Audio System Engineering
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Lecture - 05
Tutorial I

Let us do some mathematical tutorial for that the spring mass system that things, so that it will be very clear to you.

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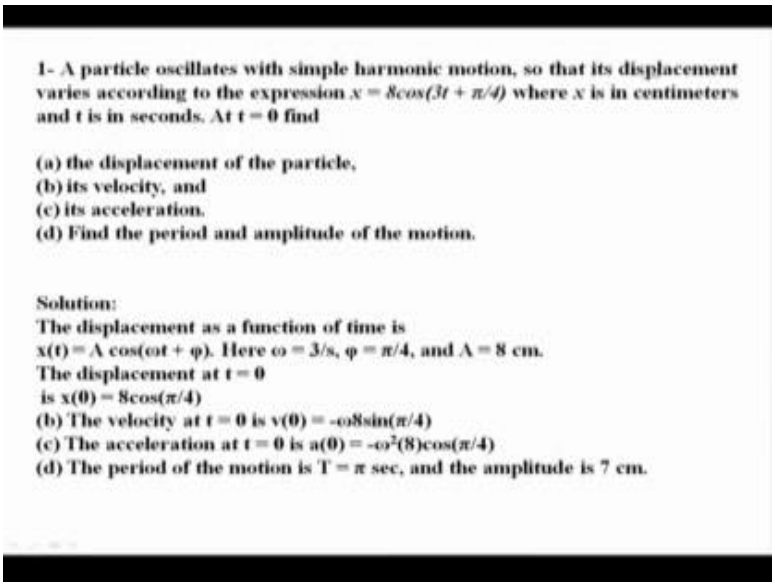

Let us consider the first problem. It say that the particle, a particle oscillates with the simple harmonic motion so that it displacement varies according to the expression of x is equal to a particle in simple harmonic motion its displacement is the form of equation 8 cos omega instead of omega I said 8 cos 3 t plus pi by 4.

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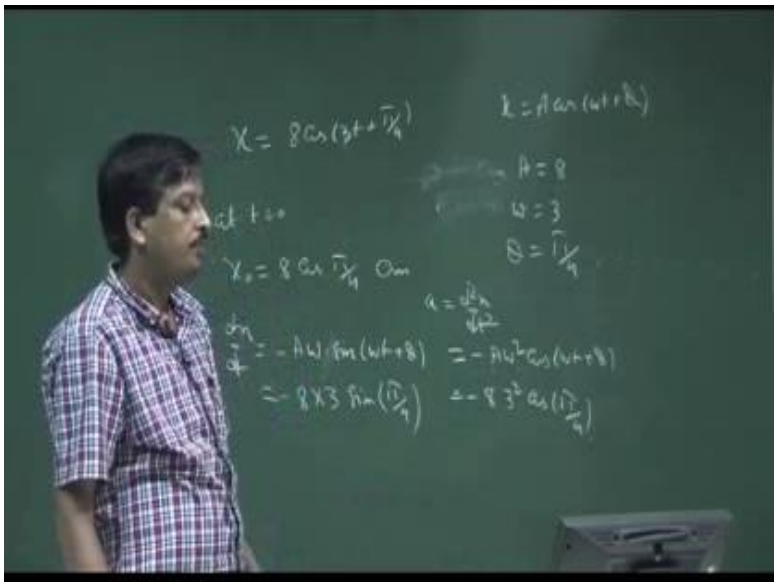

Now, if I say where x is in centimeter, x is in cm – centimeter. Now, x is in centimeter and t is in second. Now, if I say find out the displacement or displacement of particle at t equal to 0. What should be the displacement of the particle at t equal to 0? So, if you remember this x is in the form of A cos omega t plus phi. So, what is A, what is the value of A, A is nothing but 8, what is omega nothing but 3, what is phi is nothing but pi by 4. Now, if I say at t equal to 0, what should be the displacement let x 0 is equal to 8 cos t equal to 0, so it is cos pi by 4, 8 cos pi by 4, you know that pi by 4, cos pi by 4 value multiply by 8 centimeter.

Now, if I ask what is the velocity of that oscillation? So, velocity is nothing but dx d t. So, it is nothing but if I take the d x d t minus A omega sin omega t plus phi. So, what is A? It is nothing but 8, what is omega - is nothing but 3 sin t equal to 0 pi by 4. Now, if I say what is the acceleration of this oscillation a acceleration is nothing but d 2 x by dt square. So, it is nothing but a minus A omega square cos omega t plus phi. So, again A is nothing but 8 omega is nothing but c square and cos is nothing but cos pi by 4 at t equal to 0.

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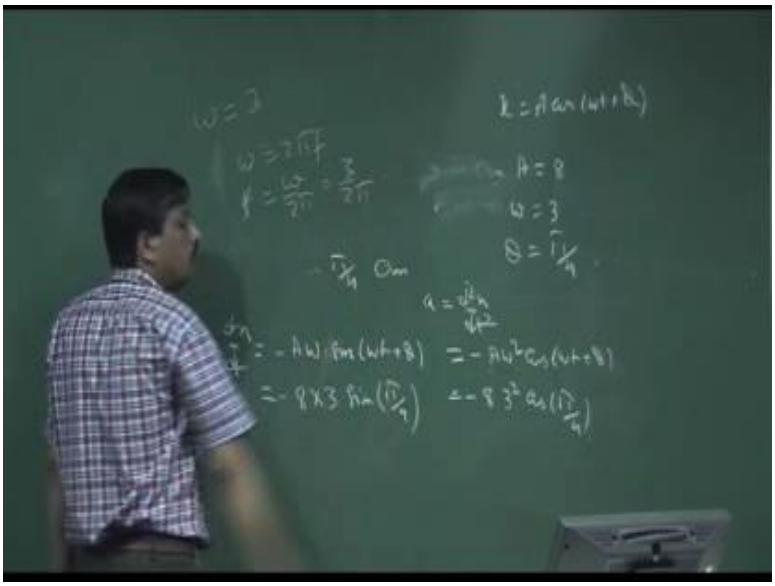

Then if I say find out the period and amplitude of the motion, if I say find out the period of the motion. What is the period of the motion? So, what is omega is nothing but 3. Now, what is f omega? It is nothing but 2 phi f, and then what is f? It is nothing but omega by 2 phi. So, it is 2 by 2 phi, so that way I can find out any parameter for those things.

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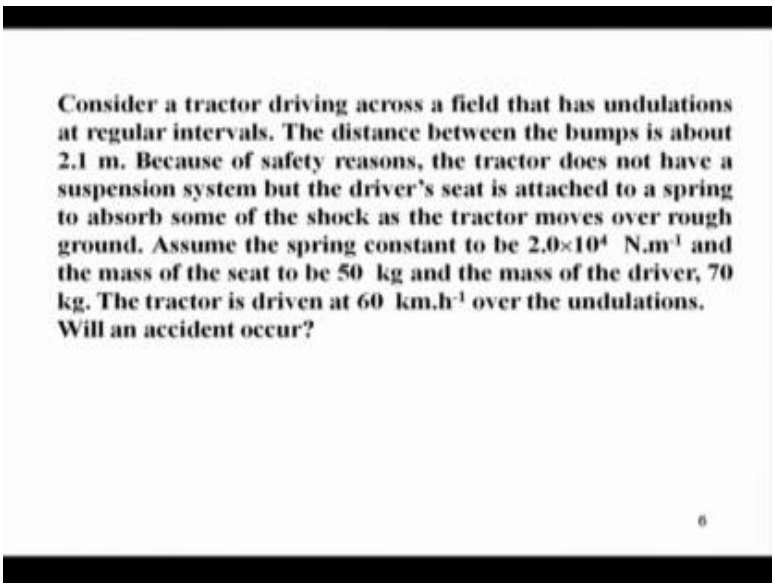

Now, let consider a problem which is little bit of complex, just read the problem from the slide, see the slide first. Consider a tractor driving across a field that has undulations at

regular interval. The distance between the bump is about 2.1 meter. Because of the safety reasons, the tractor does not have any suspension system, but the driver's seat is attached to a spring to absorb some shock as the tractor moves over the rough ground. Assume the spring constant of the driver's seat spring is 2 into 10 to the power 4 Newton meter per second, and the mass of the seat is 50 kg and the mass of the driver is 70 kg. The tractor is driven at 60 kilometer per hour over the undulation. Will an accident occur?

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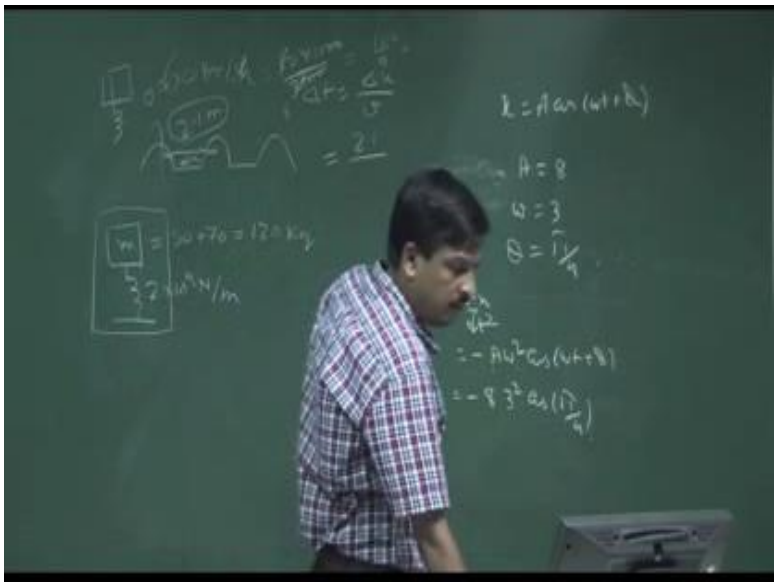

If I say will an accident occur, so after reading the statement of the problem, what you understand that there is a tractor which moves across a road or a field which has a bump one bump, another bump and another bump, bump is there. And the bump distance between the two bumps is nothing but a 2.1 meter. Now, tractor has a seat has a spring and over the seat I have a mass. So, the system is nothing but a mass spring. So, spring constant is 2 into 10 to the power 4 Newton per meter, and mass is equivalent to what is the mass seat mass plus driver mass 50 plus 70 is equal to 120 kg.

Now, I said will an accident occur. So, after you know the mechanical acceleration, what you will say that if the tractor seat is resonate with the force which is received from that bump then the driver will be uprooted from the seat, because if the system is resonate maximum power transfer will be happened. So, the driver cannot hold the seat. So, accident will be occurring. So, only I have to find out whether this spring mass system will be resonate or not due to the vertical force.

When the particle force will be happened because of the speed 60 kilometer per hour the bump will pass. So, the interval of the bump is 2.1 meter. So, the tractor will get a vertical force with the frequency in that interval, for the frequency of the tractor. So, if it is del x, and if the speed is let us v, then what is the interval between the time interval between the bump is the frequency of the vertical force because every bump provide a vertical force to the tractor. So, the frequency of that force is nothing but a interval time taken to cross this interval. So, I can say del t is nothing but a del x divided by v. What is del x 2.1 meter, what is v - 60 kilometer? So, 60, if I want to convert in meter 60 into 1000 divided by 3600. So, if I cut it, so it is nothing but 100 by 6. So, how much it will come, you can calculate that value 100 or yes 100 by 6, 100 by 6 will come. So, you can calculate.

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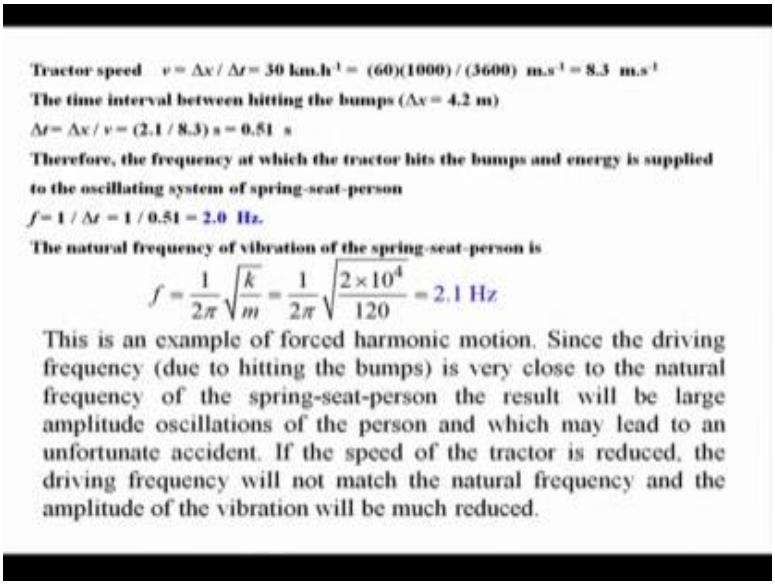

Then I get the delta d, once I get the delta t put the x value and the v value 100 by 6 then I get the delta t.

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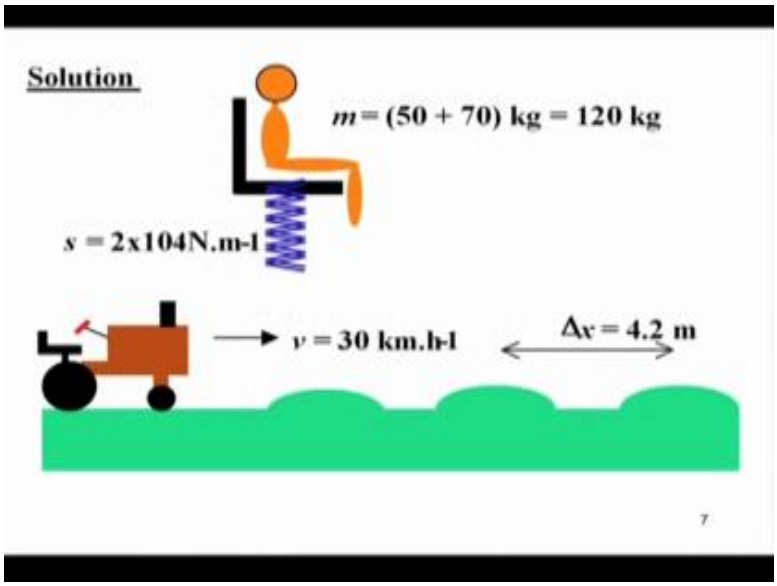

So, if the time interval is delta t then what will be the frequency, frequency is nothing but 1 by delta t, I will not completely do that problem, you have to do it. So, f is nothing but 1 by delta t.

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So, this is the frequency of the applied force f frequency of the applied force f. If the applied force frequency omega is equal to omega 0 then the resonance will be happened. So, what is omega 0, omega 0 is nothing but root over of s by m. So, what is string

constant, you know spring constant 2 into 10 to the power 4 Newton per meter divided by total m is equal to 70, so you get the omega 0.

Now, you have to see whether the omega is close to equal to omega 0 or not if it is close to omega 0 the accident will be happened; if it is not accident will not be happened ok that is why you cross a bump regular bump, you see there is a speed limit mentioned in the roadside. Please drive with 30 kilometer per hour. It is not that reduce your reduce your speed for safety once you cross the bump you get a vertical oscillation if that oscillation does not support your suspension system of the car, your car will be broke down or maximum power transfer will happen can cause the system may be damaged or an accident may be occur. So, that is why that this problem I have given you. So, this is the particle relationship between the spring mass systems to the real life problem. So, if you know the spring mass system then you can easily explain the real life problem of different situation.

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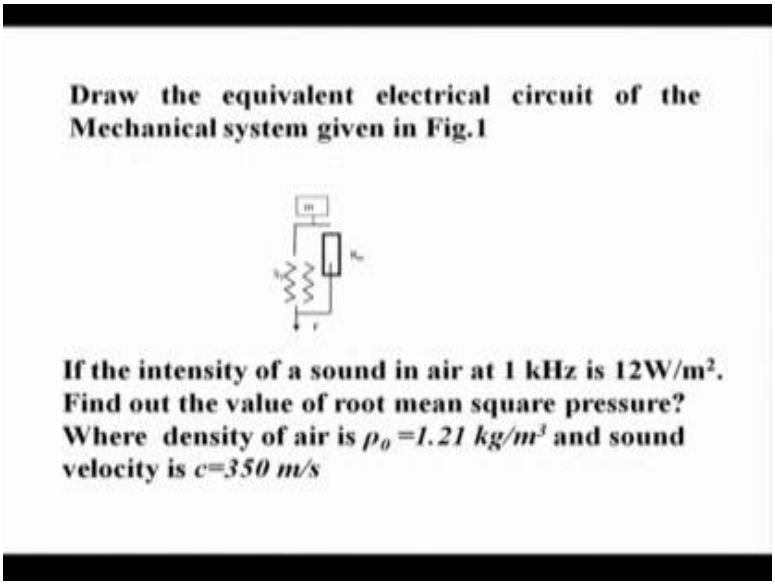

So, this is one of the problems. Similarly, let us I give another problem that is draw the mechanical equivalent circuit is not correctly drawn in slides, I will write in the board.

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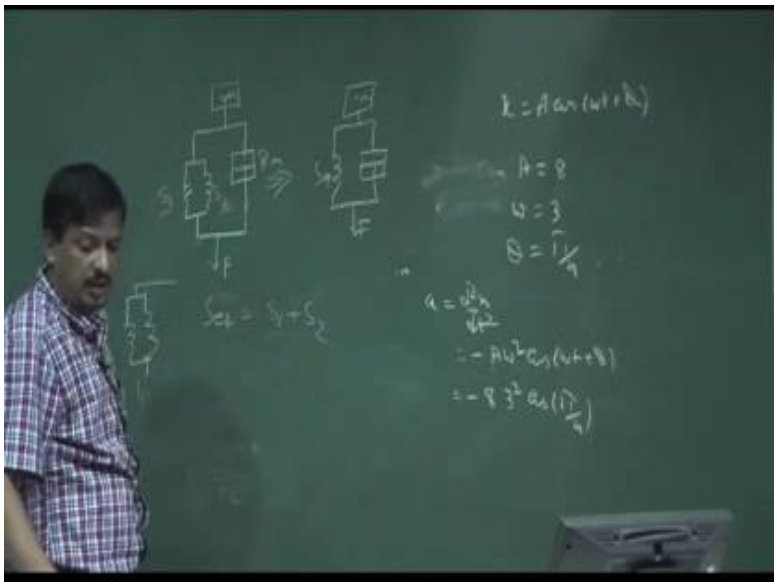

Let us I have an electrical mechanical circuits, which is m then I have an electric mechanical resistance then I have a two spring, one spring in here, another spring in here connected here and then this is applied a force in here. Now, let us this is R m, m, this is s 2, this is s 2. Draw the equivalent electrical circuits of the mechanical system given in figure 1. And find out the impedance; find out the mechanical resonance frequency of this circuit. So, suppose this R m, s 1, s 2, s 1 then there is m, I have given then can you calculate the z m, you have to calculate the omega 0, you have to the calculate draw the equivalent electrical circuits.

So, what I said a two springs, if the two springs are in parallel it is equivalent to what is s effective, if two spring are in series 1 by s 1 plus 1 by s 2; if two spring is parallel it is nothing but a s 1 plus s 2 or you know that if two spring is in parallel. Just before that I have, so it is nothing but s 1 plus s 2. So, I can replace this circuits by an equivalent mechanical circuits m, this is s effective then there is a mechanical resistance and then there is a force is applied in here. Now, if this is the mechanical circuit, what is the equivalent electrical circuit, I can easily draw it now.

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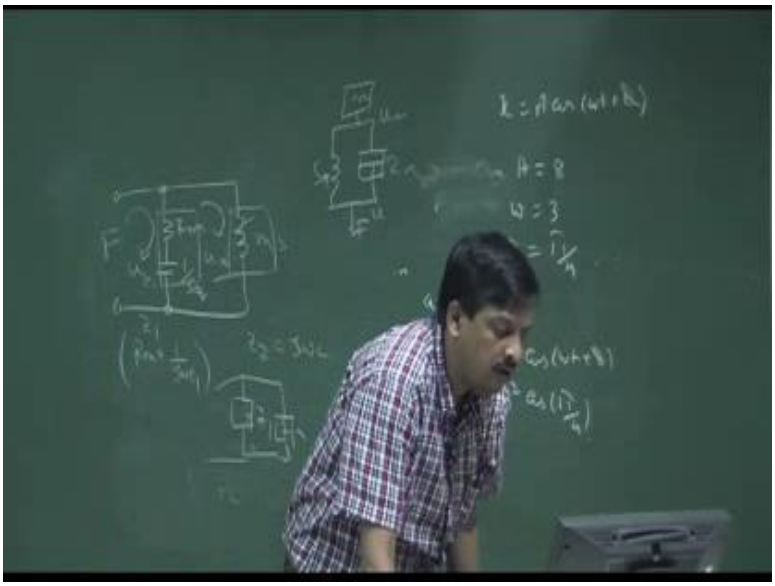

Now, it is your hand, now I can draw the volume velocity existing here is u, and here will be u m, since this is not in grounded. So, I can say it is nothing but force applied here and it produces u. So, u is nothing but a resistance plus, so it is 1 by s equivalent and it is nothing but R m, if it is R m then there will be inductance which is m which is u m. Now, can you find out the mechanical impedance of these circuits, find out the mechanical impedance between this node and this node. So, you find out the equivalent impedance of these two things equivalent impedance to a do these two things and these two are in parallel you can draw that things.

So, what is the equivalent impedance is R m plus 1 by any capacitance let it is c 1, so it is j omega c 1. These two are in series. This is let z 1 then what is z 2, z 2 is nothing but j omega L, let us this is nothing but L. So, then this z 1 and z 2 are in parallel. So, z 1 let this is z 1 load and another load is z 2 is in parallel then I can find out the equivalent impedance if the two loads are parallel. Then I can find out the resonance, resonance frequency of the circuits either 1 by a root over of L c or root over of s by s equivalent divided by mass. So, this way I can solve the problem.

So, this kind of problem people may give you that suppose there is a something that ok let this is a diaphragm mechanical that loudspeaker diaphragm. The mechanical impedance is given, mechanical resistance is given stiffness of the diaphragm is given mass of the diaphragm is given and how it is housed it is also given. So, you first draw

the equivalent mechanical circuits then you can convert to mechanical circuit to electrical circuits, so that way you can do it.

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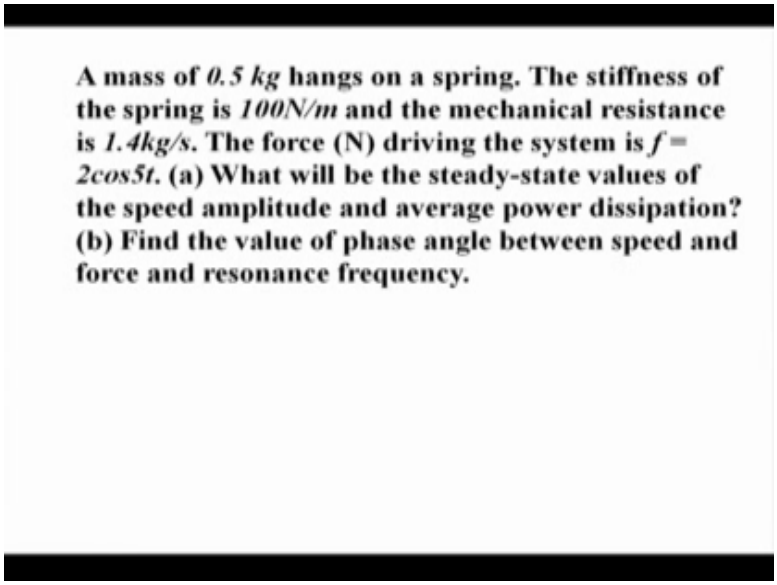

Now, suppose there is another problem if you see the problem just read the problem this problem, sorry this problem. If the intensity of the sound in air at 1 kilo hertz is 12 watt for meter square find out the value of root mean square pressure, sorry this is not in this chapter. Let us come to here. A mass of 0.5 kg hanging on a spring, the stiffness of the spring is given, the mechanical resistance is given, the force is given of the value, and force is also given, what will be the steady state value of the speed amplitude and average power dissipation. Find the value of phase angle between the speed and force and resonance frequency. So, a mass of 0.5 kg let us carefully read it a mass of 0.5 kg hanging on a spring, the stiffness of the spring is given mechanical resistance is given. So, first draw the mechanical circuits.

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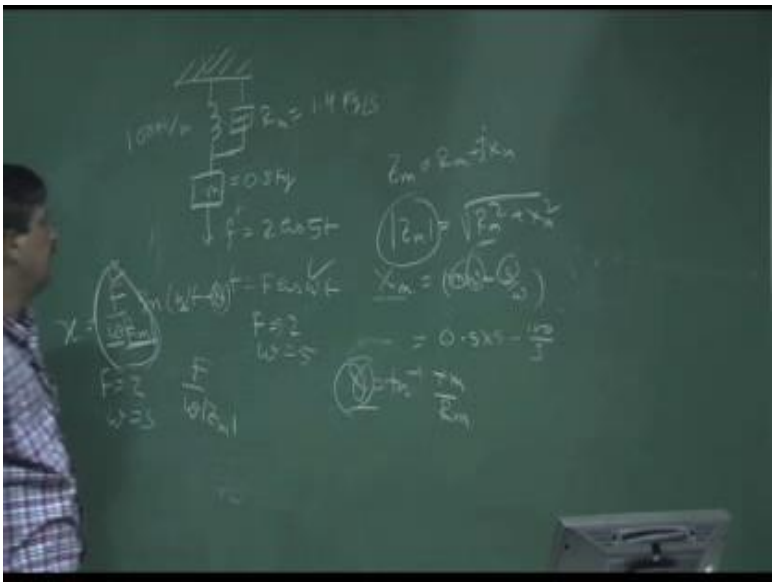

Say a mass of hanging, so spring will be there, a mass is hanging m and with the mechanical resistance is given, so R m is equal to what is the unit of R m either kg per seconds or Newton second per meter. So, R m is equal to 1.4 k g per second. Spring constant s is nothing but 100 Newton per meter. What is mass, mass is nothing but 0.5 kg then the applied force of this system f is nothing but 2 cos phi t. So, F is nothing but a F cos omega t. So, the value amplitude of the s F is equal to 2 and omega is equal to 5. So, if this mechanical oscillation is in steady state that means, force oscillation acts steady state I am not considered the tangent part so that means, the frequency of oscillation of the mechanical system is equivalent to the frequency of the force. So, what will be the steady state value of the speed amplitude and average power dissipation?

What is the speed amplitude, expression of the speed amplitude x, you know x, x is equal to we have derived all the expression we have derived. You know that expression. First you have to find out the z m, what is z m z m is nothing but a r m plus j x m. So, what is mod z m, what is mod z m mod z m is nothing but root over of R m square plus x m square. So, what is the x m again is nothing but m omega minus s by omega. So, what is m, m is given, s is given, m is given, omega is given I can find out x m. So, x m is nothing but what is m 0.5 kg. So, 0.5 kg, let us I calculate kg or gram, whatever 0.5 k g into omega is given what is omega, omega is nothing but 5, so omega is given minus s is given 100 Newton per and omega is given. So, you can convert that unit and find out the value of x m. Then you know R m find out the value of z m.

Now, what is the expression of x m, what is the expression x m, x is nothing but F by speed of the amplitude, F by omega z m real part F by omega z m sin omega t minus phi. So, I have to know the phi first, the find the value of the phase angle between the speed and force and resonance frequency value of resonance frequency and the phi I have to find out what is the pi, pi is nothing but root over a tan inverse x m by R m. So, you know the value of x m, you know the value of R m you know the value of phi. Here you know the value of z m this is mod z m, mod z m value you know the value of omega you know the value of F. So, F is equal to 2, omega is equal to 5, z m you can calculate, then omega you know. Speed amplitude, this is the speed amplitude speed amplitude is only this one F by omega mod z m is the speed amplitude. Now, phi I can calculate easily this is the phi then I can calculate power dissipation, what is the power equation.

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So, average power pi is nothing but F square into R m by two mod z m square. So, if it is that square you know the value of mod z m, you know the value of R m, you know the value of F, so you can calculate the average power dissipation. So, I can easily calculate everything. So, this problem is solved.

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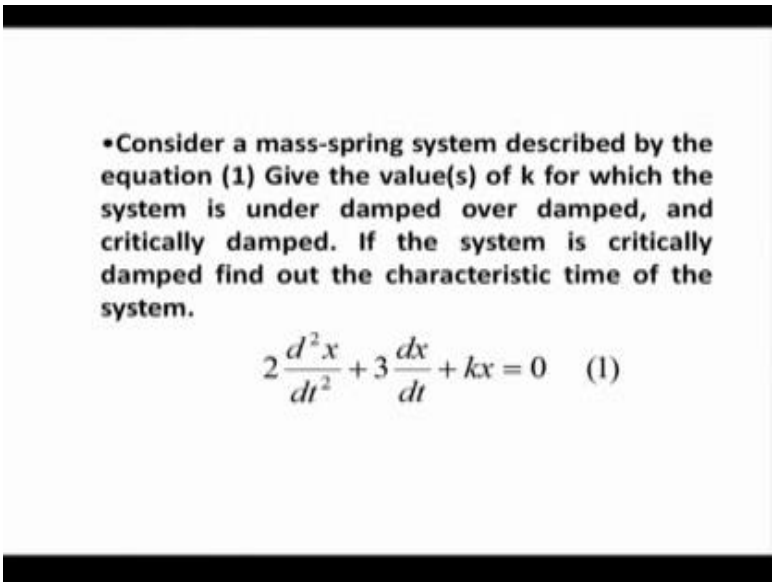

Next, there is another problem. Consider a spring-mass system described by equation one give the value of k for which the system is under damped, over damped and critically damped. If the system is critically damped find out the characteristic time of the system. So, what is given instead of circuits, the given equation of the mechanical motion 2 d 2 x by dt square plus 3 dx dt plus k x is equal to 0. So, if you know that it is nothing but m d 2 x by dt square plus R m dx dt plus s x x or whatever s or k whatever you write is equal to 0. So, this s value I do not know. So, m is equal to 2, R m is equal to 3, find out the value of k or s whatever.

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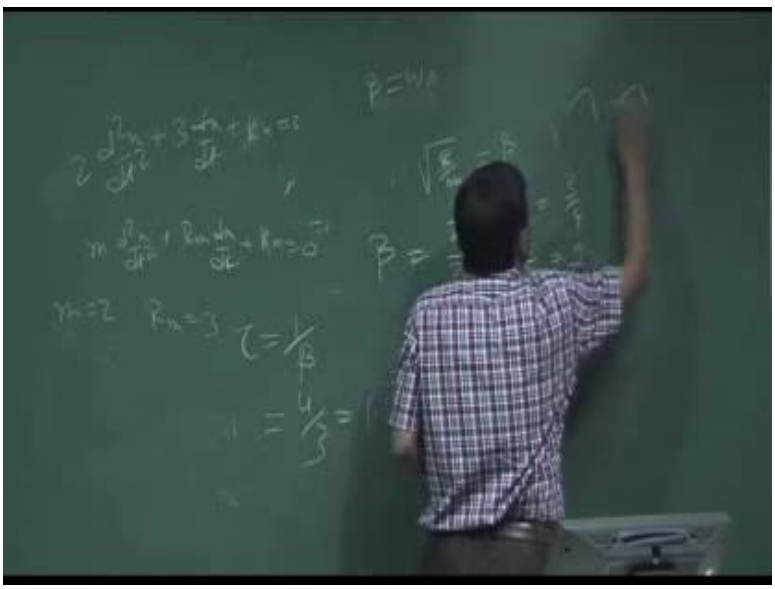

Lets I write k here k spring constant k. How do you find out the value of K, you know value of K for which is the system is under damped, over damped and critically damped. Let us consider for the critically damped condition. So, system will be critically damped if the omega is equal to omega d or omega d is equal to omega 0. So, what is omega 0 root over of k by m, find out the omega d value and then you can get that resonance things. Similarly, under damped over damped you can calculate now find out the beta value and then equate the beta value. So, what is the beta value, beta value is nothing but R m by 2 m. So, R m is nothing but 3, 2 into m is nothing but 2 c by 4.

So, I said critically damped system will be critically damped if beta is equal to omega 0. So, this is equal to beta. So, root over of k by m is equal to 3 by 4 or k by m is equal to 9 by 16, so m is 2, so I can find out the value of k. Similarly, tau what is tau it is nothing, but a one by tau is nothing, but a 1 by beta. So, it is beta is 3 by 4. So, tau is 4 by 3. So, one point something second. So, easily you can calculate. So, tractor I have given, this is not yours. Then so any mechanical oscillation something is given you can do that things.

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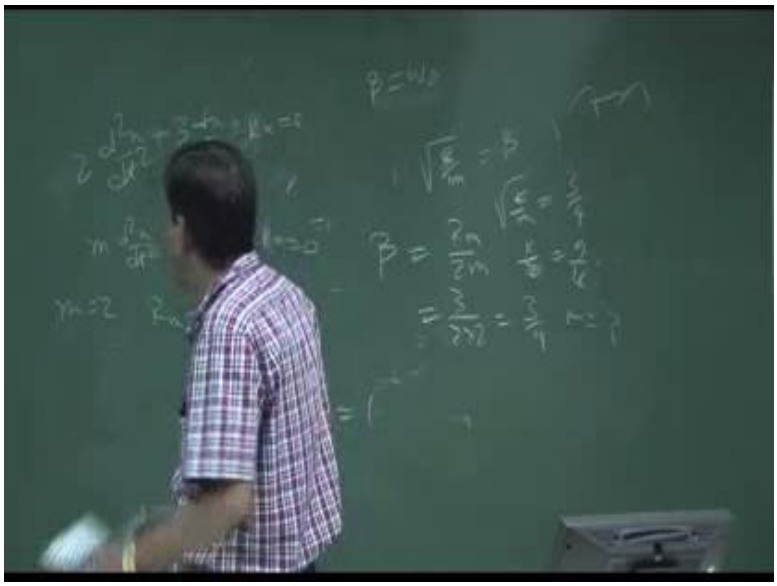

Let us I say that somebody is riding a motor cycle, and there is a bump consecutive bump and separated by some things and again say tractor problem can be given, and equation of motion will be given beta or find out the critical damp, find out the constant spring constant. Similarly, draw the mechanical equivalent mechanical circuits let us I give you a one problem let us I am not I will not do it you just try to do it.

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Let us describe the problem. Suppose that you want to install a stealer plant or ac, see stealer plant with the foundation engineering. Suppose, I have a stealer plant of mass is given mass of the stealer plant is given. To install in the arc using a spring, using spring four spring at four corners, and I installed it. Can I find out the resonance frequency of that system, can I draw the equivalent mechanical circuits of that system, can I find out that all the other characteristics of mechanical motion. So, you see that problem will be not given as a bookish problem. Practical problem will be given let say people will say I have a stealer plant, I want to install with a rubber band, and rubber bands spring constant and mechanical resistance is given that is specification is given then at are this installation is correct or not or is it vibrate. That means, if I know that the vibration frequency or suppose that I once a stealer plant is on.

So, force will be applied and that force frequency I know, if that frequency is resonate with that your system frequency then what will happen, stealer plant cannot be stable. So, I have to operate the operation frequency of the stealer plant is below of the resonance frequency or if I know the applied maximum applied force frequency then I can find out my resonance frequency should be all above of this things.

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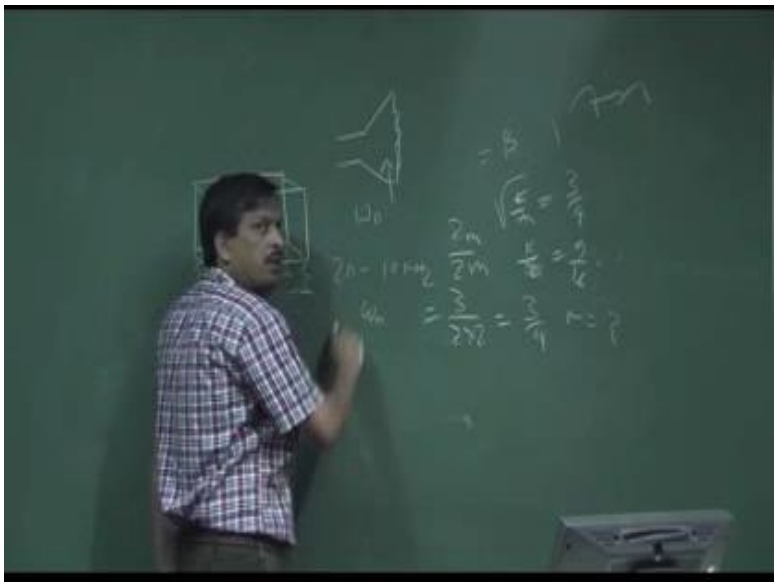

Similarly, given a practical problem suppose I want to design a loudspeaker, loudspeaker I want to design, I know the diaphragm mechanical circuits. So, I know the diaphragm resonance frequency omega 0. Now, if I want to operate this loudspeaker with 20 hertz to less 10 kilo hertz then my omega 0 should be much a above the 10 kilo hertz, so that resonance is not happened if resonance is happened then gone. So, I can easily calculate if I know the mechanical things I can easily draw the equivalent or mechanical circuits and find out the impedance, find out the resonance frequency and I know the operational frequency. So, I say resonance frequency is this, so it will. So, I will choose the mass of the diaphragm, stiffness of the diaphragm and those things depending on the design requirement.

Thank you.