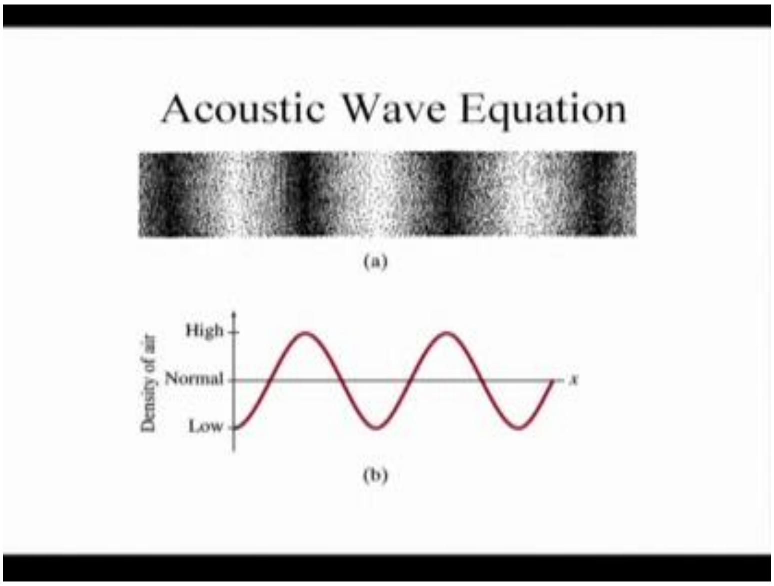
Audio System Engineering
Prof. Shyamal Kumar Das Mandal
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture-06
Acoustic Wave Equation

(Refer Slide Time: 00:33)



Morning. So, we have discussed that mechanical vibration, and their equation and equivalent electrical circuits. Now, we try to find out those wave equations, acoustic wave equations. So, now in practical if you see when think about the loudspeaker, if the loudspeaker is play in this room in the studio, what will happen, the sound will come from the loudspeaker to the person so that means, the sound is travelling through the air and reaching to the person.

Now, we have to know how the sound is propagated. If you do not know how the sound is propagate then we cannot do that acoustic strict band or acoustic how the sound is propagate, how far distance it will be attenuated, all thus properties when you want to find out you have to know how the sound is propagate. That means in mathematics or anything you know that any physical phenomena you try to model it in mathematically so that means, I have to derive the mathematical model for that wave equation.

So, I just show when the mechanical vibration produce that vibration and that vibration has to be travelled. You know that that the acoustics waves do not travel cannot travel without the medium; that means, that when acoustic wave is travel, it use the media to travel. Now, for that purpose if I want to how that acoustics wave is travel if you consider how it is 10 plus 2 you know how the acoustics waves travels; that means, some particle of the media will be move forwards and create a tensor region then it relax create that the next particle will be, so like that. So, condense and relaxation, condensation and relaxation that way sound wave is propagated.

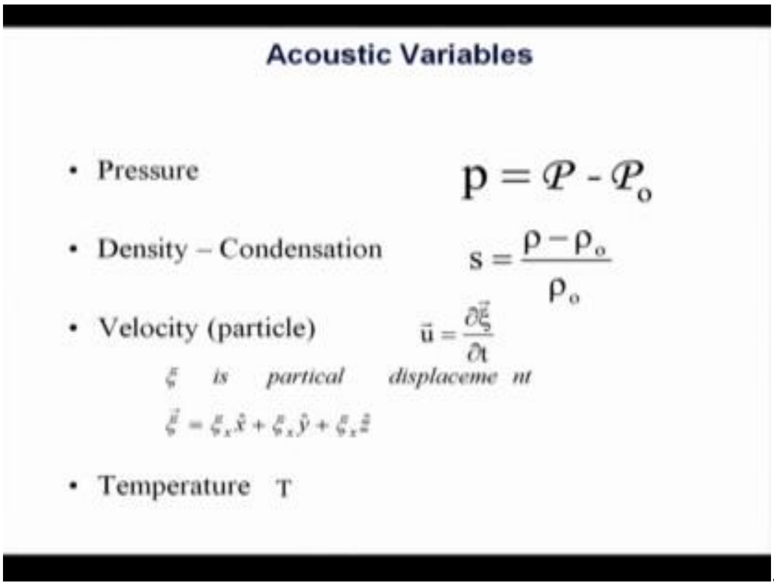
See that this picture of this computer, if you see this acoustic wave picture of this computer, so this portion is tensor, weaker, tensor, weaker, so that way the acoustics wave is propagated that is why acoustic wave is called transverse wave. Transverse wave or what kind of wave, longitudinal wave or acoustic wave is, so it is travels through a condensation and relaxation. So, how it travels, how it condensation is happen, how the relaxation is happen. So, we said that if the medium also one of the property that when the acoustic wave is travelled through a media, media does not change that it is constant.

We put that if I consider the acoustic wave forgot about that intensity of the wave is such a large, we cannot explain that phenomena that we are not considering. Consider the acoustic wave, the resonance intensity while when the acoustic wave is travels it does not change the median; that means, if it is travel through a solid, solid becomes solid, it property does not change. So, how that wave is travelled, so there will be some particle which condense and due to the restoring force, particle will be restores and next particle will be that force will be transferred to the next particle so that way if that acoustic wave travel then we have to derive that acoustic wave equation.

So, we are considering some constant and then derive the simple acoustic wave equation, so we are not deriving the acoustic wave equation which can explain the explosion of a bomb, that intensity the acoustic wave equation we are not considering. We are considering when the acoustic wave travelled through a media, the media does not change. So, when we derive the mathematical equation of the acoustic wave, we consider the fluid, because fluid exhibits some more constraint regarding that acoustic wave transmission. So, we derive that equation for fluid and that equation still valid for wave air solid and everywhere. So, simply we derive the acoustic equation for fluids.

So, now if you think, as a student that acoustic wave is travelled. What is travelling that pressure that if I produce a sound from the loudspeaker the acoustic pressure, which is produced by the diaphragm, has to be transmit to the listeners; that means acoustic pressure is transmitted through the air using condensation and relaxation to the listeners. So, we have to derive that equation how what should be equation that pressure is transmitted from the loudspeakers to the listeners. Now, instead of air, we consider let the medium is liquid, because liquid exhibit more some more constant over that we may consider the when we consider the medium property that medium property supports by the liquids and that acoustic equation which will be derived still it is valid for other medium also.

(Refer Slide Time: 05:42)



.

So, now if I consider that I have liquids I produce a sound and I said that OK.

(Refer Slide Time: 05:47)

And I said that let us consider any point of that liquid, what will be there, there will be a pressure, which pressure – total pressure capital P. And there will be a equilibrium pressure; that means, if I not applied that acoustic pressure external force which is acted acoustic pressure then this liquid has an equilibrium pressure, let us the equilibrium pressure is capital P 0. So, what should be the acoustic pressure will be the small p, which is the difference between difference between the total pressure or instantaneous pressure minus equilibrium pressure, so that is the acoustic pressure. Small p is nothing but a acoustic, normally it is nothing but a acoustic pressure. So, acoustic pressure is nothing but a instantaneous pressure; that means, pressure exhibit in this point minus the equilibrium pressure if the acoustic force is not applied what should be the pressure of at that point that is equilibrium pressure.

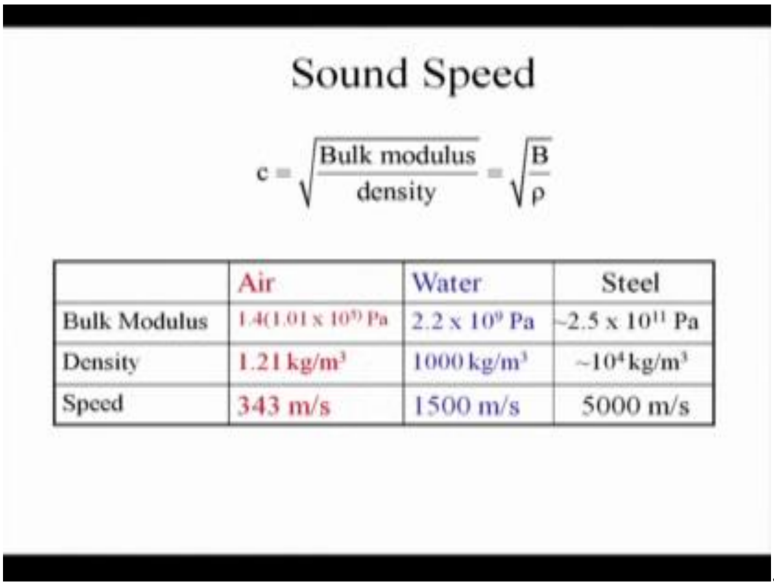
Similarly, at that point, it is a liquid that is the density. Now, if I put the pressure on that any medium, what will happen, the density will be change, let us the density will be change. Then we consider let us the same point the total means instantaneous density is rho this is instantaneous density; that means, equilibrium density plus modified the density which is changed due to the acoustic pressure, so density minus equilibrium density is equal to the instantaneous density. Instantaneous density minus equilibrium density or you can say the density divided by the rho 0 will be the condensation, which is called S. S is nothing but a condensation; so density of the medium is rho minus equilibrium density divided by rho 0 is the rho 0 is the equilibrium density is the condensation property.

Then velocity let us how the wave is propagated. The particle will move forward, create condensation then it will be come to its rest position; again next particle will be move forward, again come to rest position. So, if the particle is move forward; that means, there is a particle velocity, which creates that wave propagation. So, if the particle velocity is u I consider, the particle velocity is u then if you see in this room, if the acoustic wave is created there, so particle velocity is free to move anywhere. It will not that it will move this direction, it can free to move, suppose I create a acoustic pressure in this point, points sort, so what will happen. So, particle can move upward, can move downward, can move forward, and can move backward, all these three directions.

So, if I said that I have a three directions in this room; that means, x, y and z that means, x, y and z upward is x, y, z; let us this is x, this is y and this is z-direction, so any particle can move in this coordinate in any direction. So, u has a component that particle velocity, if the particle velocity u is nothing but a displacement of that particle with respect to time. So, particle can move any position and if the particle is displaced then the velocity is the displacement divided by the time. So, if the particle displacement is psi and d psi by d t, but psi – displacement can occur in any direction x, y, z, so I can say psi is a vector which is nothing but a x with unit vector in the x-direction, y with the unit vector in y-direction, z with the unit vector in z-direction. So, if this point is nothing but a x, y, z, so particle can move to this point which is x 1, y 1, z 1. So, in case of velocity, displacement with respect to time, so displacement with respect to time; and displacement is psi, which consists of three directions.

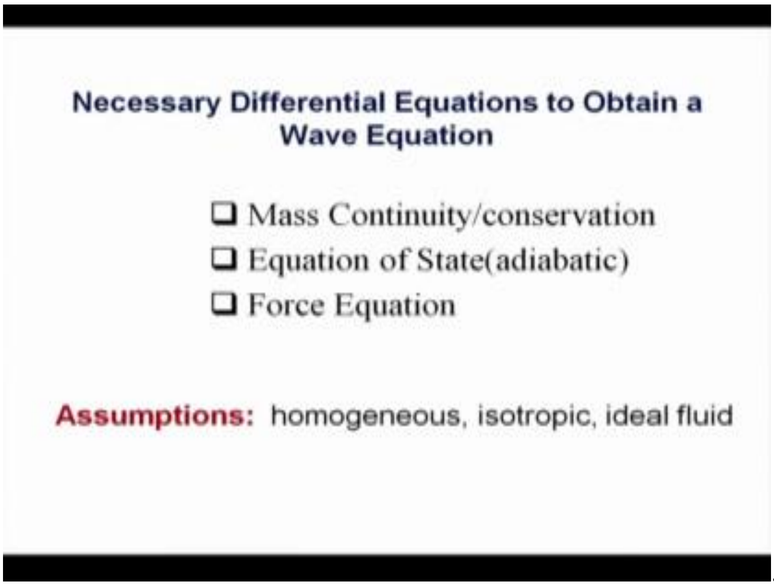
Next is temperature; think about in practical scenario when the wave is transmitted is the temperature it change. If it is support linear wave equation, we said this transmission is an adiabatic condition, so that; that means, if the acoustic wave pass through a media, temperature of the media does not change, so that means, the temperature we have constant that the temperature does not change, so that means, process is adiabatic. So, T is the temperature, which is in Kelvin scale.

(Refer Slide Time: 11:16)



Now, what is the velocity of the sound? If you see the velocity of the sound is nothing but a root over of bulk modulus divided by density of that medium. So, the velocity of the sound is root over of bulk modulus divided by the density. So, if you see the velocity of sound in air and velocity of the sound in solid and velocity of sound in water or liquids will be different, because the bulk modulus and density of the medium will be different. So, in that case, the velocity of the air, say this is a one example is given, in case of air, bulk modulus value is given, density is given, so the speed of the sound I can derived. Similarly, in case of water, bulk modulus is given then there is a density is give then I can derive the velocity of the sound. Steel, bulk modulus is given, then the density is given, you can derive the velocity of sound. So, if you see you know that the solid has the highest velocity than the liquid than the air.

(Refer Slide Time: 12:27)

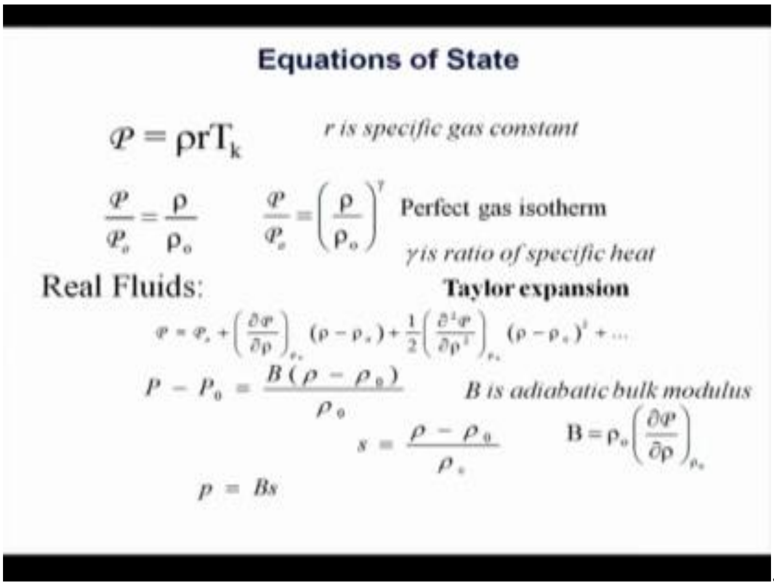


Now, we want to derive that equation, those of the preliminary part of deriving that equation. So, we will use that nomenclature when we derive that equation. Now, if I want to derive that equation, what are the consideration we said when sound is propagated in a medium, medium does not change, and equilibrium density of the medium in all point it same. So, that means, the medium is homogenous, isotropic and ideal fluid.

Ideal fluid means there is no viscous loss, there is no friction loss that part we consider, there is no loss when the acoustic wave, in practical it is not, but simply we derive those put those constant to derive that equation then we employ one after another constant in C how the equation is modified. So, for the linear wave equation, we consider the medium is homogenous, the process of sound transmission is adiabatic – there is no heat loss, and the fluid is ideal; that means, there is no friction, no viscosity, that part is not there. So, the process is isotherm.

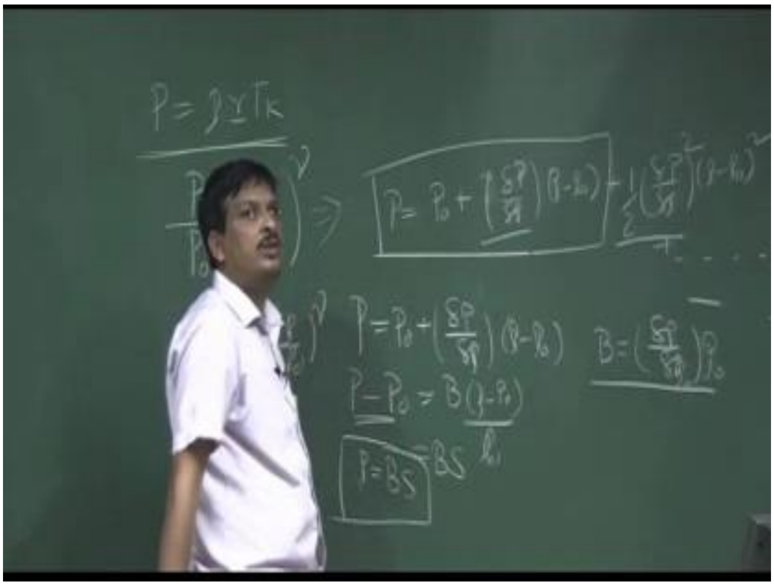
Then we employ these three simple principle one is called continuity of the mass or conservation of mass, then another is called equation of state and another is called force equation. So, those three equations will be employed during the design derivation of the pressure wave equation. So, our ultimate goal is to find out pressure wave equation, how the pressure wave is travelled in the medium that mathematical equation of that pressure wave. Now, we go for one by another.

(Refer Slide Time: 14:18)



So, let us equations of state. What is an equation of state? What we know, we know pressure – total pressure is nothing but, what is total pressure density into r into T k. What is r, r is called gas constant; density into gas constant into T k; T k is the temperature in Kelvin.

(Refer Slide Time: 14:24)



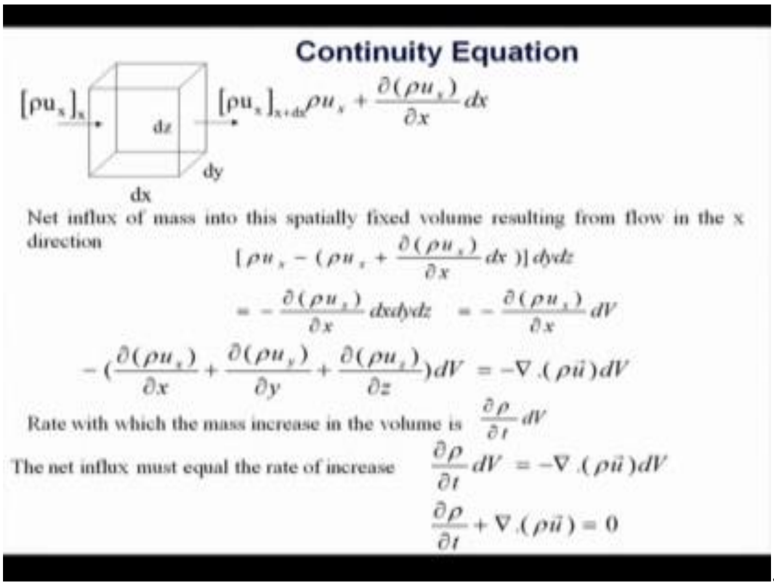
So, pressure is nothing but a density into gas constant or specific gas constant into T k. Now, if this is an isotherm condition, then we know total pressure divided by the equilibrium pressure is equal to rho by rho 0. So, in case of perfect gas condition, when isotherm condition – perfect isotherm condition, it is nothing but a root to the power gamma. This is in case either it is rho by rho 0, but in case of isotherm condition, so this gamma is nothing but a specific heat constant gamma is called specific heat constant. So, if I if this is my equation and gamma is specific heat constant, can I express this term in term of Taylor series.

Now, if I want to express this in term expand in Taylor series, then what will get, I get P is equal to total P 0 plus just express this, this is P is equal to P 0 into rho by rho 0 to the power gamma. This thing I express in Taylor series, so this is 1 plus first order term into difference of the difference of the density, then second order term that will come. So, rho P 0 plus P 0 into what will come P 0 plus 1 plus, so it will be P 0 plus del P by del rho into rho minus rho 0 plus second order term del P by del rho square into half of this into rho minus rho 0 that way it will go after all term will come, square. So, this is the first order, second order then third order, fourth order, fifth order term will come.

Now, if I say that whole thing is linear wave equation, then I consider only to the first order term. I can neglect that higher order terms, because I explaining the linear wave equation. So, in that case, total pressure is nothing but a P 0 plus del P divided by del rho into rho minus rho 0. Now, if I consider that P minus P 0 is equal to I can write B into rho minus rho 0 divided by rho 0. So, what is B, B is nothing but a del P by del rho into rho 0, because I am considering B into rho minus I dividing with the rho 0, so it will be multiply by the rho 0. This B is called bulk modulus, this is called bulk modulus.

And what is rho minus rho 0 divided by rho 0 is nothing but a B into condensation – S, B into S condensation. So, P minus P 0, what is P minus P 0, total pressure minus equilibrium pressure is nothing but a acoustic pressure, which is small p, small p is equal to B into S. small p is equal to B into S, bulk modulus into condensation. So, this is the one equation we have derived from the equation of state that acoustic pressure is nothing but a equal to bulk modulus multiply by the condensation.

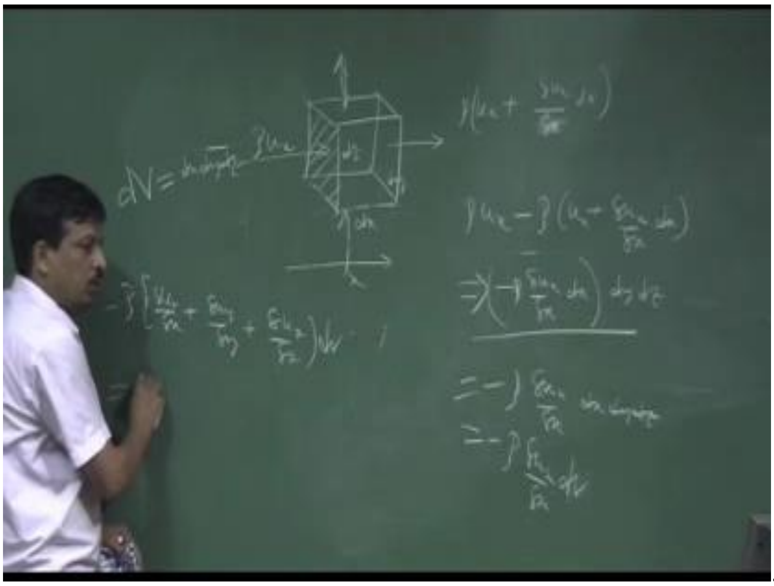
(Refer Slide Time: 19:13)



.

Now, we go for the continuity equation, what is continuity equation. What I said that acoustic wave is propagated through a condensation and relaxation, so in that case condensation and relaxation.

(Refer Slide Time: 19:33)


.

So, let us this is the space of acoustic wave. I consider a volume in this space like this. Let us this is the volume in the acoustic wave propagation space, I consider a small volume, which has d x, d y and d z; three axis d x, d y and d z, I consider a small volume. So, if it is condense, lets it is condense what will happen, what we mean condense. Some particle will enter the volume, some particle will enter in the volume, and how it is enter in the volume, because of the particle velocity.

If wave is propagated in this direction, lets wave is propagated in x-direction, what will happen, some particle will enter into the volume and who cause them to enter in the volume velocity particle velocity. So, if this is u x – particle velocity in x-direction is u x, so what is the particle density, density is rho. So, how many particles will enter in the volume depends on the density. So, density multiply by the u x is the number of particle which enter in the volume. Let us I have a volume, some particle enter in the volume, what will happen the mass of the volume will be increased, but it does not support the continuity of the mass, so that means, if some particle enter into the volume, some particle should leave out the volume. So, that means, there is a particle velocity this direction also, this surface also.

So, in that surface, if the particle velocity is rho u x and the velocity is change due to the d x; that means, rho into u x plus new velocity is del u x by d x into d x. This way we write. The change of velocity due to this d x position is nothing but a del u x d x into d x, change of velocity within a unit distance total distance is d x. So, the change of velocity with the unit distance is del u x by d x into total distance is d x. So, we always write it is del u x by del x into d x, because d x is the distance and change of velocity due to the delta change of space, it is del u x by del x into d x multiply by the rho, so that much of particle will leave to the volume.

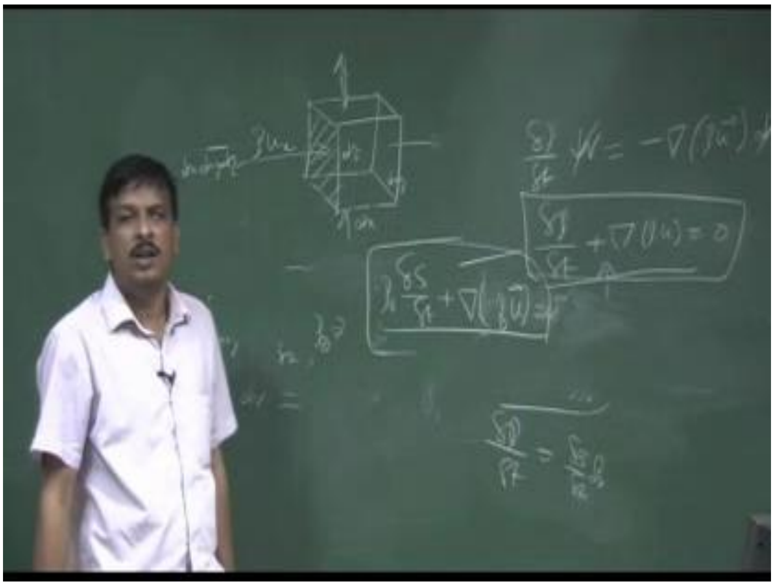
So, net particle enter in this volume - enter amount minus exit amount, so net flux of the mass into this fixed volume is nothing but a rho into u x minus rho into u plus del u x by del x into d x. So, if I do that things so this will del u x cancel, so it is nothing but a minus del u x by del x into dx. Now, what is the mass, total mass increase of the volume, this is the density change due to the particle velocity sorry that will be a rho will be there, rho term will be there; rho into del u x by del x into d x, so that multiply by the total volume is the total mass change.

So, I multiply the total mass increase or intake will be multiply by what is the x-direction, and if the x-direction is dx, what is the surface area of this. It will not single pointed. So, this is the cross section area, so that within the whole cross section area mass will be injected. So, this is I take one point. So, whole cross section area mass will be injected into area cross section area of those things. So, this into the cross section area, what is del y del z; the cross section of this things is del y multiply by the del z, so that is the total increase of the mass.

So, this amount of mass will be increases. Now, in this case, it is nothing but a rho del u x by del x into dx dy dz. So, what is dx dy dz? This is nothing but a volume, because volume is nothing but a dx dy dz. So, it is nothing but a minus rho del u x by del x into del v or I can write instead of v del v, because I take del x del y del z, so write del v, so it is del v. So, that much of mass, this is negative – minus will be increase in this volume; the total mass increase in the volume.

Similarly, the wave we will come from that this direction, the wave will come from this direction and then will go from this direction. Similarly, the wave will come from the z-direction also. So, in all six planes, the direction wave will come. So, in that case, generalized equation I can write, the total increase of the mass is nothing but a del del u x sorry rho into del u x by del x plus del u by del y plus del u z by del z into del v, this is the total.

(Refer Slide Time: 26:49)

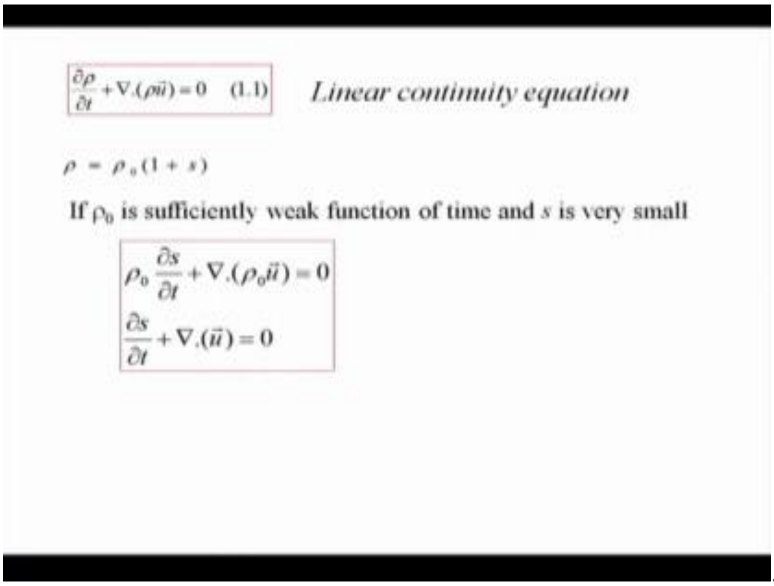


Now, what is the mass change? If the mass is change then what will be change, volume is the del v, so del v is the volume. So, mass change is nothing but a so I can write so this I can write what is del u x by d x, del u by d y del u z by del z, what is the meaning of this. This is nothing but a divergence, so it is nothing but I can write divergent of u is nothing but a del u x by del plus del u y by del y plus del u z by del z.

So, I can write this equation is nothing but a minus rho divergence of u into d v. So, I can or I can write minus divergence of rho into u, u is a vector now three-dimensional vector u x, u y, u z into del v, which is the total mass increase in that volume. So, rate at which mass is increase, now if this amount of mass is increase in here that is nothing but the rate of increase of mass with respect to time rate of increase of density. So, what will happen, so if some mass is enter here, the density of the volume will be change with respect to time, so let us initial it is 0.

Now, once it is started the mass flowing into that side, some mass will be come out that I consider. So, net is going in minus going out is the net amount of mass increase in that volume. In that case, that increase the density of the volume with respect to time, so I can write del rho by del t is the density change with respect to time into del v is the change of mass for the same volume. So, this is nothing but equal to minus del of rho u into del v. So, dv, dv is canceled, so I can write del rho by del t plus divergence of rho into u is equal to 0.

(Refer Slide Time: 29:08)



So, in that case, what will happen, so this is called linear continuity equation? I cannot say that if the if you think about it, I take a ball, liquid ball, if you squeezed it what will happen, the density of the ball will be, the volume is decreases, mass may be increases, mass has to be increases. Or if you see, I take a ball and I inject suppose I have a fixed volume in here air, and I put some more air in that volume with a fixed volume what will happen, air density will be change with respect to the flow of the air inside that volume. In which rate, I injected the volume in that that rate will change the density of the volume.

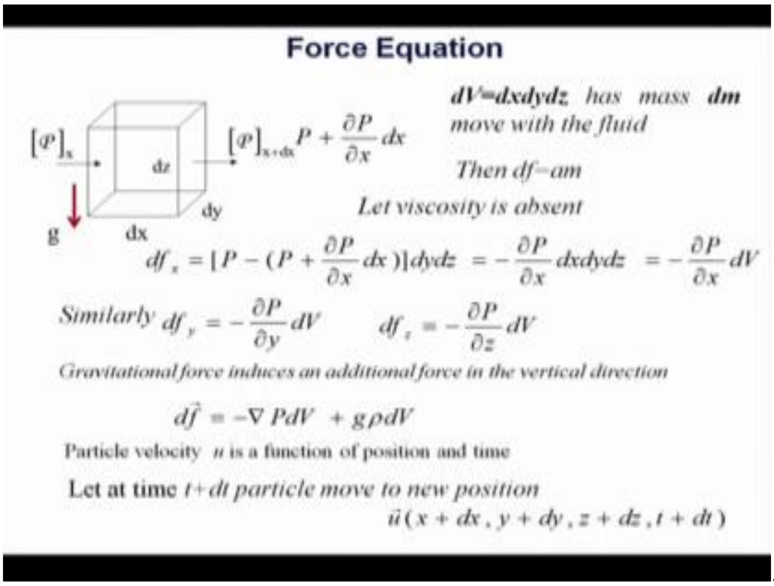
So, I can say the amount of mass per flux which is increase in this thing will be the rate of change of density. If the volume is remained constant, because I have volume I have same, I am not change the volume. So, if del v I consider even constant; that means, rate of change of particle deposited particle or amount of deposited particle will increase the density. So, in that case, the rate of change of density should be equal to the net mass flux enter into the volume. So, it is nothing but a, this equation.

So, if this is the equation, this equation is called linear continuity equation, conservation of mass. If the volume remain change, if the volume remain change, if you more particle is entered, density will be change; similarly, if I change the volume, the density will be change. So, in that case, this is the linear continuity equation. Now, if I say that rho is total density rho, so rho is nothing but a total density rho instantaneous density. So, I know one is called condensation. How we say that if I apply a acoustic force, some particle will be condensed in one point and then it will be relaxed then condense then relaxed. So, this condensation S is nothing but a total density minus equilibrium density divided by the equilibrium density.

Now, if I change this thing, this is nothing but a so I can say rho is equal to one plus S into rho 0, rho 0 S, rho is nothing but a one plus S into rho 0. So, rho 0 plus S rho 0 is equal to rho. So, rho is nothing but a one plus S into rho 0. So, if this now I put this things in this equation, if I take the derivative, so del rho by del t rho 0 is constant, so differentiation will be zero, so del S by del t into rho 0. So, I put this del S here. So, what will happen, this case that rho this equation. This equation become del S of S by del t, rho 0 will be multiplied instead of del rho by del t plus what will there rho, rho is nothing but a one plus S rho 0, so I write divergence of one plus S rho 0 into u is equal to 0.

So, now if I say if the S is very, very small, and rho 0 I said rho 0 is constant rho 0 is constant with respect to time that mean rho 0 is not a function of time, so I said rho 0 is with respect of time. And S is very, very small. If S is very, very small, I can ignore S here, so in that case, it is nothing but a rho 0 u is equal to 0. So, the equation is del rho by rho 0 into del S by del t plus divergence of rho 0 u is equal to 0. So, this is the continuity equation or I can use this equation. So, sometime I may use this equation, sometime I may use this equation later on. So, this is called continuity equation. Next equation is the mass equation or sorry force equation. This is the continuity equation or mass equation.

(Refer Slide Time: 34:04)



Next is the force equation. So, force equation, I discuss in the next class.