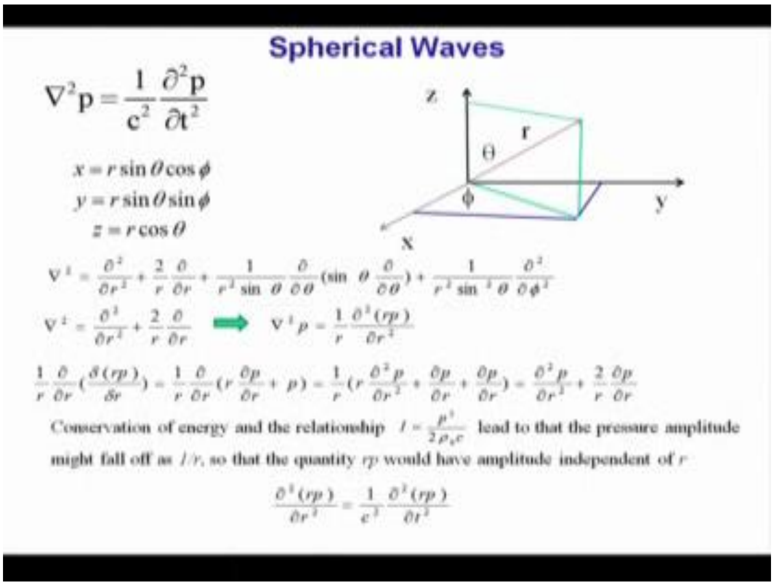
Audio System Engineering
Prof. Shyamal Kumar Das Mandal
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 09
Spherical Waves Propagation

(Refer Slide Time: 00:21)



So, in last class, we are discussing about that spherical waves propagation.

(Refer Slide Time: 00:37)

So, we are saying that let us the instead of linear waves, let us sound is propagated like a spherical sphere. So, I can say that let us this is the source of the sound, and it is propagated like a sphere. So, instead of linearly propagate, it propagate in all direction, in all three-dimensional space in all direction like a sphere, spherical wave propagation. Now, what is the difference, now I know that in Cartesian coordinate x, y, z we have consider. Now, in here, I have to converted that x, y, z to r, theta and phi, same three coordinates; instead of x, y, z, I convert that r, theta and phi. So, in that case, what we will do is we just write draw the coordinate x, y, z and we put a point here, which is r. So, this point is theta, so this is r, projection of r here, r cos theta.

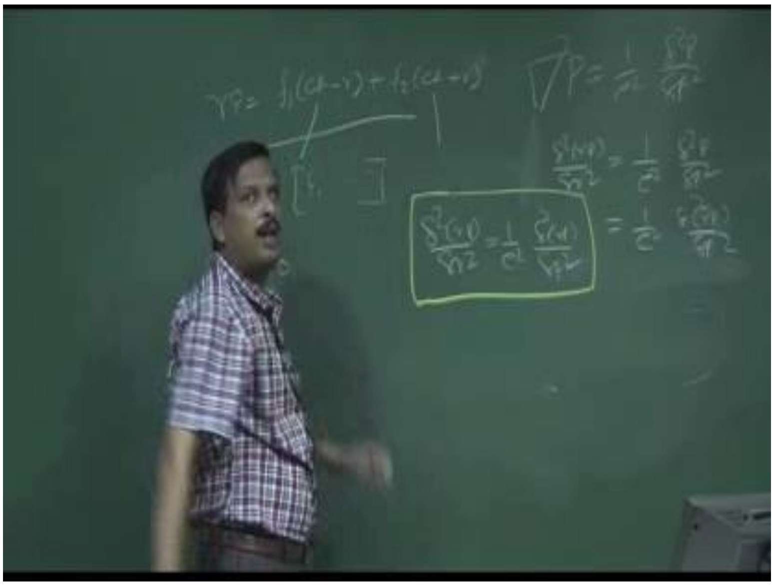
Now, if I make a projection in x, y plane of this same arc, I get r sin theta. So, the projection of r sin theta on x-axis gives me the x. So, I can write x is nothing but a r sin theta cos phi if this angle is phi. Similarly, y is nothing but a r sin theta sin phi. And z is nothing but a r cost theta. So, you know what is the wave equation we know; now wave equation is nothing but a divergence square P is equal to one by C square del square P by del t square, this is the linear wave equation. So, I can say what is the meaning of this thing, this thing is nothing but a del square by del x square plus del square by del y square plus del square by del z square. So, this is that things.

Now, I instead of x, y, z this term can be calculated from this x, del x del x del del x del del square del x square I can calculate, del square del y square I can calculate, and del square del z square I can calculate. So, eventually, this Laplacian operator will become in theta r and phi domain that will be del square by del r square first term plus 2 by r del by del r plus one by r square sin theta into sin theta into del by del theta into sin theta del del theta plus one by r square sin square theta into del square by del phi square. So, this is prove of this thing, this is details mathematically prove you can calculate in that case.

So, now if this is the case, so instead of this, I have to get this whole equation. Now, think I said sound is propagated or acoustic wave is propagated in a spherical in nature. So, in every angle, every direction, it is symmetry. If something is propagated in spherical nature; that means, I am saying it propagate in symmetric in all direction so if it is propagate symmetric in all direction, so the variation against the theta and phi will we not consider, nothing will be there. So, I can say the variation against theta and phi, no terminology will be there, so I can say these whole terminology will be vanished if I write del square P then will be only del square p by del r square plus 2 by r del p by del r. So, del square p by del r square plus 2 by r del p by del r.

Now, if I say this term can be written as 1 by r del square r p divided by del r square. I can prove it, 1 by r del square r p divided by del r square. What is this? It is nothing but a 1 by r, del by del r into r del p by del r plus t. Then I took those things also. So, 1 by r into r del square p by del r square plus del p by del r plus del p by del r 1 by r and 1 by r. So, r, r cancel, it is nothing but a del square p by del r square plus 2 by r del p by del r. So, these things and these things are same.

(Refer Slide Time: 06:35)



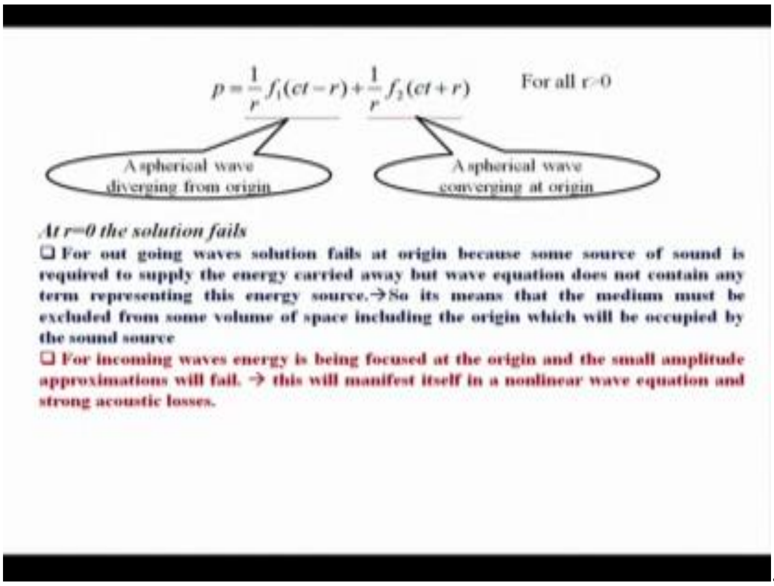
So, I can write del square P is nothing but a 1 by r del square r p divided by del r square. So, my acoustics wave equation, I just put these values in here. In case of spherical wave propagation, it becomes 1 by r del square r p divided by del r square is equal to 1 by C square del square p del p square. Now, if I say P is my function, the pressure amplitude, so I said the pressure amplitude, independent pressure amplitude is independent of r, and can I say that. The pressure amplitude P is independent of r, can I say that. So, if I say that spherical wave propagation also preserve the conservation of energy. So, now let us take a small sphere with a radius lets r 1, and the intensity of the wave is I. So, energy at this surface of the sphere, if all direction I will be same, so I multiply by the 4 pi r 1 square.

Similarly, if I take another at this distance a sphere, whose radius is r 2? So, energy here is 4 pi r 2 square into I. So, now, if I say the conservation of the energy, energy let us there is not loss in acoustic wave propagation; consider there is no other kinds of losses. So, energy in here and energy in here will be the same. So, I into 4 pi r 1 square and I into 4 pi r 2 square will be the same. Since, r 1 and r 2 is not same, these two is has to be same, then I has to be change.

So, what is I, I is nothing but a P square by two rho 0 c. So, P cannot be a parameter, which is independent of r. So, instead of P, I consider r P is a parameter, r P is pressure amplitude, amplitude which is independent of r. So, instead of P, here I consider r P is the function. So, I can write r P divided by del t square. So, this 1 by r will be not there. So, I can say in case of spherical wave propagation, wave equation become del square r P divided by del r square is equal to 1 by C square del square r P divided by del t square. So, this is the spherical wave propagation equation.

So, in that case, what is the solution in spherical wave propagation? So, I can say r P is a consistent of two functions of wave, one is called forward wave or you can say converging, and another is called divergence. Forward, forward means it is going out; it is diverging wave, in case of sphere. And if it is backward waves, so it is converging wave. So, I can write lets this r P, the two function one is f 1 into C t minus r and plus f 2 C t plus r. So, this is a converging wave, and this is a diverging wave. So, waves are diverted is a forward wave; and once it is backward wave means over converted. So, it is a diverging wave, and this is a converging wave.

(Refer Slide Time: 11:13)



Now, what is P, P is nothing but a 1 by r that function F 1 plus F 2. So, now if it is one by r, so at r equal to 0, this concept does not valid; diverging wave, converging wave at r equal to 0, P no solution is valid. So, in that case, what will happen, so solution fail? So, in case of solution fail, for outgoing wave solution fail at origin required some source of sound is required to supply the energy carried away, but wave equation does not contain any term representing this energy source. So, it means that medium must excluded from some volume from the media some volume must be excluded from the media, from the medium, so that space at origin, at origin that space, which acquired by the sound source or acoustic source.

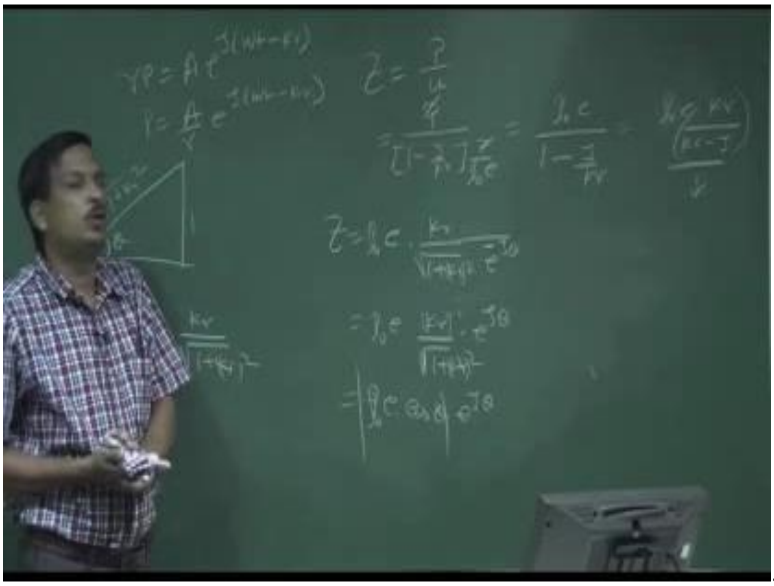
Now, in case of diverging wave, it is fail explanation is there. For incoming wave, energy is being focused at the origin and the small amplitude approximation will fail. This will manifested itself in a non-linear wave equation and strong acoustic losses. So, this is the physical meaning of if r equal to 0.

(Refer Slide Time: 12:42)

Now, let us r is not equal to 0, and we proceed. Then I can say if this is my wave equation, this is the wave equation, and then what is the solution of general solution of this wave equation. So, general solution is that r p is equal to A e to the power J omega t minus K r, K is the wave constant and r is the distance from the source. So, in that case, P is nothing but a A by r e to the power J omega t minus K r. So, this is the P. Now, if this is P, what is velocity potential phi? So, it is written that rho 0 del p by del t is equal to minus P. We said del by del t, we always write J omega, so it is rho 0 J omega phi is equal to minus P. So, phi is equal to minus P divided by J omega rho 0, velocity potential.

Now, what is the particle velocity u is nothing but a divergence of phi. So, in case of spherical coordinates, divergence, d phi by d r only. So, if I put that d phi by d r in case of P, what I will get, let us put that d phi by d r, so phi is equal to minus P by J omega 0 rho 0. So, I can write minus A by r e to the power J omega minus K r divided by J omega 0 rho 0. Now, I take the del phi by del r. What I will get, first order differential equation, so I will get I just writing 1 minus J by K r into P by rho 0 C. If you do the differentiation, you will get this thing; del phi by del r is nothing but a 1 minus J by K r P by rho 0 C, which is nothing but u.

(Refer Slide Time: 15:18)



Now, see that in case of linear wave equation that pressure wave P and phi are in the same phase, but here u is a function of some complex thing. So, if I want to calculate the impedance z, so z is nothing but a P by u; z is nothing but a P by u. So, if I calculate the z, what I will get, I will get, so P will be cancel, P by 1 minus J by K r into P by rho 0 C, so P, P cancel. So, it is nothing but a rho 0 C divided 1 minus J by K r. So, I can write it is nothing but a rho 0 C into K r by K r minus J.

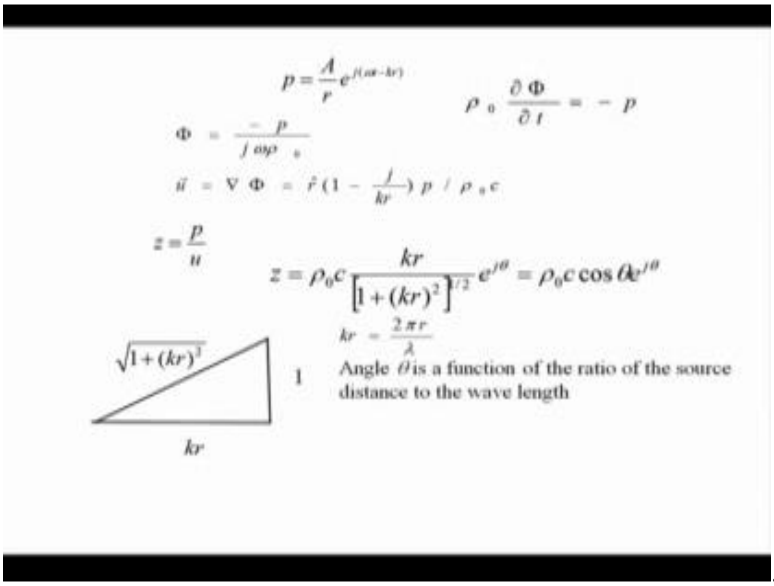
Let us consider in case of linear wave, z is nothing but a rho 0 C. Here I am saying there is another complex stock is there. Let us consider this J, create an angle so if I say representation of the J, if this is theta then if I represent this complex number in a trigonometry form, so it will K r, it will be one and it will be root over of 1 plus K r square, if this angle is theta. This is K r; this is one. So, I can write this complex in term of amplitude and theta, so I can write let us a minus J b, so I can write root over a square plus b square is the amplitude and e to the power J theta. And theta will be tan inverse b by a, lets this will be minus, this is minus, so I write minus theta. So, instead of plus theta, I write theta then it will be plus.

So, instead of one by K r into J, I can write rho C into K r by amplitude 1 plus K r square into e to the power minus J theta. Same thing, I can write rho 0 C K r by root over of 1 plus K r square into e to the power J theta, minus would be plus, when it go up. Now, what is K r by 1 plus K r square? So, we have drawn that triangle like this way. This is theta, this is one, this is K r, and this is 1 plus K r square. So, what is cos theta, cos theta is nothing but a base divided by 1 plus K r square, in that case, I can write this is nothing but a rho 0 C cos theta into e to the power J theta. So, rho 0 C cos theta into e to the power J theta, if I say the amplitude part of the z is not rho 0 C.

(Refer Slide Time: 19:51)

So, in case of spherical wave propagation, the impedance - acoustic impedance is not rho 0 C, instead of rho 0 C, it is a rho 0 C cos theta. What is the theta? Theta is the angle between the pressure and velocity.

(Refer Slide Time: 20:11)



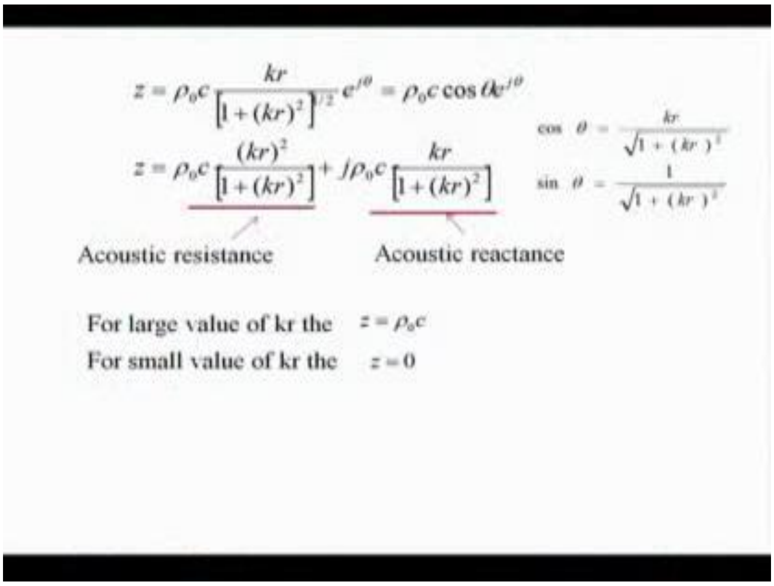
.

Now, if this is theta, what is theta, I can write cot theta is equal to K r by 1. So, cot theta is nothing but a K r. Now, what is K r, what is K r, r is the distance from the source, what is the K, K is the omega sub c, so I can write K r is nothing but a 2 pi r divided by lambda. So, I can say the theta is a function of the ratio between the distance from the source and wavelength of the acoustic source, acoustic wave. So, distance from the source and wavelength of the acoustic source.

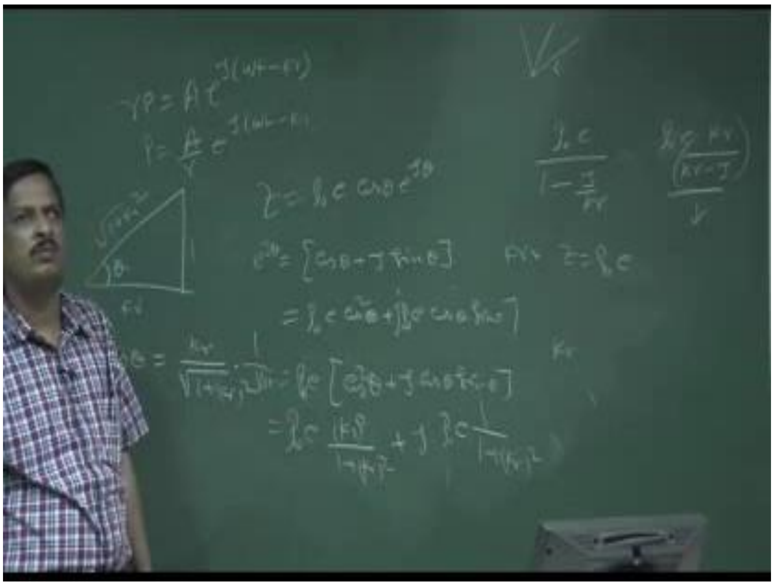
Now, let us the spherical wave is started here and going like this way. If this r is small, very small compared to this lambda, what will happen? If r is very small, that means, the distance travelled by the source, the distance travelled by the acoustic wave from the source is very small compared to the wavelength of that acoustic wave. So, compared to the wavelength of the acoustic wave, if the distance is very small, very small then the complex the theta, value of the theta will be large, theta will be increase, theta will be very large.

Now, if the distance is very far away, let us from the source compare to lambda is r is very large, that means, the acoustics wave is travelled far away from the source in that case, theta will be very small, because cot theta, so theta will be very small. So, in that case, theta will be very small means angle between the pressure wave and particle velocity or velocity wave will be almost NIL. So, in that case, z becomes rho 0, sorry rho 0 C, cos zero one. So, it is far away from the source, it is looks like a plane wave; although it is a spherical wave, it is look like a plane wave.

(Refer Slide Time: 23:16)



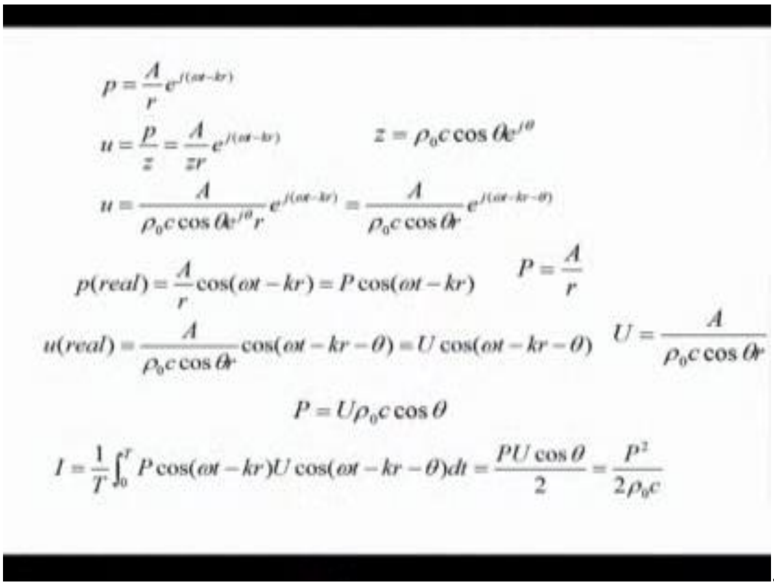
(Refer Slide Time: 23:18)



Now, I can express z, in other term also. So, you have seen z is the complex function. So, it is a two parameter function. So, z is nothing but a rho 0 C cos theta e to the power J theta. What is e to the power J theta is nothing but a cos theta J sin theta. So, I can write z is nothing but a rho 0, C cos theta into cos theta, so cos square theta plus rho 0 C cos theta J sin theta. Or I can write rho 0 C, cos square theta plus J cos theta sin theta. So, what is the value of cos theta and sin theta. I said the cos theta is nothing k r by root over of 1 plus k r square. So, I can write rho 0 C k r square divided by 1 plus k r square plus J rho 0, C into cos theta k r by 1 by 1 plus k r square and sin theta is 1 by root over of 1 plus k r square, so it is nothing but a 1 by 1 plus k r square.

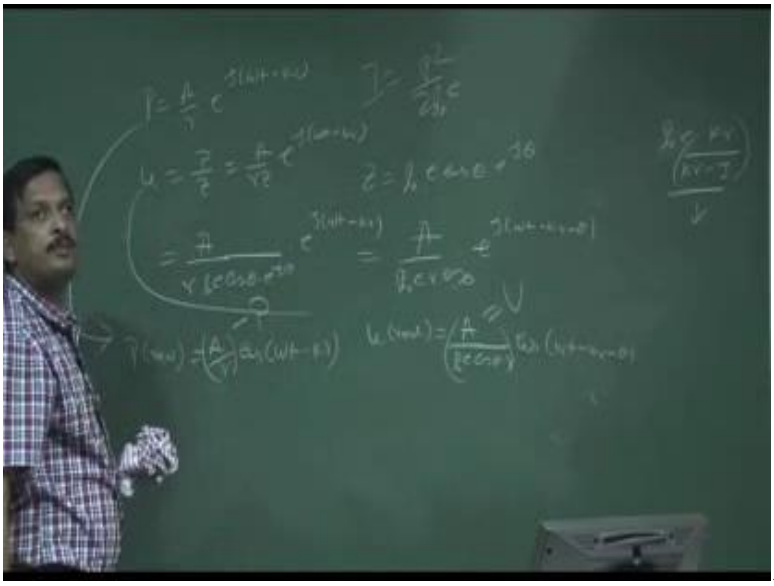
So, if you see a plus J b, so z is not a resistive not only the resistive term, there also some term, which is reactance. So, it is called acoustic resistance and it is the acoustic reactance. For large value of k r, if the k r is very large, then you see this term is 0, and this term is one. So, it is z is nothing but a so at k r is very large, z is nothing but a rho 0 C. Now, if the k r is very small, then z becomes 0, so that cannot be explained. So, the k r is very small, z become 0.

(Refer Slide Time: 25:51)



Now, that wave equation I have understand. Now, one thing how do we express the intensity of the sound. Most of the cases, most of the sound wave propagation we discuss about the spherical wave propagation. So, now we have to find out what should the expression of the intensity in case of spherical wave propagation. So, in case of plane wave propagation I is nothing but a P square by 2 rho C. I have to say whether this I expression is remain same in case of spherical wave or it is change.

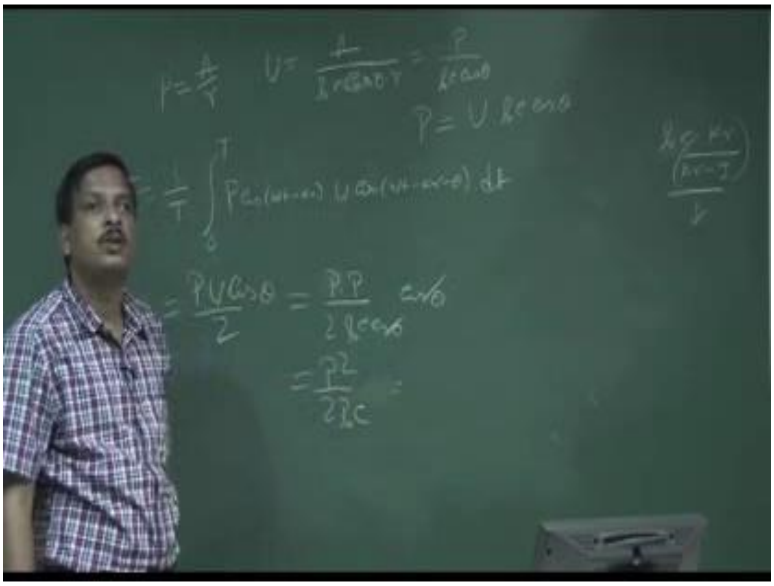
(Refer Slide Time: 26:41)

.

So, let us I consider that P is nothing but a A by r e to the power J omega t minus k r. And in u is nothing but a P by z, I can write, so it is nothing but a A by r z, z is the impedance e to the power J omega t minus k r, where z is equal to rho 0 C cos theta e to the power J theta. So, I can write u is nothing but a A by r rho 0 C cos theta e to the power J theta into e to the power J omega t minus k r. So, I can write A by rho 0 C r cos theta into e to the power J omega t minus k r minus theta, e to the power theta you can term into here, so minus theta.

Now, this is u and this is P is this. So, what is the real part of the P? If I say the real part of the P, so P real – real part of the P is nothing but A by r cos omega t minus k r. Now, what is the real part of u, u real – real part is nothing but A by rho 0 C cos theta into r that term will be there into cos omega t minus k r minus theta. Let us this term as a capital U, and this term as a capital P. So, now if that is the case, then I can write P is equal to.

(Refer Slide Time: 29:15)



So, I can write sorry I can write u is equal to small u and this is the capital U, and this is the P, P is equal to A by r, capital P is equal to A by r, and capital U is equal to A by rho 0 C cos theta into r. So, A by r is nothing but a P, capital P by rho 0 C cos theta (Refer Time: 30:05). So, I can write capital P is nothing but a capital U into rho 0 C cos theta.

Now, what is I, what is the intensity of the wave? Intensity of the wave I is nothing but a average intensity, time average zero to T pressure into velocity. So, what is pressure, pressure is capital P cos omega t minus k r into capital U cos omega t minus k r minus theta into dt. Now, do this whole integration, if you do this integration, you will get, what will get P into U cos theta divided by two. So, if I get that what is P, P is A by or I can in term of P or U any one I can replace the U in terms of P, so U is nothing but a so P into P by two rho 0 C cos theta into cos theta; cos theta, cos theta cancel. So, it is P square by 2 rho 0 C. So, it is same as linear wave intensity equation, so it is same as linear wave intensity equation.

So, next or next class, I will discuss the physical meaning of acoustic intensity propagation. What is decibel, how decibel is measure, why it is decibel, and why intensity energy is different that is we will discuss in the next class.

Thank you.