Discrete Time Signal Processing
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Lecture - 30
Structures for FIR Filters

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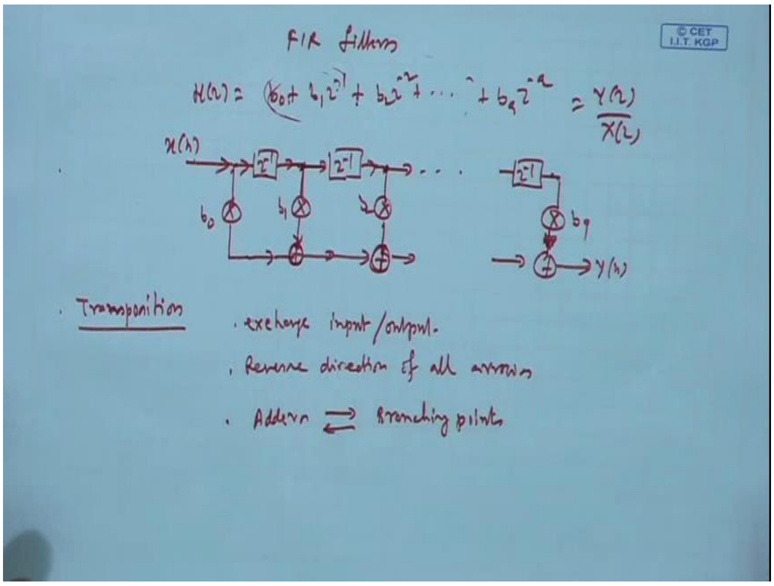

Now consider about FIR filters. Suppose you have got H z as b 0 then b 1 z inverse b 2 z inverse 2 to dot, dot, dot, b q z inverse q and this is equal to Y z by X z where, its simply Y z is X z times the entire things. So, X z into b 0 plus X z z inverse into b 1 X z z inverse in to b 2 like that and add it. This form come for easily x n that is X z into b 0, X z into z inverse then b 1. So, there is a delay then b 1 and this to be added because of this plus, that X z into b 2 z inverse 2. So, z inverse 1 into X z is already here one more z inverse. So, two delays, delay divide by 2, and then multiply by b 2. Here I take b 2 and these are to be added with this result, that is why this summation is coming here accumulation with that you are adding dot, dot, dot. Lastly it will be multiplied by. So, here we have got z to the power minus q into X z. Now here b q that will be added with whatever has been accumulated till then it is coming from this side sorry this is other way output, this is very simple.

You can get another form by using transposition. Transposition means actually transposition comes from graph theory there is a transposition theorem. We will not get that here, but that is very useful in our network analysis, but you can apply transposition here by simply following a rule that you find out the notes. There are two kinds of notes here say, adder I take adder as one note and this are called branching points. The adders then there are branching points.

By transposition what we have to do you know exchange input output, reverse direction of all arrows and adders replace by branching points adders will be replaced by branching points and branching points will be replaced by adder. If that be the case I apply to this you get another structure and this theorem says that will be equivalent to this. That is for the same input exchange, same output will produce. So, that is comes from the theorem since you are not giving a proof of a theorem, we will just apply this rules on this structure derive the transpose form and see will directly manually verify, that the equivalent that is for the same input exchange you get the same output y n. So, here the delays same as it is, but all is in the arrows are reversed this is branching point. So, branching points means adder let me draw it in another page.

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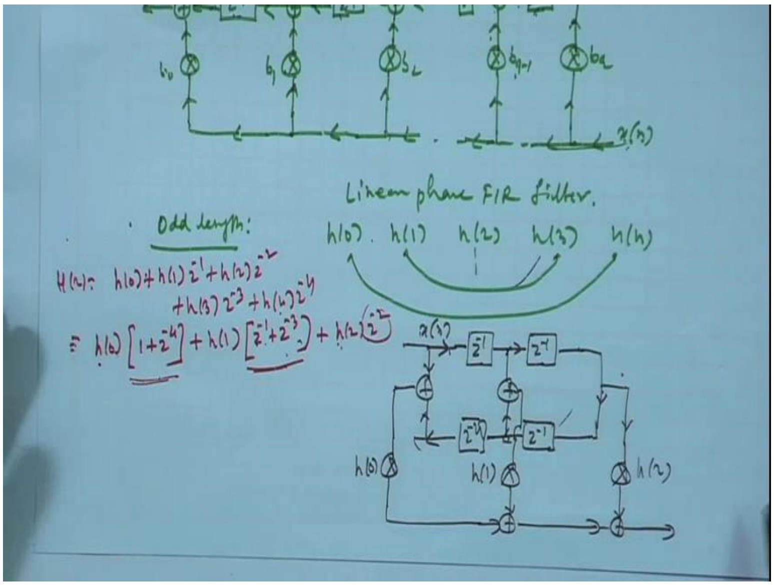

This is branching node here. So, branching node becomes adder as I told you and this arrow, this a arrow this arrow, arrows gets reversed. Arrows here (Refer Time: 04:38) in this direction, now it will go up, otherwise multiplier remains as it is b 1. Then I get the arrow will gets reversed. Earlier it was in this direction this arrow now it will change and basically branching point it will be a adder. This output since we have got verified that output will be same as the previous one for the same input exchange, instead of calling that y n which is used here. Let us call it y prime n we will saw that y prime n and y n are the same if you put this x n here also.

So, these arrows are reversed instead of going in this direction adders are branching. Previous adders are now branching point. This arrow is reversed in this direction earlier it was downward. Now this is up ward which was this, it should be this way at the same branching point. So, adders are 2 inputs, 1 output device and branching point is what 1 input 2 outputs. So, if the arrows are reversed. So, there will be 2 outgoing arrow, 1 incoming arrow so; obviously, it cannot be an adder it will be a branching point and is a branching point, there was 1 incoming 2 outgoing. So, now, after reversing the direction there will be 2 incoming 1 outgoing; that means, it cannot be branching point is as an adder, it is like that.

So, this arrow reversed in the branching adder, arrow reverse this again become branching point because it is an added. This arrow reverse, this arrow reverse, this arrow reverse, this is the input is going this way finally, taking this branching point I give the same input now, input, output have exchanged. This is the input now dot, dot, dot, dot, (Refer Time: 07:03). This is a structure these two structures are equivalent because what is happening here is the x n is going in this path, there is adder again it will be draw b q minus 1 dot, dot, dot. So, x n is going through this path b q into x n then that component gets delayed. So, b q x n, x n minus 1 there is b q X z into z inverse. There is a contribution coming from this side, they get added and then move forward, but suppose I take this contribution alone separately do not mix up with these, I will take this also separately through this path finally, I will add. So, this contribution goes like this x n into b q z inverse there again another z inverse, another z inverse, z inverse, z inverse.

So, that component which is here separately as b q into X z into z inverse q because there are q stages, there are total q stages b 1 to b q, q stages z inverse b 1, z inverse b 2, z inverse b q. So, if there are q stages that q z inverse. So, X zs into b q into if you take this that signal only do not connect this signals that consider this only separately. Then you get this part b q into z inverse q b q into z inverse q into X z this part comes up, this into X z and then x n goes through this again. So, X z into the b q minus 1 X z into is b q minus 1, this separate also goes. I am not mixing with this, suppose this goes to it z inverse, z inverse, z inverse, z inverse, total q minus 1 now 1 is gone here. So q minus 1 times z inverse. So, b q minus 1 into z inverse q minus 1 times X z dot, dot, dot, dot, x n goes this way x n into b 2 that contribution separately if you consider, x n into b 2 into z inverse into z inverse. So, b 2 X z z inverse 2 present then x n into b 1, there is x n into b 1, there only 1 z inverse. So, b 1 z inverse X z represent.

And lastly X z into this. So, X z into b 0 this is b 0 and they are all getting added. They got added here together moved, then they got added together moved, then they got added move together, move together. So, basically all are getting added and so you get this. If you want, you can carry out a transfer function analysis. This is I leave for you as an exercise Y z by X z you find out, y prime z by X z into find out. We will get the same thing that is very easy. If you apply in the case of IIR filter, this is verifying yourself. If you apply transposition to the direct form one you will get direct form 2 and if you apply transportation to direct form 2, you will get direct form 1. This you verify yourself.

Now, consider a linear phase FIR filter. In the last class, I considered them. By the way apart from this structure, FIR filter cannot cascade forms one. So there is if you factorize this into first order of factors and then if you give a complex root, then conjugate root or some has been there take those 2 factors multiply, you get second order factor and like that. So, we cascade off, is a product of various factors as the second order, first order each will give rise 1, sub filter either second order of first order you can arrange them in any order in a cascade form and that will give you H z. So, cascade form is available for FIR filter also.

Linear phase FIR filter as an example suppose you have got odd length say h 0, h 1, h 2, h 3, h 4. So, these two, this is the mirror. These two are same as so these two are same. Odd length, odd then, what is the transfer function? Transfer function is h 0 plus h 1 z inverse plus h 2 this is general form of writing z inverse 2 h 3 z inverse 3 plus h 4 z inverse 4 right, but there is equivalent to, if you have h 4 is equal to h 0. So, you take h 0 common 1 plus then take h 1 common and then h 2 z inverse 2.

So, output will be X z into this part into h 0, X z into this into h 0 plus X z into this into h 1 plus X z into this into h 2 because this is transfer function H z. So, output Y z by X z is this. So, X z into this whole thing which is equivalent to X z into the whole thing, that will be output Y z X z into the whole thing this whole thing means X z into this part multiplied by h 0. Here is X z in to this part multiply by h 1, here X z in to this part multiplied by h 2. So, this we can you do like this you see, if I proceed like this x n put a delay.

So, this is z inverse coming z inverse 2 coming put another z inverse. So, z inverse 3 coming for these and there is another delay z inverse 4. Now, X z into 1 plus z inverse 4. So, X z add z inverse 4 times x z. So, you add the two first, then this addition you will multiplied by h 0. So, this shapes the multipliers, you do not have one multiplied by h 0 into x n, another h 4 into x n minus 4 and then add. Whether you first add the two x n and x n minus 4, there is X z and z inverse for X z h, take a h 0 common. So, then you multiply by h 0, so you need h 0 or h 4 the multipliers only once not twice are unless before. So, you see one multiplier. Similarly X z into this part. So, X z into z inverse 1 and X z in to z inverse 3. So, z inverse 1 2 3 so here So, X z into z inverse, X z into z inverse 3. So, first you add the two, and then use a multiplier, second you are saving one multiplier. It should be having one multiplier for this, another for this. You are using the fact that they are same.

So, you taking them common first realizing this part X z z inverse X z z inverse 3 and then adding them and then multiplied by h 1 and then again here h 2 into z inverse 2 into x z. So, X z into z inverse 2 is already available nothing to be added. So, this I take out multiply by h 2 and then I go on adding, I get my output. Next consider case was length is even. So, h 0 to h 5, we will take.

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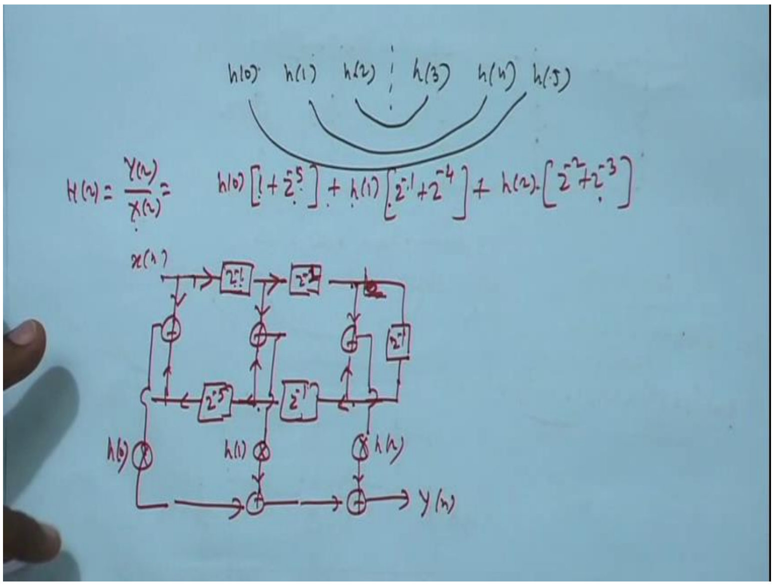

Let me write h 3, h 4, h 5. So, total 6 in this case this two should be same, this two should be same, this two should be same right. So, then you have got that means transform function is h 0 into 1 plus h this h 5 is h 0. So, that z to the power minus 5 then h 1 and h 4 they are same. So, z inverse of 1 for here z inverse 4 for there and h 2 and h 3 these are same so take h 2 z inverse 2 and z inverse 3. This is your Y z by X z transfer function, which means Y z will be X z into this part into h 0 plus X z into this part into h 1 plus X z into this part into h 2.

So, again multiply the same because you are adding this part first and multiplying only once earlier you needed multiplier by h 0 multiplier by h 5 separately. So, 6 multipliers would have required now only 3. So, what you have to do these your x n x n into 1 and z to the power minus 5.

So, one delay z inverse 1, then another delay z inverse 2, another delay, now this delay this is the interesting thing you put a delay here z inverse 2, z inverse 3, z inverse 2, z inverse 3. So, then sorry there are 1. So, z inverse 1, z inverse 1, z inverse 1, z inverse 1, z inverse 4 up to that have reached, but I have z inverse 5 also, 1 more delay. So, X z into 1 plus z inverse 5. So, 1 x n plus x n minus 5, n minus 1, n minus 2, n minus 3, n minus 4, n minus 5, perform here you take and multiplied by h 0. Then X z into this part x n minus 1 from here and x n minus 4 from here z inverse 1, 2, 3, 4 this is at first and then multiplied by h 1 and then add each adder and lastly this 1 X z into z inverse 2 that is here and X z into z inverse 3 that is here.

So, these two are added and this is again multiplied by h 2. I give you an exercise you apply transposition to these two structures for odd length and even length and get equivalent filter factors. This much I teach for filter structures.

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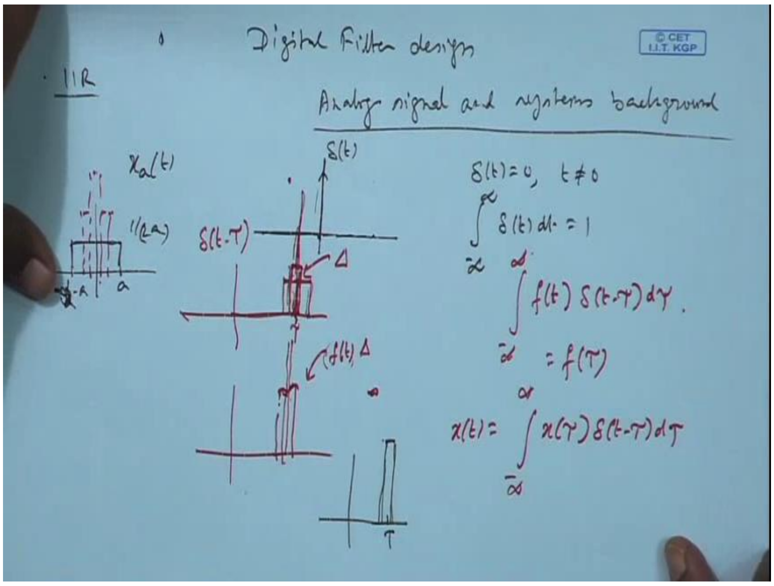

Now, we will be going for filter design, which is a very complicated not complicated, but because lot of insight and we need analogs signal processing background. We will take a quick review. So, we will now go for the digital filter design, IIR filter design for this. We require analog signal and systems background.

Now, I am not going to give a full treatment to this there is no time I will quickly review. In analogs systems you have got analog functions like x a a for analog t, but t is a continuous variable say time variable. There is one important function, delta function it is Dirac delta function impulse function, it means is going up to infinity and 0 else where there is delta t it is 0 as long as t is not 0, but it is not discrete. There are t is equal to 0 it is something, that at t is not equal 0 is not something else, then it is discrete not continuous, but it is a continuous thing.

So, the limiting function, that this will be 0 everywhere and when you approach one, it will be search area under this. I will explain area under this, it will always remain one. There is why they teach like this, the first you take a box. So, may be a minus a. So, area will be, I want to make the area 1 a minus a. So, total length is 2 a. So, this is a 1 by 2 a, 1 by 2 a. Now I want to make this width smaller as smaller because this is outside this line it is 0 everywhere. It is outside 0 or any point its 0 everywhere. So, this have be made thinner, but as a deceases 1 by 2 a increases. So, it will be like this finally, it is squeeze to 0 and this is suit to infinity in the limiting says area will be 1.

This function has another property. If you take minus infinity to infinity, any function f t say delta t minus tau d tau what will that be? So, this is like you know this box here delta t minus tau, there is a box here and this box area is 1. So, you squeeze and this will go up finally, go up and suit up to infinity area will remain 1. That is delta t minus tau around tau is around tau is start with a box. So, that within this box area is 1 this start squeezing. So, but keeping the area 1.

So, this will go up in the particular direction further go up as this width squeezes finally, go to this will go to 0, this will go to infinity, but area will remain 1 there is a minimum delta t minus tau, delta t minus tau into f t means first suppose I am not squeezed fully to 0 safety into one of this functions.

And then as you squeeze it what happens f, I mean if you first say I am going to situation like this. This is this much and I multiply this by f t. So, it will be like this, this much will be f t into this function I call capital delta. So, this capital delta into f t this height is flat. So, f t may be time variance. So, that is why it is like this and then as this delta, this becomes close to 0 and this suits up. If you still consider the area under this, it will be what? It will be this will be narrow, that it will become very narrow like this.

So, but area if this is not present area will remain 1 because of this area will be multiplied by the value of f t like, it was here. So, you have to carry out the integration, but if it was become thin enough, then the integration is not required in the value will be the roughly f tau and the tau center point what is the value? That will be say f tau into the area, area is one.

So, it will be f tau there is, it will be like this the function you know it will tau even if it is not done fully up to infinity, but this height will no longer be modulated it will be almost constant because this is so narrow. So, f t into delta with amount to multiplying this very narrow delta, capital delta with the center value of this function at tau and area will be. So, that f tau times original area that is 1, say f tau and as you squeeze it this height will go up area will remain same if tau into 1, this is f tau.

So, final in the limiting cases will be f tau. This is one property, this means any function x t can be written as x tau delta t minus tau d tau is like you know tau is varying continuous minus infinity to infinity. So, there are impulses on this axis moving as tau changes one impulse, another impulse, another impulse, another impulse like that, but there is continuously moving. Each impulse is multiplied by a constant x tau and is a linear combination. So, linear combination impulse functions say functions, this is typically x t were if I pass it through.

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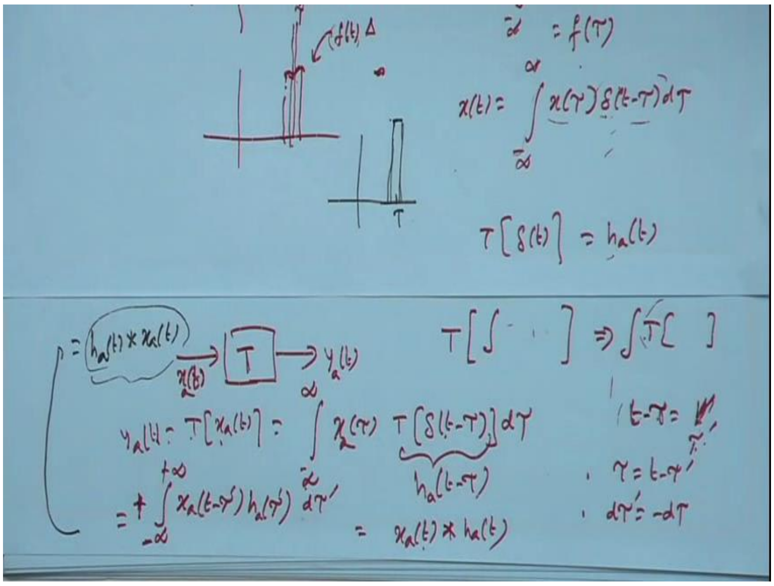

Linear and timing variant system, linear system means any operation on this integration or integration and summation. So, any operation on this summation will be summation of the operations, that is if I pass it through, system t analog system to have sorry it is X a t very sorry to have Y a t. Linear means t working on any summation, t working on any summation of signals. So, superposition of signals, they response will be summation of the individual responses. Purview summation means in this continuous summation that is why it will be integral t of this will be same as integral t of whatever you have. That is first you super impose then apply t apply this same and get response that will same as you apply t on the individual functions get their overall response and then integrate add.

So, if you do that here if you linear t X a t is very much like you know I mean the discrete case. So, you replace X a t by this and so t working on a summation of this functions, that function is delta function, delta t minus tau, tau is variable the function of the t is delta, the delta t minus tau of each tau in this integral there will be a shifted delta tau multiply by a constant x tau, factor x tau, x tau times on shifted delta shifted by tau, but tau is very continuously. So, we have shifted delta allover continuously and this if I pass through a linear time invariant system. So, after this superimposition of this shifted deltas and then apply t on it. You will get a same thing, if you bring if you apply t on this individual deltas, this is all shifted by some tau multiplied by the amplifying factor that factor constant that x tau, t on this and then you very that response about tau and get all the responses and then add. You will get the same thing because of the linearity.

So, that t if you have bring here t on these, but this is a constant actual function of time is this. So, capital t will work on this part. So, it will be your x tau, x a tau rather t working on these. If t working on this impulse 1 impulse suppose I given I (Refer Time: 28:58) delta function being input of a system I measure the output, I call it X h a t impulse response and then if it is time invariant. That is if I give the delta plus sometime tau give response full on the same time skill will be same as before, but just shifted by the same amount, delayed by the same amount tau.

So, if it is time invariant this will be h a t minus tau, which is analog convolution and t minus tau, now you can call it even you are write it as X a t star h a t. Whatever you have this, this star is analog convolution and then how to carry it out? First be an integral there is summation from minus infinity to infinity right the first guy as the first guy, but as the function of some tau local variable, integral variable then bring the next guy h a, but instead of t make it t minus tau, but tau was here.

But now in this integral t minus tau I can be called it t 1. What is t prime? Then what happens or may be tau prime say t minus tau I call it tau prime. So, tau will be t minus tau prime number 1, number 2 d tau prime is minus d tau t small t is fixed from outside you want to find out output at the particular time Y a t. So, t is fixed from outside, that t is not variable. You have to find out the output that tau is variable in the integration t minus tau substitute by tau prime, instead of tau, tau prime.

So, what is tau t minus tau prime number 1. What is d tau prime? This same as minus d tau t is constant in this because t is from outside and if tau goes to minus infinity t minus minus infinity, it goes to plus infinity and vice versa which is minus infinity. So, it will become plus infinity into minus infinity X a instead of tau, it is t minus tau prime instead of h a t minus tau t minus tau is tau prime and d tau is minus d tau prime. So, minus will go here d tau prime and then minus can be absorbed by again reversing the sign of the integral.

So, in this case it is convolution between in terms of notation, it is h because h a tau prime into X a t minus tau prime. So, h a t convault with X a t and take by what? This is more covariant for all analysis, as we did in the discrete time phase. We will continue from here in the next class, cover some more part of analog signal system before I go in to the filter design issues.

Thank you very much.