Audio System Engineering
Prof. Shyamal Kumar Das Mandal
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 07
Acoustic Wave Equation (Contd.)

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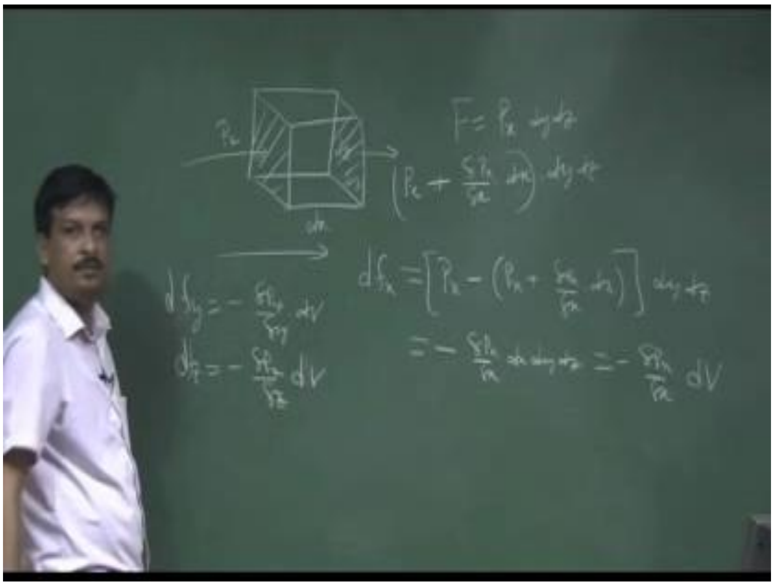

So, we have discussed to derive the two equations, equation of states and equation of mass. Now, we go for derive that equation of force. We draw the same things, the same volume will consider in the propagation space of the wave, and the medium is liquid. We have considered this is nothing but a dx, dy, dz. So, I consider that some volume I consider. Now, equation of force, what is the equation of force is nothing but a Newton second law of motion, the force is equal to mass into acceleration. So, what is force I said when that acoustic wave propagated in a medium, so if this is the plane, this plane will exposed a pressure lets the wave is propagated in this direction. So, I am saying that pressure is acting on this direction. So, I can say P x is the pressure total pressure acting on x-direction. So, total pressure is nothing but a total pressure minus equilibrium pressure will be the acoustic pressure. So, I said total pressure acting on the surface is P x.

So, what is the force, what is the definition of the pressure, force per unit area is the pressure. So, what is the area of this plane nothing but a dy into d z. So, force acting on this plane is nothing but a P x into dy dz. So, force is pressure multiplied by the area. Now, if force is propagated in this direction, sorry pressure is propagated in this direction, this here also will some pressure this boundary also some pressure this surface also a pressure and that opposite surface also express some pressure. So, that pressure is nothing but a we said P x is the pressure in here, change of pressure plus sorry P x plus change of pressure means del P x by del x. Change of pressure in a x direction multiply by the distance dx. Because I have to travel from here to here is the dx distance and rate of change of pressure in the x direction is del P x by del x. So, the pressure in this boundary will be here and also it will be multiply by the area same area will give me the force acting on this plane.

So, if the direction of the force this way, so net amount of force inside this volume will be this force minus this force. So, net amount of force del f x acting on the x-direction is nothing but a P x minus P x plus del P x by del x into dx whole multiplied by dy dz area is multiply by the area is the force. So, if I derived it, it is minus del P x by del x into dx dy d z. So, it is nothing but a minus del P x by del x into del volume P x, P y, P z is nothing but a volume. Similarly, I can get del f y is equal to minus del p y divided by del y into d v I get dz d f z is nothing but a minus del P z divided by del z into dz sorry into del V.

Similarly, y-direction and x-direction are all direction will get. So, total force acting on this volume in all three direction. So, will be is nothing but a summation of those things. So, total force is nothing but a del f or I can write del d f is equal to del f x d f x d f y plus d f z. So, it is nothing but a minus del P x by del x plus del P y by del y plus del p z by del z into del V very simple. Now, if you see. So, what is this del p x del y del p x del x del p y del y del p z del z nothing but a divergence. So, I can write minus divergent of p into dV, minus divergent of p into dV.

Now, if this is a volume on an acoustic plane lets this is a whole things is liquid in liquid this is the volume. So, these volume not only expose to the pressure force, but also expose to the gravitational force. So, gravitational force will only this way, if I consider the volume in here, so there is a gravitational force g will be there. So, what is the gravitational force, total force is nothing but a this force plus gravitational force. What is the gravitational force, we know g into mass gravitational acceleration into mass is the force f is equal a into f m into a. So, gravitational acceleration plus mass what is the mass rho into d V is the mass, g m, g m is the gravitation forces. So, this is the total force acting on this volume.

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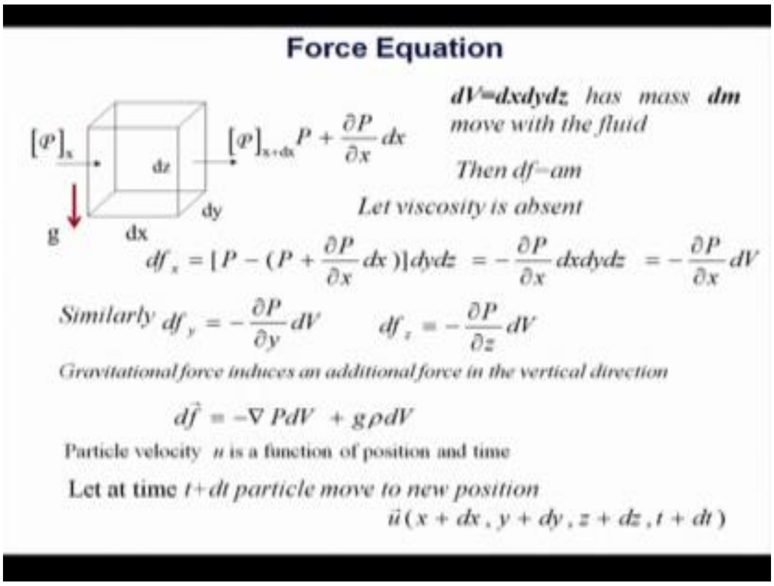

Now, if I apply the force on a particle what will happen the particle will move. So, I expose the particle I expect the particle velocity in here.

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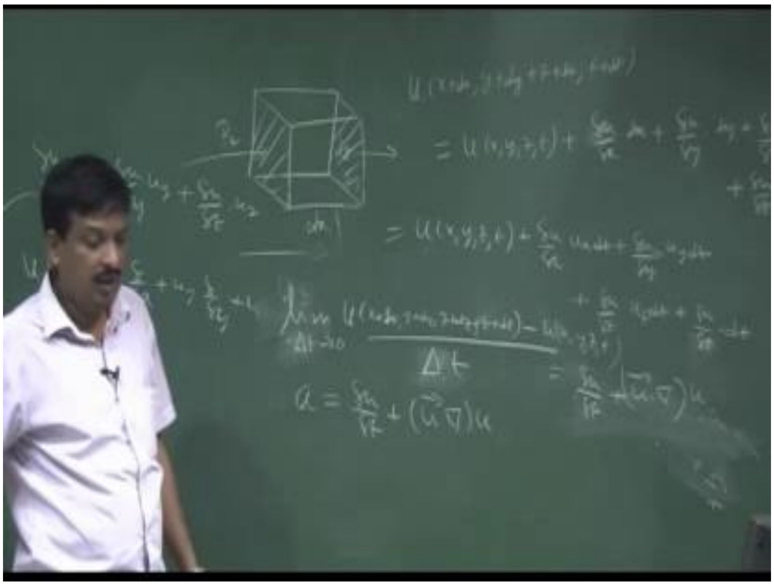

So, particle velocity is u, it will be on all three direction. So, u is function of x, y, z and t particle velocity can arise with the position of the particle and also with respect to time. So, u is the function of x, y, z and t. Now, if I consider lets this is u t and another time a take u t plus del t or d t always write d t, after little change of time not only time change of the particle, the position of the particle also change. So, here this if this is my initial u t after dt time, I will get a particle velocity which is u x plus dx plus dx z plus dx dz sorry dy and t plus dt. If the particle change with respect to time, particle velocity u is the function of x, y, z, t, if t is change then x, y, z is also change. So, at the t is equal to t plus dt, I will get new particle velocity which is u x plus dx lets dx is changes in x direction, y direction change is dy and z direction change in d z. So, new u is nothing but it u x plus dx u plus dy plus dz.

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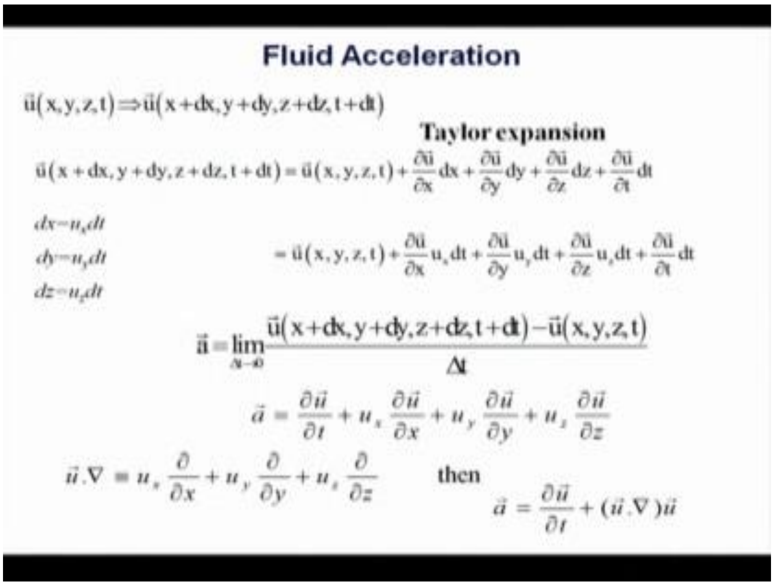

Now, if I expressed new things new particle velocity in terms of taylor expansion because of the del t time in change. So, it is nothing but a. So, I can write I can write that u x plus dx y plus dy plus z plus dz and t plus dt is nothing but a initial u x y z t plus changed due to x direction. So, change of u in x direction del u d u. So, change in x direction is del u by dx into dx change of u in x direction plus change of u in y direction plus change of u in z direction plus change of u due to time del u by del t into d t I can write I can write this thing. So, u initial velocity plus change of velocity due to x direction due to y direction due to z direction and due to time.

So, now if you see in this equation, so what is dx, what is u, u x is the velocity in x direction. So, u x is nothing but a dx by d t change of displacement part time rate of change of displacement is equal to velocity. So, that del x is nothing but a or dx is nothing but a u x into d t similarly dy is nothing but a u i into d t dz is nothing but a u z into dz d t. So, I put that things in equation, so that equation becomes u x y z t plus dx is nothing but a u x d t. So, I write del u by del x into u x d t plus del u by del y into u y d t. So, this will be y sorry this write in you write it plus del u by del z into u z d t plus d u by d t del t into d t. So, d t is common.

So, now, I can write the same equation I can write. So, this will come u x plus dx y plus dy z plus dz t plus d t minus this one, u x y z t divided by d t is equal to del u by del x u x plus del u by del z del y u u y plus del u by del z u z ok or not. Simple in that case what is this del u, del x, u x, plus del u by d t that term will be there. So, now I just delete that this term I am writing del u by del x into u x plus del u by dy into u y plus del u by del z into u z. What is this, what is u divergent is nothing but a u x del by del x plus u y del by del y plus u z del by del z. So, I can write this is nothing but u divergent u u divergent u is or not u x u x del x I consider then I d u I consider that things. So, it is nothing but a divergent of this thing.

Now, so this is nothing but a one t term will be there, which is del u by del t plus u dot divergent into u. Just change this term and d u d t term will be there d u d t term will be there. Now what is this it is a del t. Now, if I instead of d t if I write delta t delta t instead of this I write delta t then what will be there, there will be a limit delta t sorry delta t tends to 0 then it is d t. So, in limit t tends to 0, if difference or velocity divided by the time is nothing but a acceleration rate of change of velocity is the acceleration. So, I can say a acceleration is equal to del u by del t plus u divergent u there is the acceleration of the particle. Now, what is the force, force is called mass into acceleration force is nothing but a mass into acceleration. So, if this is my acceleration I have to multiplied by the mass and I get the force.

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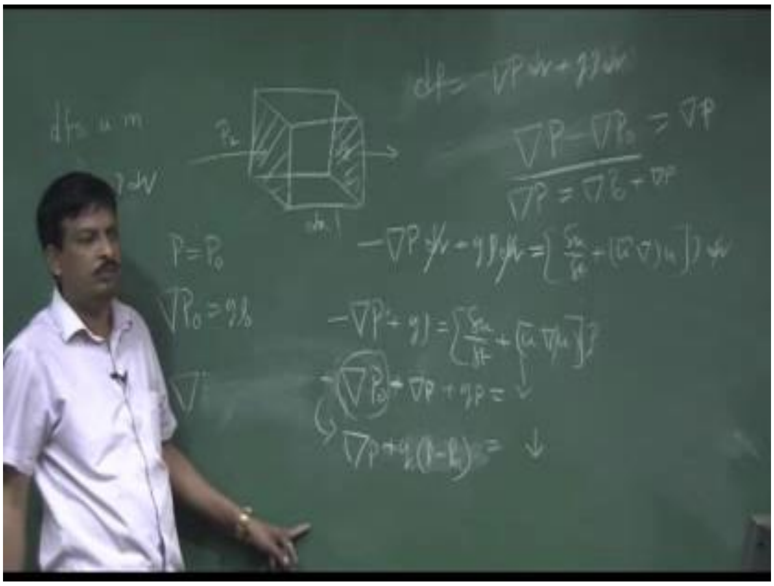

So, I can say total force d f is nothing but a acceleration into mass, what is the mass this volume rho into d v. So, it is nothing but a a into rho d v, if it is a into rho d v I can put value of a in here. So, I get it is nothing but a del u by del t plus u divergent u multiply by rho d v. And what force expression we have derived the expression of the force from that equation we derived that del f total del f total force is nothing but a divergent minus divergent of total force into d v plus gravitational acceleration into density into volume d v. So, this and this must be equal because force difference of the force which is acting on inside the volume will be same is produced by the particular acceleration inside the volume. So, if the particular acceleration is this multiply by the mass is equal to the total force. So, total force is nothing but a force applied from all direction minus force exit from in all direction.

So, in that case this and this must be equal. So, I can write that now I get the one equation what is the equation I get minus p d v plus g into rho d v is equal to del u by del t plus u divergent u whole multiplied by rho into d v. So, I can said that d v all both side d v will be cancel d v will be not acting. So, I can cancel this d v. So, I get what is the equation minus del p total pressure plus 0 is equal to del u by del t plus u divergent u whole multiplied by the rho.

Now think about if there is no acoustic pressure no acoustic pressure means total pressure is equal to p 0. So, which force is applied on only gravitational force. So, at equilibrium condition, when there is no acoustic excitation is injected, the acoustic excitation is 0 then only the gravitational force is acting. So, if I see this equation, if I said there is no acoustic excitation. In that case I can say that g rho 0, no acoustic excitation means density in equilibrium density. So, g rho 0 is nothing but a equal to divergent of p 0, because this portion is 0, no particle velocity with respect to time or with respect to position no acoustic pressure. I will say that there no acoustic force is applied. So, I can say the gravitational force only applied over there. So, g into rho 0 because at no excitation. So, rho is nothing but a rho 0 is equal to divergent of equilibrium pressure or not. So, in that case this is ok.

So, what I will get in that equation. So, g zero rho 0 is nothing but a del rho 0. So, the total pressure del p is equal to small del p rho small p plus g rho 0 at when the acoustic excitation is not there I said gravitational force multiplied by the equilibrium density is equal to del rho 0, gravitational force. So, what I said that del p is a total pressure, del p is nothing but a del p minus del rho 0 del p 0. What is that nothing but a del small p, let us small p write like this way, small p, put instantaneous pressure minus equilibrium pressure is equal to the acoustic pressure.

So, if there is a no acoustic excitation, then I say g rho 0 is nothing but a del rho 0. So, in that case I can write equilibrium pressure this del capital P is nothing but a del p 0 plus acoustic pressure is it ok or not; from this equation, I can write total pressure is nothing but a equilibrium pressure plus acoustic pressure. So, if I put this thing in this equation this equation. So, I can write del p 0 plus acoustic pressure, acoustic pressure is small p minus this will be minus, this will be minus, I just writing total pressure is nothing but a equilibrium pressure plus acoustic pressure. So, if I take the minus outside, so it will be minus, minus here and minus here both is minus.

What I said at plus g p is equal to this thing. Now I said that act there is a no acoustic pressure then del p 0 is nothing but a g rho 0. So, del p 0 is nothing but a minus g rho 0. So, it is nothing but a this equation is nothing but a acoustic pressure small p minus g rho 0 plus g p 0 is equal to same thing, so g rho 0 g p 0 and rho 0. So, If I consider is a g is plus. So, if I say this is nothing a but plus g rho minus rho 0; g is common rho minus rho 0 is equal to same things.

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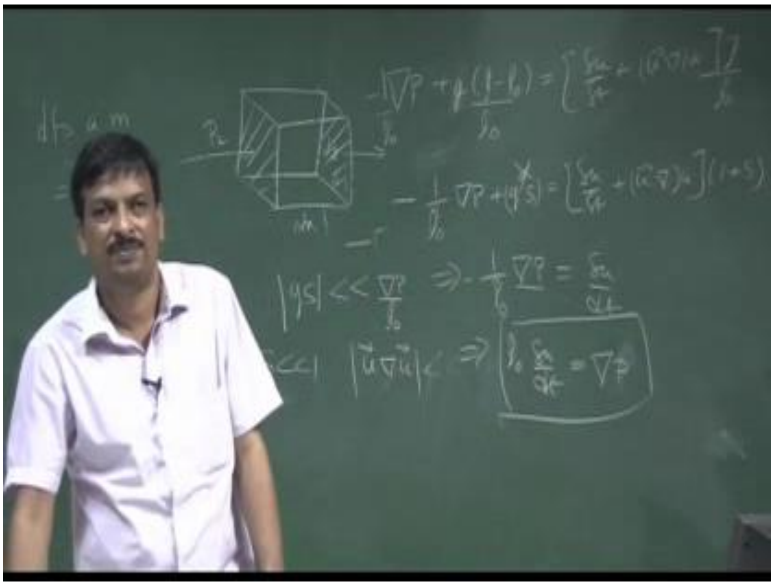

Now, g rho minus rho 0 if I divided both side by rho 0, what will happen both side I divided. So, I can write the my equation is become I said acoustic pressure minus acoustic pressure plus g into rho minus rho 0 is equal to del u by del t plus u divergent into u into rho is it ok or not. Now, I divided by 1 by rho 0 both side rho 0 p by rho 0 is. So, see that g, this is g, g into rho minus rho 0 by rho 0, we want condensation. So, it is nothing but a minus 1 by rho 0 into acoustic pressure plus g into s condensation g into s is equal to del u by del t plus u divergent u whole rho by rho 0. What is rho by rho 0 in term of s, what is s, s is nothing but a rho minus rho 0 divided by rho 0. So, rho by rho 0 is nothing but a 1 plus. So, I write 1 plus s condensation - 1 plus small s.

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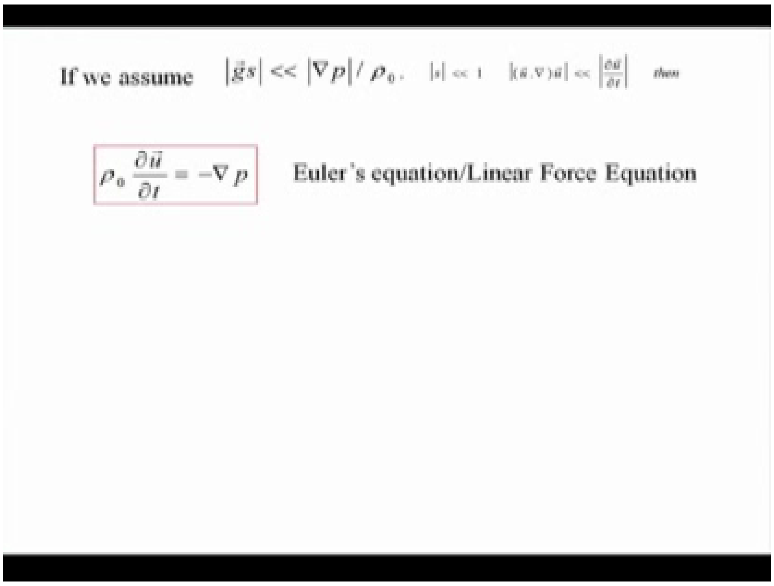

Now, consider the limitation. What are the limitation I said. Now I have to... So, there is a term I have defined. Now, I consider the limitation. What is limitation we said g into s the amplitude of g in to since this is the vectors. So, amplitude of g into s let us consider very, very less than the del p y by rho 0 that means, the amplitude of this term del p by del rho 0 acoustic pressure by rho 0 compare to g s, g s is very less compare to this term. So, I can ignore this because s is less than one or always, s is less than 1 rho minus rho 0 divided by rho 0. So, it is a is less than one. So, if it is less than 1, then I can consider g into s is a very, very weak term compare to this term. So, this term I can ignore, I can ignore this term.

Similarly, if I consider the amplitude of this vector, this vector amplitude this vector amplitude is much much less than this term. Then I consider this term is nothing but a one by rho 0 minus into divergent of p is equal to nothing but a del u by del t. So, s is I have considering s is negligible because it is much much less than 1, g s is negligible compare to this term. And this term is negligible compared to this term, why I consider that things. See that what is g into s condensation multiply the acoustics the gravitational acceleration. Now, if s is very smaller compare to less than 1, so this term is very small compare to change a pressure due to the position divided by rho 0. And this term what is this term this term is divergent of particular velocity, divergent of particular velocity means rate of change of particular velocity in all three direction.

So, with respect to time I said the positional change is very less positional change not displacement rate of change of velocity is very less in that case I get minus 1 by rho 0 into divergent of acoustic pressure is equal to differentiation of the acoustics. So, I can write or I can write rho 0 into del u by del t is nothing but a divergent of acoustic pressure. So, it is not total pressure, it is acoustic pressure small p, because we have considered that things del 0 that we have derived. So, I get acoustic pressure is a function of acoustic velocity with respect to time. So, this is called force equation, sometime it is called Euler's equation or force equation.

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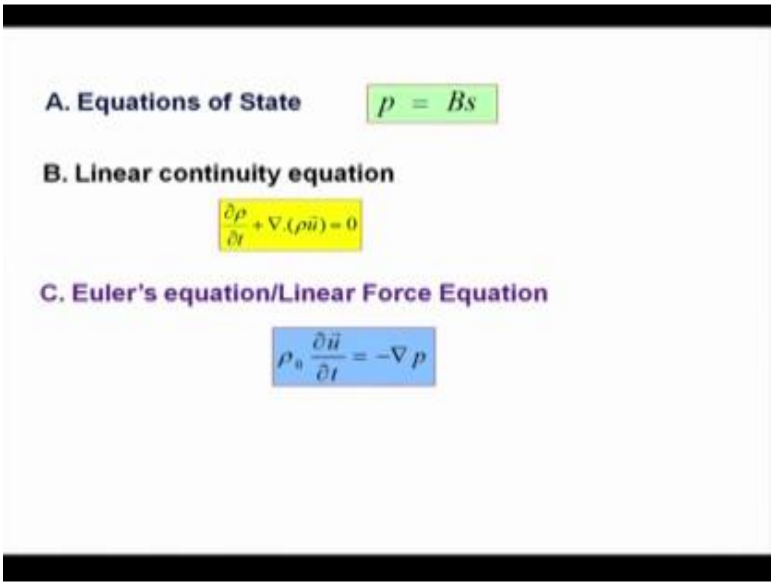

So, I get equation state, I get equation of continuity and I get equation of force. So, these three equations, if I write what are all the three equation, I get three equation acoustics of state, equation of continuity and equation of force.

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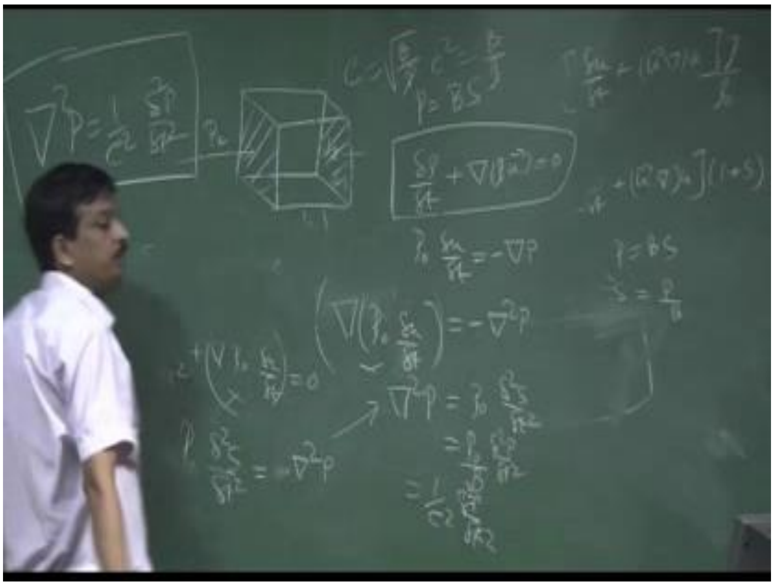

So, equation of state p equal to B into S, p is the acoustic pressure small p that I get B into S. And another continuity del p by del t plus divergent of rho u is equal to 0, that I get from the second acoustical. From the first equation I get rho 0 into del u by del t is equal to minus p. Now, what I want, I want linear wave equation. So, wave equation for what, wave equation of acoustic pressure. So, what I get how do I get acoustic pressure wave equation. So, pressure is a function of position and time. So, pressure is a function of x, y, z and t. So, I should get an equation pressure is a function of position is equal to some function of time.

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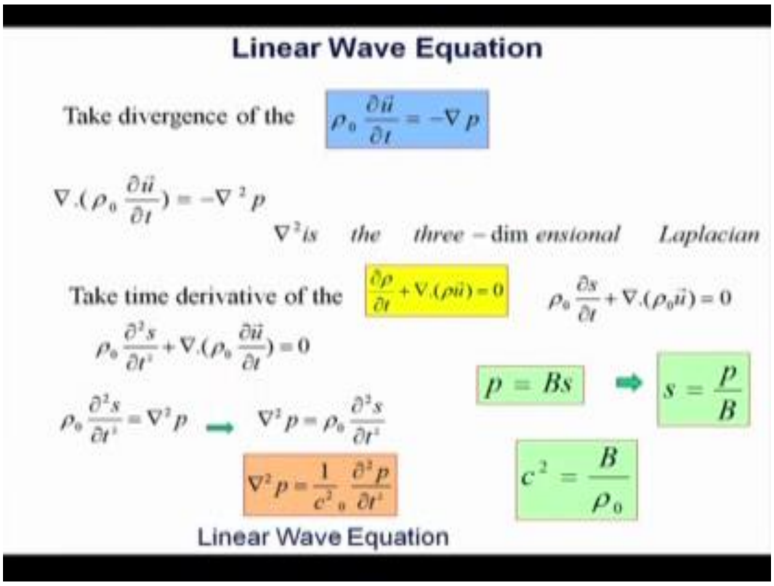

Now, if I take the only see that this equation rho 0 del u by del t minus del p this equation, equation of force. If I take the divergent in both side, so I get divergent rho 0 del u by del t is equal to minus ok or not take the divergence both side. What is this del square called, del square called three-dimensional Laplacian operator it is nothing but a d 2 x by dx plus dx plus d square by dy plus d square by dz square, so that is the Laplacian operator. So, Laplacian operator is nothing but a dx square plus dy square plus del square by del z square that is I get, instead of rho I use in term of condensation.

Now, if I take the time derivative of this equation, what I will get, I just take the time derivative of this equation both side time derivative. So, I get rho 0 del square s by d t square d square by del square by del t square plus rho 0 del rho 0 del u by del t is equal to 0 ok or not. I take the time derivative in both of this equation. So, now, if you see rho 0 del u del this term and this term are same. So, I can write rho 0 del square s by del t square is equal to minus divergence square of p or here I can write the divergence square p is nothing but a minus or minus will be not there. There will be the minus or minus will be not there, this term is same, so this term will be this term is nothing minus, so minus will be not there because this term is equal to minus I put them minus del square p, so it will that side will be plus. So, this is nothing but a del square p. So, del square p is nothing but a rho 0 del square s by del t square.

Now, what I get, I know p is equal to BS. What is s p by... So, instead of s rho 0 rho 0 del square p by del t square by B. So, what is C, C is nothing but a we said bulk modulus divided by density. So, bulk modulus divided by density is 1 by C. So, it is nothing but it a C square is equal to bulk modulus divided by density. So, it is nothing but a 1 by C square del p by del t square. So, the wave equation ultimately we have derived that del p is equal to 1 by C square del square p by del t square I have derived that wave equation. So, this is called linear wave equation.