Discrete Time Signal Processing
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Lecture - 29
Structure of IIR Filters

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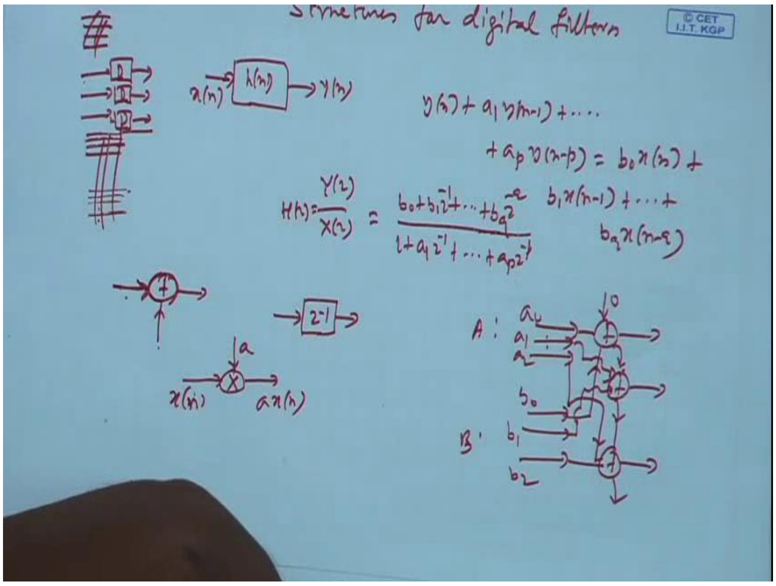

I will today consider structures, structures for digital filters. Remember, though we consider linear shift in period systems in general, we then took rational system as an x n y n. We took a special kind of linear shift invariant system, which was a rational system. That was y n plus a 1 y n minus 1 dot, dot, dot, dot, a p y n minus p it was is equal to b 0 x n plus b 1 x n minus 1 dot, dot, dot, dot, b Q x n minus Q, at with zero initial condition. It has linear at shift invariant and then we found out, it's transfer function. That is linear and shift invariant Y z by X z get that transform of h n because x n into h n was y n. So, here if you take Y z by X z there is also h z because this system is linear and shift invariant, under zero initial condition.

And then we find out to be of the rational form or ratio of two polynomials, numerator polynomial by denominator polynomial. Numerator was b 0 plus b 1 z inverse dot, dot, dot, dot, b q z inverse q denominator was 1 plus a 1 z inverse dot, dot, dot, dot, a p z inverse p. 1 b n we choose this special kind of linear shift invariant system is that, by choosing a 1 a 2 and a p this coefficient values appropriately, we can ensure that the poles there is if you factorize this denominator polynomial into faster of the factor the roots are called poles.

So, they depend on the choice of a 1 to a p by choosing a 1 to a p appropriately you can make sure that all the poles live within unit circle, which will ensure the causality and stability. Which is general not possible to ensure for an arbitrary h n, because when h n must be causal there is a h n should be 0 for less than 0 and it should be after (Refer Time: 02:42), but how to impose that? How to enforce that? How to make sure that you chosen h n daily satisfies at least the stability part? In generally it is not possible, but in this case rational system by just choosing a 1 to a p appropriately, we have the access to put poles here, ROC gone by poles. So, all the poles, unit circle that the ROC is outside that, which contribute unit circle and you can ensure stability and causality.

But another very important thing, why you choose this model is that, we can implement these things in digital electronics, using digital electronics, using some basic building blocks. One is a adder, this adder actually in digital electronics these what you write an adder, it takes 1 bit another bit 2 bits at a time and there will be an input carry that output bit and output carry. But this adder, actual is not a single bit adder and when I say this line; this is not a single bit line. This line can be a bass actually like this below 16 bits.

So 16 lines in parallel, there all margined to 1 line in this diagram, but this is a bass, this bass. So, this adder bits if that are 1 set of bits mid lines for one data, another set of bit lines for another data. Did you do addition of the LSB of this with this, then take the result out then take the carryout and then add the next two LSB, next two LSB of that, with the carry that is coming from the previous addition. Then generate new result, propagate the new carry like that you know carry of two adder. This is that kind of adder. That is in terms; of digital electronics it is like this if there is LSB, LSB of one data LSB of another data.

Suppose it is a 0, b 0. Here put the h 0, a 1, a 2, 3 bits consist of 1 what? 1 bass has got just three lines here also say b 0, b 1, b 2 and if you have to add a with b this is A, this is B, 3 bit data. 3 bit data you have to add A and B. What you do you? You add LSB with LSB as I told you with initially carry 0, result and new carry. Then again with this new carry add next 2 LSB and next 2 LSB, new result, new carry like that and again here this carry goes in, this goes in and this last bit comes in here new result, new carry. So, 3 full adders in parallel. So, this total array is indicated by this adder and this is one line, one bass of say whatever be (Refer Time: 05:35) is another, but this is only a schematic. This means a structure like this, where there will be parallel lines for the bit is of 1 data, another parallel line for the bits of another data and there is bit wise additions starting from LSB and carry propagation. This is a meaning of this.

Then another tool, that is used is this delay divided bY z inverse. This does not mean one flip flop. This means, if there are bit lines, say if there is bit lines one, another one, another one so three lines, these three lines mean actually one bass and if I delay this means every bit will be delayed by a D flip flop. This is the meaning of this delay. And another is multiplier. Multiplier normally these are called constant multipliers. So, there will be a signal line, there is a bass at this multiplier. There is a built in constant may be say a. So, it will be a times x n. So, multiplier will take all the bits of this bass, multiply that with the built in constant. So, this a is not a signal, it is built in. With that multiply and delayed the again result also the bass. Using them as a basic building blocks I will mix we can then construct this filter. These are an IIR filter we start with that and then will consider FIR filters also.

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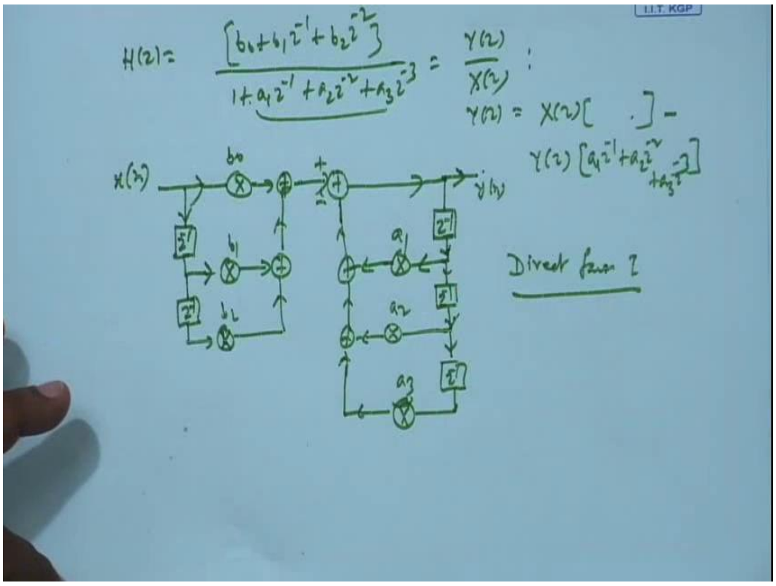

So, suppose we have got as an example H z of the form this, b 0 say b 1 z inverse just as an example say b 2 z inverse 2. You write by say 1 plus a 1 z inverse. I am taking numerator power of z inverse less than that of denominator as I told you. If it is equal to or higher than the denominator power. You have to divide the denominator by this, take the reminder on top and there will be some task like some constant in to z inverse some constant will z inverse 2 some constant in to z inverse 0, there is constant only like that.

So, there will realize separately what I am bothered about is. It difficult form when numerator degree or numerator order is less than that of the denominator. So, as an example you consider this. Three terms on the top and after one again three terms in the bottom, but this means Y z by X z right. So, X z into this which means X z into b 0 plus X z into b 1 z inverse. So, this is suppose X z there is x n. X z into b zero means x n in to b 0. Then X z into b 1 z inverse, z inverse X z means delay of x n by 1.

So, suppose there is delayed and after that multiplied by b 1, then X z b 2 z inverse 2 z inverse 2 X z means in time domain x n minus 2 already have x n minus 1. So, one more delay and then multiplied by b 2 and they have to be added this plus and this plus. So, there they get added.

So, what is generated here X z into this numerator that part is generated, now Y z by X z this means Y z is equal to we want to find out the output Y z. So, Y z in to 1 is X z in to this numerator part, that is already available here now minus Y z into this part. So, from this I have to subtract. So, adder becomes subtracted actually because how do to the subtraction taking this compliment of this number mean subtracted and then add. These are basically for (Refer Time: 10:00) I means you all know that. So, X z in to this there is already available here minus Y z into this. Suppose my Y z is here, that is y n in time domain Y z. So, that will be multiplied by a 1 z inverse. So, z inverse means 1 bit delay one cycle delay, 1 bit 1 cycle delay z inverse then multiplied by a 1 then a 2 z inverse 2. So, z inverse 2 means Y z in to z inverse 2 means y n minus 2. So, already y n minus is here.

So, this and then a 3 z inverse 3 means y n minus 3. So, already y n minus 2 is here. So, we have one more delay, sorry multiplied by a 3 and they have to be added. So, add with this and then subtract, after you do that you get Y z. So, this becomes your Y z. This relation is about simplest form of relation is called Direct form 1. I can make a more efficient version of this called direct form 2. In Direct form 2, see what is happening here is you are first doing X z into this part numerator. Then in the second part you are generating Y z in to this part and subtracting I am giving this. So, X z in to numerator,

numerator means it give rise to zeros. So, X z in to 0, 0 part first here and the poles which come from denominator that is second. That is how it realized.

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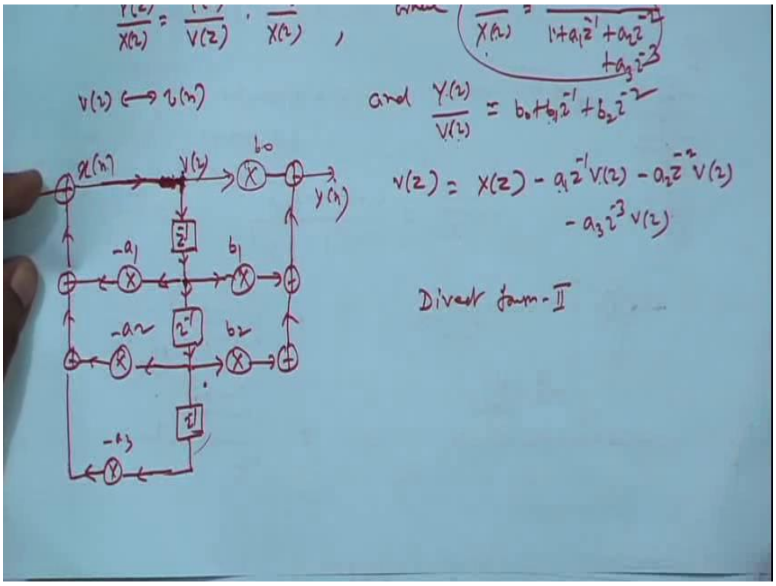

But I can interchange the 2 y. So, how and then I can have a better realization I have to take less number of adders. For that, Y z by X z, we write like this Y z by may be some V z into V z by X z where, I V z by X z I take to be either first delayed poles, so 1 by this part, V z by X z. So, I have generate, I have introduced a new variable V z, V z and V z cancels you get Y z by X z, but V z by X z I take to be this is where I take to be 1 by this. So, down this if I realize this I may handle poles only, and the other one Y z by V z, that will be the numerator the zeros. So, this will be first, pole first then zeros. There is interchanging the two here. This is equal to b 0 plus b 1 z inverse plus b 2 z inverse 2.

Now first generate V z by X z. V z in z domain means small v n in time domain. So, how to find out V z by X z, V z in to 1 is equal to X z into 1 X z minus you take a 1 z inverse with V z to the right hand side. So, this will minus. So, it is like this, V z into 1 you keep on 1 side another side you have X z into 1 X z minus a 1 z inverse V z minus again V z minus a 3 z inverse 3 V z. Suppose here is your x n and that v must be X z, x n means X z that I multiply that I delay. So, I introduce a z inverse. Suppose I multiplied by minus a 1. So, minus 1 z inverse suppose, if there is suppose somewhere rather have V z here somewhere rather have put v z here. Start from here V z. So, V z somewhere rather I got some V z, z inverse into minus a 1 that is this and V z z inverse 2 ,this one more z inverse into minus a 2 after minus has been observed in a 1, a 2, a 3. So, they have to be added now.

So, you add this. So, result you add with this and then you add with this. Whatever you get that is V z. So, now, draw this line. So, V z generated this is your V z. Now what is Y z, V z into b 0. So, if it is V z this goes in this direction also multiplied by b 0 plus b 1 z inverse V z. Now z inverse V z you see the advantage is V z in to z inverse, z inverse V z is already available. I do not have to apply another delay. Just take a line out from here multiply by b 1 and this z inverse 2, V z take a line out and multiply by b 2 and then add all of them you get your y n.

If you now compare you see, you are having the same number of multipliers and adders that is not a problem as compare to the previous structure, but here we have got adder, adder, adder, adder, 3 adders, that I am not having. I mean I am having I mean you are using delay, how many delay? 3 rather delay is optimize here, z this 3 on this side, 2 on this side, but now I am using only three delays. 2 delay this it same actually an array of d flip lop because this is a bass. So, there is legacy of register.

Another shift register, another shift register. So, here we require less number of delays here as compared to that. If you realize pole first like this said by giving X z generate V z v n and then from V z then you generate y n, V z given you generate Y n. If you do this way then, you save the number of delays. This is called direct form 2. Then there are other forms also, suppose 1 H z like that is given is of this form is given b 0 plus b 1 z inverse and p greater than q.

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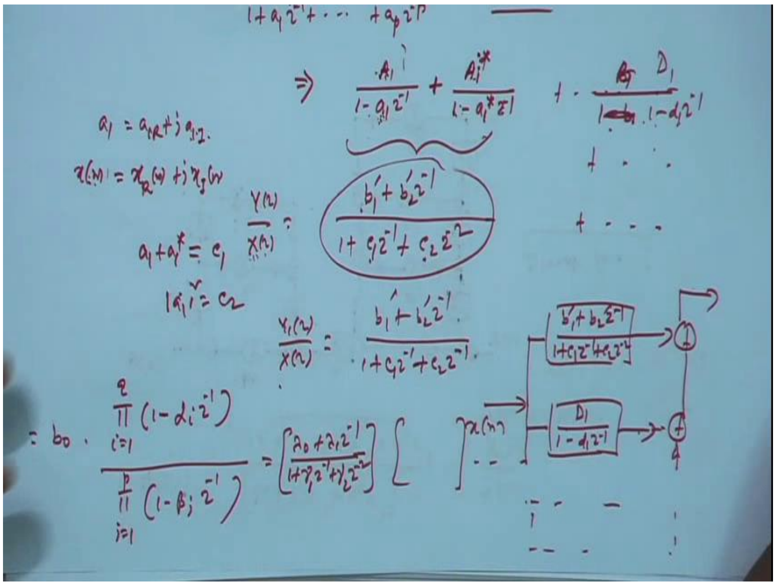

So, you can do partial fraction, partial fraction means you can have some term like say A 1 1 minus a 1 z inverse and assume first order pole here and then, you will have may be another term. If this is complex there must be a 1 star z inverse and this also should be star because then if you combine A 1 into 1, A 1 into star into 1, A 1 plus A 1 star become real because finally, my coefficients have to be real and A 1 into minus a 1 star and a 1 into a 1. If it is A 1 star small a 1 into capital A 1 star and small a 1 star into capital A 1 they conjugate up each other. So, you add them again you get real.

So, if you do this factorization if you have complex roots as I know that, since the coefficients are real not complex then if I root is complex then, we must get conjugate root also. At this coefficient because A 1 now I made up this statement that this coefficient should be the A 1 star because then if you add A 1 into 1 A 1 star into 1. If you add A 1 plus A 1 star that is real then minus A 1 a 1 star small a 1 star z inverse, z inverse you take common and again minus, minus z inverse you take common.

So, capital A 1 small a 1 star capital A 1 star small a one. So, and they are added. So, they are conjugate up each other which means, you get real. So, in such cases and there will be other terms. In such cases you do not leave it like this because if a 1 is complex then if you want to implement it. There will be multiplied by a 1, but a 1 will have it is real part and imaginary part, so a 1 R plus j a 1 I. So, even if you are multiplying some real data x n with a 1 you will have a 1 R into this and a 1 j into this right a 1 I into this.

So, two multiplies are equal. Multiplies are costly because they require more hardware. If x n also is complex. So, it has got a real part imaginary part. So, you will have, you know four multipliers. This into this minus this into this that will be like you have been x n has got suppose x R real part n plus j x imaginary part n.

If now multiply this into this then minus this into this, that will be real part. So, two multipliers and cross terms two multipliers. So, four multipliers are required. Cross term will give rise to imaginary part. If it is real a 1 R into x n 1 multiplier and a 1 I x n is another multiplier. Still two multipliers or multipliers are costly, you can avoid then by just using the fact that if there is a root at a 1 there is root a 1 star and this 2 coefficients are also conjugate up each other. So, if you add them this part will give a rise to A 1 plus A 1 star something real.

So, may be b 1 capital A 1 plus capital A 1 star that is a real quantity of complex term and it's conjugate, if you add you get real number call it b 1 and then minus z inverse if you take common as a told of a while ago. Capital A 1 into small A 1 star capital a 1 star into small a 1. So, that again those two quantities are conjugate of each other. If you add them you get real. So, may be b 1 plus I am absorbing the minus sign in the coefficients into z inverse something like this. So, this is what you get in the numerator, in the denominator you have got 1 minus small a 1 plus small a 1 star. So, suppose a 1 plus a 1 star, but we will give some name to it, say c 1. So, c 1 and c 1 is real, z inverse minus and minus plus, plus mod a 1 square these again real call it c 2.

So, c 2 z inverse 2, this is real, this is real, this is real, this is real, that we have seen. So, we will require only real multiplier plus something. So, this part, means what? This plus another terms there all equal to if you add them there is equal to Y z by X z. So, what is Y z? X z into this plus again X z into next term, X z into next term all are partial fractions and then outputs are to be added. X z into this product X z into next ratio, X z into next ratio, carryout this product and then add. X z into this means basically it is a filter, X z into this means as this is a filter it's input is X z. What is a output? If you call the output to be y 1 z, so y 1 z by X z is this.

I can put plus also no problem c 2 z inverse. So, X z into this has to be find out, X z into this, this part will call y 1 z, x z into next we will call y 2 z, x z into next we will call y 3 z and all y 1 y 2 y 3 are to be added. So, it is like this and this is second order filter and we know how to implement in Direct form 1 or Direct form 2. So, what will you have is x n you have this filter, this you realize in Direct form 1 or 2 form. This is second order filter both the numerator and denominator second order is call bi quad, bi quadratic.

Then next, if this coefficient was real then there is I mean you cannot say that because there will be another term, another you know root with conjugate value because this is real. So, if that case will be happy we just may be a term like you know B 1 capital B 1 by just some of not b 1 call it say D 1 by 1 minus d 1 z inverse and D 1 is real. So, this kind of terms you know for this you do not have combined into with another term like d 1 star by 1 minus small d 1 star z inverse like here because D 1 is real.

So, for this kind of terms this kind of terms X z into this you can call it y 2 z. So, y 2 z by X z is this, this anything can be realized then in direct form 1 or direct form 2. So, there can be terms like you know d 1 by dot, dot, dot, dot, for all the partial factors and their results you add. This called parallel form. There are parallel filters as a second order or first order and you are passing x n through which of them is giving y 1 z it is given y 2 z, y 3 z like this and then adding all the output. It is called parallel form and then can be a cascade form also. Cascade form means instead of going for partial fraction you go for a product. So, may be if you take b 0 out.

So, 1 plus b 1 by b 0 z inverse b 2 by b 0 z inverse 2 b q by b 0 z inverse q and factorize it. So, may be numerator will be factors like this 1 minus (Refer Time: 27:04) give you some name this b 1, b 2 is not I should have told. You do not confuse with the b 1 b 2 here. I just here that if you add them if you want you can make b 1 prime b 2 prime because they are different from this original b 0 b 1 right.

Now, so, numerator will be sub co efficiency. May be alpha i z inverse i is equal to 1 to how many, z inverse 1 to z inverse q. So, there should be q terms and denominator also j is equal to 1 to p 1 minus may be beta j z inverse 1. So, p times there is a power. Now here, if they have got a complex root then it's conjugate must be there, take those two, you mean otherwise if this real, instead of taking one you take two at a time. So, and what are is in your head, which to unless you are considering the complex root where you must take it conjugate fact, the root in factor conjugate root and you can convert into second order forms.

So, typically it will be a product of this kind of forms, either second order may be first order. Second order means 0 maybe 1 plus some gamma. You multiply to gamma 1 z inverse plus gamma 2 z inverse 2 and numerator will be (Refer Time: 28:54) I mean you can alpha, beta, gamma. What else is left? May be lambda, lambda 0 plus lambda 1 z inverse something like; this is one product. This one filter there will another filter this could be a first order also depending on high order them, which once you take first which once you take (Refer Time: 29:15) various possibilities and it will be a product of this kind of form. This is called cascade, there will be one filter of this taking input as x n, then next filter like this next filter again another form of that kind. This could be either second order like this of a first order where, denominator will have z inverse and numerator will have just lambda 0, no further comes or each of them you can implement in direct form 1 and direct form 2 and they are cascade.

So, what are structure will be called cascade form. These are the typical structures for IIR digital filters. For FIR filters we will consider in the next section.

Thank you.