline matrix, lower case later underline vector.

There is a if this is a class of vectors non 0 vectors. So, that for them if you do a x you get back same x, all samples multiplied by a constant lambda. Then that x is called an Eigen vector of a, and this lambda is called a Eigen value of corresponding Eigen value. It means suppose I am in a three dimensional world. So, x is 3 coordinates x 1 x 2 x 3, these are your x you get in a three dimensional world, this is x coordinate this is y coordinate z coordinate. So, it is pointing to a vector, pointing to a vector in general a into x will be another vector of three elements. So, that will be another direction, but if it is an Eigen vector, then all elements multiplying by lambda will be the output, means vector direction remain same, every value, coordinate value just gets expanded or contracted by constant factor lambda.

So, direction does not change; a times x keeps in the same direction, only value of magnitude or length of the resultant vector is different, maybe it is lambda times or either one, length changes, but direction does not changes, then x is an Eigen vector lambda is an Eigen value. Why this is important, is because suppose I give you 1 Eigen vector x 1, and it is having Eigen value lambda 1 Eigen vector. So, this satisfied. Another one, suppose this you know what is lambda 1 you know, what is x 1 you know. Another x 2 there is also another Eigen vector lambda two times x 2, then if I give you neither x 1 nor x 2, but a linear combination, you can carry out this product from this knowledge, because a into c 1 x 1, c 1 is a scalar. So, c 1 can go out a x 1. You know a x 1 is lambda 1 x 1 directly put from here, and a into c 2 x 2 c 2 can go out a x 2 which is lambda 2 x 2. So, just c 1 times this into times this. You do not have to re compute this if you know then; that is the big advantage.

Now, the question is, if I give any arbitrary vector x, if you could write it as a summation of Eigen values, Eigen vectors if you could write it, as a summation of c I x I i equal to may be 1 2 how many p. So, there can be maximum p Eigen vectors, then a into x is very easy to compute; a times this c i go out a x i is lambda a x i, c lambda i x i. You do not have to carry out this product, you just up to add them, and these are big advantage. Same thing here, but I should be able to write x, any x in this form, that depends on the Eigen vectors. In some cases you can, some cases you cannot, a to the power n give you lambda a, a to the power n alright. So, suppose I have a 1, a 1 to the power n this is also of this form. So, it will (Refer Time: 23:30) of this similar form, it will get Eigen function, Eigen sequence of this, output will be same lambda, but lambda a 1 because it is not is a 1 times a 1 to the power n.

Suppose I have given you a 2 to the power n is lambda a 2 a 2 to the power n, is not. Then if I give you a new sequence c 1 a 1 to the power n plus c 2 a 2 to the power n, what is the output if I want to find out there is if I want to convolve h with this. I really do not have to carry out one solution, I know that my operator this linear system. So, T if it is a operator t, t working on this will be response due to this, and response due to this because linear system. And response due to this means c 1 can go out, t working on a 1 to the power n and c 2 can go out t working on a 2 to the power n, and the two responses will be added. And this I know what this response is, because this Eigen function. So, I can directly put I do not have to calculate; c 1 lambda a 1 c 2 lambda a 2 a 2 to the power n.

So, this is the big advantage, giving this kind of sequence which is a linear combination of Eigen sequence is, their output can be calculated directly from then, then only. I do not have to calculate the convolution and inverse. So, many terms you know and multiplying adding and all those things, very simple. Question is if I give an arbitrary sequence x n, if I give an arbitrary sequence x n, not necessarily in this form I have combination of, can I still write it as a super imposition, there is a summation of this Eigen sequences.

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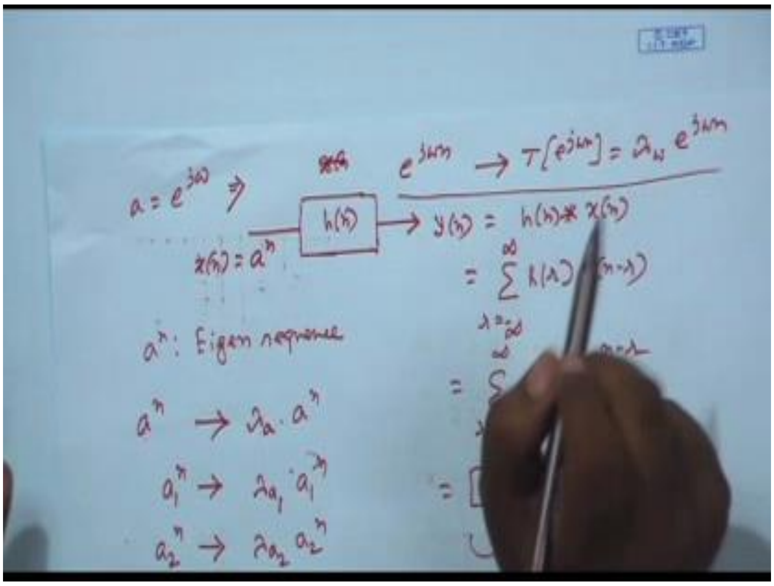

If so, then convolution of that x n with h and n becomes dam easy, we will show in the next class then if you take a to the power a as, say e to the power j omega, there is a complex number with magnitude one. Then it is possible to get or for any arbitrary x n and you know e to the power this, there is in you are giving e to the power j omega n as the input, you are getting t working on e to the power j omega n as a output, which will be some lambda, may be a function of omega e into the power j omega n, because it is an Eigen function it is of the form a to the power n e is e to the power j omega e to the power n comes back lambda omega, then this is given.

Then for your arbitrary x n also, I can write it, I can express an arbitrary x n in terms of these e to the power j omega n kind of Eigen sequence as a linear combination of then for various values of omega, and so therefore, output can be calculated easily without carrying out convolution system, more of that later in the next class.

Thank you very much.