Audio System Engineering
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Lecture - 08
Acoustic Wave Equation (Contd.)

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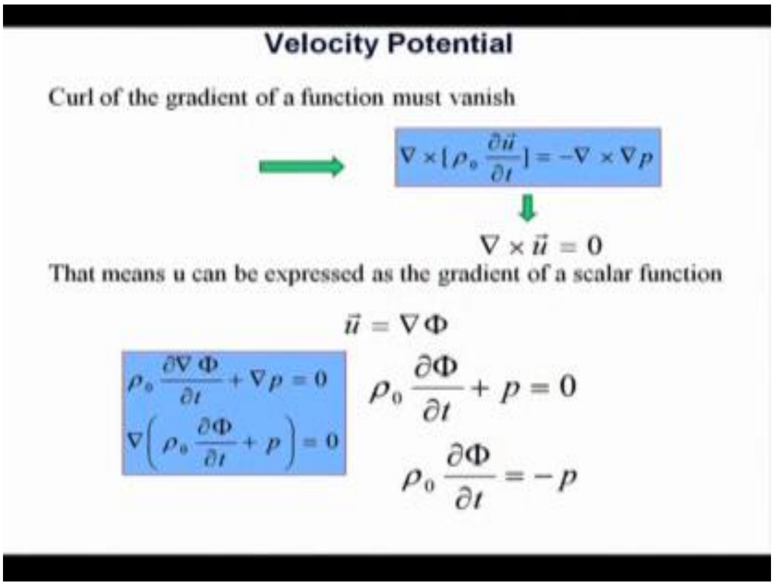

So, last class, we have derived the plane wave equation which is we have derived that plane wave equation divergent square of pressure.

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Acoustic pressure is nothing but 1 by C square del square d square of d square is right del square P by del t square this is the plane wave linear wave equation. We called linear wave equation.

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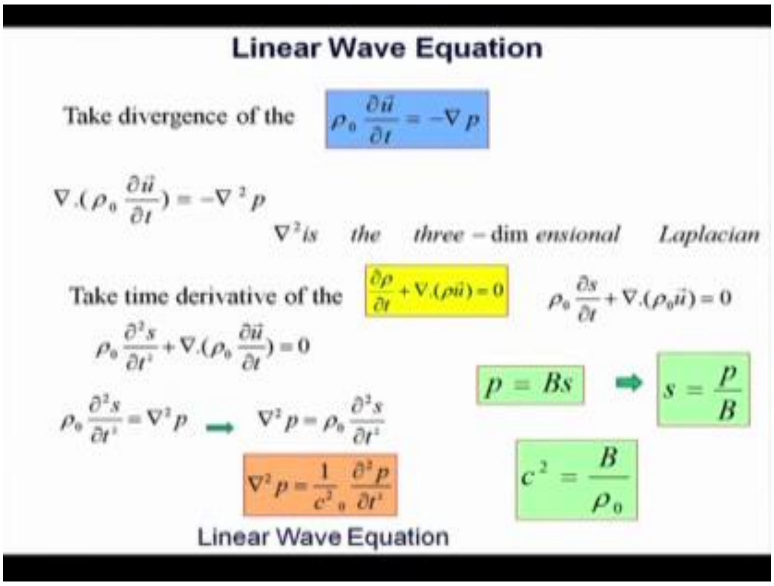

Now, interesting property we should know that we should know that the velocity we said called the velocity potential. What is velocity potential, the interesting thing is that you know that the curl of a gradient function any function if I take the gradient of the function, and then take the curl is equal to 0 that is the mathematical proof. The curl of a gradient function is equal to 0. Now, if you remember that we have derived that equation where the particle velocity rho 0 del u by del t is equal to minus p that we derived rho 0 del u by del t is equal to minus divergent of acoustic pressure.

Now, if I take the curl in both side, so I take the curl in both sides then I said curl of a divergence function is equal to 0 so that means, this is equal to 0. So, if it is this equal to 0 that this implies that the curl of u velocity curl of particle velocity is equal to 0. So, if the particle velocity is u and that curl is equal to 0 – what is the meaning physical meaning of this thing, what is curl of a velocity, curl of velocity means that particle velocity does not have any angular motion, there is no curl, if you know that what is the divergent, divergent means all it is diverted, what is the curl, curl is the rotational things. So, curl of u is equal to 0 that means we are considering a linear wave equation; there is no particle velocity in angular direction. So, there is no angular velocity in the particle velocity only the linear velocity existing particle velocity.

So, if I say the particle velocity is nothing but it can be expressed in scalar function. So, I said u is nothing but divergent of phi because particle velocity is in all direction, but it does not have any angular motion. So, the u is nothing but a divergent of scalar function; this phi is called velocity potential. Now, if u is equal to divergent of phi, then what should we do in my equation, my equation is then rho 0. So, what is that? So, rho 0 del u by del t is equal to minus P. So, it is nothing but a rho 0 del u by del t plus divergent of acoustic pressure is equal to 0.

So, what is rho 0? So, if I say I can take the divergent out or I can say what is u, u is nothing but divergent of velocity potential. So, I put it rho 0 del divergent phi del t plus divergent P is equal to 0. So, in that case, if I take the divergent out it is nothing but rho 0 del phi by del t plus P is equal to 0. If this function is equal to 0 divergent of sum function is equal to 0 that means, that rho 0 del phi by del t plus P is equal to 0. So, rho 0 by del phi by del t is equal to minus P. So, the velocity potential rho 0 del phi by del t is equal to minus P. This will be used in wave equation that is why I said the phi is the velocity potential and that will be is equal to rho 0 del phi by del t is equal to minus P.

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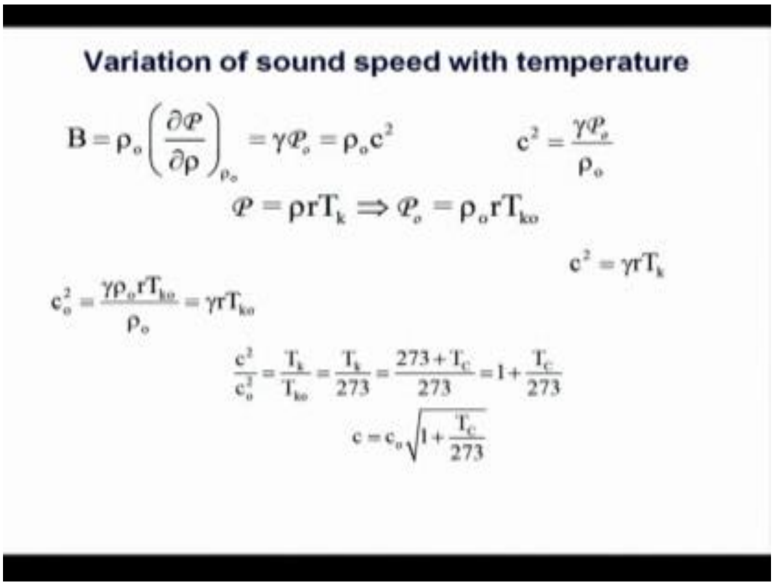

Now, what is c? If you see that velocity of the sound is it depends on temperature, velocity of sound, how it dependency on the velocity of sound.

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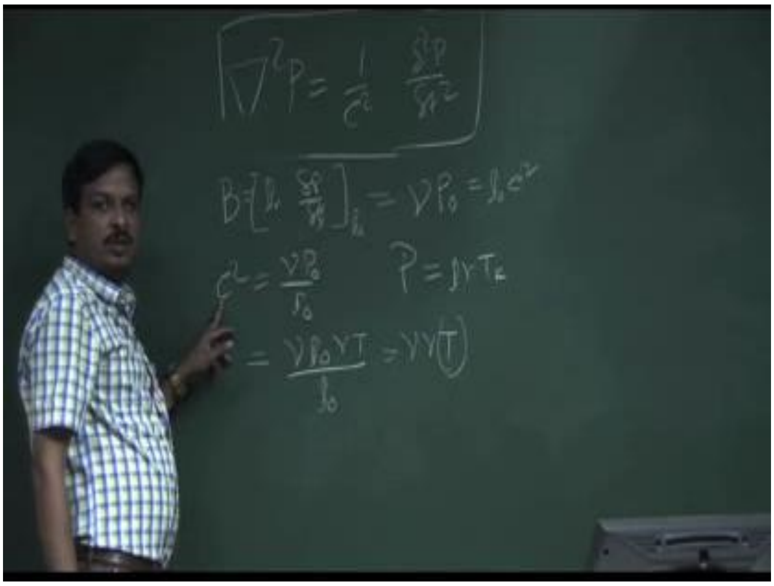

So, I say of bulk modulus of B is equal to rho 0 del p by del rho sorry at rho 0. So, it is nothing but a gamma P 0 total pressure is nothing but a rho 0 c square. So, what is c square c square is nothing but a gamma P 0 divided by rho 0. Now, you know the p is equal to what is p is equal to rho r T k gas equation from the gas equation r is the gas constant rho is the density, T is the temperature in Kelvin. So, now, in that case, so if I say that if I put the p value in here it is nothing but gamma rho 0, r T divided by rho 0. So, it is nothing but a gamma r T in case of T depending on the T c is there is. So, the velocity of the sound depends on the temperature if the temperature change velocity of the sound will be changed.

Now, we go for the, we have derived the linear wave equation. We have know that velocity potential, we know the sound speed of the sound change with temperature then we have to know how the plane wave is propagate in the medium, so that is called harmonic plane wave harmonic plane wave how the plane wave is propagated in the medium we know the equation.

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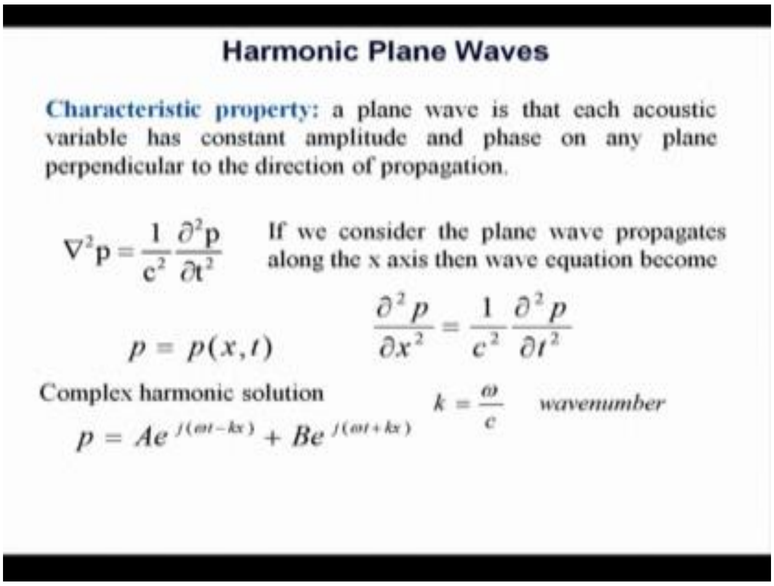

Now, how it is propagated that will be described. So, now we will go for harmonic plane wave propagation how to how to find out what should be the equation of the p if it is propagated in a medium. So, now, we have derived the equation of p and u and. We have described how the harmonic plane wave is propagated in a medium. In harmonic plane wave, the characteristics of the harmonic, plane wave is that a plane wave is that each acoustic variable has a constant amplitude and phase on any plane perpendicular to the direction of the propagation. What is the meaning, if I say the wave propagated along with the x-axis, so perpendicular to the x-axis lets the z-axis, so in the z-axis, all the acoustic variables of that plane wave are constant amplitude. So, that is the meaning.

That means, if the wave is propagated along with the x-axis perpendicular to x-axis to the z-axis. So, in the z-axis all the acoustic properties all the acoustic variable of that plane wave will remain constant. So, practical example is that if you thrown a stone in a pond, you find that there is a wave is generated. Lets the wide propagation, propagation is along the plane of the water and towards that towards the end of that towards the land. So, the wave is propagated like this way. Now, what is the perpendicular of that wave perpendicular is the z-direction. So, in perpendicular direction, at any point, the acoustic variable of all that exist in that propagation wave have the constant amplitude are the properties of plane wave.

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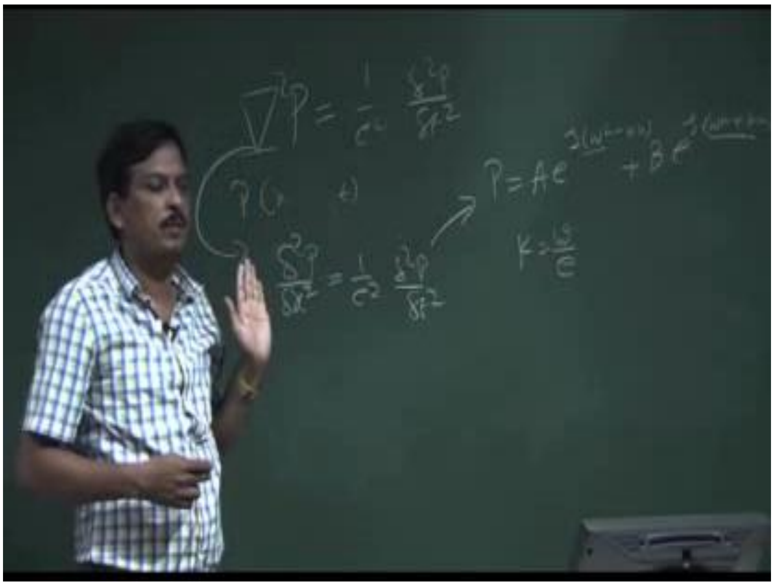

Now, we know that what is the plane wave equation we know del square P is nothing but a 1 by C square del square del square P by del t square. So, this is the plane wave equation. So, this is a second order differential equation, where P is a function of x, y, z and t so that means, P is the function of position and time. So, when a plane wave is propagated, if it is free to propagate in any direction in that. Suppose, in this room I create an acoustic sound, this sound can propagate at and direction of this room. So, if it is in any direction, so it is x, y, z any direction it can propagate, and once it propagate it has to be respect to the time. So, the pressure wave has the function of x, y, z and t.

Lets the propagation the plane wave propagated along the x-axis only, lets there is no propagation in z and y-axis. So, if the plane is propagate along the x-axis only, so P is a function of x and t only. So, in that case, this becomes del square P by del x square is equal to 1 by c square del square P by del t square. So, this is the second order differential equation. What is the solution of the differential equation, I have to find out the solution of the differential equation. What is the solution, the solution will be in the form of P is equal to A e to the power you know that second order differential equation the solution will be e to the power J omega t minus k x plus B e to the power J omega t plus k x. So, here one solution is omega t minus k x, another is omega t plus k x.

What is the meaning of this thing; meaning is that when the wave is propagated there is called forward wave and there is called backward wave. So, if it is forward wave along with the x-axis if it is the forward wave then I write minus k x; if it is backward wave, I write plus k x. What is k? k is nothing but a wave number. So, what is wave number is nothing but an omega by c, c is the velocity of the sound; omega is the frequency. So, wave number into x is the total distance travelled by the wave, wave number into x total distance travelled by the wave. So, if it is a forward wave, then it is minus; if it is backward wave then it is plus. So, distance forwarded means distance is covered; backward means distance to be covered, so that is why minus k x, plus k x. So, one is forwarded another is backward. And k is the wave number - omega by c. So, c is the velocity of the sound, omega is the frequency of the sound.

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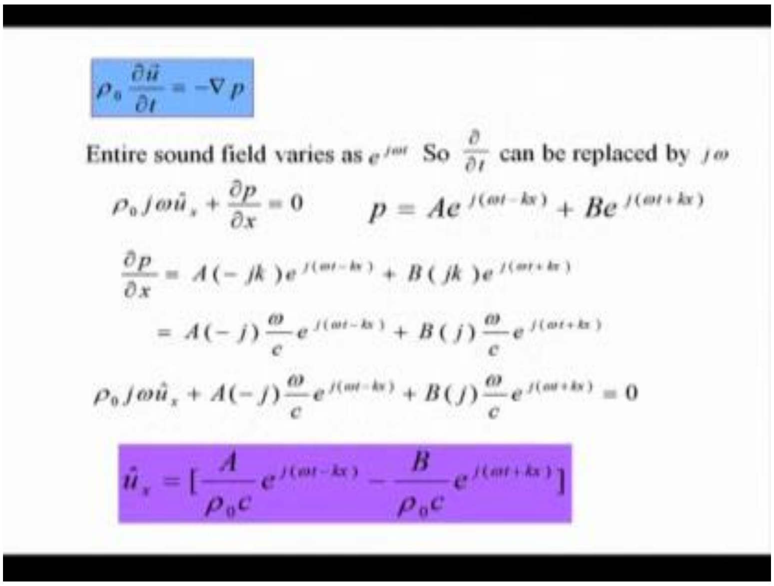

Now, you know that one of the equations of the wave is. So, this is the P now what I have to know if this is my P, what is u I have to derive particle velocity. If this is the pressure wave equation then what is the equation of the velocity wave.

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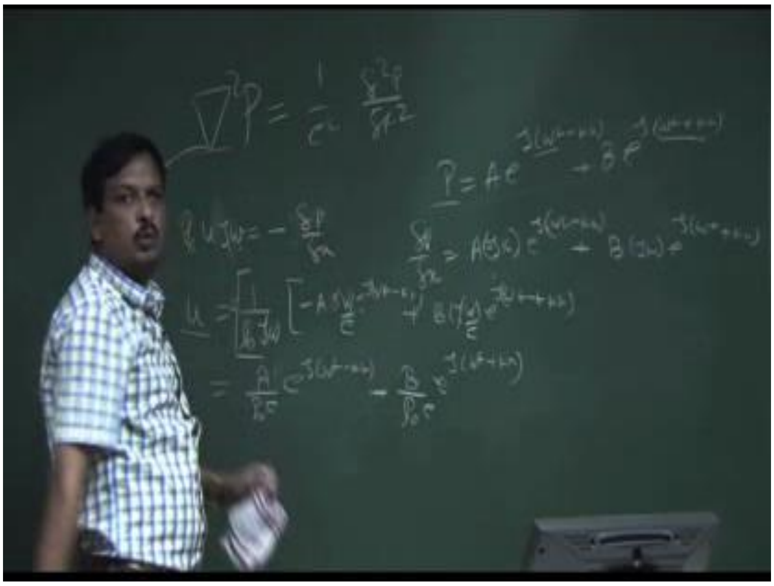

So, if I say this is the P then what is the relation of the P and u rho 0 del u by del t is equal to minus divergence of P. Now, you know del by del t this is written as a J omega because that sound entire sound fill is varying with the e to the power J omega t. So, what is the d by dt is nothing but J omega; whereas if I take the derivative it only create the J omega. So, this conversion I will use many of the cases. So, differentiation means there is no multiply by the J omega. So, in that case, if I write, so this is nothing but since I have only the x-direction. So, this is divergence means I have only x-direction. So, it is dP by dx and it is nothing but a u into J omega del by del t is J omega. So, it is rho 0 u J omega is equal to minus dP by dx.

So, what is dP by dx. If this is P, what is dP by dx differentiation against x. So, it is nothing but A J k minus e to the power J omega t minus k x plus B J k e to the power J omega t plus k x or not I take a simple differentiation of this equation. So, now, I put this value in here. So, actually u is nothing but 1 by rho 0 J omega into this function. So, it is minus A, so there is minus sign is there, so minus this into minus this into A J k e to the power J omega t minus k x plus B J k e to the power J omega t plus k x.

Now what is k, k is nothing but omega by c. So, instead of k, I will write omega by c instead of k, I will write omega by c. Now, J will be cancelled, all J will be cancelled. So, it is nothing but minus, minus plus. So, it is A omega, omega will be cancelled. So, A by rho c; omega and omega will be cancelled, minus and minus are plus, A by rho c e to the power J omega t minus k x plus B by same rho c. This will be not plus, this will be minus because this minus will get it minus B by rho c e to the power J omega t plus k x, so that means, u is nothing but 1 by rho c if I take out. So, it is nothing but A e to the power J omega t minus k x minus B e to the power minus J omega t plus k x, 1 by rho c. So, if I know the P, I know the u this is the u.

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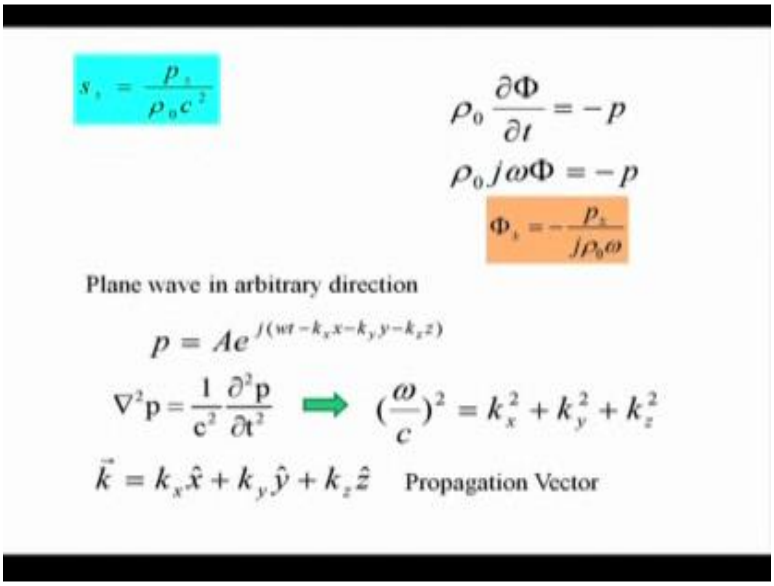

Now, forget about s, what is phi? Phi is the velocity potential; I said phi is the velocity potential.

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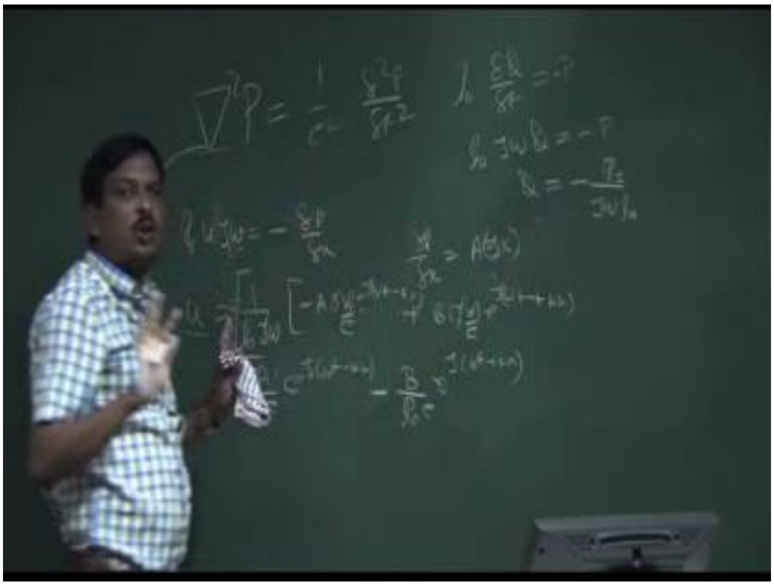

So, rho 0 del phi by del t is equal to t minus sorry minus P. So, phi is nothing but rho 0 J omega phi is equal to minus P. So, phi is minus p by J omega rho 0. And P is plus minus I am writing P is plus instead of forward wave backward wave I write plus minus; plus means back warded, minus means forwarded. So, it is nothing but phi is P by J omega rho 0. So, this is the relation between the u, phi and P, those will be used in deriving that solution.

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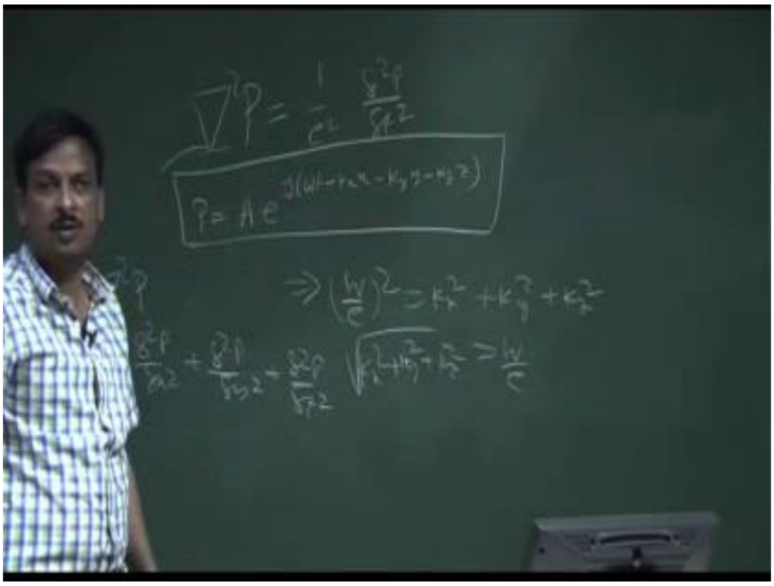

Now, what is the plane wave equation is this is the plane wave equation. Now, in case of arbitrary direction if I consider the plane wave propagated in arbitrary direction in that case the solution of P will be not in only x-direction. So, it is x, y and z-direction. So, it is nothing but A e to the power J omega t minus k x into x minus k y into y minus k z into z. So, I was saying the arbitrary direction is wave is propagated. So, the solution will be only this side one forward backward x-direction, y-direction, z-direction all arbitrary direction is propagated. So, it is e to the power J omega t minus k x x minus k y y minus k z z. So, this propagation equation P, P is equal to this only.

Now, if I put the solution in this equation what I will get, I will get I put this solution in this equation. So, P what is mean by del square P is nothing but d 2 P by dx square plus d 2 P by dy square plus d 2 P by dz square. If I take this, I will get omega by c whole square is nothing but k x square sorry k x square plus k y square plus k z square. So, I said that if I put this A value in this equation, I will get omega by c whole square is equal to k x square k y square k z square; that means, as if k x square plus k y square plus k z square root over is equal to nothing but omega by c.

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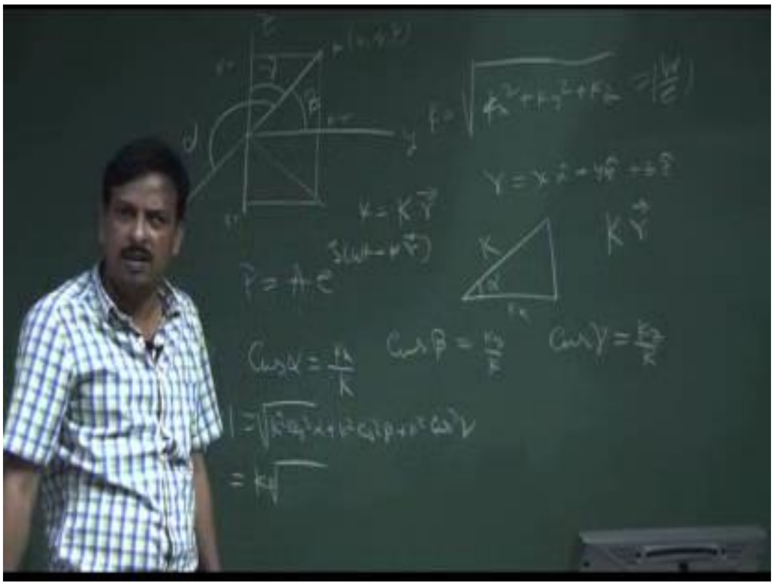

So, if I consider, lets I have a wave, I have acoustic wave and which propagation vector. So, I have acoustic wave in all three direction. Lets this is y-direction, this is x-direction, and this is z-direction. So, lets the wave is propagated in this field. So, I have a three direction y-direction and x, y plane, x-direction and y-direction. So, our propagation vector is here which is k. So, in propagation vector has amplitude as if in this room I have a point in here. So, from one corner if this corner is origin, origin to here I have an amplitude and has an angle with x-axis, y-axis and z-axis. So, propagation vector this k has an angle with z-axis, has an angle with y-axis, has an angle with x-axis. So, if I put this thing. So, I have an angle with z-axis I have a angle with x-axis, this has a angle with y-axis, so that three axis I have an angle.

So, lets the angles are beta, lets alpha, gamma. So, what is the amplitude of k, amplitude is nothing but root over of k x square plus k y square plus k z square, those are the points distance from z-axis is k z, if I take a projection in here. So, distance from this point in here is k x and here is k y. So, amplitude this is nothing but omega by c, we have already proved. So, it has an amplitude and has a position x, y, z. So, I can write the point k is nothing but an amplitude k and a position vector r and a position vector r, r is the position vector. So, r has a position vector. So, r has an x position x-direction x position, y-direction y position, if x, y, z and z-direction z position. So, this is the co ordinate system. So, the amplitude and the position is the k, k is written like that way.

So, I can write that instead of e to the power minus k x, k y, k z, I can write P is nothing but A e to the power J omega t minus k into r; k x x, k y y, so that has a position k which is omega by c. This amplitude is k is nothing but omega by c into r is the position vector I write amplitude and position vector. Now, what is r if I said that k has an angle with x-axis alpha? So, I can write what is cos alpha, cos alpha nothing but k x by k. If I have a trigonometric system if the angle is alpha this is k and this is k x, so cos alpha is nothing but k x by k. Similarly, cos beta is nothing but k y by k; cos gamma is nothing but k z by k, I can write instead of k x, k y, k z, so I can write mod amplitude mode k is the amplitude. So, it is nothing but root over of k x square k x square plus k y square plus k z square. So, it is nothing but k into cos square alpha. So, this is k into cos square alpha plus k square cos square beta plus k square cos square gamma. So, it is nothing but k into root over of cos square alpha cos square beta cos square gamma.

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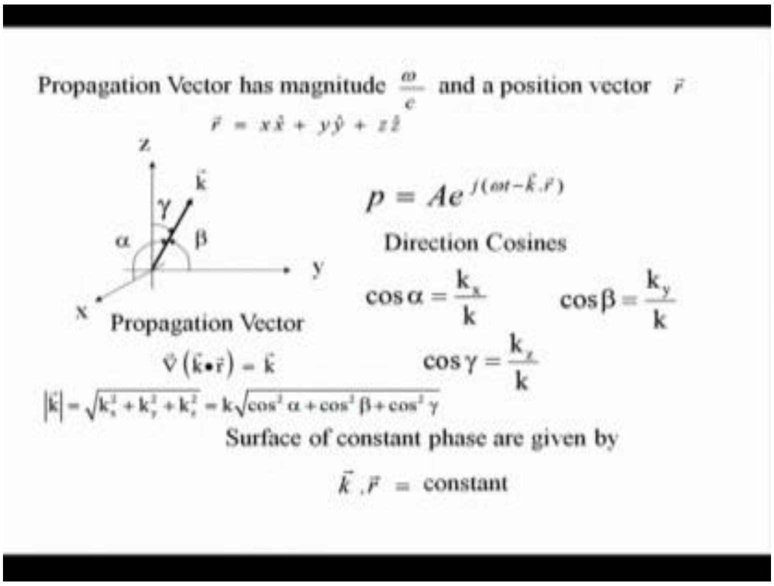

So, now if I say k into r is a wave vector, vector directional vector this is called propagation vector. So, this propagation vector is equal to constant, characteristics of the plane wave says that that perpendicular direction to the propagation is nothing but constant k into constant.

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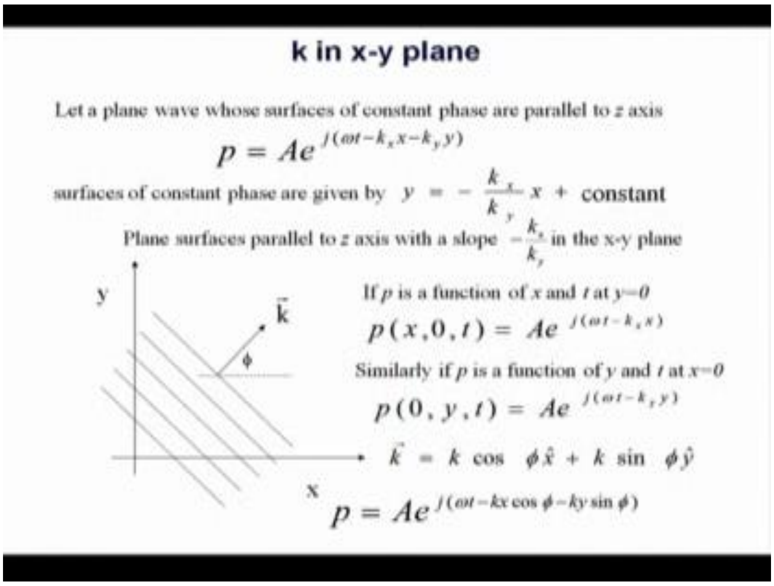

So, if it is constant then let us consider a situation, where let us say my plane wave surface is parallel to z-axis, surface of the plane wave is parallel to the z-axis; that means, the plane wave is propagated along with x y plane. So, if I you thrown a stone to the pond, so whole the water level is the x, y plane. How it is propagated the surface constant of that wave is perpendicular to the z-axis. So, if this is the pond water level then the wave propagation is the perpendicular the surface constant of the wave is perpendicular to the x y plane. So, it is nothing but parallel to z-axis.

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So, let us plane wave whose surface constant r parallel to z-axis that constant so that means, plane wave is propagate along the x, y plane. So, there is no propagation in z-direction. So, in that case, P is nothing but A e to the power J omega t minus k x x minus k y y, z-direction there is no propagation. So, parallel to the z-axis surface constant is parallel to the z-axis now in that case what is this is k x x k y y. So, I said that k dot r should be a constant so; that means, minus k x x or k x x plus k y y k y y which is nothing but k dot r propositional vector is equal to constant. So, in that case what is k y y is nothing but minus k x x plus constant plus c. So, it is a nothing but y is nothing but minus k x by k y into x plus c see the equation y equal to m x plus c is equation of a straight line what is m m is nothing but tangent that is the oblique, oblique angle on the x y plane angle. So, I have a - this equation; that means, if my have line like this. So, this is the angle region m is represent the angle. So, in that case, so I can write that wave is propagated along the x y plane with the angle k x by k y.

So, wave is propagated in real life situation. If I throw a stone to the pond wave is propagated like this x, y. This is x y plane, but it is not necessary that without any angle wave is propagated in that x y plane with a angle which is defined by k x by k y and wave surface is always if I cut the wave. So, surface is always perpendicular to the z-axis we said. So, wave is propagated along the x y plane with the angle k x by k y. So, now, if I draw the pictures, I cannot draw the three dimensional a picture, lets this is my x y plane, this plane is the black board is an x, y plane. So, this is x-axis, this is y-axis. So, this plane is x y plane. So, wave is propagated along the x y plane with a angle which is defined by this is wave propagation, lets this is the wave propagation with the angle which is phi, phi is now 10 phi is nothing but minus k x by k y.

So, with angle phi, so I can write that this cos phi is nothing but what is cos phi cos phi is I can write k x is nothing but k cos phi this angle phi or this is x-axis projection is k cos phi what is y-axis projection k y is nothing but k sin phi. So, whole k k can be written as k is nothing but k cos phi in x direction plus k sin phi in y direction k. So, what is the equation of the plane wave equation? So, it is nothing but p is equal to a e to the power J omega J omega t minus k cos phi x direction k cos phi x direction minus k sin phi y direction. So, I get the equation of propagation. So, that way we can derive the plane wave propagation equation it may be x y plane it may be y z plane. So, in that case of y z plane x will not be there. So, we are discussing about the plane wave propagation and we derived the expression of plane wave propagation equation now we have to know have calculate the energy of the plane wave propagation. So, energy for the plane wave propagations.

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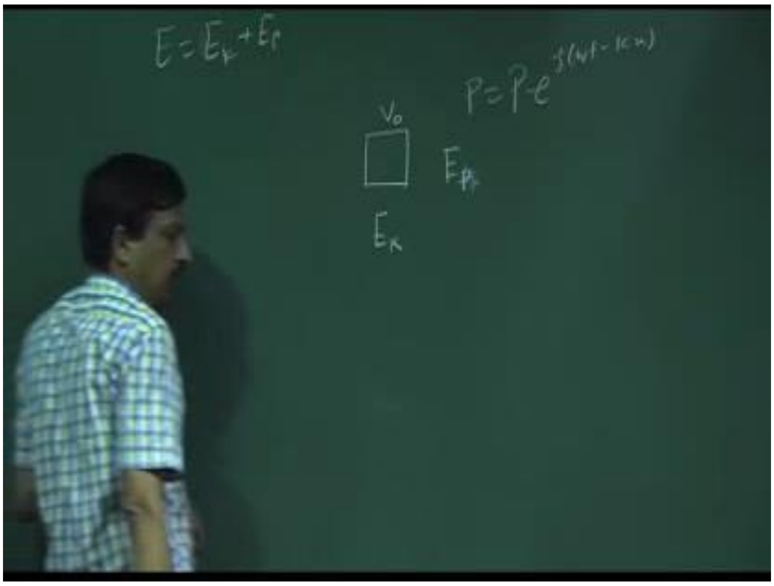

So, you know that let us consider that plane wave propagated along the x-axis. So, along the x-axis, propagation pressure equation P is equal to pressure amplitude e to the power J omega t minus k x this is the propagation pressure equation. Now for that we have derived that energy. Now, if I all the plane wave is propagated if when the wave is propagated it is nothing but when the pressure wave is propagated through a media how the energy is transferred.

So, let us consider a small volume on the propagation field V 0, V which is V 0. So, you know that it is compressed and decompressed. So, when the wave will propagate this volume will be compressed and the volume will be expanding. So, any compression or expansion has a potential energy. So, that creates the potential energy which is E p. Similarly, when the wave is propagated through a media the particle has a velocity. So, that creates an kinetic energy E k. So, the total energy in the plane wave propagation e is nothing but kinetic energy plus potential energy now we have derived the expression for the kinetic energy and potential energy.

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Now, if you know you know that that what is kinetic energy E k, if the volume velocity is u particle sorry particle velocity is u then or the volume sorry volume velocity is u then kinetic energy is nothing but half m v square of half u square. Now, if I consider the small volume v zero on that propagation plane that this is the small volume V 0. So, v zero will be compressed if it is compressed some potential energy will be stored if it is decompressed potential energy will be released. So, this is the kinetic energy.

So, the potential energy E p is nothing but change of volume. Let us this is V 0 initial volume; V 0 to V change of volume due to the pressure applied on to that volume P dV. And this is negative sign indicate that the energy if the volume is compressed then the force is acting on that volume the pressure divided by pressure into unitary area is the force cross sectional area. So, force is acting on the volume and volume is decreased. So, that decrease even the force is increased the volume is decreased. So, direction of the f is increased then volume decreased. So, what will happen? So, potential energy will be increased and the volume is decreased. So, that is why I create a negative sign that change of volume and applied force is in reverse. So, that is why I create a negative I give a negative sound. So, due to the change of volume the potential energy will be created.

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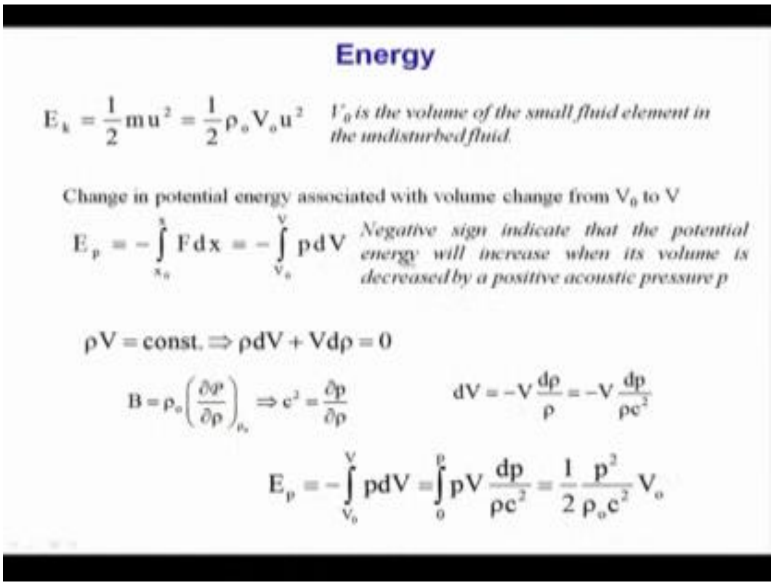

So, if you see the slide negative sign is indicate that the potential energy will be increased when it is volume is decreased by a positive acoustic pressure p. So, now, if you remember the p v rho v rho into v is what is nothing but mass. So, rho v is nothing but constant rho v is constant mass is constant now if I take the derivative with respective d and v. So, I can write rho into del v plus v into del rho into del v plus v into del rho is equal to 0. So, from here another equation is bulk modulus B is nothing but rho 0 del p divided by del rho. This is the bulk modulus which is nothing but b by rho 0 c velocity of the sound c is nothing but root over of bulk modulus B divided by rho.

So, I can say c square is nothing but a B square by rho 0, B by rho 0. So, b by rho 0 is nothing but a c square. So, I can write the del p by del rho is nothing but a c square. Now, if I put that, so I can say the del v del v is equal to what del v is equal to minus v del rho divided by rho. So, del rho here I can write del rho del rho is nothing but a 1 by c square into del p. So, I can write this is nothing but minus V del p divided by rho 0 rho c square rho c square. So, I put that things is in here this equation. So, it is minus, minus, minus positive. So, del p I put in the value of del p is the one this things. So, positive, minus and minus is positive, so it is plus. So, it is nothing but a if it is V 0 to V then what if I convert to del p by del c square. So, it will be 0 to p pressure, so 0 to p, P into V del p by rho c square, so which is nothing but a half of p square by rho 0 c square into V 0.

So, if it is that then what is the total energy? So, total energy is nothing but kinetic energy plus potential energy is equal to half m u square plus half p square by rho 0 c square V 0. Now, what is m, m is nothing but rho 0 into V 0 small volume mass of that volume is rho 0 into V 0. So, it is half rho 0 into V 0 into u square plus half rho p square by p square by rho 0 c square into V 0. So, I can say E by V 0 is nothing but half, let us rho 0 take this side. So, it is nothing but a u square plus p square by rho square c square u square by u square plus p square by rho square c square. So, this is the E by V 0. What is E by V 0 is nothing but energy density instantaneous energy density E I, E by V 0 instantaneous this is the instantaneous energy. So, energy divided by the volume is nothing but instantaneous energy density and it unit will be joules per meter cube.

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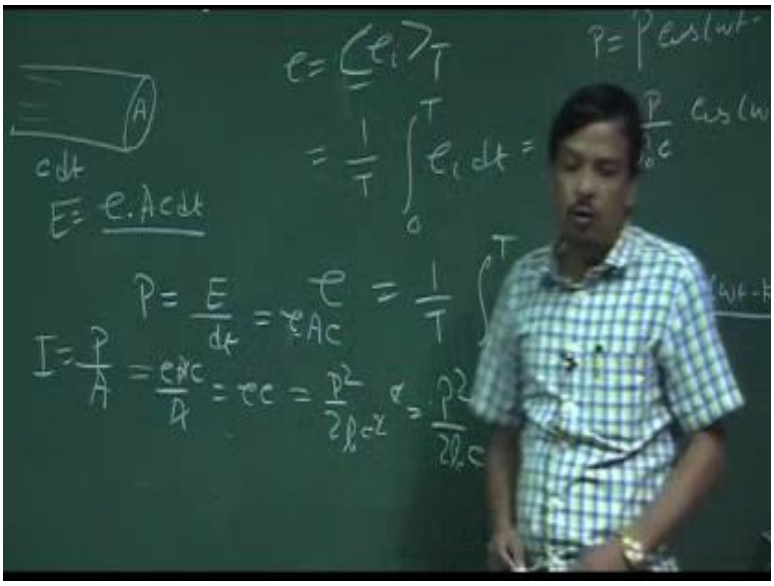

Now, we are not taking the instantaneous energy density we always take the average energy density. So, what is average energy density I can write average energy density e is nothing but time average of instantaneous energy density. So, time average of instantaneous energy density. So, this is nothing but 1 by T 0 to T e i d t. So, what is e i, e i is nothing but u square plus p square by rho square c square. So, u square plus p square by rho square c square. Now, take some consideration. What is relation between the u p you know the p is nothing but plus minus rho c into u pressure is nothing but plus minus rho 0 c into u. Now, if I say the pressure equation p is nothing is the pressure amplitude p plus cos omega t minus k x then what is u is nothing but pressure amplitude p by rho 0 c cos omega t minus k x.

Now, if I put the two values in here, either I can covert in the term of p or in term of u. So, let us consider in term of p. So, it u square is nothing but a p square by rho square c square. So, it is nothing but a if I put the value, so it is nothing but a p square c square this two will be cancelled. So, it is nothing but p square by rho 0 c square. So, I put the value in here, so 1 by T 0 to T p square. So, p square means p square by rho 0 c square. So, it is nothing but a 1 by T 0 to t pressure amplitude square cos square omega t dt minus k x divided by rho 0 c square dt, which will be. If I take the integration it is p square by 2 rho 0 c square. So, if it is p square by 2 rho 0 c square is the average energy density. Still, we are not getting the energy, power and intensity.

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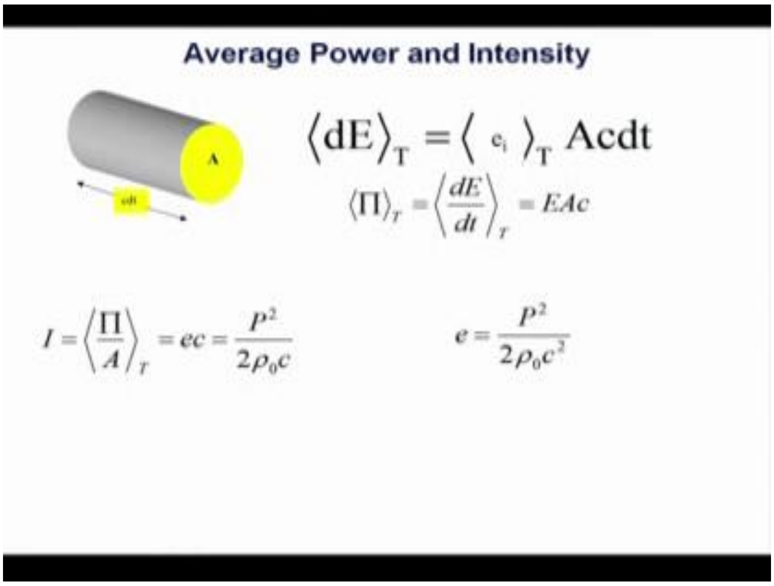

So, what is energy this is average energy density e. Now, let us consider that lets the wave is propagate along with this black board. So, let us consider a volume which the cross sectional area A. So, energy density inside let us consider a pipe. So, you are saying that energy density inside the pipe is nothing but p square by 2 rho c square. So, how much energy will be flow in one second. So, energy density if the energy density is known then if I multiply the volume of that pipe for one second pipe volume then I can get the energy total energy. So, let us for dt time if the velocity of the sound is c, so length of the pipe will be c into d t. Within the d t time c into d t length will be covered. And if the cross sectional area is A, then the volume is a c dt. And total energy e is nothing but average energy multiply by that volume is the total energy.

So, what is power, power is nothing but energy per unit time. So, energy by dt. So, power is nothing but e A into c, are you understand or not, see that this board also this slides. So, I am saying I want to find out the energy flow for that plane wave propagation. So, I find out the expression of the energy density which is p square by 2 rho 0 c. Now, if I know the energy density lets I consider for the dt time how much energy will be flow with in this volume whose cross sectional area is A. So, it is nothing but c dt into A. So, energy density multiply by the volume is nothing but total energy by volume. So, volume I multiplied. Now, if I gain the total energy what is the power energy per unit time? So, energy divided by dt. So, it is nothing but e into A c, it not capital e small e, e into average energy density multiply by A into c.

Now what is intensity, intensity means the power per unit area. So, intensity I is nothing but P by A. So, if it is P by A then intensity is nothing but e A c divided by A. So, A, A cancelled it is e into c. So, what is e value? P square by 2 rho c square. So, P square by two rho 0 c square into c. So, it is nothing but p square by 2 rho 0 c. So, intensity of the plane wave propagation is nothing but P square by 2 rho 0 c.

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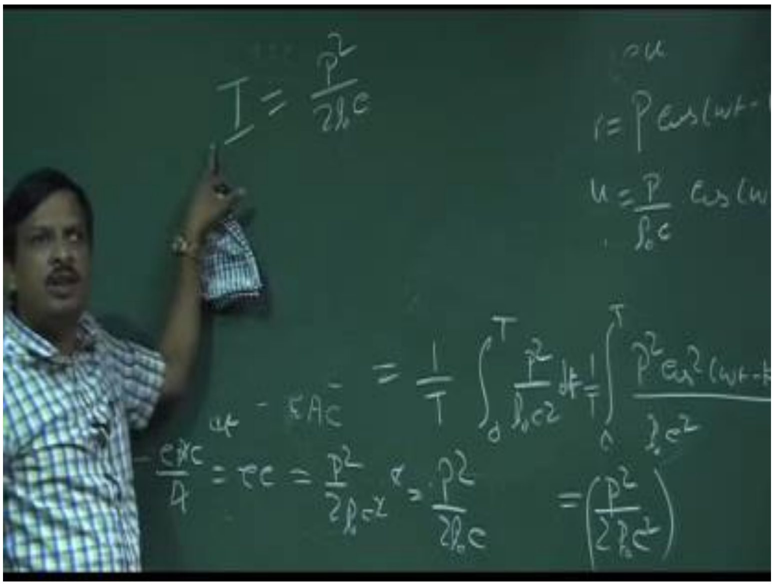

So, intensity if I consider for the plane wave propagation of the sound then always if I know the sound pressure then intensity of the sound is nothing but P square by 2 rho c and power intensity multiply by the area, so that is the power. So, P square by 2 rho c is the intensity.

Next, we will discuss about the spherical wave propagation, and then we will consider that how intensity will be calculate for the spherical wave propagation, and what is the difference between the plane wave propagation, then intensity in dB then perception of the sound in dB perception of the sound will talk about. And then we complete this chapter.

Thank you.